PNS SCHOOL OF ENGINEERING & TECHNOLOGY

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LECTURER'S NOTE

ON

CIRCUIT AND NETWORK THEORY

(THEORY – 2)

FOR

3RD SEMESTER, ELECTRICAL ENGINEERING

(AS PER SCTE&VT SYLLABUS)

PREPARED BY

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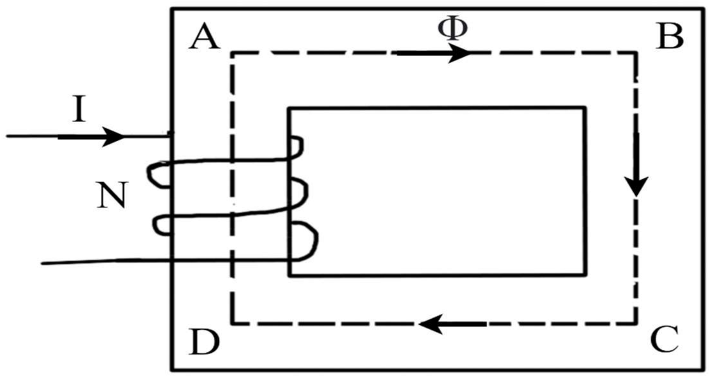
HOD ELECTRICAL

CHAPTER 1: MAGNETIC CIRCUIT

Introduction:

A magnetic circuit is a closed path that is followed by magnetic field lines of magnetic flux. Materials having high permeability such as soft steel, iron, etc are used in the magnetic circuit. The materials with a high degree of permeability will offer very low resistance to the flow of the magnetic flux.

Let's consider a coil as shown in the figure below.

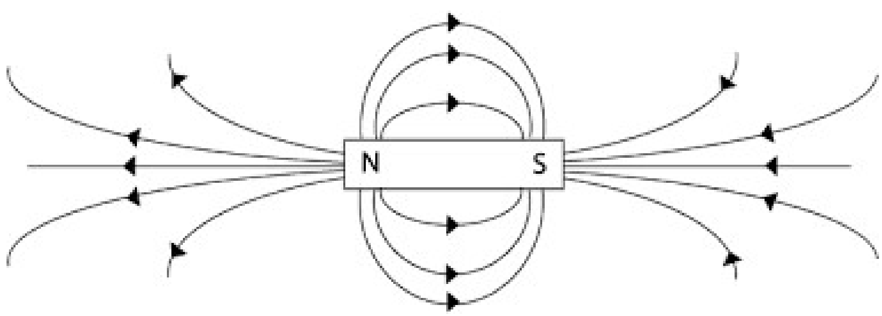


There are 'N' numbers of turns present in the coil and this coil is wound on a rectangular iron core. As we pass current 'I' through the coil, magnetic flux Φ is set up in the iron core. The flux will follow a path ABCDA and the direction of the magnetic flux is given by the "Right-hand thumb rule."

The magnetic flux Φ depends upon the magnitude of the current 'I' and the number of turns 'N' of the coil.

Magnetising flux (Φ):

Magnetic flux is the total amount of imaginary lines representing the direction of magnetic field such that the tangent at any point is the direction of the field vector at that point. Its unit is Weber (Wb).



Magneto motive Force (MMF):

The force which drives flux through a magnetic circuit is known as MMF. It is the product of current and the number of turns in a coil.

MMF =NI   Ampere turns (AT)

Magnetic field strength or intensity (H):

It is defined as the magneto motive force per unit length of the magnetic flux path. It is denoted by 'H'.

$$H = \frac{MMF}{l} = \frac{NI}{l} \quad \text{Ampere turns/meter (AT/m)}$$

Flux density (B):

It is defines as the flux passing per unit area through a plane at right angle to the flux. It is denoted by 'B'.

$$B = \frac{\Phi}{A} \quad \text{Weber/meter square } (Wb / m^2) \text{ or Tesla (T)}$$

Magnetic Permeability (µ):

Permeability of a material describes its ability to support or concentrate magnetic flux. It means, it is a measure of the ease with which the material can be magnetized. It is denoted by µ. Its unit is Henry/meter (H/m).

Permeability of free space is known as 'Absolute permeability' and donated by ' μ_0'.

$$\mu_0 = 4 \pi * 10^{-7} \quad \text{Henry/meter (H/m)}$$

Permeability of all other material is

$$\mu = \mu_0 * \mu_r$$

Where, μ_r is known as relative permeability. It is simply a numeric which expresses the degree by which the material is a better conductor of magnetic flux than free space.

Reluctance (S):

It is a measure of the opposition offered by a magnetic circuit to the setting up of the magnetic flux. It is denoted by 'S'. It is also defined as the ratio of MMF to flux. It is directly proportional to length and inversely proportional to X-sectional area of the magnetic path.

$$S \propto \frac{l}{a}$$

Or $S = \dfrac{l}{\mu a}$ (H^{-1})

Also $S = \dfrac{\Phi}{MMF}$ (AT/Wb)

Permeance (P):

The reciprocal of reluctance is called as Permeance. It allows the magnetic flux inside the magnetic material. It is denoted by 'P'.

$$P = \frac{1}{S} = \frac{\mu a}{l} \quad \text{(Wb/AT)}$$

Relation between B & H:

The magnetic flux density 'B' produced in a material is directly proportional to applied filedintensity 'H'.

$$B \propto H$$

Or $B = \mu H = \mu_0 * \mu_r H$

Analogy between electric and Magnetic Circuits:

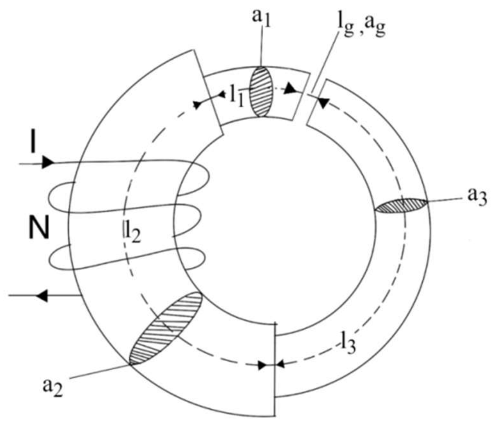
Similarities

Magnetic circuit		Electric circuit	
1	Closed path for magnetic flux is called Magnetic circuit.	1	Closed path for electric current is called as Electric circuit.
2	Flux (**Φ**)	2	Current **(I)**
3	Magneto motive Force **(MMF)**	3	Electro motive Force **(EMF)**
4	Reluctance **(S)**	4	Resistance **(R)**
5	Permeance **(P)**	5	Conductance **(G)**
6	Permeability **(μ)**	6	Conductivity **(σ)**
7	Flux density **(B)**	7	Current density **(J)**
8	Magnetic field Intensity **(H)**	8	Electric field Intensity **(E)**

Dissimilarities

Magnetic circuit		Electric circuit	
1	Actually flux does not flow but it set up in magnetic field.	1	Current flow in an electric circuit.
2	For magnetic flux there is no perfect insulation.	2	For current there is large no. of insulators. For example: Rubber, glass, mica etc.
3	The reluctance of magnetic circuit is not constant for entire circuit.	3	The resistance of electric circuit almost constant for entire circuit.
4	Once the magnetic flux is set-up in a magnetic circuit, no energy is expanded.	4	Energy is expanded, means energy can be transferred to the other circuits by using Transformer.

Series Magnetic Circuit:



In a series magnetic circuit, the same amount of flux Φ flows through each part of the circuit and this is comparable to the series electric circuit where the same amount of current flows through the circuit.

The figure below shows a composite magnetic circuit which is a series circuit that is composed of parts having different dimensions and different materials.

The series of magnetic circuits consist of three different materials along with an air gap. The different materials in the circuit have their own relative permeability. Also, the different parts have their differences in the cross-sectional area and hence the flux density will also be different in all these parts.

Total Reluctance:

$$S = S_1 + S_2 + S_3 + S_g$$

Or $\quad S = \frac{l_1}{\mu_1 a_1} + \frac{l_2}{\mu_2 a_2} + \frac{l_3}{\mu_3 a_3} + \frac{l_g}{\mu_0 a_g}$

Total MMF = Flux × Total reluctance

Or $\quad MMF = \Phi * S$

Or $\quad MMF = \Phi \left[\frac{l_1}{\mu_1 a_1} + \frac{l_2}{\mu_2 a_2} + \frac{l_3}{\mu_3 a_3} + \frac{l_g}{\mu_0 a_g}\right]$

Or $\quad MMF = \frac{\Phi l_1}{\mu_1 a_1} + \frac{\Phi l_2}{\mu_2 a_2} + \frac{\Phi l_3}{\mu_3 a_3} + \frac{\Phi l_g}{\mu_0 a_g}$

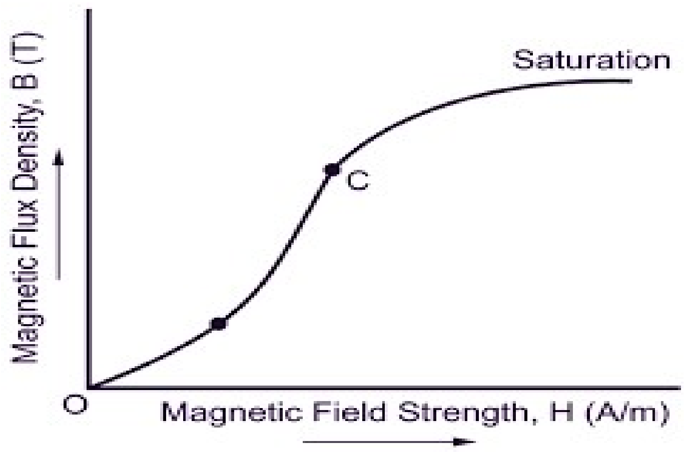
Or $\quad MMF = \frac{B_1 l_1}{\mu_1} + \frac{B_2 l_2}{\mu_2} + \frac{B_3 l_3}{\mu_3} + \frac{B_g l_g}{\mu_0}$

Or $\quad MMF = H_1 l_1 + H_2 l_2 + H_3 l_3 + H_g l_g$

B-H curve:

The curve showing the relation between magnetic flux density 'B' and magnetic field strength 'H' is known as B-H curve.

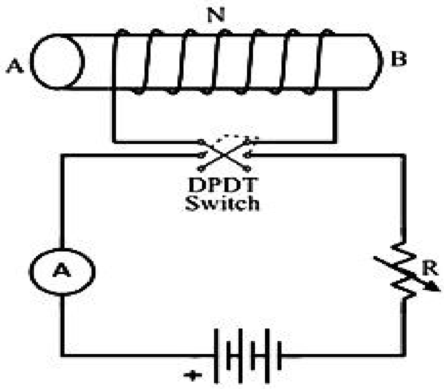
The following figure shows the general shape of B-H curve of a magnetic material. The nonlinearity of the curve shows that the relative permeability μ_r of a magnetic material is not constant but varies depending upon the magnetic flux density B.

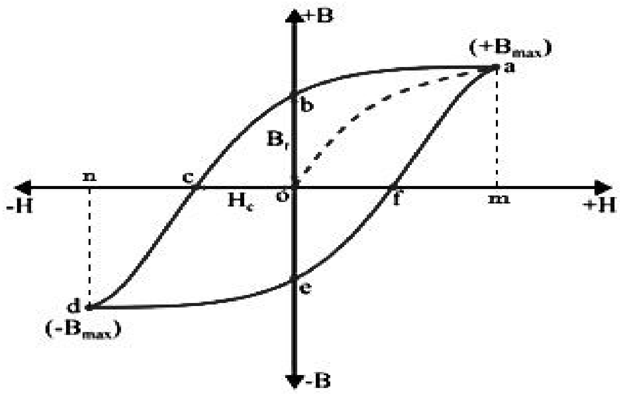


Magnetic Hysteresis:

The phenomenon of lagging of magnetic flux density 'B behind the magnetic field strength 'H' in a magnetic material subjected to the cycle of magnetization called as magnetic hysteresis.

Hysteresis Loop:





Let us take an unmagnetized bar of iron AB and magnetize it by placing it within the field of a solenoid. The field ‘H’ produced by the solenoid is called magnetic field strength. The value of ‘H’ can be increased or decreased by increasing or decreasing the current through the coil.

Let ‘H’ is increased from zero to a certain maximum value and the corresponding values of flux density ‘B’ be noted. If we plot the relation between ‘H’ and ‘B’, a curve like ‘oa’ is obtained. The material becomes magnetically saturated for ‘H’ = ‘om’ and at that time the flux density ‘B’ = ‘B_{max}’

If ‘H’ is now decreased gradually by decreasing the solenoid current, flux density ‘B’ will not decrease along ‘oa’, as might be expected but will decrease less rapidly along ‘ab’. When ‘H’ is zero, ‘B’ is not but has a definite value. It means that on removing the magnetic strength ‘H’, the iron bar is not completely demagnetized. This value of ‘B’ i.e. ‘ob’ measures retentivity or remanence of the material and is called remanent or residual flux density ‘B_r’.

To demagnetize the iron bar, we have to apply the magnetic strength in the reverse direction. When ‘H’ is reversed by reversing solenoid current, then ‘B’ is reduced to zero at a point ‘c’ where the value of ‘H’ is ‘oc’. This value of ‘H’ is required to wipe off the residual magnetism is known as coercive force ‘H_c’ and is a measure of the coercivity of the material. When the flux density has been reduced to zero and value of ‘H’ is further increased in the reverse direction, the iron bar again reaches a state of magnetic saturation, represented by point ‘n’. By taking ‘H’ back from its value corresponding to negative saturation ‘H’ = ‘on’ to its value for positive saturation ‘H’ = ‘om’, a curve ‘defa’ is obtained.

It is seen that ‘B’ always lag behind ‘H’. This lagging of ‘B’ behind ‘H’ is given the name Hysteresis. The closed loop ‘abcdefa’ which is obtained when iron bar is taken through one complete cycle of magnetization is known as Hysteresis loop.

QUESTION BANK

Short questions with answer:

Q: What is magnetic circuit?

A: The closed path followed by magnetic flux is called magnetic circuit.

Q: Define magnetic flux. The magnetic lines of force produced by a magnet are called magnetic flux &it is denoted as 'Φ' and its unit is Weber.

Q: Define magnetic flux density.

A: It is the flux per unit area at right angles to the flux it is denoted by 'B' and unit is Weber/m^2.

Q: Define magneto motive force.

A: The force which drives flux through a magnetic circuit is known as MMF. It is the product of current and the number of turns in a coil. MMF=NI ampere turns (AT).

Q: Define reluctance.

A: It is a measure of the opposition offered by a magnetic circuit to the setting up of the magnetic flux. It is denoted by 'S'. It is also defined as the ratio of MMF to flux. It is directly proportional to length and inversely proportional to X-sectional area of the magnetic path.

Q: What is retentivity?

A: The property of magnetic material by which it can retain the magnetism even after the removal of inducing source is called retentivity.

Q: Define permeance.

A: The reciprocal of reluctance is called as Permeance. It allows the magnetic flux inside the magnetic material. It is denoted by 'P'.

Q: Define permeability.

A: Permeability of a material describes its ability to support or concentrate magnetic flux. It means, it is a measure of the ease with which the material can be magnetized. It is denoted by μ. Its unit is Henry/meter (H/m).

Q: Define relative permeability.

A: It is equal to the ratio of flux density produced in that material to the flux density produced in air by the same magnetizing force $\mu_r = \mu/\mu_0$.

Long questions:

Q: Explain B-H Curve and Hysteresis loop.

Q: What are the similarities and dissimilarities between magnetic and electric circuits?

CHAPTER 2: COUPLED CIRCUIT

Self-Inductance:

When the current in a coil is changing, the magnetic flux linking the same coil changes and an EMF is induced in the coil. This induced EMF is proportional to the rate of change of current.

$$V \propto \frac{di}{dt} = L\frac{di}{dt} \quad \text{... (1)}$$

Here the constant of proportionality 'L' is known as Self-inductance.

Again according to Faraday's law of electromagnetic induction, the induced EMF in a coil having 'N' turns is given by

$$V = N\frac{d\Phi}{dt} \quad \text{... (2)}$$

Equation (1) & (2) give

$$L\frac{di}{dt} = N\frac{d\Phi}{dt}$$

Or $$L = N\frac{d\Phi}{dt} \quad \text{... (3)}$$

If the rate of change of flux with current is constant then equation (3) becomes

$$L = N\frac{\Phi}{i} \quad \text{... (4)}$$

Hence unit of Self-inductance is weber/ampere (Wb/A).

As we know that

$$\Phi = \frac{MMF}{S}, \quad MMF = Ni \quad \& \quad S = \frac{l}{\mu a}$$

Equation (4) gives

$$L = \frac{N*MMF}{i*S} = \frac{N*Ni}{i*\frac{l}{\mu a}}$$

Or $$L = \frac{N^2 \mu a}{l} \quad \text{... (5)}$$

Hence unit of Self-inductance is also Henery (H).

Mutual Inductance:

According to Faraday's law of electromagnetic induction, voltage induced in coil 2 due to a change in flux in coil 1 is given by

$$V_2 = N_2 \frac{d\Phi_{12}}{dt} \qquad \text{... (6)}$$

Here N_2 refers to No. of turns in coil 2 & Φ_{12} refers to change in flux in coil 1 linked to coil 2.

Since Φ_{12} is related to i_1, so V_2 is proportional to rate of change of current of coil 1 i.e. i_1.

$$V_2 \propto \frac{di_1}{dt} = M \frac{di_1}{dt} \qquad \text{... (7)}$$

Here the constant of proportionality 'M' is known as Mutual inductance.

Equation (6) & (7) give

$$M \frac{di_1}{dt} = N_2 \frac{d\Phi_{12}}{di_1}$$

Or $$M = N_2 \frac{d\Phi_{12}}{di_1} \qquad \text{... (8)}$$

If the rate of change of flux Φ_{12} with current i_1 is constant then equation (8) becomes

$$M = N_2 \frac{\Phi_{12}}{i_1} \qquad \text{... (9)}$$

Similarly, for coil 1 Mutual inductance is given by

$$M = N_1 \frac{\Phi_{21}}{i_2} \qquad \text{... (10)}$$

Total flux of coil 1

$$\Phi_1 = \frac{\text{MMF of coil 1}}{\text{Reluctance}}$$

Or $$\Phi_1 = \frac{N_1 i_1}{S} \qquad \text{... (11)}$$

Let the flux linked with coil 2 i.e. Φ_{12} is the K_{th} fraction of flux Φ_1

Hence $$\Phi_{12} = K\,\Phi_1 = K \frac{N_1 i_1}{S} \qquad \text{... (12)}$$

Putting the value of Φ_{12} in equation (9)

$$M = \frac{N_2 K N_1 i_1}{i_1 S}$$

Or $$M = \frac{K N_1 N_2}{S} \quad \text{... (13)}$$

Similarly, equation (10) becomes

$$M = \frac{N_1 K N_2 i_2}{i_2 S}$$

Or $$M = \frac{K N_1 N_2}{S} \quad \text{... (14)}$$

Coefficient of coupling:

If a portion of magnetic flux established by one circuit, inter links with a second coil, then two circuits have said to be coupled magnetically.

Let the fractional part of Φ_1 which links with N_2 is Φ_{12}/ Φ_1 and the fractional part of Φ_2 which links with N_1 is Φ_{21}/ Φ_2.

