PNS School of Engineering & Technology

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Internal Assessment Examination-2022(3rd Semester)

Subject : Th-1 -Engineering Math-III

Branch : Electrical & ETC Engineering

Time : $1\frac{1}{2}$ Hours F.M. : 20

1. *Answer the following questions (any FIVE).* [2 x 5]

(a) Solve : $(x+y) + i(x-y) = 12+6i$

(b) Find reciprocal of $1+i$

(c) Find rank of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(d) Compute $\frac{1}{D^2+4}(\sin 3x)$

(e) Solve $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$

(f) Find $\Gamma\left(\frac{3}{2}\right)$

(g) Form a partial differential equation corresponding to

$Z = f(x^2-y^2)$

(h) Find $L\{e^{3t}\sin 5t\}$

2. *Answer any TWO questions -* [5 x2]

(a) Show that $\left(\frac{1+\sin\theta + i\cos\theta}{1+\sin\theta - i\cos\theta}\right)^n = \cos\left(\frac{n\pi}{2} - n\theta\right) + i\sin\left(\frac{n\pi}{2} - n\theta\right)$

(b) Test consistency of following system and if possible solve it.

$2x-3y+7z=5,\ 3x+y-3z=13,\ 2x+19y-47z=32.$

(c) Find P.I for $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 4\cos^2 x$

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SOLUTIONS:

(1)(a) $(x+y)+i(x-y) = 12+6i$

$\Rightarrow x+y = 12$ ——— ①

and $x-y = 6$ ——— ②

$Eq^n(1)+Eq^n(2) \Rightarrow 2x = 18 \Rightarrow x = 9$

$Eq^n(1)-Eq^n(2) \Rightarrow 2y = 6 \Rightarrow y = 3$

$\therefore x = 9,\ y = 3$ (Ans)

(b) Reciprocal of $(1+i) = \frac{1}{1+i}$

$$= \frac{1\times(1-i)}{(1+i)(1-i)} = \frac{(1-i)}{(1)^2-(i)^2} = \frac{1-i}{1-i^2}$$

$$= \frac{1-i}{1+1} \quad (\because i^2 = -1)$$

$$= \frac{1}{2} - \frac{1}{2}i \quad \text{(Ans)}$$

(c) Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Now $|A| = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 1\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = 1(1-0) = 1 \neq 0$

$\therefore \exists$ a 3rd order non vanishing minor

Hence $\rho(A) = 3$ (Ans)

(d) $\frac{1}{D^2+4}(\sin 3x) = \frac{1}{-3^2+4}(\sin 3x)$

$$= \frac{1}{-9+4}(\sin 3x)$$

$$= -\frac{1}{5}(\sin 3x) \quad \text{(Ans)}$$

1(e)

Soln $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0 \Rightarrow [D^2 - 5D + 6]y = 0$ — ①

Here $F(D) = D^2 - 5D + 6$, $f(x) = 0$.

Auxiliary equation for (i) is $F(m) = 0$

$\Rightarrow m^2 - 5m + 6 = 0 \Rightarrow m = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 1 \cdot 6}}{2 \times 1} = \frac{5 \pm 1}{2} =$

$\frac{5+1}{2}$ or $\frac{5-1}{2} = 3, 2$.

$\because$ Roots are 2, 3

Hence $C.F = C_1 e^{2x} + C_2 e^{3x}$

$P.I = \frac{1}{F(D)} f(x) = \frac{1}{F(D)} (0) = 0$

$\therefore$ $C.S = C.F + P.I \Rightarrow y = C_1 e^{2x} + C_2 e^{3x}$ (Ans)

1(f) $\Gamma\left(\frac{3}{2}\right)$

$= \Gamma\left(1+\frac{1}{2}\right)$

$= \Gamma\left(\frac{1}{2}+1\right) = \frac{1}{2}\Gamma\left(\frac{1}{2}\right)$

$= \frac{1}{2} \cdot \sqrt{\pi} = \frac{\sqrt{\pi}}{2}$ (Ans)

$\left(\because \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}\right)$

1(8) Soln $z = f(x^2-y^2)$ ——— (i)

$p = \frac{\partial z}{\partial x} = \frac{\partial}{\partial x} f(x^2-y^2) = f'(x^2-y^2) \cdot \frac{\partial}{\partial x}(x^2-y^2)$

$= f'(x^2-y^2) \cdot 2x$

$\therefore p = 2x \cdot f'(x^2-y^2)$ ——— (2)

Again $q = \frac{\partial z}{\partial y} = \frac{\partial}{\partial y} f(x^2-y^2) = f'(x^2-y^2) \cdot \frac{\partial}{\partial y}(x^2-y^2)$

$= f'(x^2-y^2) \cdot (-2y)$

$\Rightarrow q = -2y f'(x^2-y^2)$ ——— (3)

Now $\frac{p}{q} = \frac{2x f'(x^2-y^2)}{-2y f'(x^2-y^2)} = -\frac{x}{y}$

$\Rightarrow py = -qx \Rightarrow py + qx = 0$ which is the required partial differential equation corresponding to given equation. (Ans)

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1(h) $L(\sin 5t) = \frac{5}{s^2+5^2} = \frac{5}{s^2+25}$

$\therefore L\{e^{3t} \cdot \sin 5t\} = \frac{5}{(s-3)^2+25}$

(by shifting property)

$= \frac{5}{s^2-6s+34}$ (Ans)

