MATHS/1

1- MATRIX & DETERMINANT

A) MATRIX:

(i) A Rectangular Array of numbers arranged in some complete Rows & in complete columns is called a matrix.

(ii) Horizontal lines are called rows & vertical lines are called columns.

(iii) Number of Rows X Number of columns is called order of a matrix.

(iv) A matrix is denoted by capital letters of English alphabet i.e A, B, C ----
Also by A_1, A_2 ------
order of a matrix "A" is denoted by O(A).

(v) Elements of a matrix are included within a pair of square bracket or a pair of open bracket. Elements are separated from each other by space not by comma. Element belonging to ith row & jth column is denoted by a_{ij}

Hence a 3×2 matrix will be

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}_{3\times 2}.$$

(vi) Examples of matrices are $[1]_{1\times 1}$,
$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2}$, $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times 3}$, $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{3\times 1}$, $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{2\times 2}$

(B) TYPES OF MATRICES:

(i) **Rectangular matrix:** A matrix whose number of rows and number of columns are not equal is called Rectangular matrix.

Ex: $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 4 \end{bmatrix}_{2\times 3}$

(ii) **Square matrix:** → A matrix whose number of rows and number of columns are equal is called a square matrix.

Ex $[a]_{1\times 1}$, $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, $\begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$

(iii) **Row matrix:** A matrix having only one row is called a row matrix.

Ex: $[2]_{1\times 1}$, $[1 \;\; 0 \;\; 5]_{1\times 3}$

(iv) **Column matrix** A matrix having only one column is called column matrix.

Ex $[3]_{1\times 1}$ $\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}_{3\times 1}$

(v) **Zero or null matrix:** A matrix whose all elements are zero (0) is called a zero matrix.

Ex: $[0]_{1\times 1}$ $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{2\times 3}$

(vi) **UNIT MATRIX / Identity matrix:** → A square matrix whose leading diagonal elements are all 1 and other elements are zeros is called unit matrix.

→ unit matrix of order n is denoted by I_n.

Ex: $I_1 = [1]_{1\times 1}$, $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

vii) **Singular matrix:** A square matrix A is said to be singular if $\boxed{|A| = 0}$

Ex $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ is a singular matrix.

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For $|A| = \begin{vmatrix} 1 & 2 \\ 3 & 6 \end{vmatrix} = (1)(6) - (3)(2) = 6 - 6 = 0$

viii) **Non Singular or Regular matrix:**
A square matrix A is said to be non-singular if $\boxed{|A| \neq 0}$

Ex: $A = \begin{bmatrix} 1 & 3 \\ 4 & 0 \end{bmatrix}$ is non-singular for $|A| = \begin{vmatrix} 1 & 3 \\ 4 & 0 \end{vmatrix}$
$= (1)(0) - (4)(3) = 0 - 12 = -12 \neq 0$.

ix) **Horizontal matrix:** A matrix whose number of rows is less than the number of column is called a Horizontal matrix

Ex $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}_{1\times 3}$ $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 4 \end{bmatrix}_{2\times 3}$

(x) **Vertical matrix:** A matrix whose number of rows is greater than that number of columns is called Vertical matrix.

Ex $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{3\times 1}$, $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 0 & -1 \end{bmatrix}_{3\times 2}$.

xi) **Diagonal matrix:** A square matrix whose all ~~non dia~~ non diagonal elements are zero is called diagonal matrix.

Ex: $\begin{bmatrix} 3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 6 \end{bmatrix}$, $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$, $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 6 \end{bmatrix}$

xii) **Scalar matrix:** A diagonal matrix ~~in which~~ all main diagonal elements are equal is called a ~~diagonal~~ scalar matrix.

Ex $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$,

xiii) Lower triangular matrix:
A square matrix is said to be lower triangular if all the elements above the main diagonal are zero.

Ex: $\begin{bmatrix} 1 & 0 & 0 \\ 4 & 2 & 0 \\ 6 & 5 & 1 \end{bmatrix}$

(xiv) upper triangular matrix: A square matrix is said to be upper triangular if all the elements below the main diagonal are zero.

Ex: $\begin{bmatrix} 3 & 2 & 6 & 7 \\ 0 & 6 & 6 & 8 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

C: ALGEBRA OF MATRIX:

① Equality: (i) Two matrices are said to be equal if they have $\boxed{\text{same order}}$ and their corresponding elements are equal.
(ii) If A and B are two equal matrices then we write it as $A=B$.
(iii) Mathematically $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} e & f \\ g & h \end{bmatrix} \Leftrightarrow a=e, b=f$
$c=g$ & $d=h$.

② Addition: (i) If A & B are two matrices of same order then $A+B$ is defined.
(ii) $A+B$ is the matrix obtained by Adding elements of A with corresponding elements of B. Note that $A-B = A+(-B)$.
(iii) Ex: If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 7 & -2 \\ 1 & 0 \end{bmatrix}$ then

$$A+B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 7 & -2 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1+7 & 2-2 \\ 3+1 & 4+0 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 4 & 4 \end{bmatrix}$$

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③ SCALAR MULTIPLICATION

i) If A is a matrix and α is any scalar then αA is the matrix obtained by multiplying α in each elements of A.

(ii) Ex: Let $A = \begin{bmatrix} 2 & 0 \\ -1 & 3 \end{bmatrix}$ then $3A = 3 \cdot \begin{bmatrix} 2 & 0 \\ -1 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 3\times 2 & 3\times 0 \\ 3\times(-1) & 3\times 3 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ -3 & 9 \end{bmatrix}$$

Conversely, $\begin{bmatrix} 6 & 0 \\ -3 & 9 \end{bmatrix} = 3\begin{bmatrix} 2 & 0 \\ -1 & 3 \end{bmatrix}$

(iii) Hence $\begin{bmatrix} \alpha a_{11} & \alpha a_{12} \\ \alpha a_{21} & \alpha a_{22} \end{bmatrix} = \alpha \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ (Taking common α)

Ⓓ Transpose

(i) If A is a matrix then transpose of A is denoted by A' or A^T and defined as A^T is the matrix obtained by writting rows of A as columns or vice-versa.

(ii) Note that transpose of Transpose of a matrix is the matrix itself i.e $(A^T)^T = A$

(iii) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times 3}$ then $A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}_{3\times 2}$.

Ⓔ Minor and co-factor:

(i) Minor of a_{ij} is denoted by M_{ij} and M_{ij} is equal to the determinant obtained by suppressing elements in ith. row & jth column.