The coefficient of coupling between two coils is given by

$$K = [(\frac{\Phi_{12}}{\Phi_1})(\frac{\Phi_{21}}{\Phi_2})]^{0.5} \quad \text{... (15)}$$

Putting the value of Φ_{12} & Φ_{21} from equation (9) & (10)

Or $$K = [(\frac{Mi_1}{\Phi_1 N_2})(\frac{Mi_2}{\Phi_2 N_1})]^{0.5}$$

Or $$K = [(\frac{Mi_1}{\Phi_1 N_1})(\frac{Mi_2}{\Phi_2 N_2})]^{0.5}$$

Or $$K = [(\frac{Mi_1}{L_1})(\frac{Mi_2}{L_2})]^{0.5} \qquad \text{Putting } \frac{i_1}{\Phi_1 N_1} = \frac{1}{L_1} \ \& \ \frac{i_2}{\Phi_2 N_2} = \frac{1}{L_2}$$

Or $$K = [\frac{M^2}{L_1 L_2}]^{0.5}$$

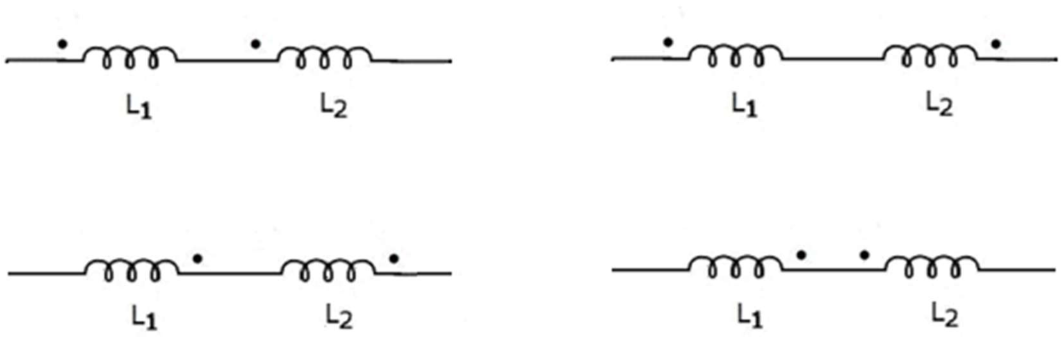
Or $$K = \frac{M}{\sqrt{L_1 L_2}} \quad \text{... (16)}$$

The coefficient of coupling may range from about 0.01 to about 0.98.

Dot Convention:

In a circuit the voltage induced due to mutual inductance may aid or oppose the voltage induced due to self-inductance. This depends on the relative positive direction of currents, the relative modes of windings of the coils involved and the actual physical placement of one winding with respect to other. Instead of showing actual mode of the winding, dot convention is used to yield the same information.

Coupled Inductor in Series:



Series Aiding

Series Opposing

For Series Aiding:

In this case, the total voltage induced in each of the two inductors is partly due to its self-inductance and partly due to mutual inductance.

$$V_1 = L_1 \frac{di}{dt} + M \frac{di}{dt} = (L_1 + M) \frac{di}{dt}$$

$$V_2 = L_2 \frac{di}{dt} + M \frac{di}{dt} = (L_2 + M) \frac{di}{dt}$$

Total Voltage

$$V = V_1 + V_2$$

Or $$V = (L_1 + M) \frac{di}{dt} + (L_2 + M) \frac{di}{dt}$$

Or $$V = (L_1 + M + L_2 + M) \frac{di}{dt}$$

Or $V = (L_1 + L_2 + 2M)\frac{di}{dt}$ ... (17)

Let the total inductance is L_T, then the total voltage

$$V = L_T\frac{di}{dt} \quad \text{... (18)}$$

Equation (17) & (18) give

$$L_T\frac{di}{dt} = (L_1 + L_2 + 2M)\frac{di}{dt}$$

Or $L_T = (L_1 + L_2 + 2M)$ ... (19)

For Series Opposing:

In this case the mutually induced voltage opposes the self-induced voltage.

$$V_1 = L_1\frac{di}{dt} - M\frac{di}{dt} = (L_1 - M)\frac{di}{dt}$$

$$V_2 = L_2\frac{di}{dt} - M\frac{di}{dt} = (L_2 - M)\frac{di}{dt}$$

Total Voltage

$$V = V_1 + V_2$$

Or $V = (L_1 - M)\frac{di}{dt} + (L_2 - M)\frac{di}{dt}$

Or $V = (L_1 - M + L_2 - M)\frac{di}{dt}$

Or $V = (L_1 + L_2 - 2M)\frac{di}{dt}$ ... (20)

Let the total inductance is L_T, then the total voltage

$$V = L_T\frac{di}{dt} \quad \text{... (21)}$$

Equation (20) & (21) give

$$L_T\frac{di}{dt} = (L_1 + L_2 - 2M)\frac{di}{dt}$$

Or $L_T = (L_1 + L_2 - 2M)$ ... (22)

Example 1: When two coils are connected in series, their effective inductance is found to be 10 H. When the connections of one coil are reversed, the effective inductance is 6 H. If the co-efficient of coupling is 0·6, calculate the self-inductance of each coil and the mutual inductance.

Solution:

$10 = L_1 + L_2 + 2_M$ ... (23)

$6 = L_1 + L_2 - 2_M$ ... (24)

Subtracting equation (24) from (23), we get,

$4 = 4M$ or $M = 1$ H

Putting M = 1 H in eq. (23), we have,

$L_1 + L_2 = 8$ ... (25)

Also $L_1L_2 = \frac{M^2}{K^2} = \frac{1^2}{0.6^2} = 2.78$

Now $(L_1 - L_2)\ 2 = (L_1 + L_2)\ 2 - 4L_1L_2 = (8)2 - 4 \times 2{\cdot}78 = 52{\cdot}88$

$L_1 - L_2 = \sqrt{52.88} = 7{\cdot}27$ ... (26)

Solving equation (25) and (26),

$L_1 = 7{\cdot}635$ H and $L_2 = 0{\cdot}365$ H

Example 2: The total inductance of two coils, A and B, when connected in series, is 0.5 H or 0.2H, depending upon the relative direction of the currents in the coils. Coil A, when isolated from coil B, has a self-inductance of 0.2 H. Calculate (i) the mutual inductance between the two coils, (ii) the self-inductance of coil B, (iii) the coupling factor between the coils, and (iv) the two possible values of the induced e.m.f. in coil A when the current is decreasing at 1000 A/s in the series circuit.

Solution: (i) Combined inductance of two coils, $L = L_1 + L_2 \pm 2_M$

For series-aiding,

$L_1 + L_2 + 2M = 0.5$ ... (27)

For series-opposing,

$L_1 + L_2 - 2M = 0.2$ ... (28)

Subtracting equation (28) from (27), we have,

$4M = 0.3$

$M = 0.075\ H$

(ii) Adding equation (28) & (27), we have,

$2(L_1 + L_2) = 0.7$

Or $2(0.2 + L_2) = 0.7$

$L_2 = 0.15\ H$

(iii) Coefficient of coupling is given by

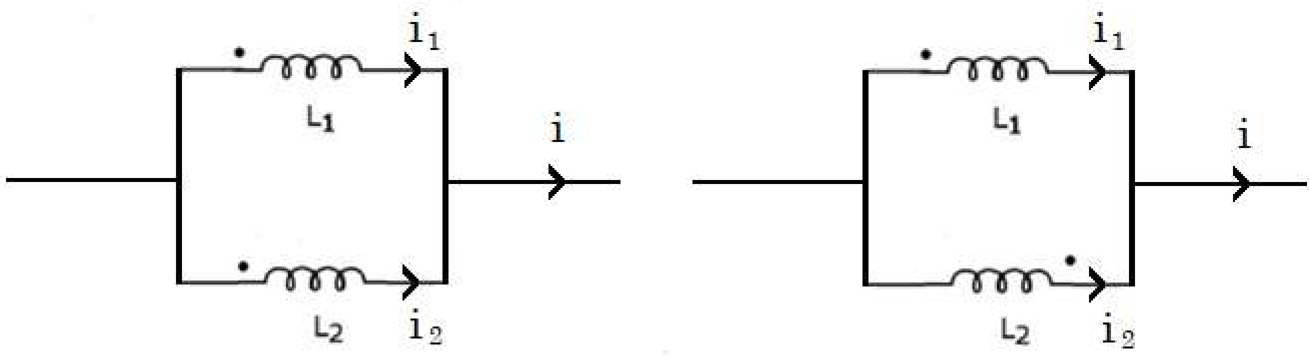
$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{0.075}{\sqrt{0.2 * 0.15}} = 0.433$$

(iv) $V_1 = L_1 \frac{di}{dt} \pm M \frac{di}{dt}$

$$V_1 = L_1 \frac{di}{dt} + M \frac{di}{dt} = 0.2 \times 1000 + 0.075 \times 1000 = 275\ V$$

$$V_1 = L_1 \frac{di}{dt} - M \frac{di}{dt} = 0.2 \times 1000 - 0.075 \times 1000 = 125\ V$$

Coupled Inductor in Parallel:



Parallel Aiding Parallel Opposing

For Parallel Aiding:

$i = i_1 + i_2$

Or $$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} \qquad \dots (29)$$

Since inductors are in parallel, voltages across them are equal, say 'V'

$$V = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \qquad = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

Or $$L_1 \frac{di_1}{dt} - M \frac{di_1}{dt} = L_2 \frac{di_2}{dt} - M \frac{di_2}{dt}$$

Or $$(L_1 - M) \frac{di_1}{dt} = (L_2 - M) \frac{di_2}{dt}$$

Or $$\frac{di_1}{dt} = \left(\frac{L_2 - M}{L_1 - M}\right) \frac{di_2}{dt} \qquad \text{... (30)}$$

Putting the value of $\frac{di_1}{dt}$ in equation (29)

$$\frac{di}{dt} = \left(\frac{L_2 - M}{L_1 - M}\right) \frac{di_2}{dt} + \frac{di_2}{dt}$$

Or $$\frac{di}{dt} = \left(\frac{L_2 - M}{L_1 - M} + 1\right) \frac{di_2}{dt} \qquad \text{... (31)}$$

Let the total inductance is L_T, then the voltage

$$V = L_T \frac{di}{dt} \qquad \text{... (32)}$$

Again as we know that this voltage 'V' is also equal to th voltage across any inductor.

$$L_T \frac{di}{dt} = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \qquad \text{... (33)}$$

Putting the value of $\frac{di_1}{dt}$ from equation (24)

$$L_T \frac{di}{dt} = L_1 \left(\frac{L_2 - M}{L_1 - M}\right) \frac{di_2}{dt} + M \frac{di_2}{dt}$$

Or $$L_T \frac{di}{dt} = L_1 \left(\frac{L_2 - M}{L_1 - M} + M\right) \frac{di_2}{dt}$$

Or $$L_T \frac{di}{dt} = \left(\frac{L_1 L_2 - L_1 M + L_1 M + M^2}{L_1 - M}\right) \frac{di_2}{dt}$$

Or $$\frac{di}{dt} = \frac{1}{L_T} \left(\frac{L_1 L_2 - M^2}{L_1 - M}\right) \frac{di_2}{dt} \qquad \text{... (34)}$$

Equation (31) & (34) give

$$\left(\frac{L_2 - M}{L_1 - M} + 1\right) \frac{di_2}{dt} = \frac{1}{L_T} \left(\frac{L_1 L_2 - M^2}{L_1 - M}\right) \frac{di_2}{dt}$$

Or $$\left(\frac{L_2 - M + L_1 - M}{L_1 - M}\right) = \frac{1}{L_T} \left(\frac{L_1 L_2 - M^2}{L_1 - M}\right)$$

Or $$(L_1 + L_2 - 2M) = \frac{1}{L_T} (L_1 L_2 - M^2)$$

Or $$L_T = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} \qquad \text{... (35)}$$

For Parallel Opposing:

Similar to the above derivation

$$L_T = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M} \qquad \text{... (36)}$$

Example 3: Two coils of self-inductances 150 mH and 250 mH and of mutual inductance 120 mH are connected in parallel. Determine the equivalent inductance of the combination if (i) mutual flux helps the individual flux and (ii) mutual flux opposes the individual flux.

Solution. Here, L1 = 0.15 H ; L2 = 0.25 H ; M = 0.12 H

(i) Equivalent inductance LT of the parallel combination when mutual flux helps the individual flux is

$$L_T = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} = L_T = \frac{0.15*0.25 - 0.12^2}{0.15 + 0.25 - 2*0.12} = 0.144 \text{ H.}$$

(ii) Equivalent inductance LT of the parallel combination when the mutual flux opposes the individual flux is

$$L_T = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} = L_T = \frac{0.15*0.25 - 0.12^2}{0.15 + 0.25 + 2*0.12} = 0.036 \text{ H.}$$

QUESTION BANK

Short questions with answer:

Q: What is Self-inductance?

A: The property of a coil that opposes any change in the amount of current flowing through it is called its self-inductance.

Q: What is Mutual-inductance?

A: Mutual inductance is the property of two coils by the virtue of which each opposes any change in the value of current flowing through the other by developing an induced EMF.

Q: Define Coefficient of Coupling.

A: The coefficient of coupling (k) between two coils is defined as the fraction of magnetic flux produced by the current in one coil that links the other.

Q: Define Dot Convention.

A: In a circuit the voltage induced due to mutual inductance may aid or oppose the voltage induced due to self-inductance. This depends on the relative positive direction of currents, the relative modes of windings of the coils involved and the actual physical placement of one winding with respect to other. Instead of showing actual mode of the winding, dot convention is used to yield the same information.

Long questions:

Q: The mutual inductance between two coils in a radio receiver is 100 mH. One coil has 100 mH of selfinductance. What is the self-inductance of the other if coefficient of coupling between the coils is 0·5 ?

Q: The self-inductances of two coils are L1 = 150 mH, L2 = 250 mH. When they are connected in series with their fluxes aiding, their total inductance is 620 mH. When the connection to one of the coils is reversed (they are still in series), their total inductance is 180 mH. How much mutual inductance exists between them ?

Q: Two coils of self-inductances 5 H and 8 H are connected in series with their fluxes aiding. If the coefficient of coupling between the coils is 0·45, find the total inductance of the circuit.

Q: The self-inductances of three coils are L_A = 20 H, L_B = 30 H and L_C = 40 H. The coils are connected in series in such a way that fluxes of L_A and L_B add, fluxes of L_A and L_C are in opposition and fluxes of L_B and L_C are in opposition. If M_{AB} = 8 H, M_{BC} = 12 H and M_{AC} = 10 H, find the total inductance of the circuit.

CHAPTER 3: CIRCUIT ELEMENTS AND ANALYSIS

Active Elements:

The elements which are capable of providing or delivering energy to the circuit are known as active element. Example– Voltage source, Current source

Passive Elements:

A passive element is one which receives electrical energy and then either converts it into heat (resistance) or stores in an electric field (capacitance) or magnetic field (inductance). Example– Resistor, Inductor and Capacitor.

Unilateral Elements:

The element whose V-I characteristics changes on reversal of polarity of applied voltage is known as Unilateral elements. Example– Diode.

Bilateral Elements:

The element whose V-I characteristics remains same on reversal of polarity of applied voltage is known as Bilateral elements. Example– Resistor, Inductor and Capacitor.

Linear elements:

Linear elements are those through which the flow of current changes linearly with the changing of the applied voltage across them. Example– Resistor.

Non-Linear Elements:

Non-Linear Elements are those through which, the flowing current does not change linearly with the changing of the applied voltage across them. Example– Diode.

Mesh Analysis:

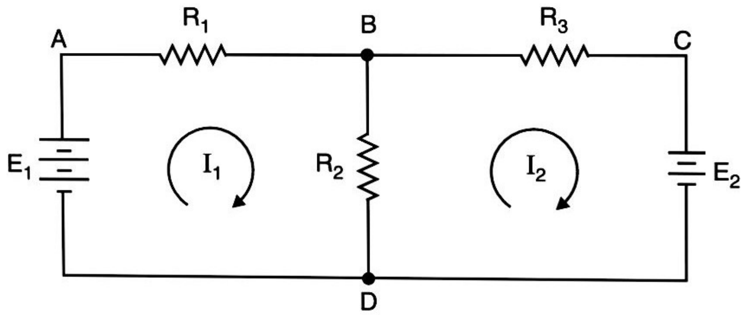
In this method, Kirchhoff's voltage law is applied to a network to write mesh equations in terms of mesh currents instead of branch currents. Each mesh is assigned a separate mesh current. This mesh current is assumed to flow clockwise around the perimeter of the mesh without splitting at a junction into branch currents. Kirchhoff's voltage law is then applied to write equations in terms of unknown mesh currents. The branch currents are then found by taking the algebraic sum of the mesh currents which are common to that branch.

Steps Involved in Mesh Analysis:

Each mesh is assigned a separate mesh current. For convenience, all mesh currents are assumed to flow in clockwise direction. For example, in the given figure, meshes ABDA and BCDB have been assigned mesh currents I_1 and I_2 respectively. The mesh currents take on the appearance of a mesh fence and hence the name mesh currents.

If two mesh currents are flowing through a circuit element, the actual current in the circuit element is the algebraic sum of the two. Thus in the figure, there are two mesh currents I_1 and I_2

flowing in R_2. If we go from B to D, current is $I_1 - I_2$ and if we go in the other direction (i.e. from D to B), current is $I_2 - I_1$.



Kirchhoff's voltage law is applied to write equation for each mesh in terms of mesh currents. Remember, while writing mesh equations, rise in potential is assigned positive sign and fall in potential negative sign.

If the value of any mesh current comes out to be negative in the solution, it means that true direction of that mesh current is anticlockwise i.e. opposite to the assumed clockwise direction.

Applying Kirchhoff's voltage law to the given figure, we have,

Mesh ABDA.

$$-I_1R_1 - (I_1 - I_2)R_2 + E_1 = 0$$

Or $\quad I_1(R_1 + R_2) - I_2R_2 = E_1 \quad \ldots (1)$

Mesh BCDB.

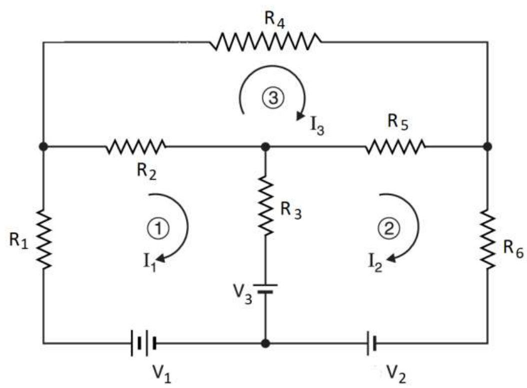
$$-I_2R_3 - E_2 - (I_2 - I_1)R_2 = 0$$

Or $\quad -I_1R_2 + (R_2 + R_3)I_2 = -E_2 \quad \ldots (2)$

Solving eq. (1) and eq. (2) simultaneously, mesh currents I_1 and I_2 can be found out. Once the mesh currents are known, the branch currents can be readily obtained. The advantage of this method is that it usually reduces the number of equations to solve a network problem.

Mesh Equation by Inspection:

We have seen above that mesh current analysis involves lengthy mesh equations. Here is a shortcut method to write mesh equations simply by inspection of the circuit. Consider the circuit shown below. The circuit contains resistances and independent voltage sources and has three meshes. Let the three mesh currents be I_1, I_2 and I_3 flowing in the clockwise direction.



Loop 1: Applying KVL to this loop, we have,

$E_1 = I_1R_{11} - I_2R_{12} - I_3R_{13}$ ... (3)

Here E_1 = Algebraic sum of e.m.f.s in Loop (1) in the direction of I_1

Or $E_1 = V_1 - V_3$

R_{11} = Sum of resistances in Loop (1)

Or R_{11} = Self resistance of Loop (1)

Or $R_{11} = R_1 + R_2 + R_3$

R_{12} = Total resistance common to Loops (1) and (2)

Or $R_{12} = R_3$

R_{13} = Total resistance common to Loops (1) and (3)

Or $R_{13} = R_2$

It may be seen that the sign of the term involving self-resistances is positive while the sign of common resistances is negative. It is because the positive directions for mesh currents were all chosen clockwise. Although mesh currents are abstract currents, yet mesh current analysis offers the advantage that resistor polarities do not have to be considered when writing mesh equations.