2(a) LHS $= \cancel{\cos\theta +} \left(\dfrac{1+\sin\theta + i\cos\theta}{1+\sin\theta - i\cos\theta}\right)^n$

$$= \left[\frac{1+\cos(\frac{\pi}{2}-\theta) + i\sin(\pi/2-\theta)}{1+\cos(\frac{\pi}{2}-\theta) - i\sin(\frac{\pi}{2}-\theta)}\right]^n$$

$$= \left[\frac{1+\cos A + i\sin A}{1+\cos A - i\sin A}\right]^n$$ put $\frac{\pi}{2}-\theta = A$

$$= \left[\frac{1+(\cos A + i\sin A)}{1+(\cos A + i\sin A)^{-1}}\right]^n$$ (using De-moivre's thm)

$$= \left[\frac{1+z}{1+z^{-1}}\right]^n$$ put $z = \cos A + i\sin A$

$$= \left[\frac{1+z}{1+\frac{1}{z}}\right]^n = \left[\frac{(1+z)}{(1+z)/z}\right]^n = \left[(1+z)\times\frac{z}{(1+z)}\right]^n$$

$$= z^n = \left[\cos A + i\sin A\right]^n = \cos nA + i\sin nA$$

$$= \cos n\left(\frac{\pi}{2}-\theta\right) + i\sin n\left(\frac{\pi}{2}-\theta\right)$$

$$= \cos\left(\frac{n\pi}{2}-n\theta\right) + i\sin\left(\frac{n\pi}{2}-n\theta\right) = RHS$$

2(b) Soln Given system is

$$\left.\begin{aligned} 2x-3y+7z &= 5 \\ 3x+y-3z &= 13 \\ 2x+19y-47z &= 32 \end{aligned}\right\} \text{——} (1)$$

Co-efficient matrix

$$A = \begin{bmatrix} 2 & -3 & 7 \\ 3 & 1 & -3 \\ 2 & 19 & -47 \end{bmatrix}$$

Augmented matrix

$$K = \left[\begin{array}{ccc:c} 2 & -3 & 7 & 5 \\ 3 & 1 & -3 & 13 \\ 2 & 19 & -47 & 32 \end{array}\right]$$

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$$= \left[\begin{array}{ccc:c} 2 & -3 & 7 & 5 \\ 1 & 4 & -10 & 8 \\ 2 & 19 & -47 & 32 \end{array}\right] \quad R_2 \to R_2 - R_1$$

$$= \left[\begin{array}{ccc:c} 1 & 4 & -10 & 8 \\ 2 & -3 & 7 & 5 \\ 2 & 19 & -47 & 32 \end{array}\right] \quad R_1 \leftrightarrow R_2$$

$$\left[\begin{array}{cccc|c} 1 & 4 & -10 & : & 8 \\ 0 & -11 & 27 & : & -11 \\ 0 & 11 & -27 & : & 16 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 2R_1 \end{array}$$

$R_3 \to R_3 + R_2$

$$\left[\begin{array}{cccc|c} 1 & 4 & -10 & : & 8 \\ 0 & -11 & 27 & : & -11 \\ 0 & 0 & 0 & : & 5 \end{array}\right] \text{ ——— } (2).$$

For A, All 3rd order minor vanishes.
But $\exists$ a 2nd order non-vanishing minor which is $|A_1| = \begin{vmatrix} 1 & 4 \\ 0 & -11 \end{vmatrix} = -11 \neq 0$

$\therefore \rho(A) = r = 2$

For K, $\exists$ a 3rd order non-vanishing minor which is $|A_2| = \begin{vmatrix} 4 & -10 & 8 \\ -11 & 27 & -11 \\ 0 & 0 & 5 \end{vmatrix}$

$= 5 \begin{vmatrix} 4 & -10 \\ -11 & 27 \end{vmatrix}$ Expanding w.r.t R_3

$= 5(108 - 110) = -10 \neq 0$

$\therefore \rho(K) = r' = 3$

Here $r \neq r'$

Hence Given system is inconsistent (Ans)

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2(c) Sol^n $\dfrac{d^2y}{dx^2} + 3\dfrac{dy}{dx} + 2y = 4\cos^2 x$

$\Rightarrow [D^2 + 3D + 2]y = 4\cos^2 x$ —— (1) $\quad D^n \equiv \dfrac{d^n}{dx^n}$

Here $F(D) = D^2+3D+2$, $f(x) = 4\cos^2 x$

$$P.I = \frac{1}{F(D)} f(x) = \frac{1}{D^2+3D+2}(4\cos^2 x)$$

$$= \frac{1}{D^2+3D+2}\left[2\cdot(2\cos^2 x)\right]$$

$$= \frac{1}{D^2+3D+2}\left[2\cdot(1+\cos 2x)\right]$$

$$= \frac{1}{D^2+3D+2}\left[2 + 2\cos 2x\right]$$

$$= \frac{1}{D^2+3D+2}(2) + \frac{1}{D^2+3D+2}2\cos 2x$$

$$= 2\frac{1}{D^2+3D+2}(1) + 2\frac{1}{D^2+3D+2}(\cos 2x)$$

$$= 2 \cdot \frac{1}{D^2+3D+2}\left(e^{0\times x}\right) + 2 \cdot \frac{1}{-2^2+3D+2} \cos 2x$$

$$= 2 \times \frac{1}{0^2+3\times 0+2}(1) + \frac{2}{3D-2} \cos 2x$$

$$= 1 + \frac{2(3D+2)}{(3D-2)(3D+2)} \cos 2x$$

$$= 1 + \frac{2(3D+2)}{9D^2 - 4} \cos 2x$$

$$= 1 + \frac{2(3D+2)}{9(-2^2) - 4} \cos 2x$$

$$= 1 + \frac{2(3D+2)}{-36-4} \cos 2x$$

$$= 1 - \frac{1}{20}(3D+2) \cos 2x$$

$$= 1 - \frac{1}{20}\left[-6 \sin 2x + 2 \cos 2x\right]$$

$$= 1 + \frac{1}{10}\left[3 \sin 2x - \cos 2x\right] \text{ (Ans)}$$

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