Internal Assessment Examination - 2022
3rd Semester Mechanical
Subject : Strength of Material (Th-2)

Questions And Answers :

Q.1 : a) Define stress and strain and write its Units ?

Ans : Stress : The force of resistance per unit area, offered by a body against deformation is known as stress.

Mathematically, $\boxed{\sigma = \frac{P}{A}}$ where, σ = Stress
P = external force or load
A = Area of cross-sectional area

Units : In S.I ⇒ N/m^2 or N/mm^2
In CGS ⇒ $dyne/cm^2$

Strain : When a body is subjected to some external force, there is some change of dimension of the body. The ratio of change of dimension of the body to the original dimension is known as strain. Mathematically $\boxed{e = \frac{\text{Change in dimension}}{\text{Original dimension}}}$

Units : It has no units

b) State Hooke's law and express Mathematically ?

Ans : Hooke's Law : When a material is loaded within elastic limit, the stress is proportional to the strain produced by the stress. This means the ratio of the stress to the corresponding strain is a constant within the elastic limit. This constant is known as Modulus of elasticity or Modulus of Rigidity or Elastic Modulus.

Mathematically, Stress $\propto$ Strain

or $\sigma \propto e$

or $\sigma = E \times e$

or $\boxed{E = \frac{\sigma}{e}}$.

c) Define poission's Ratio and Express Mathematically ?

Ans : Poission's Ratio :
The ratio of lateral strain to the longitudinal strain is a constant for a given material, when the material is stressed within the elastic limit. This ratio is called Poission's Ratio and it is denoted by μ or $\frac{1}{m}$.

Mathematically, $\boxed{\mu = \frac{1}{m} = \frac{\text{Lateral Strain}}{\text{Longitudinal Strain}}}$

d) Define Strain Energy and What do you mean by Straining effect ?

Ans: Strain energy: Whenever a body is strained, the energy is absorbed in the body. The energy, which is absorbed in the body due to straining effect, is known as strain energy.

Straining effect: The straining effect may be due to
* gradually applied load
* Suddenly applied load
* Load with impact.

e) Define Hoop Stress and Longitudinal Stress ?

Ans: Hoop Stress: The stress acting along the circumference of the cylinder is called circumferential stress or Hoop stress.

Mathematically, $\boxed{\sigma_h = \frac{p \cdot d}{2t}}$ ∵ p = internal fluid pressure
d = internal diameter
t = thickness of the cylinder

Longitudinal Stress: The stress acting along the length of the cylinder is called longitudinal stress.

Mathematically, $\boxed{\sigma_L = \frac{p \cdot d}{4t}}$

f) Define Principal plane and Principal Stress ?

Ans: Principal plane: The plane, which have no shear stress are known as principal plane. Hence principal planes are the planes of zero shear stress.

Principal Stress: The normal stress, acting on a principal plane, are known as principal stress.

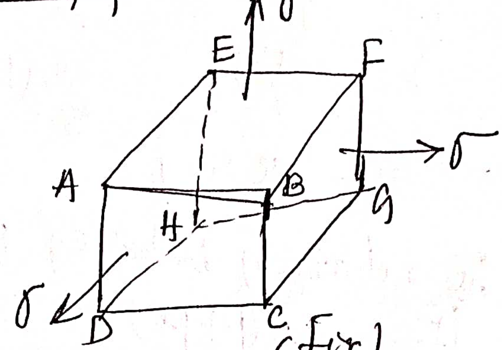
g) What do you mean by SF, BM, SFD and BMD ?

Ans: SF → It represent Shear force
BM → It represent Bending Moment
SFD → It represent Shear force diagram.
BMD → It represent Bending Moment diagram

Q.2: a) Establish relationship between three elastic Constants i/e E, G, K.

Ans: Relation Between E & K

Fig. ABCDEFGH which is Subjected to three Mutually perpendicular tensile stress of equal intensity.



(Fig)

Let L = Length of Cube

dL = change in length of the Cube

E = Young's modulus of the material Cube

σ = tensile stress acting on the faces

μ = poisson's ratio

∴ volume of Cube = $V = L^3$

Let us Consider the strain of one of the sides of the Cube say AB under the action of the three mutually perpendicular stresses. This side will suffer the following three strains:

* Strain AB due to stresses on the faces AEHD and BFGC. This strain is tensile and is equal to $\frac{\sigma}{E}$
* Strain AB due to stresses on the faces AEFB and DHGC. This is Compressive lateral strain and is equal to $-\mu\frac{\sigma}{E}$
* Strain of AB due to stresses on the faces ABCD and EFGH. This is also compressive lateral strain and is equal to $-\mu\frac{\sigma}{E}$

Hence, the total strain of AB is given by

$$\frac{dL}{L} = \frac{\sigma}{E} - \mu\frac{\sigma}{E} - \mu\frac{\sigma}{E} = \frac{\sigma}{E}(1-2\mu) \quad \text{---} (1)$$

∴ original volume of Cube, $V = L^3$ ——— (2)

If dL is the change in length, then dV is the change in volume

Differentiating Eq(2) w.r.t 'L'

$$\cancel{\frac{dV}{dL}} = dV = 3L^2 \cdot dL \quad \text{---} (3)$$

Dividing Eq(3) by eq(2), We get

$$\frac{dV}{V} = \frac{3L^2 \cdot dL}{L^3} = 3 \cdot \frac{dL}{L}$$

or. $\frac{dV}{V} = 3 \cdot \frac{\sigma}{E}(1-2\mu)$, using eq(i)