Loop 2: Applying KVL to this loop, we have,

$E_2 = -I_1R_{21} + I_2R_{22} - I_3R_{23}$ ... (4)

Here E_2 = Algebraic sum of e.m.f.s in Loop (2) in the direction of I_2

Or $E_2 = V_2 + V_3$

R_{21} = Total resistance common to Loops (2) and (1)

Or $R_{21} = R_3$

R_{22} = Sum of resistances in Loop (2)

Or R_{22} = Self resistance of Loop (2)

Or $R_{22} = R_3 + R_5 + R_6$

R_{23} = Total resistance common to Loops (2) and (3)

Or $R_{23} = R_5$

Loop 3: Applying KVL to this loop, we have,

$E_3 = -I_1R_{31} - I_2R_{32} + I_3R_{33}$ ... (5)

Here E_3 = Algebraic sum of e.m.f.s in Loop (3) in the direction of I_3

Or $E_3 = 0$

R_{31} = Total resistance common to Loops (3) and (1)

Or $R_{21} = R_2$

R_{32} = Total resistance common to Loops (3) and (2)

Or $R_{32} = R_6$

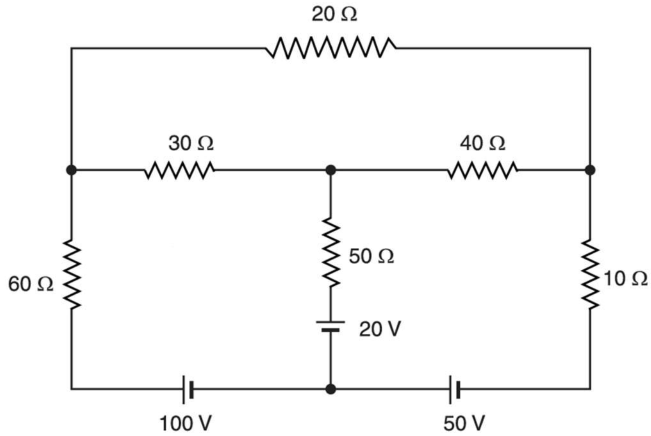
R_{23} = Sum of resistances in Loop (3)

Or R_{22} = Self resistance of Loop (3)

Or $R_{33} = R_2 + R_4 + R_5$

Solving eq. (3), (4) and (5) simultaneously, mesh currents I_1, I_2 and I_3 can be found out. Once the mesh currents are known, the branch currents can be readily obtained.

Example 1: Calculate the current in each branch of the circuit shown in Figure given below.

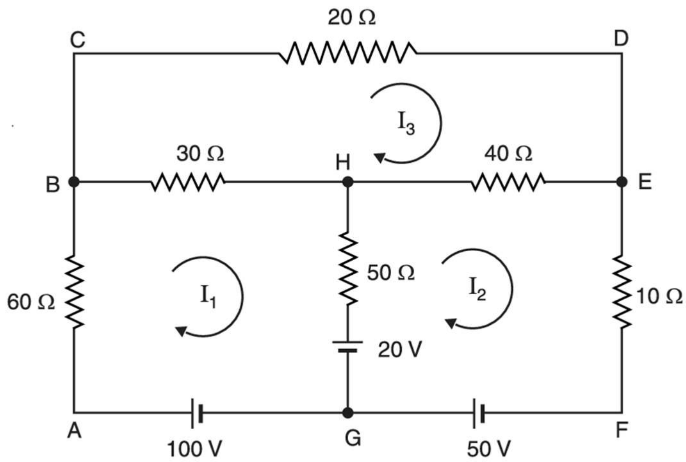


Solution: Assign mesh currents I_1, I_2 and I_3 to meshes ABHGA, HEFGH and BCDEHB respectively as shown below.

By inspecting the circuit,

Mesh ABHGA**:** Applying KVL, we have,

$140I_1 - 50I_2 - 30I_3 = 80$ ... (6)



Mesh GHEFG: Applying KVL, we have,

$-50I_1 + 100I_2 - 40I_3 = 70$ ... (7)

Mesh BCDEHB: Applying KVL, we have,

$-30I_1 - 40I_2 + 90I_3 = 0$ ... (8)

Solving eq. (3), (4) and (5) simultaneously,

$I_1 = 1{\cdot}65$ A, $I_2 = 2{\cdot}12$ A and $I_3 = 1{\cdot}5$ A

Current in 60 Ω = I_1 = 1·65 A from A to B

Current in 30 Ω = $I_1 - I_3$ = 1·65 – 1·5 = 0·15 A from B to H

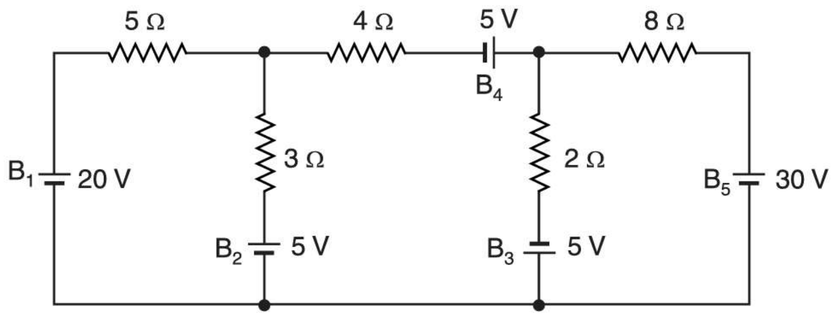
Current in 50 Ω = $I_2 - I_1$ = 2·12 – 1·65 = 0·47 A from G to H

Current in 40 Ω = $I_2 - I_3$ = 2·12 – 1·5 = 0·62 A from H to E

Current in 10 Ω = I_2 = 2·12 A from E to F

Current in 20 Ω = I_3 = 1·5 A from C to D

Example 2: By using mesh resistance matrix, determine the current supplied by each battery in the circuit shown.



Solution: Since there are three meshes, let the three mesh currents be I_1, I_2 and I_3, all assumed to be flowing in the clockwise direction.

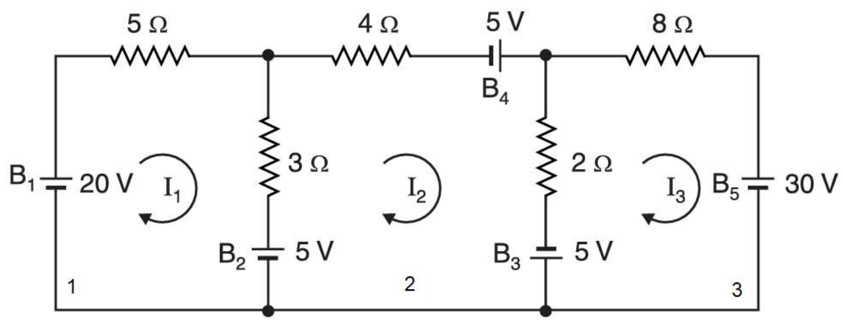
By inspecting the circuit, the three mesh equations are

For mesh 1,

$8I_1 - 3I_2 = 15$ ... (9)

For mesh 2,

$-3I_1 + 9I_2 - 2I_3 = 15$ ... (10)



For mesh 3,

$-2I_2 + 10I_3 = -35$ ... (11)

Solving eq. (9), (10) and (11) simultaneously,

$I_1 = 2{\cdot}56$ A, $I_2 = 1{\cdot}82$ A and $I_3 = -3{\cdot}13$ A

The negative sign with I_3 indicates that actual direction of I_3 is opposite to that assumed in the figure...

Current supplied by battery $B_1 = I_1 = 2{\cdot}56$ A

Current supplied by battery $B_2 = I_1 - I_2 = 2{\cdot}56 - 1{\cdot}82 = 0{\cdot}74$ A

Current supplied by battery $B_3 = I_2 + I_3 = 1{\cdot}82 + 3{\cdot}13 = 4{\cdot}95$ A

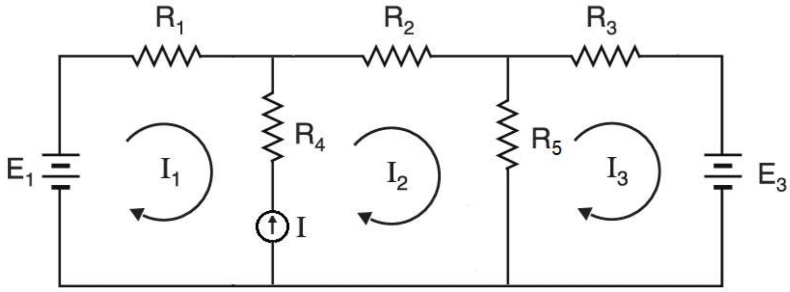
Current supplied by battery $B_4 = I_2 = 1{\cdot}82$ A

Current supplied by battery $B_5 = I_3 = 3{\cdot}13$ A

Super mesh analysis:

When two meshes having the current source as a common element then a super mesh is formed. In this analysis, the current source will be present inside of the super mesh. Hence the no. of meshes can be reduced by one for every current source that is present in the given complex network. We can ignore the single mesh if the current element is placed on the perimeter of the electrical circuit. By incorporating two mesh currents, one mesh equation is formed, which relates to the current source. The current source in the resultant mesh equation is equal to one of the mesh currents minus the other.

Consider the following circuit.



As shown in the circuit, there is a current source is common to mesh 1 and 2. Hence a super mesh can be formed by combining these two meshes and ignoring the branch consisting of current source.

The super mesh equation is

$$E_1 - I_1R_1 - I_2R_2 - (I_2 - I_3)\, R5 = 0 \quad \text{... (12)}$$

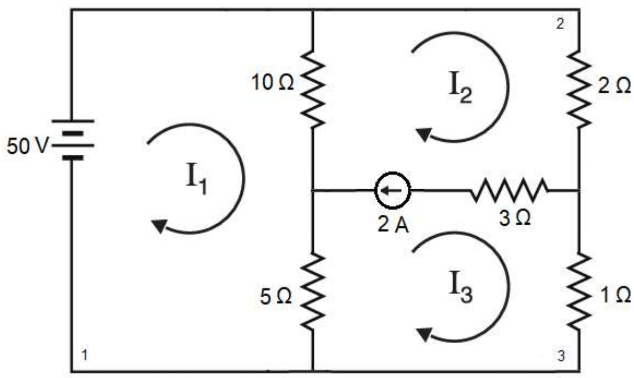
The current driven by I, can be calculated as

$$I = (I_2 - I_1) \quad \text{... (13)}$$

The other mesh equation is

$$-E_3 - (I_3 - I_2) - I_3R_3 = 0 \quad \text{... (14)}$$

Example 3: Determine the current in the 5 **Ω** resistor in the network given. below.



Solution: From the mesh1, we get

$50 = 10(I_1 – I_2) + 5(I_1 – I_3)$

Or $15I1 – 10I2 – 5I3 = 50$ ... (15)

From the mesh 2 & 3, we can form a super mesh and the super mesh equation is

$10(I_2 – I_1) + 2I_2 + I_3 + 5(I_3 – I_1) = 0$

$–15I_1 + 12I_2 + 6I_3 = 0$ ... (16)

The current source is equal to the difference between I_2 and I_3 mesh currents

i.e. $I_2 – I_3 = 2$ ... (17)

Now solving the three equations (15), (16) and (17), we get

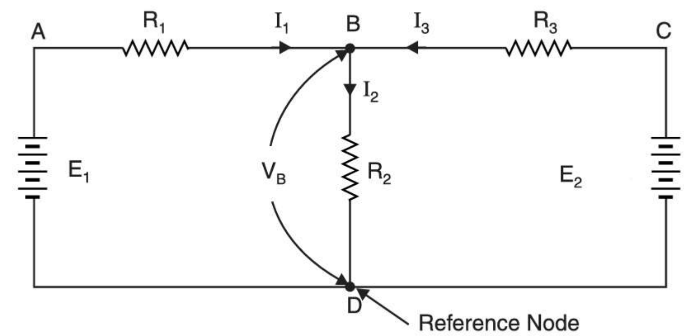
$I_1 = 19.99$ A, $I_2 = 17.33$ A and $I_3 = 15.33$ A

Current in the 5 Ω resistor = $I_1 - I_3 = 19.99 – 15.33 = 4.66$

Nodal Analysis:

In this method the branch currents in the circuit can be found by Kirchhoff's laws. This method essentially aims at choosing a reference node in the network and then finding the unknown voltages at the independent nodes w.r.t. reference node. For a circuit containing N nodes, there will be N–1 node voltages, some of which may be known if voltage sources are present.

Steps Involved in Nodal Analysis:



Consider the circuit shown above, one of the nodes (Remember a node is a point in a network where two or more circuit elements meet) is taken as the reference node. The potentials of all the points in the circuit are measured w.r.t. this reference node. In the figure A, B, C and D are four nodes and the node D has been taken as the reference node. The fixed-voltage nodes are called dependent nodes. Thus A and C are fixed nodes. The voltage from D to B is V_B and its magnitude depends upon the parameters of circuit elements and the currents through these elements. Therefore, node B is called independent node. Once we calculate the potential at the independent node (or nodes), each branch current can be determined because the voltage across each resistor will then be known.

The voltage VB can be found by applying Kirchhoff's current law at node B.

$$I_1 + I_3 = I_2 \qquad \text{... (12)}$$

In mesh ABDA, the voltage drop across R_1 is $E_1 – V_B$.

$$I_1 = \frac{E_1 - V_B}{R_1}$$

In mesh CBDC, the voltage drop across R_3 is $E_2 – V_B$.

$$I_3 = \frac{E_2 - V_B}{R_3}$$

Also $I_2 = \dfrac{V_B}{R_2}$

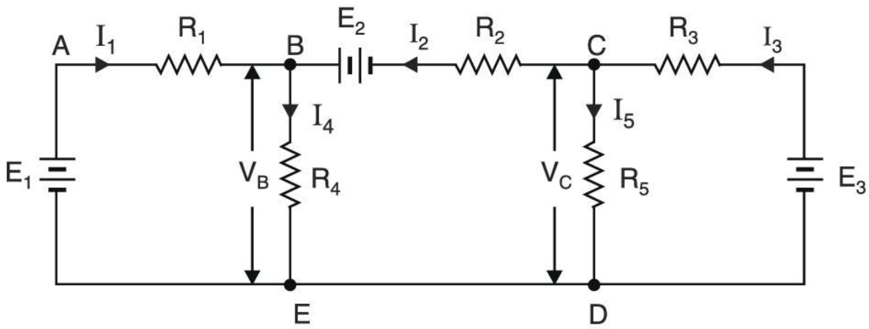
Putting the values of I_1, I_2 and I_3 in eq. (12), we get,

$$\frac{E_1 - V_B}{R_1} + \frac{E_2 - V_B}{R_3} = \frac{V_B}{R_2}$$

All quantities except V_B are known. Hence V_B can be found out. Once V_B is known, all branch currents can be calculated. It may be seen that nodal analysis requires only one equation for determining the branch currents in this circuit.

We can mark the directions of currents at will. If the value of any current comes out to be negative in the solution, it means that actual direction of current is opposite to that of assumed.

With Two Independent Nodes:



The network has two independent nodes B and C. We take node D (or E) as the reference node. We shall use Kirchhoff's current law for nodes B and C to find V_B and V_C. Once the values of V_B and V_C are known, we can find all the branch currents in the network.

Each current can be expressed in terms of e.m.f.s, resistances (or conductances), VB and V_C.

$$E_1 = V_B + I_1R_1 \qquad \therefore I_1 = \frac{E_1 - V_B}{R_1}$$

$$E_3 = V_C + I_3R_3 \qquad \therefore I_3 = \frac{E_3 - V_C}{R_3}$$

$$E_2 = V_B - V_C + I_2R_2 \quad \therefore I_2 = \frac{E_2 - V_B + V_C}{R_2}$$

Similarly,

$$I_4 = \frac{V_B}{R_4} \qquad \& \qquad I_5 = \frac{V_C}{R_5}$$

At node B:

$$I_1 + I_2 = I_4$$

$$\text{Or} \quad \frac{E_1 - V_B}{R_1} + \frac{E_2 - V_B + V_C}{R_2} = \frac{V_B}{R_4} \qquad \text{... (18)}$$

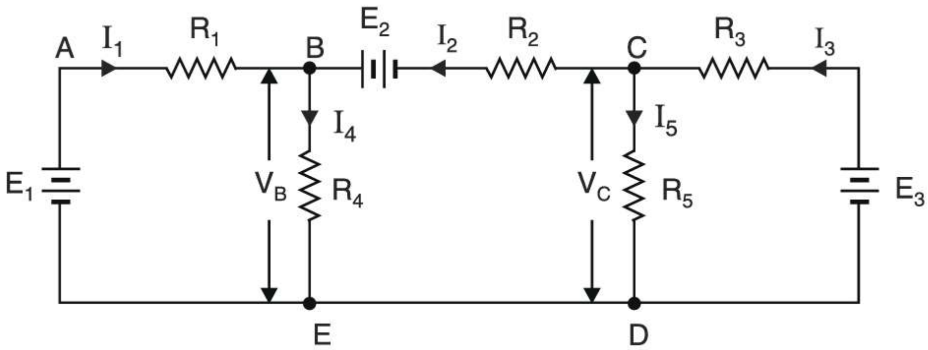
At node C:

$$I_2 + I_5 = I_3$$

$$\frac{E_2 - V_B + V_C}{R_2} + \frac{V_C}{R_5} = \frac{E_3 - V_C}{R_3} \qquad \text{... (19)}$$

From equations (13) and (14), we can find V_B and V_C since all other quantities are known. Once we know the values of V_B and V_C, we can find all the branch currents in the network.

Nodal Equation by Inspection:



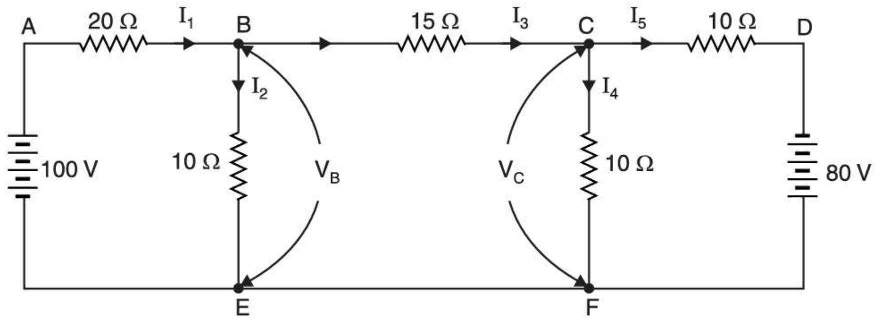
There is a shortcut method for writing node equations similar to the form for mesh equations. Consider the circuit with two independent nodes B and C as shown in the above figure. The node equations in shortcut form for nodes B and C can be written as under

$$V_B\left(\frac{1}{R_1} + \frac{1}{R_4} + \frac{1}{R_2}\right) - \frac{V_C}{R_2} - \frac{E_1}{R_1} - \frac{E_2}{R_2} = 0 \qquad \text{... (20)}$$

$$V_C\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_5}\right)-\frac{V_B}{R_2}+\frac{E_2}{R_2}-\frac{E_3}{R_5}=0 \quad \text{... (21)}$$

The voltage of self-node is always positive and is multiplied with algebraic sum of reciprocal of each resistance connected to it, the voltage of other independent node is negative and multiplied with the reciprocal of the resistance through which it is connected to self-node and the voltage source connected to self-node is also multiplied with the reciprocal of the resistance through which it is connected to self-node. The polarity of the voltage source is negative if the positive side is connected to self-node and vice-versa.

Example 4: Find the currents in the various branches of the circuit shown in figure below by nodal analysis.