(ii) Cofactor of a_{ij} is denoted by C_{ij} and defined by $\boxed{C_{ij} = (-1)^{i+j} M_{ij}}$

(F) Adjoint of a matrix: If A is a matrix then adjoint of A is denoted by Adj.(A) and defined by $\boxed{Adj.(A) = C^T}$ where C is the co-factor matrix

(G) INVERSE OF A matrix: (A^{-1})

(i) If A is a square matrix such that $|A| \neq 0$ then A^{-1} exist. and

$$\boxed{A^{-1} = \frac{adj.(A)}{|A|}}$$

(ii) The square matrix whose inverse exist is called Invertible i.e if $|A| \neq 0 \Rightarrow A^{-1}$ exist $\Rightarrow A$ is invertible.

(iii) If A is a square matrix of order 'n' and B is inverse of A then

$$\boxed{A.B = B.A = I_n}$$

(iv) If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $\boxed{A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}$

(Apply this formula for 2 mark problems)

(4) FROM ALGEBRA: PRODUCT OF TWO MATRICES

(i) If $O(A) = m \times n$ & $O(B) = n \times p$ then $A \cdot B$ is weldefined & $O(AB) = m \times p$.

(ii) How we multiply (through example)

Let $A = \begin{bmatrix} 1 & -1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 5 & 0 & 2 \\ 1 & 1 & 2 & -1 \end{bmatrix}$.

then $O(A) = 2\times3$, $O(B) = 3\times4$. Hence $A.B$ exist & $O(AB) = 2\times4$

Now $AB = \begin{bmatrix} 1 & -1 & 0 \\ 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 5 & 0 & 2 \\ 1 & 1 & 2 & -1 \end{bmatrix}$

$$= \begin{bmatrix} 1\times1+(-1)\times0+0\times1 & 1\times2+(-1)\times5+0\times1 & 1\times(-1)+(-1)\times0+0\times2 & 1\times3+(-1)\times2+0\times(-1) \\ 3\times1+2\times0+1\times1 & 3\times2+2\times5+1\times1 & 3\times(-1)+2\times0+1\times2 & 3\times3+2\times2+1\times(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -3 & -1 & 1 \\ 4 & 17 & -1 & 12 \end{bmatrix}_{2\times4} \quad \text{(Ans)}$$

⟨H⟩ SOLUTION OF SIMULTANEOUS LINEAR EQUATION BY MATRIX METHOD

This Method will be explained through examples.

⟨I⟩ DETERMINANT

(a) Second order determinant

$$\boxed{\begin{vmatrix} a & b \\ c & d \end{vmatrix} = (a)(d) - (b)(c)}$$

(b) 3rd order determinant:

$$\boxed{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix} \quad \text{Expanding w.r.t } R_1}$$

sign formula:

$$\begin{array}{cc} & \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} & \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix} \end{array}$$

Note that No. of elements in an nth. order determinant $= n^2$.

and No. of terms in an nth. order determinant $= \underline{|n}$.

MATH-I/8:

(J) CRAMER'S RULE (DETERMINANT METHOD):

STEP-i Given system of linear equations are

$$\left.\begin{array}{l} a_1x+b_1y+c_1z=d_1 \\ a_2x+b_2y+c_2z=d_2 \\ a_3x+b_3y+c_3z=d_3 \end{array}\right\} \text{——} (1)$$

step-ii find determinant of the system Δ by

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \neq 0$$

step-iii: find Δx, Δy, Δz by

$$\Delta x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}, \quad \Delta y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$$

$$\Delta z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

step-iv: find x, y, z by

$$\boxed{x = \frac{\Delta x}{\Delta}, \quad y = \frac{\Delta y}{\Delta}, \quad z = \frac{\Delta z}{\Delta}}$$

Note:
(a) when $\Delta = 0$, cramer's rule is not applicable.
(b) If $\Delta = 0$, but atleast one of $\Delta x, \Delta y, \Delta z$ is not zero then the system has no solution.
(c) If $\Delta = \Delta x = \Delta y = \Delta z = 0$ then the system has infinite no. of solution.

NOTE

$$\left.\begin{array}{l} a_1x+b_1y+c_1z=0 \\ a_2x+b_2y+c_2z=0 \\ a_3x+b_3y+c_3z=0 \end{array}\right\} \text{——} (1)$$

then eliminating x, y, z from (1) we have

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

(K) PROPERTIES OF DETERMINANT:

(i) If two rows or columns of a determinant are identical then the determinant vanishes.

(ii) The values of a determinant remains unchanged if rows are written as columns or vice-versa

(iii) If two adjacent rows (or columns) of a determinant are interchanged then the sign of the determinant changes keeping its absolute value fixed.

(iv) If each elements in any row (or column) is multiplied by same factor then the determinant is multiplied by that factor.

(v) If each elements of any row (or column) consist of two or more than two terms, then the determinant can be expressed as sum of two or more determinants.

(vi) If all elements of any row (or column) be increased or decreased by any equimultiple of the corresponding element of one or more of the other rows (or columns) then the value of the determinant remains unchanged.

$R_i \to R_i + KR_j$, $C_i \to C_i + KC_j$, $R_i \leftrightarrow R_j$

$C_i \leftrightarrow C_j$.

NOTE: All rows (or columns) should not replace simultaneously.

PROBLEMS:

① Construct a 2x3 matrix having elements $a_{ij} = i+j$

Soln: $a_{ij} = i+j, \forall\ i,j$ —— ①

~~Let~~ The required 2x3 matrix is

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} = \begin{bmatrix} 1+1 & 1+2 & 1+3 \\ 2+1 & 2+2 & 2+3 \end{bmatrix} \text{ using ①}$$

$$= \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix} \text{ (Ans).}$$

② If $A = \begin{pmatrix} 0 & 2 & 3 \\ 2 & 1 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 7 & 6 & 3 \\ 1 & 4 & 5 \end{pmatrix}$ then find $3A-B$.

Soln $A = \begin{pmatrix} 0 & 2 & 3 \\ 2 & 1 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 7 & 6 & 3 \\ 1 & 4 & 5 \end{pmatrix}$

$$3A-B = 3\times\begin{pmatrix} 0 & 2 & 3 \\ 2 & 1 & 4 \end{pmatrix} - \begin{pmatrix} 7 & 6 & 3 \\ 1 & 4 & 5 \end{pmatrix} = \begin{pmatrix} 0 & 6 & 9 \\ 6 & 3 & 12 \end{pmatrix} - \begin{pmatrix} 7 & 6 & 3 \\ 1 & 4 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} -7 & 0 & 6 \\ 5 & -1 & 7 \end{pmatrix} \text{ (Ans)}$$

③ ~~Construct~~ A matrix has 30 elements. Then what are its possible order.