We know $K = \frac{\sigma}{\left(\frac{dV}{V}\right)} = \frac{\sigma}{\frac{3\sigma}{E}(1-2\mu)} = \frac{E}{3(1-2\mu)}$

or $\boxed{E = 3K(1-2\mu)}$

Similarly, the relation between 'E' and 'G' is

$$\boxed{E = 2G(1+\mu)}$$

$\therefore E = 3K(1-2\mu)$ —— ①

& $E = 2G(1+\mu)$ —— ②

$\Rightarrow 1+\mu = \frac{E}{2G}$ —— ③

& $1-2\mu = \frac{E}{3K}$ —— ④

Multiplying Eq③ by 2 and add with eq④.

$$2 + \cancel{2\mu} = \frac{2E}{2G}$$

$$1 - \cancel{2\mu} = \frac{E}{3K}$$

$$3 = \frac{E}{G} + \frac{E}{3K} = \frac{E(3K+G)}{3KG}$$

or, $\boxed{E = \frac{9 \cdot K \cdot G}{G+3K}}$ or $\boxed{\frac{9}{E} = \frac{1}{K} + \frac{3}{G}}$

b) If thin cylindrical shell of 120 cm diameter, 1.5 cm thick and 6 m long is subjected to internal fluid pressure of 2.5 N/mm². If the value of E = 2×10⁵ N/mm² and Poisson's Ratio = 0.3. Calculate:

(i) Change in diameter

(ii) Change in length

(iii) Change in volume.

Ans.: Given :

$d = 120\,cm = 1200\,mm$

$t = 1.5\,cm = 15\,mm$

$L = 6\,m = 6000\,mm$

$p = 2.5\,N/mm^2$

$E = 2 \times 10^5\,N/mm^2$

$\mu = 0.3$

(i) Change in diameter $= \delta d = \frac{p d^2}{2tE}\left[1 - \frac{\mu}{2}\right]$

or. $\delta d = \frac{2.5 \times (1200)^2}{2 \times 15 \times 2 \times 10^5}\left[1 - \frac{0.3}{2}\right]$

or. $\boxed{\delta d = 0.51\,mm}$ Ans

(ii) change in length $= \delta L = \frac{p.d.L}{2.t.E}\left[\frac{1}{2} - \mu\right]$

or. $\delta L = \frac{2.5 \times 1200 \times 6000}{2 \times 15 \times 2 \times 10^5}\left[\frac{1}{2} - 0.3\right]$

or. $\boxed{\delta L = 0.6\,mm}$ Ans

(iii) Change in volume $= \delta V = V\left[2 \times \frac{\delta d}{d} + \frac{\delta L}{L}\right]$

or $\delta V = \frac{\pi}{4} d^2 L\left[2 \times \frac{\delta d}{d} + \frac{\delta L}{L}\right]$

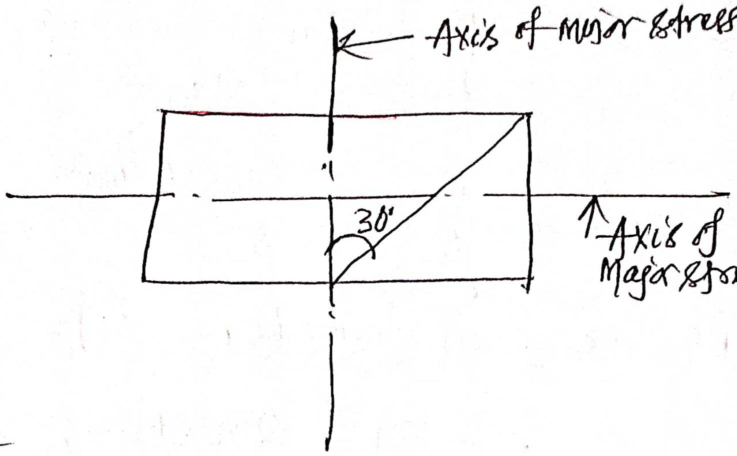
or. $\delta V = \frac{\pi}{4}(1200)^2 \times 6000\left[2 \times \frac{0.51}{1200} + \frac{0.6}{6000}\right]$

or. $\delta V = 64,497,000\,mm^3$

or. $\boxed{\delta V = 6449.7\,cm^3}$ Ans

c) The tensile stresses at a point across two mutually perpendicular planes are 120 N/mm² and 60 N/mm². Determine the normal, tangential and resultant stresses on a plane inclined at 30° to the axis of Minor Stress.

Ans:



Given:

$\sigma_1 = 120 N/mm^2$

$\sigma_2 = 60 N/mm^2$

$\theta = 30°$

We know,

$\therefore$ Normal Stress $= \sigma_n =$

$$\sigma_n = \left(\frac{\sigma_1 + \sigma_2}{2}\right) + \left(\frac{\sigma_1 - \sigma_2}{2}\right) \cos 2\theta$$

or. $$\sigma_n = \left(\frac{120+60}{2}\right) + \left(\frac{120-60}{2}\right) . \cos(2 \times 30°)$$

or. $$\boxed{\sigma_n = 105 N/mm^2}$$ — Ans

We know.

$\therefore$ Tangential Stress $= \sigma_t = \left(\frac{\sigma_1 - \sigma_2}{2}\right) . \sin 2\theta$

or. $$\sigma_t = \left(\frac{120-60}{2}\right) . \sin(2 \times 30°)$$

or. $$\boxed{\sigma_t = 25.98 N/mm^2}$$ — Ans

$\therefore$ Resultant Stress $= \sigma_r = \sqrt{(\sigma_n)^2 + (\sigma_t)^2}$

or. $$\sigma_r = \sqrt{(105)^2 + (25.98)^2}$$

or. $$\boxed{\sigma_r = 108.16 N/mm^2}$$ — Ans