Solution: By inspecting the circuit, nodal equations are

At node B:

$$V_B\left(\frac{1}{20}+\frac{1}{10}+\frac{1}{15}\right)-\frac{V_C}{15}-\frac{100}{20}=0$$

Or $\frac{13V_B}{60}-\frac{V_C}{15}=5$

Or $13V_B - 4V_C = 300$ ... (22)

At node c:

$$V_C\left(\frac{1}{15}+\frac{1}{10}+\frac{1}{10}\right)-\frac{V_B}{15}+\frac{80}{10}=0$$

Or $\frac{8V_C}{30}-\frac{V_B}{15}=8$

Or $V_B - 4V_C = 120$ ... (23)

By solving equations (22) and (23)

$V_B = 15$ V and $V_C = -\ 26{\cdot}25$ V

Current $I_1 = \dfrac{100 - V_B}{20} = 4{\cdot}25$ A

Current $I_2 = \dfrac{V_B}{10} = 1{\cdot}5$ A

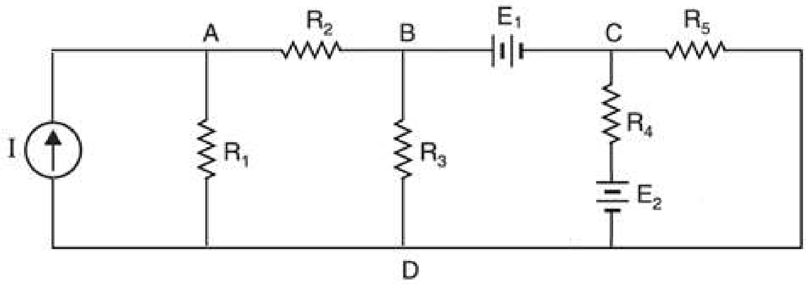
Current $I_3 = \dfrac{V_B - V_C}{15} = 2{\cdot}75$ A

Current $I_4 = \dfrac{V_C}{10} = -\ 2{\cdot}625$ A

Current $I_5 = \dfrac{V_C + 80}{10} = 5{\cdot}375$ A

Super Node Analysis:

In this method, the two adjacent nodes that are connected by a voltage source are reduced to a single node and then the equations are formed by applying Kirchhoff s current law as usual.



At node A: $$V_A\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - \frac{V_B}{R_2} = I \qquad \dots (24)$$

As there is a voltage source V_X between node B and C, they form a super node.

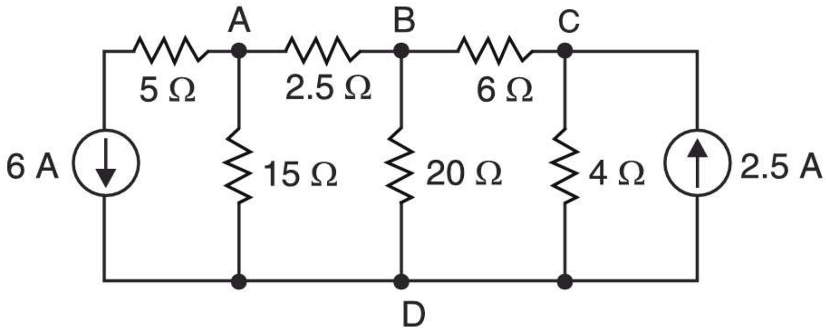
Super node equation is

$$V_B\left(\frac{1}{R_2} + \frac{1}{R_3}\right) - \frac{V_A}{R_2} + V_C\left(\frac{1}{R_4} + \frac{1}{R_5}\right) - \frac{E_2}{R_4} \qquad \dots (25)$$

Voltage between node B and C

$$V_B - V_C = E_1 \qquad \text{... (26)}$$

Example 5: Solve the circuit shown in figure below by using nodal analysis.



Solution: By inspecting the circuit, nodal equations are

At node A:

$$V_A\left(\frac{1}{15} + \frac{1}{2.5}\right) - \frac{V_B}{2.5} + 6 = 0$$

Or $\quad 0{\cdot}467\, V_A - 0{\cdot}4\, V_B = -6 \qquad$ ... (27)

At node B:

$$V_B\left(\frac{1}{2.5} + \frac{1}{20} + \frac{1}{6}\right) - \frac{V_A}{2.5} - \frac{V_C}{6} = 0$$

Or $\quad -0{\cdot}4\, V_A + 0{\cdot}617\, V_B - 0{\cdot}167\, V_C = 0 \qquad$ ... (28)

At node C:

$$V_C\left(\frac{1}{6} + \frac{1}{4}\right) - \frac{V_B}{6} - 2.5 = 0$$

Or $\quad -0{\cdot}167\, V_B + 0{\cdot}417\, V_C = 2.5 \qquad$ ... (29)

From equations (24), (25) and (26), $V_A = -30$ V, $V_B = -20$ V and $V_C = -2$ V

Current in 15 Ω = 30/15 = 2 A
Current in 20 Ω = 20/20 = 1 A
Current in 4 Ω = 2/4 = 0.5 A
Current in 6 Ω = 18/6 = 3 A

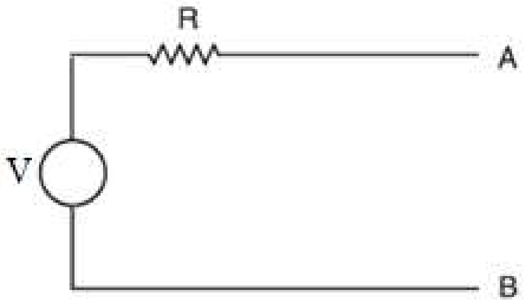
Current in 2.5 Ω = 10/2.5 = 4 A

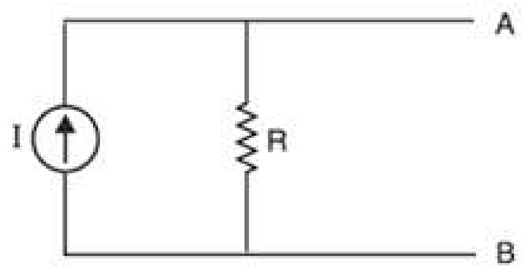
Current in 5 Ω = 4 + 2 = 6 A

Source Transformation Technique:

A given voltage source with a series resistance can be converted into an equivalent current source with a parallel resistance. Conversely a current source with a parallel resistance can be converted into an equivalent voltage source with a series resistance.

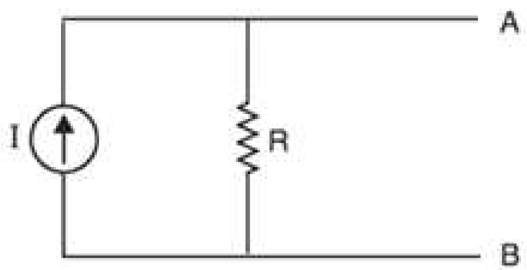
Voltage source to Current Source:

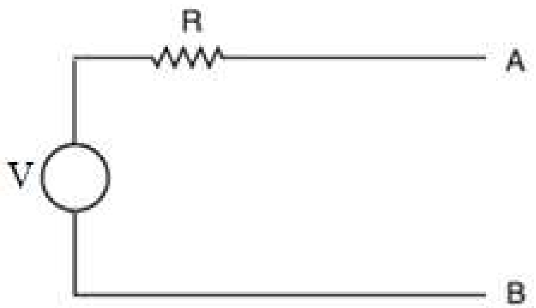




I = V/R

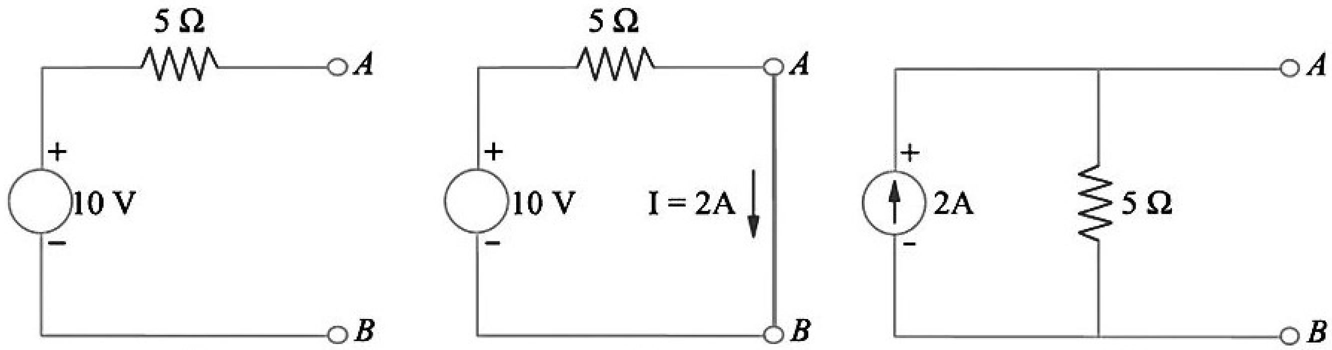
Current Source to Voltage source:





V = IR

Example 6: Convert the voltage source of below figure (a) into an equivalent current source.



(a) (b) (c)

Solution: Current obtained by putting a short across terminals A and B is 10/5 = 2 A. Hence, the equivalent current source is as in figure (c).

QUESTION BANK

Short Questions With Answer:

Q: What are Active Elements?

A: The elements which are capable of providing or delivering energy to the circuit are known as active element. Example– Voltage source, Current source

Q: What are Passive Elements?

A: A passive element is one which receives electrical energy and then either converts it into heat (resistance) or stores in an electric field (capacitance) or magnetic field (inductance). Example– Resistor, Inductor and Capacitor

Q: What are Unilateral Elements?

A: The element whose V-I characteristics changes on reversal of polarity of applied voltage is known as Unilateral elements. Example– Diode.

Q: What are Bilateral Elements?

A: The element whose V-I characteristics remains same on reversal of polarity of applied voltage is known as Bilateral elements. Example– Resistor, Inductor and Capacitor

Q: What are linear Elements?

A: Linear elements are those through which the flow of current changes linearly with the changing of the applied voltage across them. Example– Resistor

Q: What are Non-Linear Elements?

A: Non-Linear Elements are those through which, the flowing current does not change linearly with the changing of the applied voltage across them. Example– Diode.

Long Questions:

Q: Use mesh analysis to find the current in each resistor in Fig. Q1.

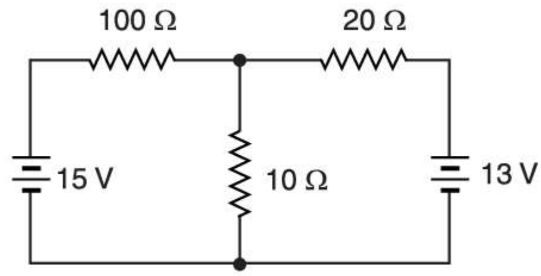


Fig Q1

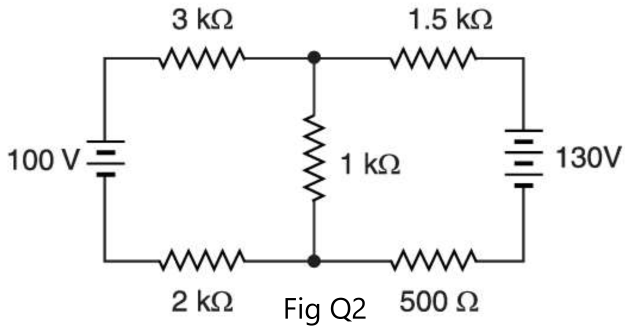


Fig Q2

Q: Using mesh analysis, find the voltage drop across the 1 kΩ resistor in Fig. Q2.

Q: Using mesh analysis, find the currents in 50 Ω, 250 Ω and 100 Ω resistors in the circuit shown in Fig. Q3.

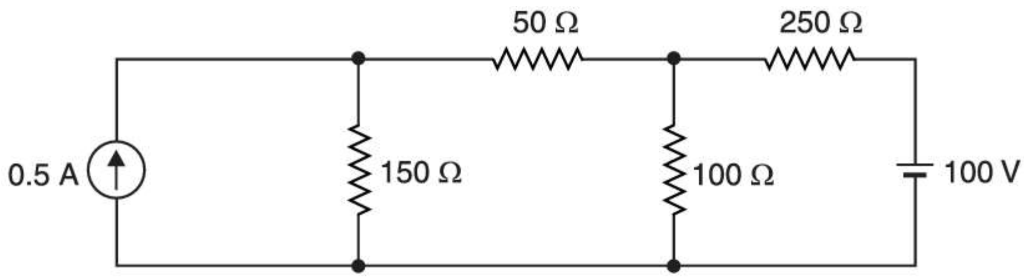


Fig Q3

Q: For the network shown in Fig. Q4, find the mesh currents I_1, I_2 and I_3.

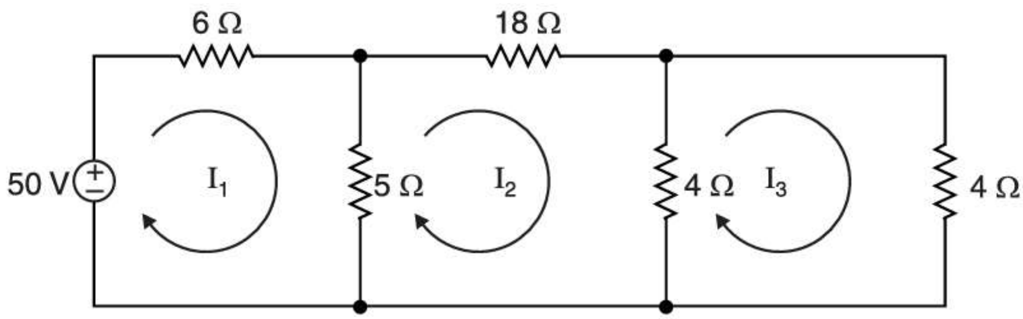


Fig Q4

Q: In the network shown in Fig. Q5, find the magnitude and direction of current in the various branches by mesh current method.

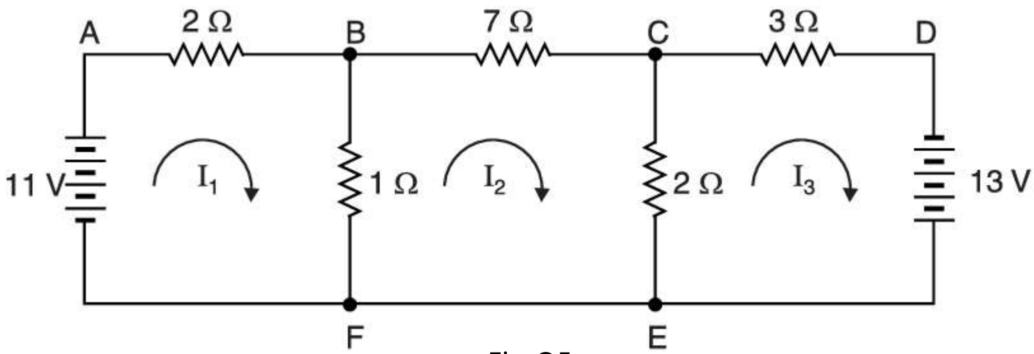


Fig Q5

Q: Using nodal analysis, find the voltages at nodes A, B and C w.r.t. the reference node shown by the ground symbol in Fig Q6.

Q: Using nodal analysis, find the current flowing in the battery in Fig. Q7.

Q: By performing an appropriate source conversion, find the voltage across 120 Ω resistor in the circuit shown in Fig. Q8.

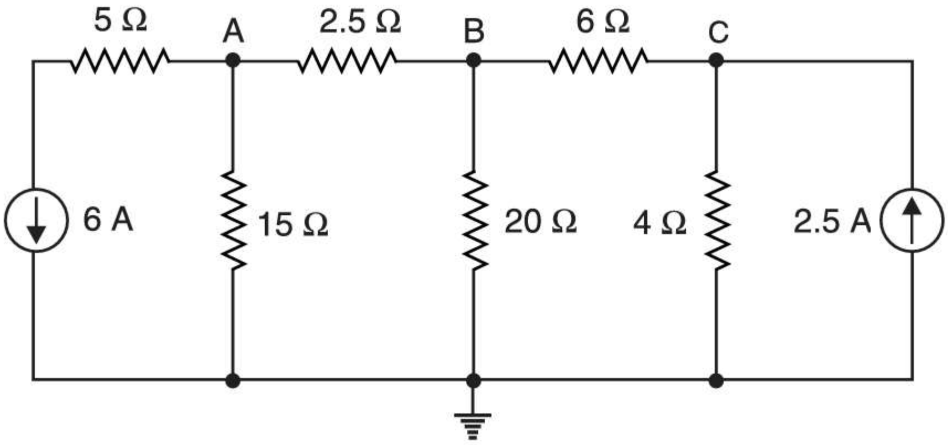


Fig Q6

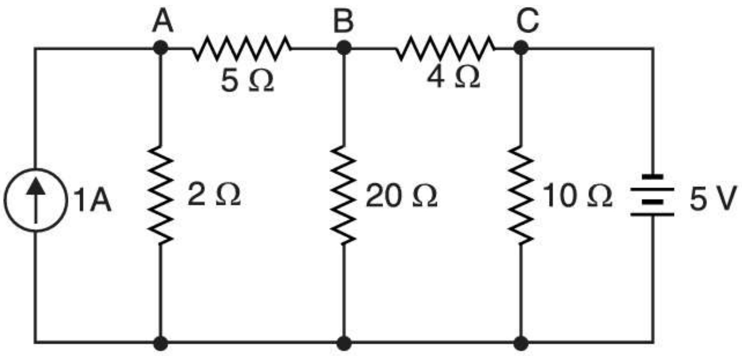


Fig Q7

Q: By performing an appropriate source conversion, find the voltage across 120 Ω resistor in the circuit shown in Fig. Q9.

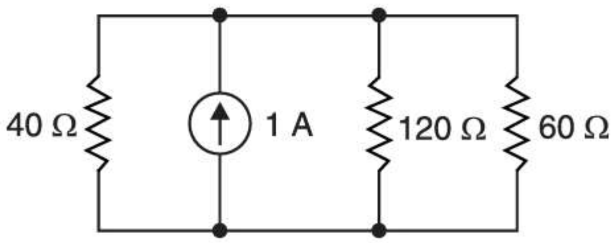


Fig Q8

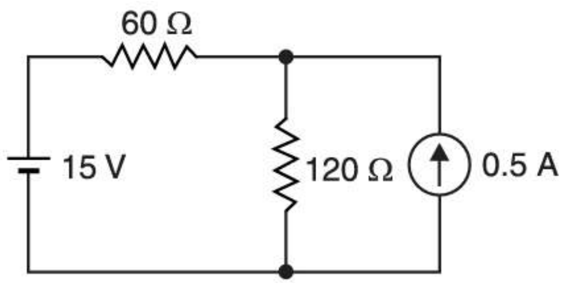


Fig. Q9

CHAPTER 4: NETWORK THEOREMS

Star to Delta and Delta to Star Transformation:

Delta to Star Transformation:

Consider three resistors R_{AB}, R_{BC} and R_{CA} connected in delta to three terminals A, B and C as shown in the below figure. Let the equivalent star-connected network have resistances R_A, R_B and R_C. Since the two arrangements are electrically equivalent, the resistance between any two terminals of one network is equal to the resistance between the corresponding terminals of the other network.

Let us consider the terminals A and B of the two networks.

Resistance between A and B for star = Resistance between A and B for delta.