Soln: $30 = 1\times30 = 2\times15 = 3\times10 = 5\times6 = 6\times5 = 10\times3 = 15\times2 = 30\times1$.

Hence possible order of the matrix will be $1\times30, 2\times15, 3\times10, 5\times6, 6\times5, 10\times3, 15\times2, 30\times1$.

④ If $A = \begin{pmatrix} 5 & 3 \\ 12 & 7 \end{pmatrix}$ then prove that $A^2 - 12A - I_2 = 0$.

Soln $A = \begin{pmatrix} 5 & 3 \\ 12 & 7 \end{pmatrix}$, $I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$A^2 = A\times A = \begin{pmatrix} 5 & 3 \\ 12 & 7 \end{pmatrix} \times \begin{pmatrix} 5 & 3 \\ 12 & 7 \end{pmatrix} = \begin{pmatrix} 61 & 36 \\ 144 & 85 \end{pmatrix}$$

Now $LHS = A^2 - 12A - I_2 = \begin{pmatrix} 61 & 36 \\ 144 & 85 \end{pmatrix} - 12\begin{pmatrix} 5 & 3 \\ 12 & 7 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$= \begin{pmatrix} 61 & 36 \\ 144 & 85 \end{pmatrix} - \begin{pmatrix} 60 & 36 \\ 144 & 84 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = 0 = RHS \ \square$$

⑤ If $A = \begin{bmatrix} 2 & -1 & 1 \\ 0 & 1 & 2 \\ 1 & 0 & 2 \end{bmatrix}$ then find A^2.

Solⁿ: $A = \begin{bmatrix} 2 & -1 & 1 \\ 0 & 1 & 2 \\ 1 & 0 & 2 \end{bmatrix}$

$$A^2 = A \cdot A = \begin{bmatrix} 2 & -1 & 1 \\ 0 & 1 & 2 \\ 1 & 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 & 1 \\ 0 & 1 & 2 \\ 1 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times2+(-1)\times0+1\times1 & 2\times(-1)+(-1)\times1+1\times0 & 2\times1+(-1)\times2+1\times2 \\ 0\times2+1\times0+2\times1 & 0\times(-1)+(1)\times1+2\times0 & 0\times1+1\times2+2\times2 \\ 1\times2+0\times0+2\times1 & 1\times(-1)+0\times1+2\times0 & 1\times1+0\times2+2\times2 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -3 & 2 \\ 2 & 1 & 4 \\ 4 & -1 & 5 \end{bmatrix} \text{(Ans)}$$

6) Find Inverse of $\begin{bmatrix} 4 & -2 \\ 3 & 1 \end{bmatrix}$

Method-1 (For Long Type)

Let $A = \begin{bmatrix} 4 & -2 \\ 3 & 1 \end{bmatrix}$. $|A| = \begin{vmatrix} 4 & -2 \\ 3 & 1 \end{vmatrix} = (4)(1)-(3)(-2) = 4+6 = 10$

$\therefore |A| \neq 0 \Rightarrow A^{-1}$ exist.

Let M_{ij} be the minor of a_{ij} & C_{ij} be the co-factor of a_{ij}

then $C_{ij} = (-1)^{i+j} M_{ij}$ —— ①

using ① $C_{11} = M_{11} = |1| = 1$

$C_{12} = -M_{12} = -|3| = -3$

$C_{21} = -M_{21} = -|-2| = 2$

$C_{22} = M_{22} = |4| = 4$

Now $C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix}$

$\text{Adj.}(A) = C^T = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$

$A^{-1} = \frac{Adj(A)}{|A|} = \frac{1}{10}\begin{bmatrix}1 & 2\\ -3 & 4\end{bmatrix} = \begin{bmatrix}1/10 & 1/5\\ -3/10 & 2/5\end{bmatrix}$ (Ans)

Method-2 (For 1, 2 mark)

Let $A = \begin{bmatrix}4 & -2\\ 3 & 1\end{bmatrix}$, $|A| = \begin{vmatrix}4 & -2\\ 3 & 1\end{vmatrix} = (4)(1)-(3)(-2) = 10 \neq 0$

Hence A^{-1} exist.

Now $A^{-1} = \frac{1}{|A|}\begin{bmatrix}1 & 2\\ -3 & 4\end{bmatrix} = \frac{1}{10}\begin{bmatrix}1 & 2\\ -3 & 4\end{bmatrix} = \begin{bmatrix}1/10 & 1/5\\ -3/10 & 2/5\end{bmatrix}$ (Ans)

(7) Show that $\begin{bmatrix}1 & 2 & 3\\ -1 & 0 & 0\\ 0 & 1 & 3\end{bmatrix}$ is invertible.

Soln: Let $A = \begin{bmatrix}1 & 2 & 3\\ -1 & 0 & 0\\ 0 & 1 & 3\end{bmatrix}$

Now $|A| = \begin{vmatrix}1 & 2 & 3\\ -1 & 0 & 0\\ 0 & 1 & 3\end{vmatrix} = -(-1)\begin{vmatrix}2 & 3\\ 1 & 3\end{vmatrix}$ (Expanding w.r.t R_2)

$= 2\times3 - 1\times3 = 6-3 = 3 \neq 0$

$\therefore |A| \neq 0 \Rightarrow A^{-1}$ exist $\Rightarrow A$ is invertible. $\square$

(8) Find Inverse of the matrix $\begin{bmatrix}1 & 2 & 3\\ 2 & 1 & 4\\ 1 & 0 & 2\end{bmatrix}$ and verify your Answer.

Sol: Let $A = \begin{bmatrix}1 & 2 & 3\\ 2 & 1 & 4\\ 1 & 0 & 2\end{bmatrix}$

$|A| = \begin{vmatrix}1 & 2 & 3\\ 2 & 1 & 4\\ 1 & 0 & 2\end{vmatrix} = 1\begin{vmatrix}2 & 3\\ 1 & 4\end{vmatrix} + 2\begin{vmatrix}1 & 2\\ 2 & 1\end{vmatrix}$ (Expanding w.r.t R_3)

$= 8-3 + 2(1-4) = 5-6 = -1 \neq 0$

$|A| \neq 0 \Rightarrow A^{-1}$ exist.