Or $R_A + R_B = R_{AB} \parallel (R_{BC} + R_{CA})$

Or $$R_A + R_B = \frac{R_{AB}(R_{BC}+R_{CA})}{(R_{AB}+R_{BC}+R_{CA})} \quad \ldots (1)$$

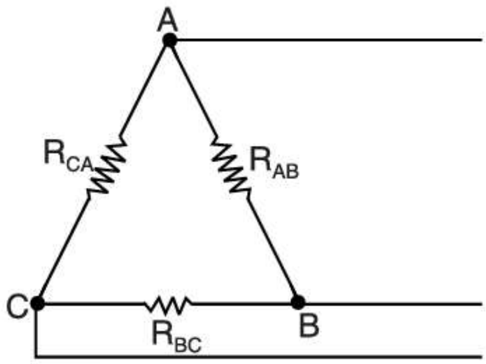


Fig. T1

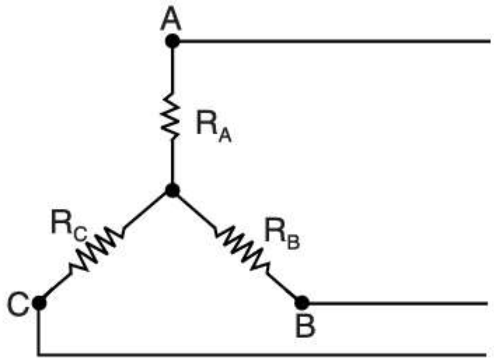


Fig. T2

Similarly,

$$R_B + R_C = \frac{R_{BC}(R_{CA}+R_{AB})}{(R_{AB}+R_{BC}+R_{CA})} \quad \ldots (2)$$

And $$R_C + R_A = \frac{R_{CA}(R_{AB}+R_{BC})}{(R_{AB}+R_{BC}+R_{CA})} \quad \ldots (3)$$

Subtracting eq. (2) from eq. (1) and adding the result to eq. (3), we have,

$$R_A = \frac{R_{AB}R_{CA}}{R_{AB}+R_{BC}+R_{CA}} \quad \ldots (4)$$

Similarly,

$$R_B = \frac{R_{BC}R_{AB}}{R_{AB}+R_{BC}+R_{CA}} \quad \text{... (5)}$$

And $$R_C = \frac{R_{CA}R_{BC}}{R_{AB}+R_{BC}+R_{CA}} \quad \text{... (6)}$$

Thus to find the star resistance that connects to terminal A, divide the product of the two delta resistors connected to A by the sum of the delta resistors.

Star to Delta Transformation:

Dividing eq. (4) by (5), we have,

$$\frac{R_A}{R_B} = \frac{R_{CA}}{R_{BC}}$$

Or $$R_{CA} = \frac{R_A R_{BC}}{R_B}$$

Dividing eq. (4) by (6), we have,

$$\frac{R_A}{R_C} = \frac{R_{AB}}{R_{BC}}$$

Or $$R_{AB} = \frac{R_A R_{BC}}{R_c}$$

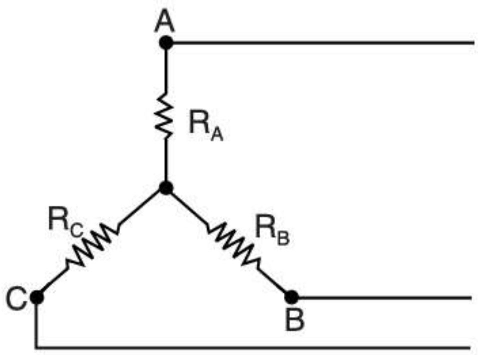


Fig. T3

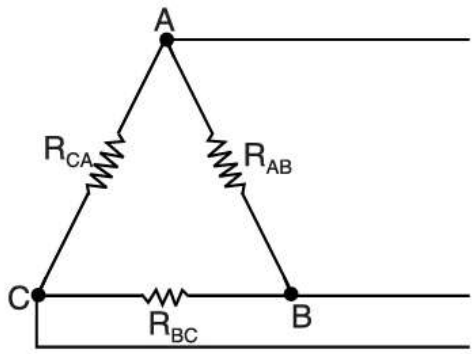


Fig. T4

Substituting the values of R_{CA} and R_{AB} in eq. (4), we have,

$$R_{BC} = R_B + R_C + \frac{R_B R_C}{R_A} \quad \text{... (7)}$$

Similarly,

$$R_{CA} = R_C + R_A + \frac{R_C R_A}{R_B} \quad \text{... (8)}$$

And $R_{AB} = R_A + R_B + \frac{R_A R_B}{R_C}$ ... (9)

Example 1: A network of resistors is shown in fig. E1.1. Find the resistance (i) between terminals A and B (ii) B and C and (iii) C and A

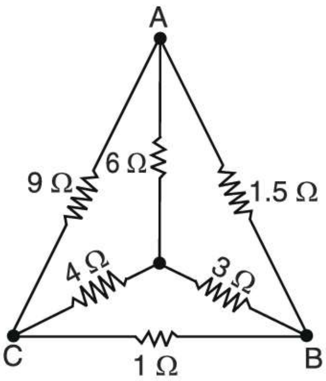


Fig. E1.1

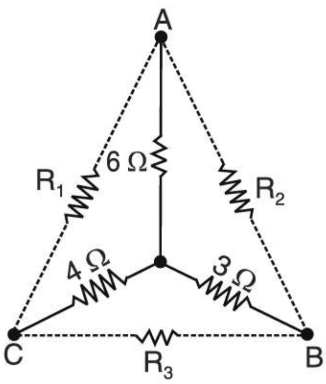


Fig. E1.2

Solution: The star-conn nces 6 Ω, 3 Ω and wn separately. These star-connected resistances can be converted into equivalent delta-connected resistances R_1, R_2 and R_3 as shown in fig. E 1.2.

R1 = 4 + 6 + (4 × 6/3) = 18 Ω

R2 = 6 + 3 + (6 × 3/4) = 13·5 Ω

R3 = 4 + 3 + (4 × 3/6) = 9 Ω

These delta-connected resistances R_1, R_2 an R_3 come in parallel with the original delta connected resistances in fig. E1.3

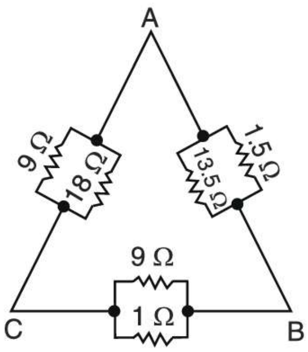


Fig. E1.3

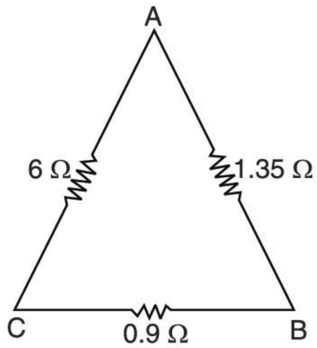


Fig. E1.4

The parallel resistances in each leg of delta can be replaced by a single resistor where as shown in fig. E1.4.

$R_{AC} = 9 \times 18/27 = 6\ \Omega$

R_{BC} = 9 × 1/10 = 0·9 Ω

R_{AB} = 1·5 × 13·5/15 = 1·35 Ω

Resistance between A and B = 1·35 Ω || (6 + 0·9) Ω = 1·35 × 6·9/8·25 = 1·13 Ω

Resistance between B and C = 0·9 Ω || (6 + 1·35) Ω = 0·9 × 7·35/8·25 = 0·8 Ω

Resistance between A and C = 6 Ω || (1·35 + 0·9) Ω = 6 × 2·25/8·25 = 1.636 Ω

Example 2: Determine the resistance between points A and B in the network shown below.

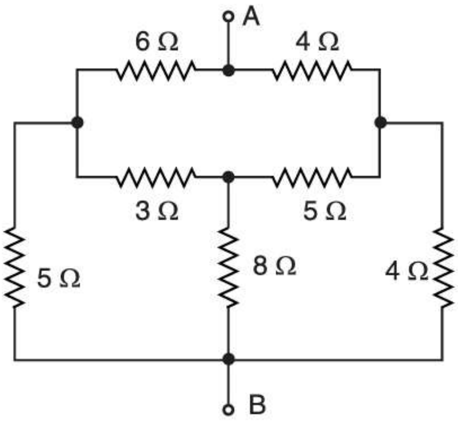


Fig. E2.1

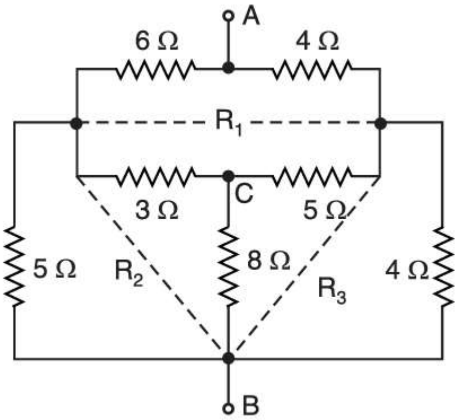


Fig. E2.2

Solution: The 3 Ω, 5 Ω and 8 Ω form star network and can be replaced by delta network as shown in fig. E2.2 where

$$R_1 = 3 + 5 + \frac{3*5}{8} = 9{\cdot}875\ \Omega$$

$$R_2 = 3 + 8 + \frac{3*8}{5} = 15{\cdot}8\ \Omega$$

$$R_3 = 5 + 8 + \frac{5*8}{3} = 26{\cdot}3\ \Omega$$

Referring to fig. E 2.2, 5 Ω resistor is in parallel with R_2 (= 15·8 Ω) and their combined resistance is 3·8 Ω. Similarly, 4 Ω resistor is in parallel with R_3 (= 26·3 Ω) and their combined resistance is 3·5 Ω. The circuit then reduces to the one shown in fig. E2.3.

Referring to fig. E 2.3, 6 Ω, 4 Ω and 9·875 Ω form a delta network and can be replaced by star network where

$$R_6 = \frac{6*4}{6+4+9.875} = 1.2\ \Omega,\quad R_7 = \frac{9.875*4}{6+4+9.875} = 1.99\ \Omega \text{ and } R_7 = \frac{9.875*6}{6+4+9.875} = 2.98\ \Omega$$

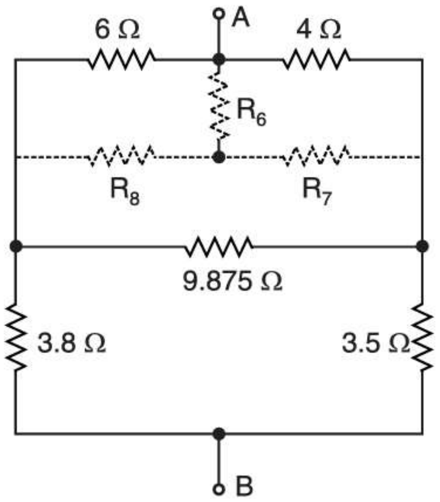


Fig. E2.3

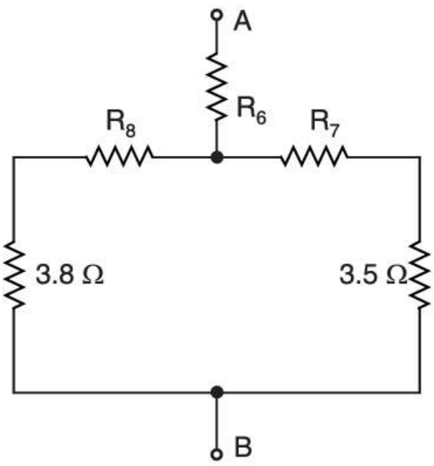


Fig. E2.4

Therefore, the circuit shown in fig. E2.3 reduces to the one shown in fig. E2.4. It is clear that

$R_{AB} = (3{\cdot}8 + R8) \,||\, (R7 + 3{\cdot}5) + R6$

$= (3{\cdot}8 + 2{\cdot}98) \,||\, (1{\cdot}99 + 3{\cdot}5) + 1{\cdot}2$

$= (6{\cdot}78 \,||\, 5{\cdot}49) + 1{\cdot}2 = 4{\cdot}23\ \Omega$

Superposition Theorem:

In a linear, bilateral network containing more than one energy source, the current which flows at any point is the sum of all the currents which would flow at that point if each source where considered separately and all the other sources replaced for the time being by resistances equal to their internal resistances.

Procedure:

The procedure for using this theorem to solve d.c. networks is as under:

(*i*) Select one source in the circuit and replace all other ideal voltage sources by short circuits and ideal current sources by open circuits.

(*ii*) Determine the current through the desired element/branch due to single source selected in step (*i*).

(*iii*) Repeat the above two steps for each of the remaining sources.

(*iv*) Algebraically, add all currents through the element/branch under consideration. The sum is the actual current through that element/branch when all the sources are acting simultaneously.

Example 3: By superposition theorem, find the current in resistance R in fig. E3.1.

Solution: In fig. E3.1, battery E2 is replaced by a short so that battery E1 is acting alone. It is clear that resistances of 1Ω (= R) and 0.04Ω are in parallel across points A and C.

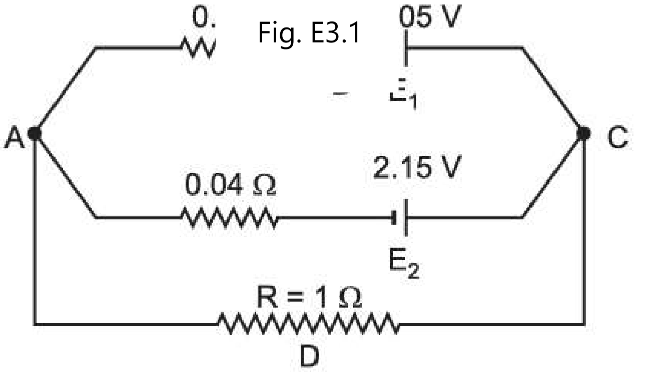


Fig. E3.1

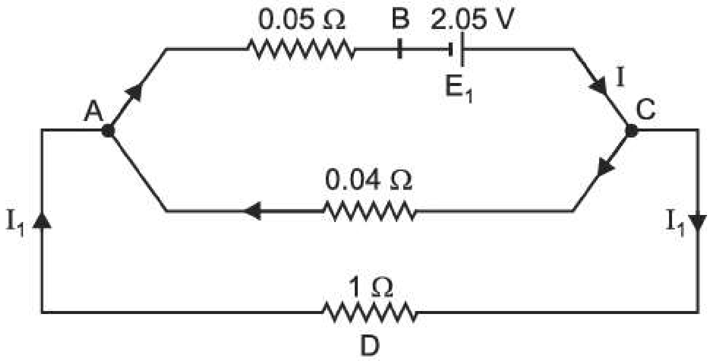


Fig. E3.2

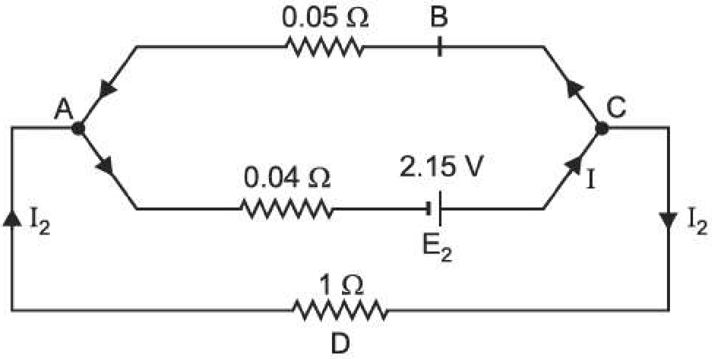


Fig. E3.3

$$R_{AC} = 1\Omega \parallel 0.04\Omega = \frac{1*0.04}{1+0.04} = 0.038\ \Omega$$

This resistance (i.e., R_{AC}) is in series with 0.05 Ω.

Total resistance to battery E1 = 0.038 + 0.05 = 0.088 Ω

Current supplied by battery E1 is

$$I = \frac{E_1}{0.088} = \frac{2.05}{0.088} = 23.2\ A$$

The current I (= 23.2A) is divided between the parallel resistances of 1Ω (= R) and 0.04Ω.

Current in 1Ω (= R) resistance is

$$I_1 = 23.2 * \frac{0.04}{1+0.04} = 0.892\ A \text{ from C to A.}$$

In Fig. E 3.3, battery E_1 is replaced by a short so that battery E_2 is acting alone.

Total resistance offered to battery E_2

$$= (1\Omega \parallel 0.05\Omega) + 0.04\Omega$$

$$= \frac{1*0.05}{1+0.05} + 0.04 = 0.088\ \Omega$$

Current supplied by battery E_2 is

$$I = \frac{E_2}{0.088} = \frac{2.15}{0.088} = 24.4\ A$$

The current I (= 24.4A) is divided between two parallel resistances of 1Ω (= R) and 0.05Ω.

Current in 1Ω (= R) resistance is

$$I_2 = 24.4 * \frac{0.05}{1+0.05} = 1.16 \text{ A from C to A.}$$

Current through 1Ω resistance when both batteries are present

$$I = I_1 + I_2 = 0.892 + 1.16 = 2.052 \text{ A.}$$

Example 4: Using superposition theorem, find the current in the each branch of the network shown in Fig. E4.1.

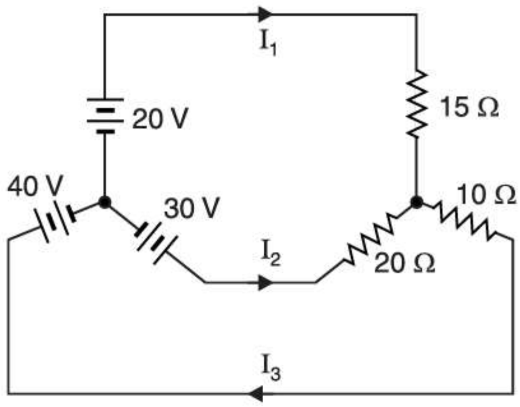


Fig. E4.1

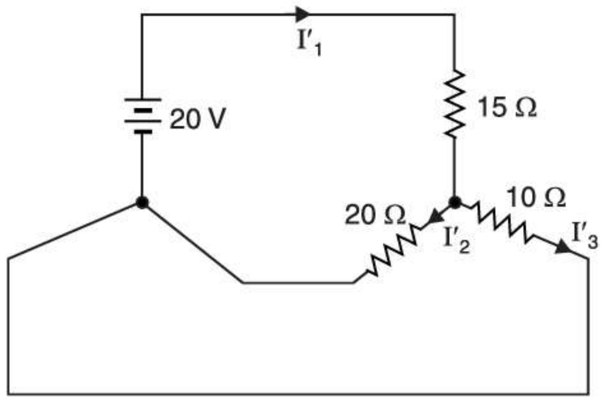


Fig. E4.2

Solution: Since there are three sources of e.m.f., three circuits are required for analysis by superposition theorem. In Fig. E4.2, it is shown that only 20 V source is acting.

Total resistance across source

$$= 15 + \frac{20*10}{20+10} = 21.67 \ \Omega.$$

Total circuit current, I'_1 = 20/21.67 = 0.923 A

Current in 20 Ω, I'_2 = 0.923 × 10/30 = 0.307A

Current in 10 Ω, I'_3 = 0.923 × 20/30 = 0.616 A

In Fig. Fig. E4.3, only 40V source is acting in the circuit.

Total resistance across source

$$=10 + \frac{20*15}{20+15} = 18.57 \ \Omega.$$

Total circuit current, I_3'' = 40/18.57 = 2.15A

Current in 20 Ω, I_2'' = 2.15 × 15/35 = 0.92 A

Current in 15 Ω, I_1'' = 2.15 × 20/35 = 1.23 A

In Fig. E4.4, only 30 V source is acting in the circuit.

Total resistance across source

$$= 20 + \frac{10*15}{10+15} = 26 \ \Omega.$$

Total circuit current, $I_2''' = 30/26 = 1.153$ A

Current in 15 Ω, $I_1''' = 1.153 \times 10/25 = 0.461$ A

Current in 10 Ω, $I_3''' = 1.153 \times 15/25 = 0.692$ A

The actual values of currents I_1, I_2 and I_3 shown in Fig. E4.1 can be found by algebraically adding the component values.

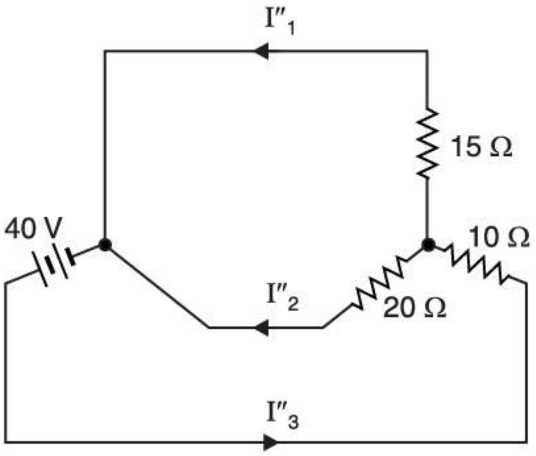


Fig. E4.3

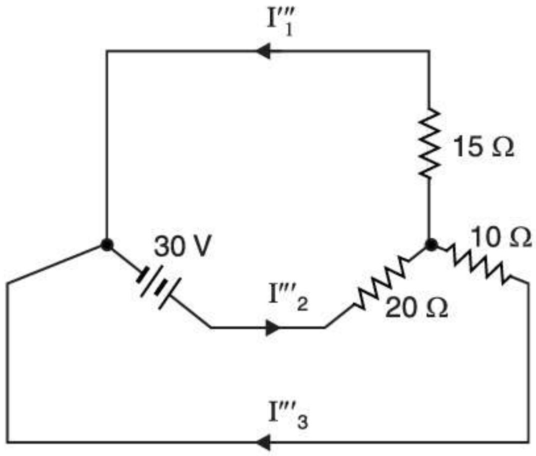


Fig. E4.4

$$I_1 = I_1' - I_1'' - I_1''' = 0.923 - 1.23 - 0.461 = -0.768 \text{ A}$$

$$I_2 = -I_2' - I_2'' + I_2''' = -0.307 - 0.92 + 1.153 = -0.074 \text{ A}$$

$$I_3 = I_3' - I_3'' + I_3''' = 0.616 - 2.15 + 0.692 = -0.842 \text{ A}$$

The negative signs with I_1, I_2 and I_3 show that their actual directions are opposite to that assumed in Fig. E4.1.

Example 5: Use superposition theorem to find the voltage V in Fig. E5.1.

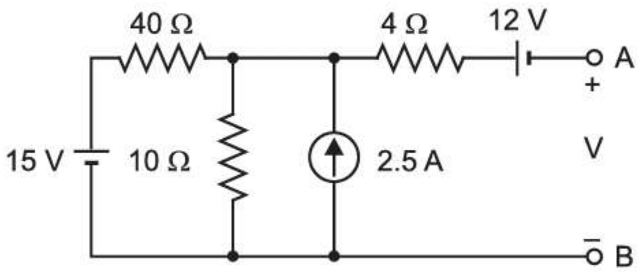


Fig. E5.1

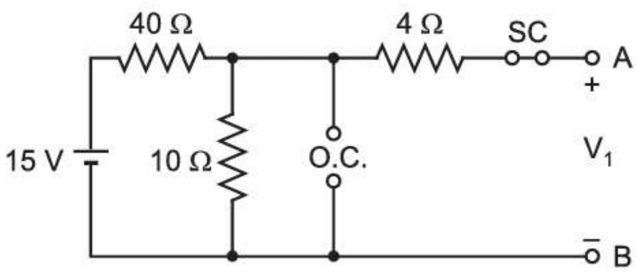


Fig. E5.2

Solution: In Fig. E5.2, 12 V battery is replaced by a short and 2.5A current source by an open so that 15V battery is acting alone. Therefore, voltage V_1 across open terminals A and B is

V_1 = Voltage across 10Ω resistor

By voltage-divider rule, V_1 is given by

$$V_1 = 15 * \frac{10}{10+40} = 3 \text{ V.}$$

In Fig. E5.3, 15 V and 12 V batteries are replaced by shorts so that 2.5A current source is acting alone. Therefore, voltage V_2 across open terminals A and B is

V_2 = Voltage across 10 Ω resistor

By current-divider rule, current in 10 Ω = $2.5 * \frac{40}{10+40} = 2$ A.

$V_2 = 2 \times 10 = 20$ V.

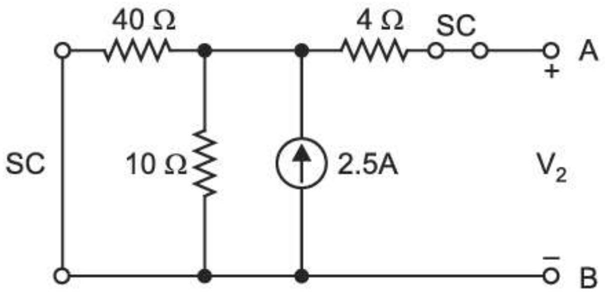


Fig. E5.3

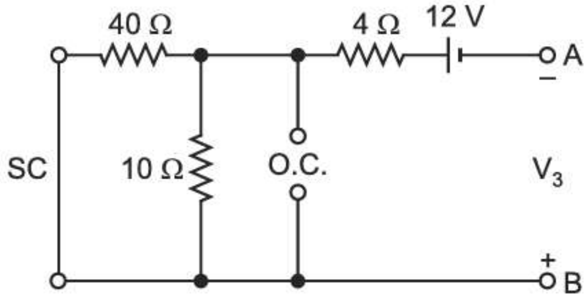


Fig. E5.4

In Fig. E5.4, 15 V battery is replaced by a short and 2.5 A current source by an open so that 12V battery is acting alone. Therefore, voltage V_3 across open terminals A and B is

$V_3 = -12$V

The minus sign is given because the negative terminal of the battery is connected to point A and positive terminal to point B.

Voltage across open terminals AB when all sources are present is

$V = V_1 + V_2 + (-V_3) = 3 + 20 - 12 = 11$ V.

Example 6: Use superposition theorem to find current I in the circuit shown in Fig. E6.1. All resistances are in ohms.

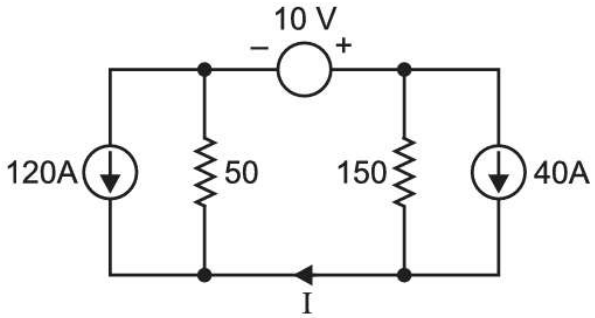


Fig. E6.1

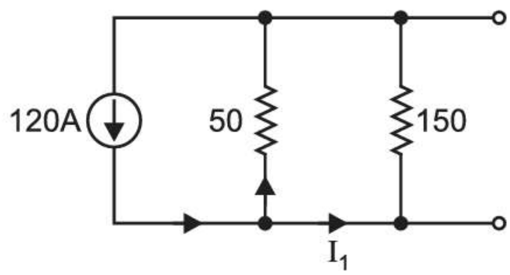


Fig. E6.2

Solution: In Fig. E6.2, the 10V voltage source has been replaced by a short and the 40A current source by an open so that now only 120A current source is acting alone. By current-divider rule, I_1 is given by

$$I_1 = 120 * \frac{50}{50+150} = 30 \text{ A}.$$

In Fig. E6.3, 40A current source is acting alone, 10 V voltage source being replaced by a short and 120A current source by an open. By current-divider rule, I2 is given by

$$I_2 = 40 * \frac{150}{50+150} = 30 \text{ A}.$$

In Fig. E6.4, 10V voltage source is acting alone. By Ohm's law, I_3 is given by

$$I_3 = \frac{10}{50+150} = 0.05 \text{ A.}$$

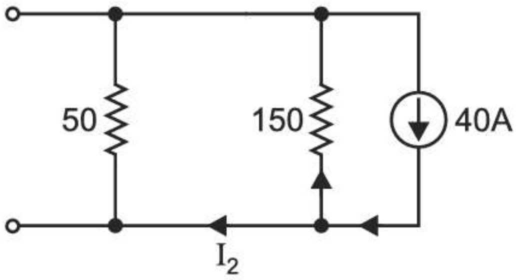


Fig. E6.3

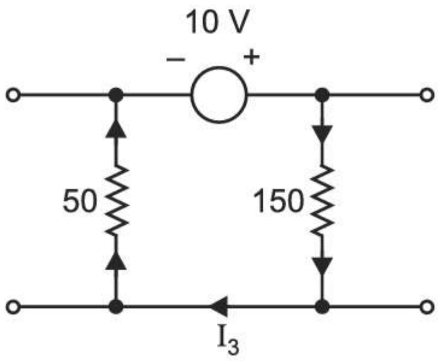


Fig. E6.4

Currents I_1 and I_2, being equal and opposite, cancel out so that

$I = I_3 = 0.05$ A.

Thevenin's Theorem:

Any linear, bilateral network having terminals A and B can be replaced by a single source of e.m.f. V_{Th} in series with a single resistance R_{Th}, where the e.m.f. V_{Th} is the voltage obtained across terminals A and B with load, if any removed i.e. it is open-circuited voltage between terminals A and B and the resistance R_{Th} is the resistance of the network measured between terminals A and B with load removed and sources of e.m.f. replaced by their internal resistances. Ideal voltage sources are replaced with short circuits and ideal current sources are replaced with open circuits.

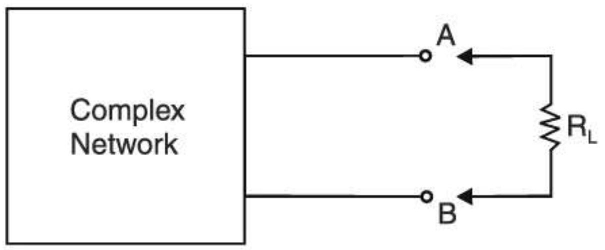


Fig. T5

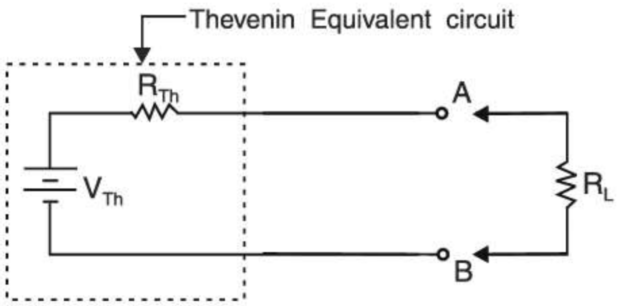


Fig. T6

Explanation:

Consider the circuit shown in Fig. T7. As far as the circuit behind terminals AB is concerned, it can be replaced by a single source of e.m.f. V_{Th} in series with a single resistance R_{Th} as shown in Fig. T10.

Finding V_{Th}:

The e.m.f. V_{Th}, is the voltage across terminals AB with load (i.e. R_L) removed as shown in Fig. T8. With R_L disconnected, there is no current in R_2 and V_{Th} is the voltage appearing across R_3.

$$V_{Th} = \text{Voltage across } R_3 = \frac{V}{R_1+R_3} * R_3$$

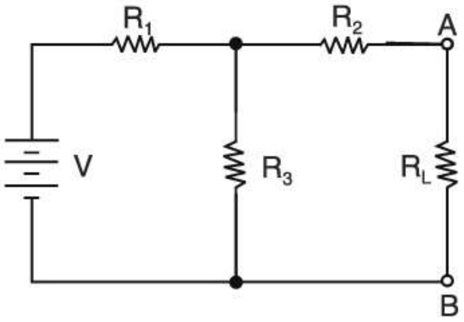


Fig. T7

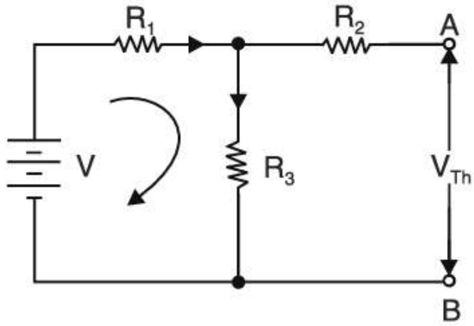


Fig. T8

Finding R_{Th}:

To find R_{Th}, remove the load R_L and replace the battery by a short-circuit because its internal resistance is assumed zero. Then resistance between terminals A and B is equal to R_{Th} as shown in Fig. T9. Obviously, at the terminals AB in Fig. T9, R_1 and R_3 are in parallel and this parallel combination is in series with R_2.

$$R_{Th} = R_2 + \frac{R_1 * R_3}{R_1 + R_3}$$

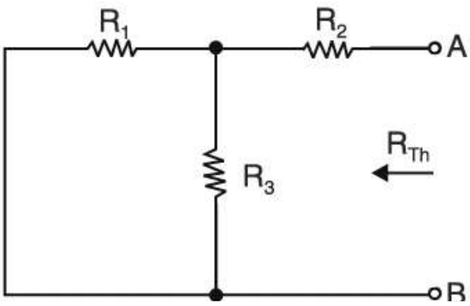


Fig. T9

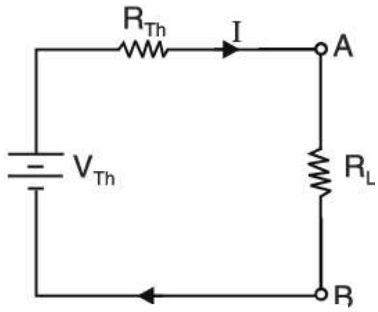


Fig. T10

When load R_L is connected between terminals A and B [See Fig. T10], then current in R_L is given by $I = \frac{V_{Th}}{R_{Th} + R_L}$

Procedure for Finding Thevenin Equivalent Circuit:

Open the two terminals (i.e., remove any load) between which you want to find Thevenin equivalent circuit.

Find the open-circuit voltage between the two open terminals. It is called Thevenin voltage V_{Th}.

Determine the resistance between the two open terminals with all ideal voltage sources shorted and all ideal current sources opened (a non-ideal source is replaced by its internal resistance). It is called Thevenin resistance R_{Th}.

Connect V_{Th} and R_{Th} in series to produce Thevenin equivalent circuit between the two terminals under consideration.

Place the load resistor removed in first step across the terminals of the Thevenin equivalent circuit. The load current can now be calculated using only Ohm's law and it has the same value as the load current in the original circuit.

Example 7: Using Thevenin's theorem, find the current in 6 Ω resistor in Fig. E7.1

Solution. Since internal resistances of batteries are not given, it will be assumed that they are zero. We shall find Thevenin's equivalent circuit at terminals AB in Fig. E7.1.

V_{Th} = Voltage across terminals AB with load (i.e. 6 Ω resistor) removed as shown in Fig. E7.2.

$$V_{Th} = 4{\cdot}5 - \left(\frac{4.5 - 3}{4+5}\right) \times 4 = 3{\cdot}83 \text{ V}$$

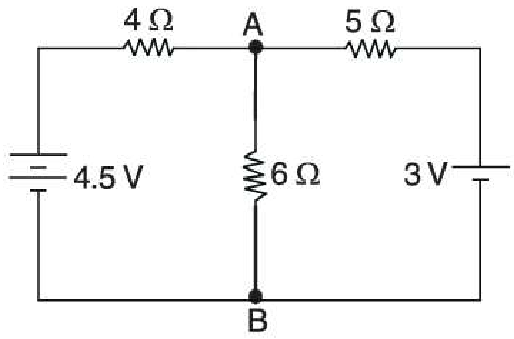


Fig. E7.1

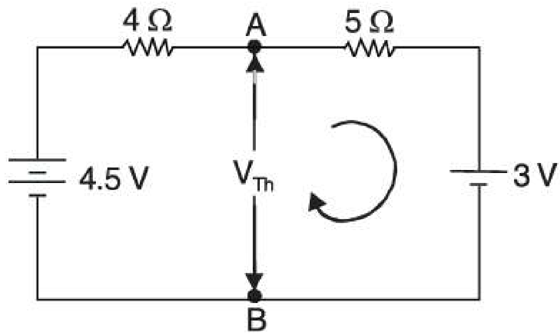


Fig. E7.2

R_{Th} = Resistance at terminals AB with load (i.e. 6 Ω resistor) removed and battery replaced by a short as shown in Fig. E7.3

$$R_{Th} = \frac{4*5}{4+5} = 2.22\ \Omega.$$

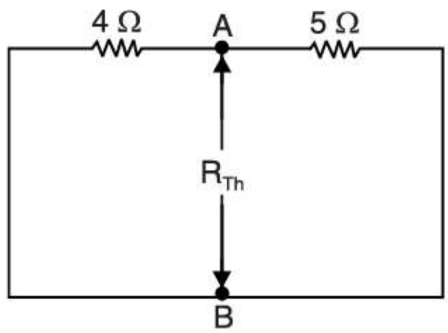


Fig. E7.3

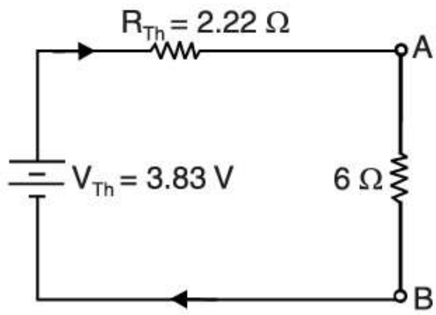


Fig. E7.4

Thevenin's equivalent circuit at terminals AB is V_{Th} (= 3·83 V) in series with R_{Th} (= 2·22 Ω). When load (i.e. 6 Ω resistor) is connected between terminals A and B, the circuit becomes as shown in Fig. E7.4.

$$\text{Current in 6 Ω resistor} = \frac{V_{Th}}{R_{Th}+R_L} = \frac{3.83}{2.22+6} = 0.466\text{A}$$

Example 8: Using Thevenin's theorem, find the current through resistance R connected between points a and b in Fig. E8.1.

Solution:

Finding V_{Th}: Thevenin voltage V_{Th} is the voltage across terminals ab with resistance R (=10Ω) removed as shown in Fig. E8.2. It can be found by Maxwell's mesh current method.

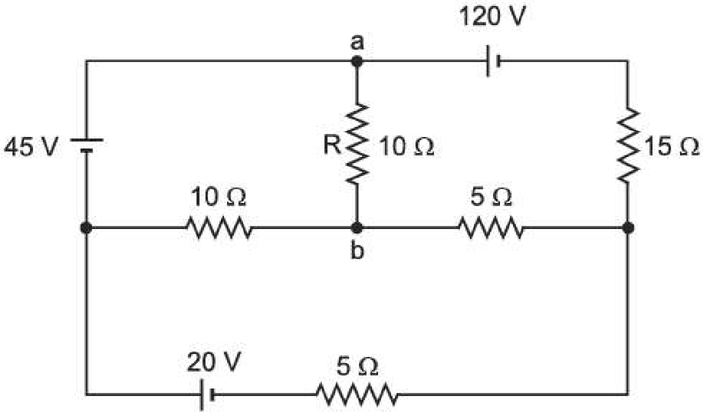


Fig. E8.1

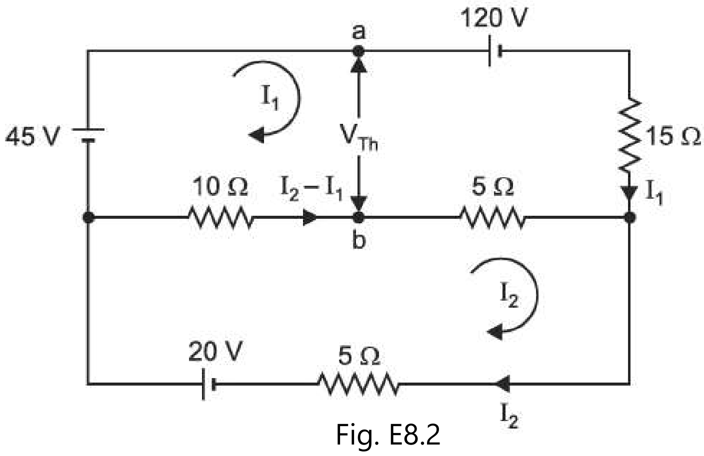


Fig. E8.2

Mesh 1:

$$45 - 120 - 15I_1 - 5(I_1 - I_2) - 10(I_1 - I_2) = 0$$

Or $\quad 30I_1 - 15I_2 = -75$

Mesh 2:

$$-10(I_2 - I_1) - 5(I_2 - I_1) - 5I_2 + 20 = 0$$

Or $\quad -15I_1 + 20I_2 = 20$

From the above two eqs, $I_1 = -3.2$A & $I2 = -1.4$ A

Now,

$$Va - 45 - 10(I_2 - I_1) = Vb$$

or $\quad Va - Vb = 45 + 10(I_2 - I_1) = 45 + 10[-1.4 - (-3.2)] = 63V$

$$V_{Th} = Vab = Va - Vb = 63V$$

Finding R_{Th}: Thevenin resistance R_{Th} is the resistance at terminals ab with resistance R (=10Ω) removed and batteries replaced by a short as shown in Fig. E8.3. Using laws of series and parallel resistances, the circuit is reduced to the one shown in Fig. E8.4.