Now Let M_{ij} be the minor and C_{ij} be the co-factor of a_{ij}. Then $C_{ij} = (-1)^{i+j} M_{ij}$ —— (1)

using (1) $C_{11} = M_{11} = \begin{vmatrix}1 & 4\\ 0 & 2\end{vmatrix} = 2$

$C_{12} = -M_{12} = -\begin{vmatrix}2 & 4\\ 1 & 2\end{vmatrix} = 0$

$C_{13} = M_{13} = \begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix} = -1$, $C_{22} = M_{22} = \begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix} = -1$

$C_{21} = -M_{21} = -\begin{vmatrix} 2 & 3 \\ 0 & 2 \end{vmatrix} = -4$, $C_{23} = -M_{23} = -\begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix} = 2$

$C_{31} = M_{31} = \begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} = 5$, $C_{32} = -M_{32} = -\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} = 2$

$C_{33} = M_{33} = \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = -3.$

Now $C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} = \begin{bmatrix} 2 & 0 & -1 \\ -4 & -1 & 2 \\ 5 & 2 & -3 \end{bmatrix}$

$Adj(A) = C^T = \begin{bmatrix} 2 & -4 & 5 \\ 0 & -1 & 2 \\ -1 & 2 & -3 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \cdot Adj(A) = \frac{1}{-1} \begin{bmatrix} 2 & -4 & 5 \\ 0 & -1 & 2 \\ -1 & 2 & -3 \end{bmatrix} = \begin{bmatrix} -2 & 4 & -5 \\ 0 & 1 & -2 \\ 1 & -2 & 3 \end{bmatrix}$

Verification: $A \cdot A^{-1} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 1 & 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} -2 & 4 & -5 \\ 0 & 1 & -2 \\ 1 & -2 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 1\times(-2)+2\times0+3\times1 & 1\times4+2\times1+3\times(-2) & 1\times(-5)+2\times(-2)+3\times3 \\ 2\times(-2)+1\times0+4\times1 & 2\times4+1\times1+4\times(-2) & 2\times(-5)+1\times(-2)+4\times3 \\ 1\times(-2)+0\times0+2\times1 & 1\times4+0\times1+2\times(-2) & 1\times(-5)+0\times(-2)+2\times3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3$$

This verifies our Answer.

(9) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 1 \end{bmatrix}$ then verify that $(AB)^T = B^T \cdot A^T$.

Soln $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 1 \end{bmatrix}$

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$O(A) = 2\times3$ & $O(B) = 3\times2$. Hence AB exist.

Now $AB = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2 & 5 \\ -2 & 7 \end{bmatrix}$

$(AB)^T = \begin{bmatrix} 2 & -2 \\ 5 & 7 \end{bmatrix}$ ——— (1)

Now $A^T = \begin{bmatrix} 1 & 3 \\ 2 & -2 \\ 3 & 1 \end{bmatrix}$, $B^T = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 1 \end{bmatrix}$

$O(B^T) = 2\times3$, $O(A^T) = 3\times2$. Hence $B^T.A^T$ exist.

Now $B^T.A^T = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & -2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 5 & 7 \end{bmatrix}$ ——— (2)

from (1) & (2) $(AB)^T = B^T.A^T$ □

(10) Find Adjoint of $\begin{bmatrix} 1 & 1 & -1 \\ 2 & -1 & 2 \\ 1 & 3 & -2 \end{bmatrix}$

Sol^n Let $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -1 & 2 \\ 1 & 3 & -2 \end{bmatrix}$. Let M_{ij} be the minor and C_{ij} be the co-factor of a_{ij}. Then

$C_{ij} = (-1)^{i+j} M_{ij}$ ——— (1)

using (1)

$C_{11} = M_{11} = \begin{vmatrix} -1 & 2 \\ 3 & -2 \end{vmatrix} = (-1)(-2) - (3)(2) = 2-6 = -4$

$C_{12} = -M_{12} = -\begin{vmatrix} 2 & 2 \\ 1 & -2 \end{vmatrix} = -\{(2)(-2) - (1)(2)\} = -\{-4-2\} = 6$

$C_{13} = M_{13} = \begin{vmatrix} 2 & -1 \\ 1 & 3 \end{vmatrix} = (2)(3) - (1)(-1) = 6+1 = 7$

$C_{21} = -M_{21} = -\begin{vmatrix} 1 & -1 \\ 3 & -2 \end{vmatrix} = -\{(1)(-2) - (3)(-1)\} = -1$

$C_{22} = M_{22} = \begin{vmatrix} 1 & -1 \\ 1 & -2 \end{vmatrix} = (1)(-2) - (1)(-1) = -1$

$C_{23} = -\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} = -\{(1)(3)-(1)(1)\} = -2.$

$C_{31} = M_{31} = \begin{vmatrix} 1 & -1 \\ -1 & 2 \end{vmatrix} = 2-1 = 1$

$C_{32} = -M_{32} = -\begin{vmatrix} 1 & -1 \\ 2 & 2 \end{vmatrix} = -4$

$C_{33} = M_{33} = \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = -3$

Now $C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} = \begin{bmatrix} -4 & 6 & 7 \\ -1 & -1 & -2 \\ 1 & -4 & -3 \end{bmatrix}$

~~Adjoint~~ $\text{Adj.}(A) = C^T = \begin{bmatrix} -4 & -1 & 1 \\ 6 & -1 & -4 \\ 7 & -2 & -3 \end{bmatrix}$ (Ans)

(ii) Find Inverse of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 1 & 0 & 2 \end{bmatrix}$

Soln Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 1 & 0 & 2 \end{bmatrix}$. Now $|A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 1 & 0 & 2 \end{vmatrix}$

$= 1\begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} + 2\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix}$ (Expanding w.r.t R_3)

$= 5-6 = -1 \neq 0$. Hence A^{-1} exist.

Now let M_{ij} be the minor and C_{ij} be the co-factor of a_{ij}. Then $C_{ij} = (-1)^{i+j} M_{ij}$ —— (1)

using (1)

$C_{11} = M_{11} = \begin{vmatrix} 1 & 4 \\ 0 & 2 \end{vmatrix} = 2$

$C_{12} = -M_{12} = -\begin{vmatrix} 2 & 4 \\ 1 & 2 \end{vmatrix} = 0$

$C_{13} = M_{13} = \begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix} = -1$

$c_{21} = -M_{21} = -\begin{vmatrix} 2 & 3 \\ 0 & 2 \end{vmatrix} = -4$

$c_{22} = M_{22} = \begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix} = -1$

$c_{23} = -M_{23} = -\begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix} = 2.$

$c_{31} = M_{31} = \begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} = 5$

$c_{32} = -M_{32} = -\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} = 2$

$c_{33} = M_{33} = \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = -3$

Now $C = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} = \begin{bmatrix} 2 & 0 & -1 \\ -4 & -1 & 2 \\ 5 & 2 & -3 \end{bmatrix}$

$\text{Adj}(A) = C^T = \begin{bmatrix} 2 & -4 & 5 \\ 0 & -1 & 2 \\ -1 & 2 & -3 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \cdot \text{Adj}(A) = \frac{1}{-1}\begin{bmatrix} 2 & -4 & 5 \\ 0 & -1 & 2 \\ -1 & 2 & -3 \end{bmatrix} = \begin{bmatrix} -2 & 4 & -5 \\ 0 & 1 & -2 \\ 1 & -2 & 3 \end{bmatrix}$

(12) Find Minors and co-factors of $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 0 \\ -2 & 2 & -1 \end{bmatrix}$ (Ans)

Soln Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 0 \\ -2 & 2 & -1 \end{bmatrix}$

Let M_{ij} be the minor of a_{ij}. then

$M_{11} = \begin{vmatrix} 1 & 0 \\ 2 & -1 \end{vmatrix} = -1$

$M_{12} = \begin{vmatrix} 3 & 0 \\ -2 & -1 \end{vmatrix} = -3$

$M_{13} = \begin{vmatrix} 3 & 1 \\ -2 & 2 \end{vmatrix} = 8$

$M_{21} = \begin{vmatrix} 2 & 3 \\ 2 & -1 \end{vmatrix} = -8$

$M_{22} = \begin{vmatrix} 1 & 3 \\ -2 & -1 \end{vmatrix} = 5.$

$M_{23} = \begin{vmatrix} 1 & 2 \\ -2 & 2 \end{vmatrix} = 6$

$M_{31} = \begin{vmatrix} 2 & 3 \\ 1 & 0 \end{vmatrix} = -3$

$M_{32} = \begin{vmatrix} 1 & 3 \\ 3 & 0 \end{vmatrix} = -9$

$M_{33} = \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = -5$

Let c_{ij} be the cofactor of a_{ij}. then $c_{ij} = (-1)^{i+j} M_{ij}$ — ①

using ①

$c_{11} = M_{11} = -1$

$c_{12} = -M_{12} = 3$

$c_{13} = M_{13} = 8$

$c_{21} = -M_{21} = 8$

$c_{22} = M_{22} = 5$

$c_{23} = -M_{23} = -6$

$c_{31} = M_{31} = -3$

$c_{32} = -M_{32} = 9$

$c_{33} = M_{33} = -5$

(Ans)

(13) Find A & B where $2A+B = \begin{bmatrix} 2 & 2 & 5 \\ 5 & 4 & 3 \\ 1 & 1 & 4 \end{bmatrix}$ &

$A-2B = \begin{bmatrix} 1 & 6 & 5 \\ 5 & 2 & -1 \\ -2 & -2 & 2 \end{bmatrix}$

Soln

$$2A+B = \begin{bmatrix} 2 & 2 & 5 \\ 5 & 4 & 3 \\ 1 & 1 & 4 \end{bmatrix} \quad \text{——— (1)}$$

$$\& \quad A-2B = \begin{bmatrix} 1 & 6 & 5 \\ 5 & 2 & -1 \\ -2 & -2 & 2 \end{bmatrix} \quad \text{——— (2)}$$

$$\text{Eq}^n(1) \times 1 \Rightarrow 2A+B = \begin{bmatrix} 2 & 2 & 5 \\ 5 & 4 & 3 \\ 1 & 1 & 4 \end{bmatrix}$$

$$\text{Eq}^n(2) \times 2 \Rightarrow \underset{(-)}{2A} \underset{(+)}{-4B} = - \begin{bmatrix} 2 & 12 & 10 \\ 10 & 4 & -2 \\ -4 & -4 & 4 \end{bmatrix}$$

substracting

$$5B = \begin{bmatrix} 0 & -10 & -5 \\ -5 & 0 & 5 \\ 5 & 5 & 0 \end{bmatrix} \Rightarrow B = \frac{1}{5} \begin{bmatrix} 0 & -10 & -5 \\ -5 & 0 & 5 \\ 5 & 5 & 0 \end{bmatrix}$$

$$\Rightarrow B = \begin{bmatrix} 0 & -2 & -1 \\ -1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Now from (2) $A = \begin{bmatrix} 1 & 6 & 5 \\ 5 & 2 & -1 \\ -2 & -2 & 2 \end{bmatrix} + 2B$

$$= \begin{bmatrix} 1 & 6 & 5 \\ 5 & 2 & -1 \\ -2 & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -4 & -2 \\ -2 & 0 & 2 \\ 2 & 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

$\therefore A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & -2 & -1 \\ -1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ (Ans)

14) Solve by matrix Method: $x+2y=3,\ 3x+y=4$.

Soln Given equations are $\left.\begin{aligned} x+2y&=3 \\ 3x+y&=4 \end{aligned}\right\}$ —— (1)

The above system in matrix form can be written as $AX = B \Rightarrow X = A^{-1}B$ —— (2)

where $A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 3 \\ 4 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}$

Now $|A| = \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = (1)(1)-(2)(3) = 1-6 = -5 \neq 0$.

Hence A^{-1} exist.

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix} = \frac{1}{-5}\begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -1/5 & 2/5 \\ 3/5 & -1/5 \end{bmatrix}$$

From (2) $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1/5 & 2/5 \\ 3/5 & -1/5 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$\Rightarrow x=1,\ y=1$

$\therefore x=1, y=1$ is the required solution.

15) Solve by matrix method: $x+2y+3z=8,\ 2x+y+z=8,\ x+y+2z=6$

Ans: Given system is

$$\left.\begin{aligned} x+2y+3z&=8 \\ 2x+y+z&=8 \\ x+y+2z&=6 \end{aligned}\right\} \text{—— (1)}$$

System (1) in matrix form is $AX = B \Rightarrow X = A^{-1}B$ —— (2)

Where $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 8 \\ 8 \\ 6 \end{bmatrix}$

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$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & 1 \\ 1 & 1 & 2 \end{vmatrix} = 1(2-1) - 2(4-1) + 3(2-1) = 1-6+3 = -2 \neq 0$$

$\Rightarrow A^{-1}$ exist.