R_{Th} = Resistance at terminals ab in Fig E8.4.

$$R_{Th} = 10\Omega \,||\, [5\Omega + (15\Omega \,||\, 5\Omega)] = 10\Omega \,||\, (5\Omega + 3.75\Omega) = \frac{14}{3}\,\Omega$$

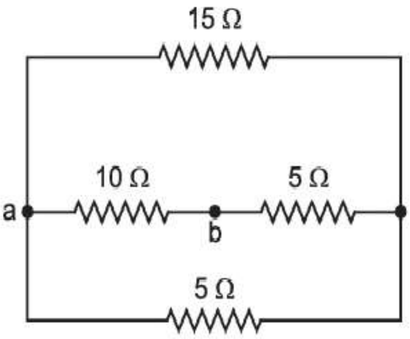


Fig. E8.3

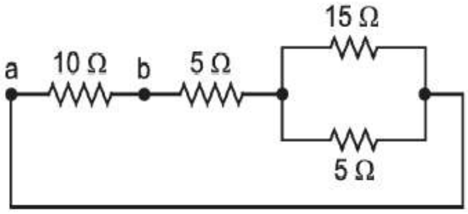


Fig. E8.4

$$\text{Current in R } (= 10\Omega) = \frac{V_{Th}}{R_{Th}+R_L} = \frac{63}{(14/3)+10} = 4.295 \text{ A.}$$

Example 9: Calculate the power which would be dissipated in a 50 Ω resistor connected across xy in the network shown in Fig. E9.1.

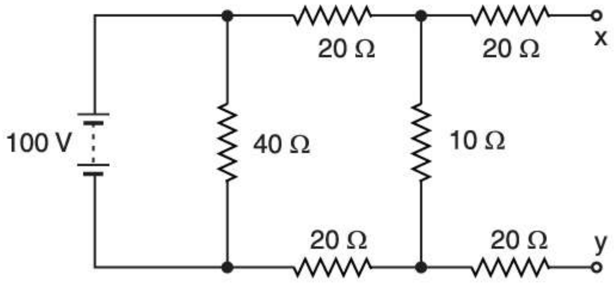


Fig. E9.1

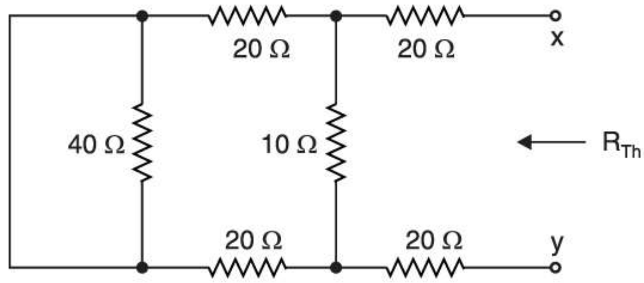


Fig. E9.2

Solution. We shall find Thevenin equivalent circuit to the left of terminals xy. With xy terminals open, the current in 10 Ω resistor is given by

$$I = \frac{100}{20+10+20} = 2 \text{ A}$$

Open circuit voltage across xy is given by

$$V_{Th} = I \times 10 = 2 \times 10 = 20V$$

In order to find R_{Th} replace the battery by a short since its internal resistance is assumed to be zero [See Fig. E9.2].

R_{Th} = Resistance looking into the terminals xy in Fig. E9.2.

$$R_{Th} = 20 + [(20 + 20) \,||\, 10] + 20$$

Or $$R_{Th} = 20 + \frac{40*10}{40+10} + 20 = 20 + 8 + 20 = 48\ \Omega$$

Therefore, Thevenin's equivalent circuit behind terminals xy is V_{Th} (= 20V) in series with R_{Th} (= 48 Ω).

When load R_L (= 50 Ω) is connected across xy, the circuit becomes as shown in Fig. E9.3

Current I in 50 Ω resistor is

$$I = \frac{V_{Th}}{R_{Th}+R_L} = \frac{20}{48+50} = \frac{20}{98} \text{ A}$$

Power dissipated in 50 Ω resistor is

$$P = I^2R_L = (20/98)^2*50 = 2.08 \text{ W}$$

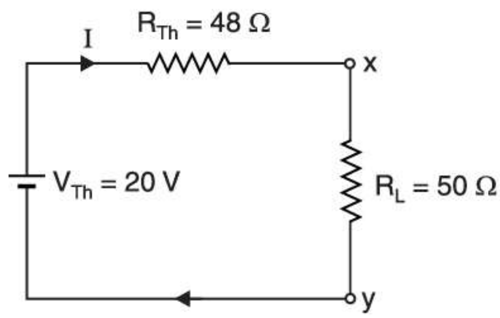


Fig. E9.3

Example 10: A Wheatstone bridge ABCD has the following details: AB = 10 Ω, BC = 30 Ω, CD = 15 Ω and DA = 20 Ω. A battery of e.m.f. 2 V and negligible resistance is connected between A and C with A positive. A galvanometer of 40 Ω resistance is connected between B and D. Using Thevenin's theorem, determine the magnitude and direction of current in the galvanometer.

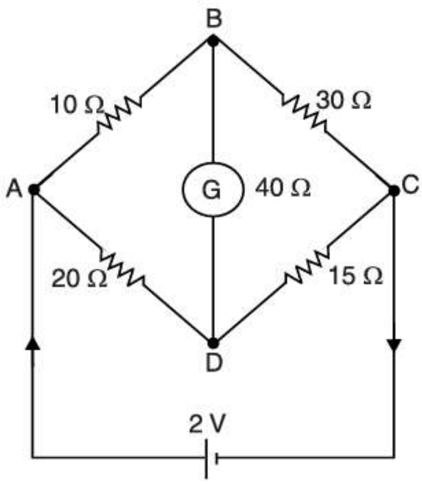


Fig. E10.1

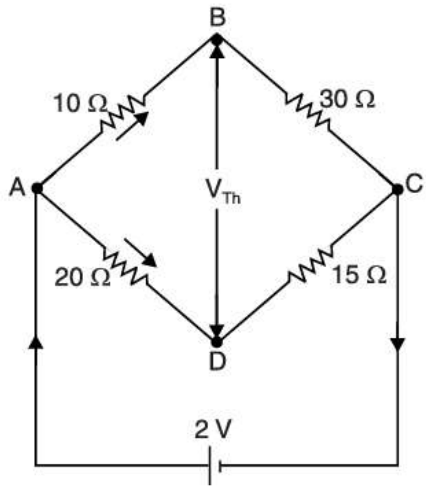


Fig. E10.2

Solution: We shall find Thevenin's equivalent circuit at terminals BD in Fig. E10.1.

Finding V_{Th}: To find V_{Th} at terminals BD, remove the load (i.e. 40 Ω galvanometer) as shown in Fig. E10.2). The voltage between terminals B and D is equal to V_{Th}.

$$\text{Current in branch ABC} = \frac{2}{10+30} = 0.05 \text{ A}$$

P.D. between A and B, $V_{AB} = 10 \times 0.05 = 0.5$ V

$$\text{Current in branch ADC} = \frac{2}{20+15} = 0.0571 \text{ A}$$

P.D. between A and D, $V_{AD} = 0.0571 \times 20 = 1.142$ V

P.D. between B and D, $V_{Th} = V_{AD} - V_{AB} = 1.142 - 0.5 = 0.642$ V

Obviously, point B is positive w.r.t. point D i.e. current in the galvanometer, when connected between B and D, will flow from B to D.

Finding R_{Th}: In order to find R_{Th}, remove the load (i.e. 40 Ω galvanometer) and replace the battery by a short (as its internal resistance is assumed zero) as shown in Fig. E10.1. Then resistance measured between terminals B and D is equal to R_{Th}.

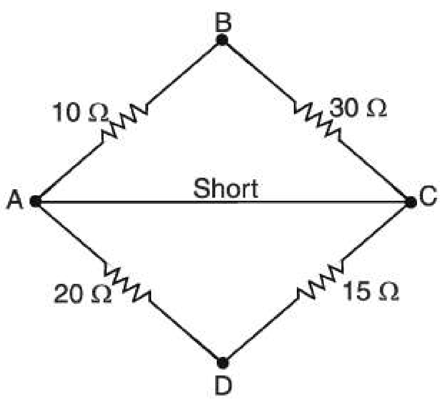


Fig. E10.3

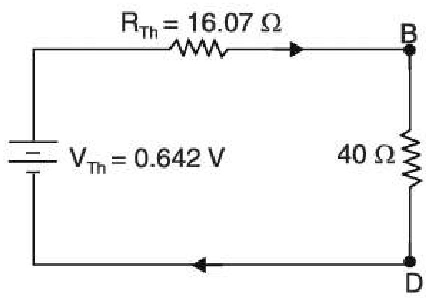


Fig. E10.4

R_{Th} = Resistance at terminals BD in Fig. E10.3

$$R_{Th} = \frac{10*30}{10+30} + \frac{20*15}{20+15} = 7.5 + 8.57 = 16.07\ \Omega$$

Thevenin's equivalent circuit at terminals BD is V_{Th} (= 0.642 V) in series with R_{Th} (= 16·07 Ω). When galvanometer is connected between B and D, the circuit becomes as shown in Fig. E10.4

$$\text{Galvanometer current} = \frac{V_{Th}}{R_{Th}+R_L} = \frac{0.642}{16.07+40} = 11.5 \times 10^{-3}\ \text{A} = 11.5\ \text{mA from B to D.}$$

Norton's Theorem:

Any linear, bilateral network having two terminals A and B can be replaced by a current source I_N in parallel with a resistance R_N where the current source I_N is equal to the current that would flow through AB when A and B are short-circuited and the resistance R_N is the resistance of the network measured between A and B with load removed and the sources of e.m.f. replaced by their internal resistances. Ideal voltage sources are replaced with short circuits and ideal current sources are replaced with open circuits.

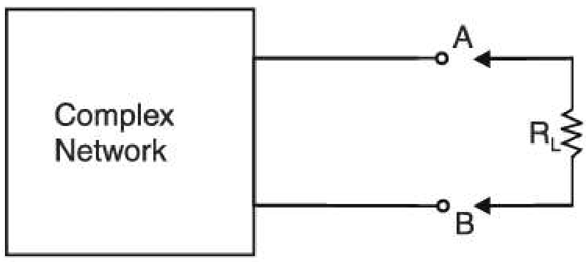


Fig. T11

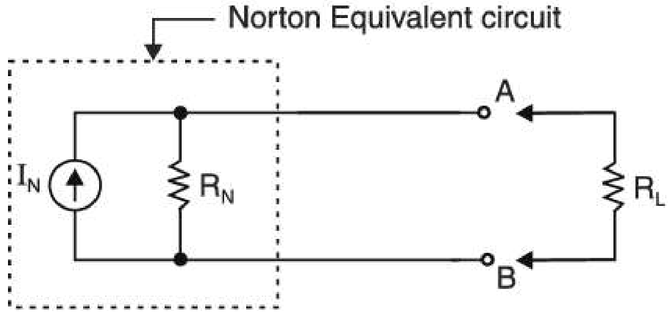


Fig. T12

Explanation:

Consider the circuit shown in Fig. T13. As far as the circuit behind terminals AB is concerned, it can be replaced by a current source I_N in parallel with a resistance R_N as shown in Fig. T16. The current source I_N is equal to the current that would flow through AB when terminals A and B are short-circuited as shown in Fig. T14.

Finding V_{Th}:

The load on the source when terminals AB are short-circuited is given by

$$R' = R_1 + \frac{R_2*R_3}{R_2+R_3} = \frac{R_1R_2+R_2R_3+R_1R_3}{R_2+R_3}$$

Source current,

$$I' = \frac{V}{R'} = \frac{V(R_2+R_3)}{R_1R_2+R_2R_3+R_1R_3}$$

Short-circuit current, I_N = Current in R_2 in Fig. T14.

$$I_N = I' * \frac{R_3}{R_2+R_3} = \frac{VR_3}{R_1R_2+R_2R_3+R_1R_3}$$

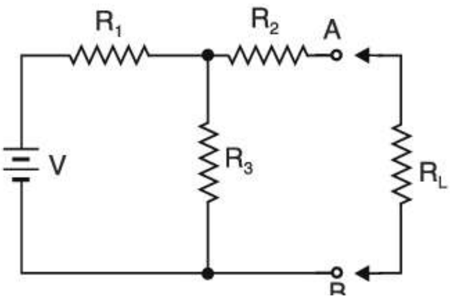


Fig. T13

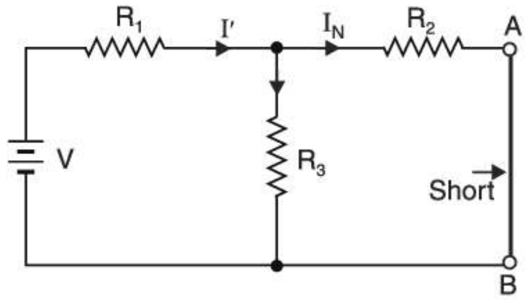


Fig. T14

Finding R_{Th}:

To find R_N, remove the load R_L and replace battery by a short because its internal resistance is assumed zero [See Fig. T15].

R_N = Resistance at terminals AB in Fig. T15.

$$R_N = R_2 + \frac{R_1 * R_3}{R_1 + R_3}$$

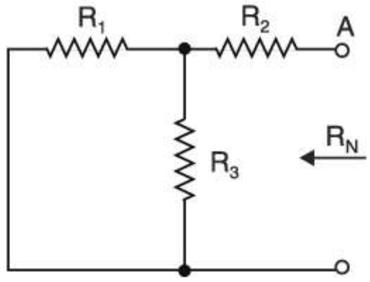


Fig. T15

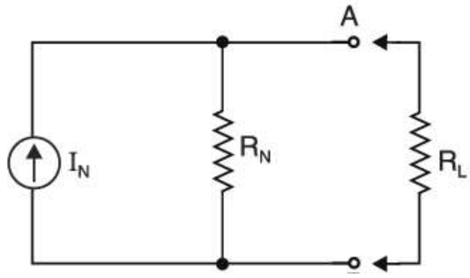


Fig. T16

Thus the values of I_N and R_N are known. The Norton equivalent circuit will be as shown in Fig. T16. The load current can be found by

$$I_L = \frac{I_N * R_N}{R_N + R_L}$$

Procedure for Finding Norton Equivalent Circuit:

Open the two terminals (i.e. remove any load) between which we want to find Norton equivalent circuit.

Put a short-circuit across the terminals under consideration. Find the short-circuit current flowing in the short circuit. It is called Norton current I_N.

Determine the resistance between the two open terminals with all ideal voltage sources shorted and all ideal current sources opened (a non-ideal source is replaced by its internal resistance). It is called Norton's resistance R_N. It is easy to see that $R_N = R_{Th}$.

Connect I_N and R_N in parallel to produce Norton equivalent circuit between the two terminals under consideration.

Place the load resistor removed in first step across the terminals of the Norton equivalent circuit. The load current can now be calculated by using current-divider rule. This load current will be the same as the load current in the original circuit.

Example 11: Using Norton’s theorem, calculate the current in the 5 Ω resistor in the circuit shown in Fig. E11.1.

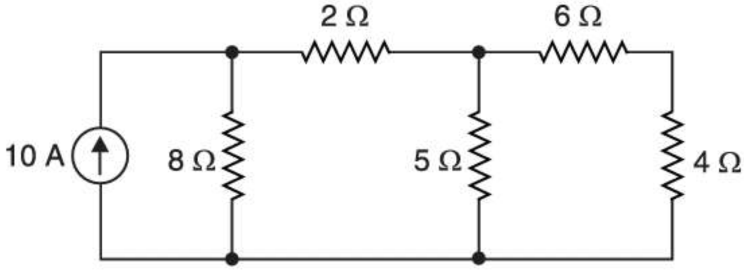


Fig. E11.1

Solution. Short the branch that contains 5 Ω resistor in Fig. E 11.1. The circuit then becomes as shown in Fig. E11.2. Referring to Fig. E11.2, the 6 Ω and 4 Ω resistors are in series and this series combination is in parallel with the short.

Therefore, these resistors have no effect on Norton current and may be considered as removed from the circuit. As a result, 10 A divides between parallel resistors of 8 Ω and 2 Ω.

Norton current, I_N = Current in 2 Ω resistor

$$I_N = \frac{10*8}{2+8} = 8\text{ A}.$$

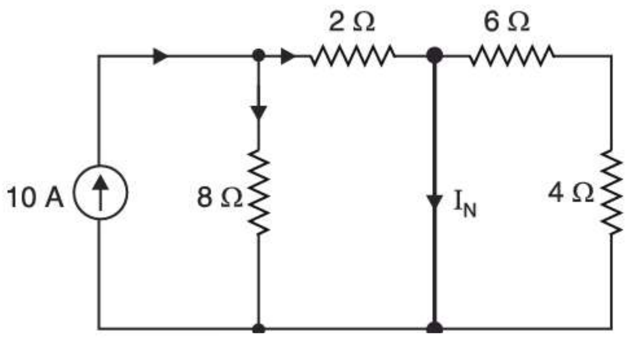


Fig. E11.2

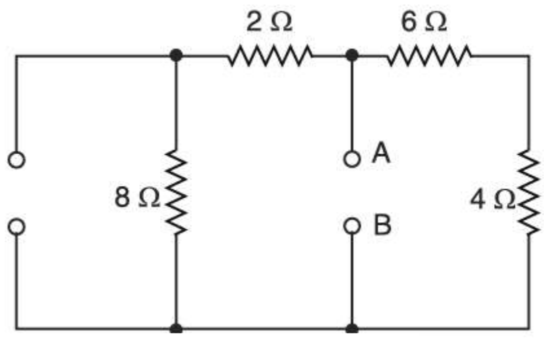


Fig. E11.3

In order to find Norton resistance R_N, open circuit the branch containing the 5 Ω resistor and replace the current source by an open in Fig. E11.1. The circuit then becomes as shown in Fig. E11.3

Norton resistance, R_N = Resistance at terminals AB in Fig. E11.3.

$$R_N = (2 + 8) \,||\, (4 + 6) = 10 \,||\, 10 = \frac{10*10}{10+10} = 5\ \Omega$$

Therefore, Norton equivalent circuit consists of a current source of 8 A in parallel with a resistance of 5 Ω as shown in Fig. E11.4. Now the branch containing 5 Ω resistor is connected across the output terminals of Norton’s equivalent circuit as shown in Fig. E 11.5.