Let M_{ij} be the minor & c_{ij} be the co-factor of a_{ij}.

Then $\quad c_{ij} = (-1)^{i+j} M_{ij}$ —— ③

using ③

$c_{11} = M_{11} = \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 2-1 = 1$ | $c_{21} = -M_{21} = -\begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = -1$

$c_{12} = -M_{12} = -\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = -3$ | $c_{22} = M_{22} = \begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix} = -1$

$c_{13} = M_{13} = \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 1$ | $c_{23} = -M_{23} = -\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = 1$

$c_{31} = M_{31} = \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} = -1, \quad c_{32} = -M_{32} = -\begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} = 5$

$c_{33} = M_{33} = \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = -3$

Now $\quad C = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} = \begin{bmatrix} 1 & -3 & 1 \\ -1 & -1 & 1 \\ -1 & 5 & -3 \end{bmatrix}$

$Adj(A) = C^T = \begin{bmatrix} 1 & -1 & -1 \\ -3 & -1 & 5 \\ 1 & 1 & -3 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{|A|} Adj(A) = -\frac{1}{2}\begin{bmatrix} 1 & -1 & -1 \\ -3 & -1 & 5 \\ 1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} -1/2 & 1/2 & 1/2 \\ 3/2 & 1/2 & -5/2 \\ -1/2 & -1/2 & 3/2 \end{bmatrix}$

From (2) $\quad X = A^{-1}B$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1/2 & 1/2 & 1/2 \\ \frac{3}{2} & 1/2 & -5/2 \\ -1/2 & -1/2 & 3/2 \end{bmatrix} \begin{bmatrix} 8 \\ 8 \\ 6 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$

$\Rightarrow x = 3, y = 1, z = 1$ is the solution.

(16) Find x & y if $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

Sol^n: $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} x+3y \\ 2x-y \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

$\Rightarrow x+3y = 4$ —— (1)

& $2x - y = 1$ —— (2)

Eq^n (1) $\times 2 \Rightarrow 2x + 6y = 8$

Eq^n (2) $\times 1 \Rightarrow \underline{\underset{(-)}{2x} \underset{(+)}{-} y = \underset{(-)}{1}}$

$\Rightarrow 7y = 7 \Rightarrow y = 1.$

From (1) $x+3y=4 \Rightarrow x+3\times 1 = 4 \Rightarrow x = 4-3 = 1$

$\therefore x = 1, y = 1$ (Ans)

(17) Solve by cramer's Rule: $2x+y+2z=2$, $3x+2y+z-2=0$ & $-x+y+3z=6$

Sol^n Given system is $2x+y+2z=2$, $3x+2y+z-2=0$ & $-x+y+3z=6$

i.e
$$\left.\begin{aligned} 2x+y+2z&=2 \\ 3x+2y+z&=2 \\ -x+y+3z&=6 \end{aligned}\right\} \text{—— (1)}$$

Determinant of the system $\Delta = \begin{vmatrix} 2 & 1 & 2 \\ 3 & 2 & 1 \\ -1 & 1 & 3 \end{vmatrix}$

$= 2(6-1) - 1(9+1) + 2(3+2) = 10 - 10 + 10 = 10 \neq 0$

Now $\Delta_x = \begin{vmatrix} 2 & 1 & 2 \\ 2 & 2 & 1 \\ 6 & 1 & 3 \end{vmatrix} = 2(6-1) - 1(6-6) + 2(2-12) = -10$

$\Delta_y = \begin{vmatrix} 2 & 2 & 2 \\ 3 & 2 & 1 \\ -1 & 6 & 3 \end{vmatrix} = 2(6-6) - 2(9+1) + 2(18+2) = 20$

$\Delta_z = \begin{vmatrix} 2 & 1 & 2 \\ 3 & 2 & 2 \\ -1 & 1 & 6 \end{vmatrix} = 2(12-2) - 1(18+2) + 2(3+2) = 10$

$\therefore x = \frac{\Delta_x}{\Delta} = \frac{-10}{10} = -1$, $y = \frac{\Delta_y}{\Delta} = \frac{20}{10} = 2$, $z = \frac{\Delta_z}{\Delta} = \frac{10}{10} = 1$

$\therefore x = -1, y = 2, z = 1$ is the solution.

(18) Solve by Determinant Method: $4x-y=9,\ 5x+2y=8$.

Soln Given system is
$$\left.\begin{aligned} 4x-y&=9 \\ \&\quad 5x+2y&=8 \end{aligned}\right\} \text{—(1)}$$

Determinant of the system $\Delta = \begin{vmatrix} 4 & -1 \\ 5 & 2 \end{vmatrix} = 13 \neq 0$

Now $\Delta x = \begin{vmatrix} 9 & -1 \\ 8 & 2 \end{vmatrix} = 26,\ \Delta y = \begin{vmatrix} 4 & 9 \\ 5 & 8 \end{vmatrix} = -13.$

$$x = \frac{\Delta x}{\Delta} = \frac{26}{13} = 2 \ \& \ y = \frac{\Delta y}{\Delta} = \frac{-13}{13} = -1$$

$\therefore\ x=2,\ y=-1$ is the solution (Ans.)

(19) Solve $\begin{vmatrix} a-1 & 2 \\ 3 & a \end{vmatrix} = 0$

Soln $\begin{vmatrix} a-1 & 2 \\ 3 & a \end{vmatrix} = 0 \Rightarrow a(a-1)-6=0 \Rightarrow a^2-a-6=0$

$$\Rightarrow a = \frac{-(-1) \pm \sqrt{(-1)^2-4\cdot1\cdot(-6)}}{2} = \frac{1 \pm \sqrt{25}}{2} = \frac{1 \pm 5}{2}$$

$$= \frac{1+5}{2} \text{ or } \frac{1-5}{2} = 3 \text{ or } -2 \text{ (Ans)}$$

(20) Find a if $\begin{vmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \\ 1 & 2 & a-1 \end{vmatrix} = 0$

Soln $\begin{vmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \\ 1 & 2 & a-1 \end{vmatrix} = 0$

$\Rightarrow a-1=3 \qquad (\because a_{11}=a_{31}=1,\ a_{12}=a_{32}=2)$

$\Rightarrow a = 3+1 = 4$ (Ans)

(22) Find $\begin{vmatrix} \sin x & \cos x \\ -\cos x & \sin x \end{vmatrix}$

Soln $\begin{vmatrix} \sin x & \cos x \\ -\cos x & \sin x \end{vmatrix} = \sin^2 x + \cos^2 x = 1$

(23) Without expanding find $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 5 & 7 & 9 \end{vmatrix}$

Sol^n $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 5 & 7 & 9 \end{vmatrix}$

$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 0 & 0 & 0 \end{vmatrix}$ $R_3 \to R_3 - (R_1 + R_2)$

$= 0$ (Ans)

(24) Without Expanding find $\begin{vmatrix} 1 & 2 & 4 \\ 2 & 3 & 1 \\ 3 & 5 & 6 \end{vmatrix}$

Sol^n $\begin{vmatrix} 1 & 2 & 4 \\ 2 & 3 & 1 \\ 3 & 5 & 6 \end{vmatrix}$ $R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 3R_1$

$= \begin{vmatrix} 1 & 2 & 4 \\ 0 & -1 & -7 \\ 0 & -1 & -6 \end{vmatrix}$

$= 1\begin{vmatrix} -1 & -7 \\ -1 & -6 \end{vmatrix} = 6 - 7 = -1$ (Ans)

(25) Find minimum value of $\begin{vmatrix} \sin x & \cos x \\ -\cos x & 1+\sin x \end{vmatrix}$

Sol^n Let $P(x) = \begin{vmatrix} \sin x & \cos x \\ -\cos x & 1+\sin x \end{vmatrix}$

$= \sin x \cdot (1+\sin x) - (\cos x)(-\cos x)$

$= \sin x + \sin^2 x + \cos^2 x = \sin x + 1$

$\therefore P(x) = 1 + \sin x$

$\because \min.(\sin x) = -1 \Rightarrow \min\{P(x)\} = 1 + (-1) = 0$ (Ans)

(26) Prove that $\begin{vmatrix} a-b & 1 & a \\ b-c & 1 & b \\ c-a & 1 & c \end{vmatrix} = \begin{vmatrix} a & 1 & b \\ b & 1 & c \\ c & 1 & a \end{vmatrix}$

Sol^n LHS $= \begin{vmatrix} a-b & 1 & a \\ b-c & 1 & b \\ c-a & 1 & c \end{vmatrix}$

$C_1 \to C_1 - C_3$

$$= \begin{vmatrix} -b & 1 & a \\ -c & 1 & b \\ -a & 1 & c \end{vmatrix} = (-) \begin{vmatrix} b & 1 & a \\ c & 1 & b \\ a & 1 & c \end{vmatrix}$$

$$= \begin{vmatrix} 1 & b & a \\ 1 & c & b \\ 1 & a & c \end{vmatrix} \quad (C_1 \leftrightarrow C_2)$$

$$= - \begin{vmatrix} 1 & a & b \\ 1 & b & c \\ 1 & c & a \end{vmatrix} \quad (C_2 \leftrightarrow C_3)$$

$$= \begin{vmatrix} a & 1 & b \\ b & 1 & c \\ c & 1 & a \end{vmatrix} \quad (C_1 \to C_2)$$

$= \text{RHS} \quad \square$

(27) Prove that $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$

Sol^n LHS $= \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix}$

$C_1 \to C_1 - C_2$
$C_2 \to C_2 - C_3$

$$\begin{vmatrix} a-b & b-c & c \\ a^2-b^2 & b^2-c^2 & c^2 \\ bc-ca & ca-ab & ab \end{vmatrix}$$

$$= \begin{vmatrix} a-b & b-c & c \\ (a+b)(a-b) & (b+c)(b-c) & c^2 \\ -c(a-b) & -a(b-c) & ab \end{vmatrix}$$

$$= (a-b)(b-c)\begin{vmatrix} 1 & 1 & c \\ a+b & b+c & c^2 \\ -c & -a & ab \end{vmatrix}$$ (Taking common $a-b, b-c$ from C_1, C_2)

$$= (a-b)(b-c)\begin{vmatrix} 0 & 1 & c \\ a-c & b+c & c^2 \\ a-c & -a & ab \end{vmatrix}$$ $C_1 \to C_1 - C_2$

~~$R_2 \to R_2 - R_3$~~

$$= (a-b)(b-c)(c-a)\begin{vmatrix} 0 & 1 & c \\ -1 & b+c & c^2 \\ -1 & -a & ab \end{vmatrix}$$ (Taking $c-a$ from C_1)

$$=(a-b)(b-c)(c-a)\begin{vmatrix} 0 & 1 & c \\ 0 & a+b+c & c^2-ab \\ -1 & -a & ab \end{vmatrix}$$ $R_2 \to R_2 - R_3$

$$= (a-b)(b-c)(c-a)(-1)\begin{vmatrix} 1 & c \\ a+b+c & c^2-ab \end{vmatrix}$$ Expanding w.r.t C_1

$$= -(a-b)(b-c)(c-a)\{c^2-ab-ac-bc-c^2\}$$

$$= (a-b)(b-c)(c-a)(ab+bc+ca) = \text{RHS} \quad \square$$

(28) prove that $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} = xyz\left(1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$.

Soln: LHS $= \begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} = \begin{vmatrix} x(1+\frac{1}{x}) & x\cdot\frac{1}{x} & x\cdot\frac{1}{x} \\ y\cdot\frac{1}{y} & y(1+\frac{1}{y}) & y\cdot\frac{1}{y} \\ z\cdot\frac{1}{z} & z\cdot\frac{1}{z} & z(1+\frac{1}{z}) \end{vmatrix}$

$$= x\cdot y\cdot z\begin{vmatrix} 1+\frac{1}{x} & \frac{1}{x} & \frac{1}{x} \\ \frac{1}{y} & 1+\frac{1}{y} & \frac{1}{y} \\ \frac{1}{z} & \frac{1}{z} & 1+\frac{1}{z} \end{vmatrix}$$ taking x, y, z from R_1, R_2, R_3

$$= xyz\begin{vmatrix} 1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z} & 1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z} & 1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z} \\ \frac{1}{y} & 1+\frac{1}{y} & \frac{1}{y} \\ \frac{1}{z} & \frac{1}{z} & 1+\frac{1}{z} \end{vmatrix}$$ $R_1 \to R_1+R_2+R_3$

$$= xyz\left(1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)\begin{vmatrix} 1 & 1 & 1 \\ \frac{1}{y} & 1+\frac{1}{y} & \frac{1}{y} \\ \frac{1}{z} & \frac{1}{z} & 1+\frac{1}{z}\end{vmatrix}$$ (Taking $1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$ from R_1)

$$= xyz\left(1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)\begin{vmatrix} 0 & 0 & 1 \\ -1 & 1 & \frac{1}{y} \\ 0 & -1 & 1+\frac{1}{z}\end{vmatrix}$$ $C_1 \to C_1 - C_2$, $C_2 \to C_2 - C_3$

$$= xyz\left(1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)1\begin{vmatrix} -1 & 1 \\ 0 & -1\end{vmatrix}$$ Expanding w.r.t R_1

$$= xyz\left(1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)1\{1-0\} = xyz\left(1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right) = RHS \ \square.$$

(29) Given that the equations $x = cy+bz$, $y = az+cx$ and $z = bx+ay$, where x, y, z are not all zero. Then prove that $a^2+b^2+c^2+2abc = 1$ by determinant method.

Soln Given equations are

$x = cy+bz$ i.e $x-cy-bz = 0$ ——— ①

$y = az+cx$ i.e $cx-y+az = 0$ ——— ②

and $z = bx+ay$ i.e $bx+ay-z = 0$ ——— ③

Eliminating x, y, z from ① ② & ③ we have.

$$\begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1\end{vmatrix} = 0 \Rightarrow 1\begin{vmatrix} -1 & a \\ a & -1\end{vmatrix} - (-c)\begin{vmatrix} c & a \\ b & -1\end{vmatrix} + (-b)\begin{vmatrix} c & -1 \\ b & a\end{vmatrix} = 0$$

$\Rightarrow 1\{(-1)(-1)-(a)(a)\} + c\{(c)(-1)-(a)(b)\} - b\{(c)(a)-(b)(-1)\} = 0$

$\Rightarrow 1\{1-a^2\} + c\{-c-ab\} - b\{ac+b\} = 0$

$\Rightarrow 1-a^2-c^2-abc-abc-b^2 = 0$

$\Rightarrow 1-a^2-b^2-c^2-2abc = 0 \Rightarrow a^2+b^2+c^2+2abc = 1 \ \square.$

HOME TASK

(30) Construct a 2×3 matrix having elements $a_{ij} = \frac{i}{j}$.

(31) Construct a 3×3 matrix having elements $a_{ij} = i^2+j^2$

(32) Construct a 2×4 matrix having elements $a_{ij} = (i+j)^2$

(33) If $A = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2\end{bmatrix}$, verify $A^2-3A+2I = 0$

DECEMBER

MATH-I/26

(34) If $A = \begin{bmatrix} 2 & -1 & 1 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$ then find A^2 & A^3.

(35) If $A = \begin{bmatrix} 0 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 7 & 6 & 3 \\ 1 & 4 & 5 \end{bmatrix}$ find $3A - B$

(36) What are the possible order of a matrix having elements 26.

(37) Solve if $2A + 3B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ & $A - B = \begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix}$

(38) Show that the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ is singular

(39) Show that the matrix $\begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 3 \\ 0 & 0 & 4 \end{bmatrix}$ is non singular.

(40) Show that the matrix $\begin{bmatrix} 1 & 2 & 5 \\ 3 & 1 & 7 \\ 0 & 0 & 3 \end{bmatrix}$ is invertible.

(41) Show that the matrix $\begin{bmatrix} 1 & 5 \\ 2 & 10 \end{bmatrix}$ is not invertible.

(42) ~~Solve~~ Find Inverse of

(a) $\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 7 \\ 5 & 9 \end{bmatrix}$

(43) Find Inverse of (a) $\begin{bmatrix} -2 & 2 & 3 \\ 1 & 4 & 2 \\ -2 & -3 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$

(44) If $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 1 & 5 \\ 1 & 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 5 \\ 3 & 7 \\ 4 & 9 \end{bmatrix}$ then verify that $(AB)^T = B^T \cdot A^T$.

(45) Find Adjoint of $\begin{bmatrix} -2 & 2 & 3 \\ 1 & 4 & 2 \\ -2 & -3 & 1 \end{bmatrix}$

(46) Find minors and co-factors of $\begin{bmatrix} 1 & 3 & 0 \\ 0 & 5 & 1 \\ 1 & -1 & 7 \end{bmatrix}$

(47) Solve if (a) $\begin{bmatrix} 12 & 3 \\ 2 & 6 \end{bmatrix} = \begin{bmatrix} x+y & 3 \\ 2 & x-y \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(c) $\begin{bmatrix} x & y & z \end{bmatrix} - \begin{bmatrix} -4 & 3 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 1 & 0 \end{bmatrix}$

(48) solve by matrix method

(a) $2x-3y=1,\ x+y=2$

(b) $x-y+z=4,\ 2x+y-3z=0,\ x+y+z=2$

(c) ~~$2x-y+2=0,\ 3x+4y-3=0$~~

(d) ~~$x+y-z=6,\ 2x-3y+z=1,\ 2x-4y+2z=1$~~

(49) Give an exampe of 2x3 matrix.

(50) If $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ x & 0 & 0 \end{vmatrix}$ then find the value of x.

(51) For which value of x the given determinant $\begin{vmatrix} x & 1 \\ 2x & 2 \end{vmatrix} = 0$

(52) ~~Find maximum value of the given determinant~~ $\begin{vmatrix} \sin^2 x & \sin x \cdot \cos x \\ -\cos x & \sin x \end{vmatrix}$

(53) If $\begin{vmatrix} a & b & c \\ b & a & b \\ x & b & c \end{vmatrix} = 0$ then find x.

(54) Solve using ~~cramers rule:~~

(a) $2x-3y=7$
$3x-2y=3$

(b) ~~$3x+2y+6z=1$~~
~~$2x-3y+4z=3$~~
~~$4x-3y+7z=4$~~

(c) ~~$x+y+z=3$~~
~~$2x+3y+4z=9$~~
~~$x+2y-4z=-1$~~

(55) page 87 4(i), (iii), ~~page 89/No5, page 90/No7,~~ page 91/No8
page 92/9, ~~page 92/10, page 93/11, page 93/No12,~~
page 94/No13, ~~page 95/No14,~~
See from book.