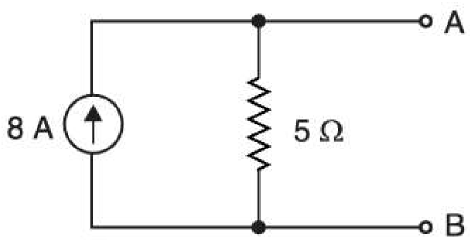


Fig. E11.4

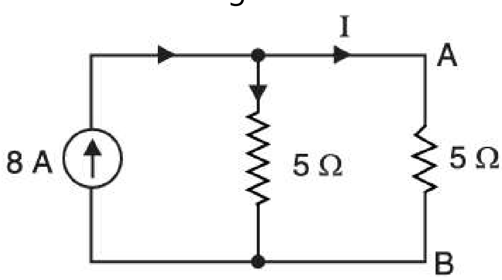


Fig. E11.5

By current-divider rule, the current I in 5 Ω resistor is

$$I = \frac{8*5}{5+5} = 4 \text{ A.}$$

Example 12: Find Norton equivalent circuit for Fig. E12.1. Also solve for load current and load voltage.

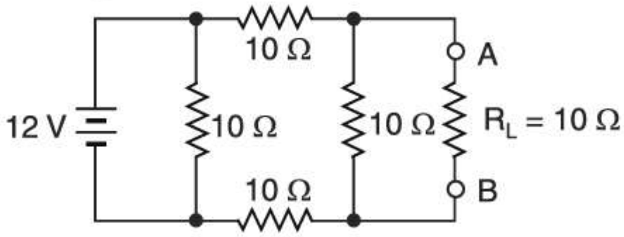


Fig. E12.1

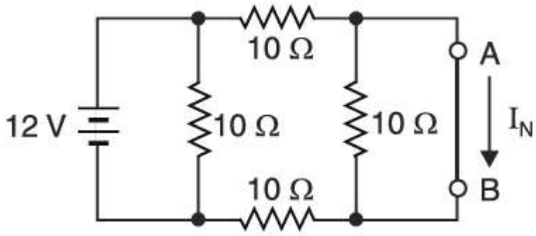


Fig. E12.2

Solution: Short the branch that contains R_L (= 10 Ω) in Fig. E12.1. The circuit then becomes as shown in Fig. E12.2. The resistor that is in parallel with the short has no effect. Therefore, this resistor may be considered as removed from the circuit shown in Fig. E12.2. The circuit then contains two 10 Ω resistors in series.

Norton current,

$$I_N = \frac{12}{10+10} = 0.6 \text{ A.}$$

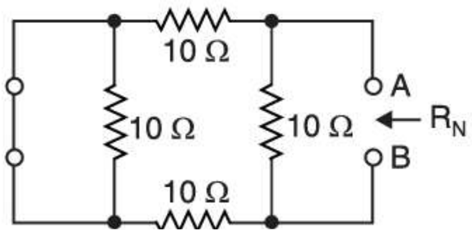


Fig. E12.3

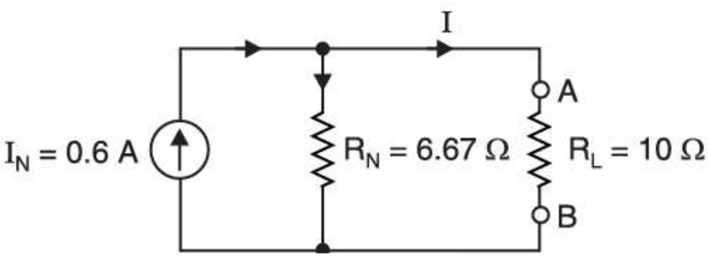


Fig. E12.4

In order to find Norton resistance R_N, open circuit the branch containing R_L and replace the voltage source by a short in Fig. E12.1. The circuit then becomes as shown in Fig. E 12.3.

Norton resistance, R_N = Resistance at terminals AB in Fig. E12.3.

$$R_N = (10 + 10) \| 10 = \frac{20*10}{20+10} = 6.67 \ \Omega$$

Therefore, Norton equivalent circuit consists of a current source of 0·6 A in parallel with a resistance of 6·67 Ω. When the branch containing R_L (=10 Ω) is connected across the output terminals of Norton equivalent circuit, the circuit becomes as shown in Fig. E12.4.

By current-divider rule, the current I in RL is

$$I = \frac{0.6*6.67}{6.67+10} = 0.24 \text{ A.}$$

Voltage across $R_L = I*R_L$ = 0·24 * 10 = 2·4 V.

Example 13: Find the Norton current for the unbalanced Wheatstone bridge in Fig. 13.1.

Solution: The Norton current is found by shorting the load terminals as shown in Fig. E13.2. First determine the total current and then use Ohm's law to find current in the four resistors. Once the currents in the four resistors are known, Kirchhoff's current law can be used to determine Norton current I_N.

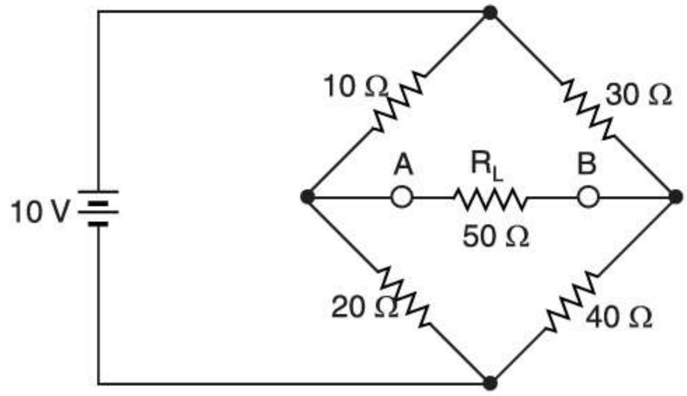


Fig. E13.1

Fig. E13.3 shows the equivalent circuit of Fig. E13.2. The total circuit resistance R_T to 10 V source is

$$R_T = (10 \parallel 30) + (20 \parallel 40) = \frac{10*30}{10+30} + \frac{20*40}{20+40} = 7{\cdot}5 + 13{\cdot}33 = 20{\cdot}83\ \Omega$$

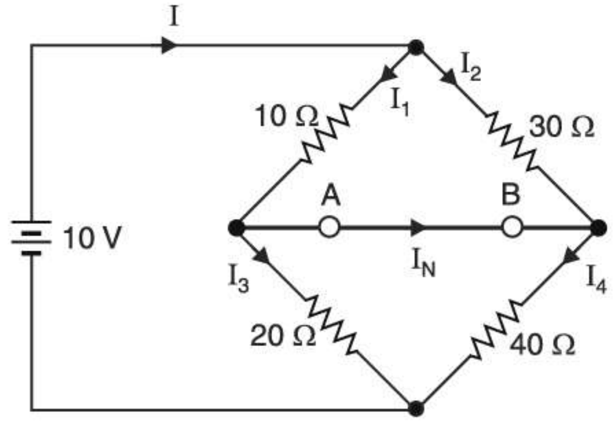


Fig. E13.2

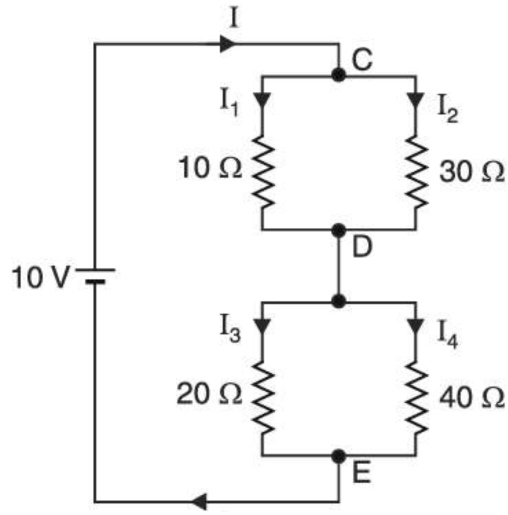


Fig. E13.3

Total circuit current, $I = 10/20.83 = 0.48$ A.

Referring to Fig. 3.167 (ii), we have,

$V_{CD} = I * R_{CD} = 0{\cdot}48 \times 7{\cdot}5 = 3{\cdot}6$ V

$V_{DE} = I * R_{DE} = 0{\cdot}48 \times 13{\cdot}33 = 6{\cdot}4$ V

$I_1 = V_{CD}/10 = 3.6/10 = 0.36$ A, $I_2 = V_{CD}/30\ 3.6/30 = 0.12$ A.

$I_3 = V_{DE}/20 = 6.4/20 = 0.32$ A, $I_4 = V_{DE}/40 = 6.4/40 = 0.16$ A.

Referring to Fig. 13.2, it is now clear that I_1(= 0·36 A) is greater than I_3(= 0·32 A). Therefore, current I_N will flow from A to B and its value is

$I_N = I_1 - I_3 = 0{\cdot}36 - 0{\cdot}32 = 0{\cdot}04$ A.

Example 14: Two batteries, each of e.m.f. 12 V, are connected in parallel to supply a resistive load of 0·5 Ω. The internal resistances of the batteries are 0·12 Ω and 0·08 Ω. Calculate the current in the load and the current supplied by each battery.

Solution: Fig. 14.1 shows the conditions of the problem. If a short circuit is placed across the load, the circuit becomes as shown in Fig. 14.2. The total short circuit current is given by

$$I_N = \frac{12}{0.12} + \frac{12}{0.02} = 100 + 150 = 250 \text{ A}$$

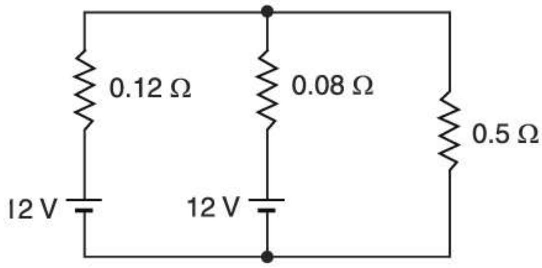


Fig. E14.1

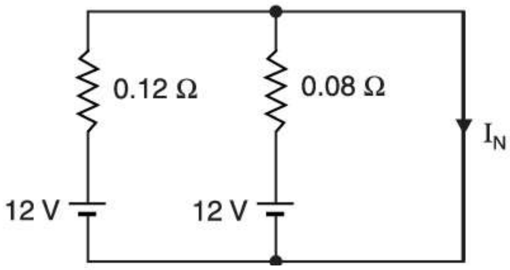


Fig. E14.2

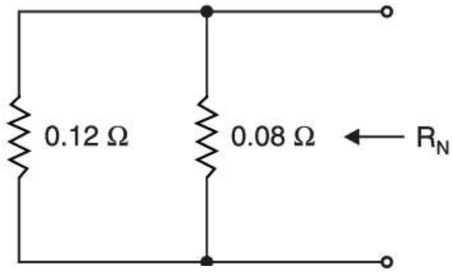


Fig. E14.3

In order to find Norton resistance R_N, open circuit the load and replace the batteries by their internal resistances. The circuit then becomes as shown in Fig. E14.3. Then resistance looking into the open-circuited terminals is the Norton resistance.

$$\text{Norton resistance, } R_N = 0{\cdot}12 \parallel 0{\cdot}08 = \frac{0.12*0.08}{0.12+0.08} = 0.048\ \Omega.$$

Therefore, Norton equivalent circuit consists of a current source of 250 A in parallel with a resistance of 0·048 Ω .When load (= 0·5 Ω) is connected across the output terminals of Norton equivalent circuit, the circuit becomes as shown in Fig. E14.4. By current-divider rule, the current I in load (= 0·5 Ω) is given by

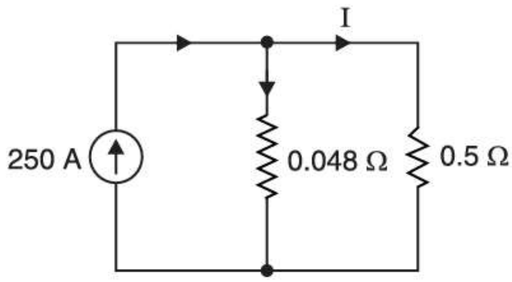


Fig. E14.4

$$I = \frac{250*0.48}{0.48+0.5} = 21.9 \text{ A.}$$

Battery terminal voltage, $I*R_L = 21{\cdot}9 \times 0{\cdot}5 = 10{\cdot}95$ V.

$$\text{Current in first battery} = \frac{12-10.95}{0.12} = 8.8 \text{ A.}$$

$$\text{Current in second battery} = \frac{12-10.95}{0.08} = 13.1 \text{ A.}$$

Maximum Power Transfer Theorem:

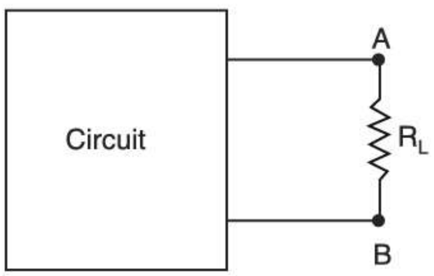


Fig. T16

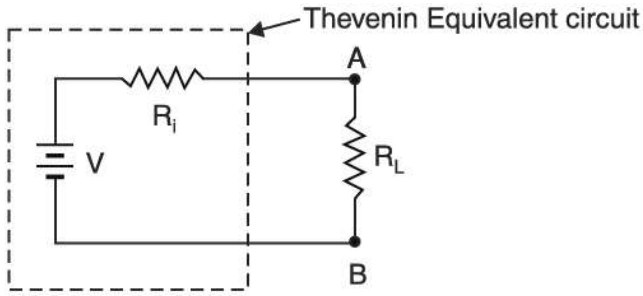
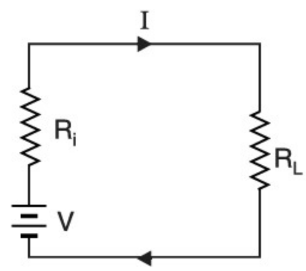


Fig. T17

In d.c. circuits, maximum power is transferred from a source to load when the load resistance is made equal to the internal resistance of the source as viewed from the load terminals with load removed and all e.m.f. sources replaced by their internal resistances.

Fig. T16 shows a circuit supplying power to a load R_L. The circuit enclosed in the box can be replaced by Thevenin's equivalent circuit consisting of Thevenin voltage $V = V_{Th}$ in series with Thevenin resistance $R_i(=R_{Th})$ as shown in Fig. T17. Clearly, resistance R_i is the resistance measured between terminals AB with R_L removed and e.m.f. sources replaced by their internal resistances. According to maximum power transfer theorem, maximum power will be transferred from the circuit to the load when R_L is made equal to R_i, the Thevenin resistance at terminals AB.

Proof of Maximum Power Transfer Theorem:



Consider a voltage source V of internal resistance Ri delivering power to a load R_L. We shall prove that when $R_L = R_i$, the power delivered to R_L is maximum. Referring to Fig. T18, we have,

Circuit current,

$$I = \frac{V}{R_i + R_l}$$

Power delivered to load,

$$P = I^2 R_L = [V/(R_i + R_L)]^2 R_L$$

For a given source, generated voltage V and internal resistance R_i are constant. Therefore, power delivered to the load depends upon R_L. In order to find the value of R_L for which the value of P is maximum, differentiate P w.r.t. R_L and set the result equal to zero.

$$\text{Thus,} \quad \frac{dP}{dR_L} = V^2 \left[\frac{(R_i + R_L)^2 - 2R_L(R_i + R_L)}{(R_i + R_L)^4} \right] = 0$$

Or $\quad (R_L + R_i)^2 - 2 R_L (R_L + R_i) = 0$

Or $\quad (R_L + R_i)(R_L + R_i - 2R_L) = 0$

Or $\quad (R_L + R_i)(R_i - R_L) = 0$

Since $R_L + R_i$ cannot be zero,

$$R_i - R_L = 0$$

Or $R_L = R_i$

Or Load resistance = Internal resistance of the source

Thus, for maximum power transfer, load resistance R_L must be equal to the internal resistance R_i of the source.

Maximum Power,

$P_{max} = I^2 R_L$

Or $P_{max} = [V/(R_i + R_L)]^2 R_L$

Or $P_{max} = [V/(R_L + R_L)]^2 R_L$

Or $P_{max} = [V/(2R_L)]^2 R_L$

Or $P_{max} = V^2/4R_L$

Example 15: Find the value of resistance R to have maximum power transfer in the circuit shown in Fig. E 15.1. Also obtain the amount of maximum power. All resistances are in ohms.

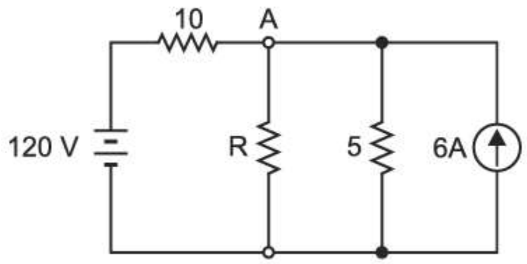


Fig. E15.1

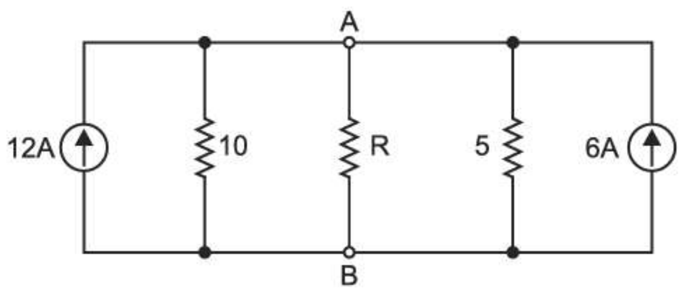


Fig. E15.2

Solution: To find the desired answers, we should find VTh and RTh at the load (i.e. R) terminals. For this purpose, we first convert 120V voltage source in series with 10Ω resistance into equivalent current source of 120/10 = 12A in parallel with 10Ω resistance. The circuit then becomes as shown in Fig. E15.2

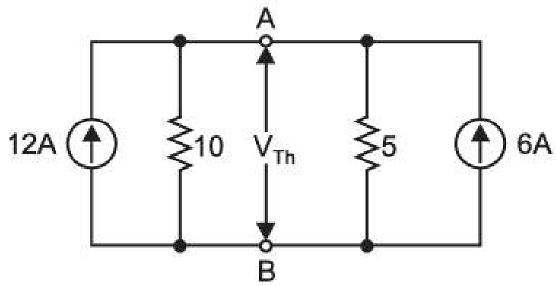


Fig. E15.3

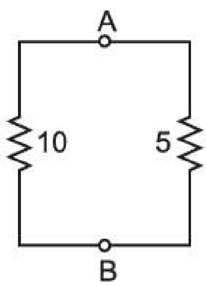


Fig. E15.4

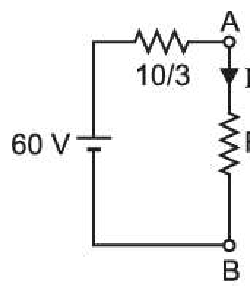


Fig. 15.5

To find V_{Th}, remove R (i.e. load) from the circuit in Fig. E15.2 and the circuit becomes as shown in Fig. E15.3.Then voltage across the open-circuited terminals AB is V_{Th}. Referring to Fig. E15.3 and applying KCL, we have,

$$\frac{V_{Th}}{10} + \frac{V_{Th}}{5} = 12 + 6$$

Or $V_{Th} = 60$ V.

In order to find R_{Th}, remove R and replace the current sources by open in Fig. E15.2. Then circuit becomes as shown in Fig. E15.4 Then resistance at the open-circuited terminals AB is R_{Th}.

$$R_{Th} = 10\Omega \,||\, 5\Omega = \frac{10*5}{10+5} = \frac{10}{3}\ \Omega.$$

When R is connected to the terminals of Thevenin equivalent circuit, the circuit becomes as shown in Fig. E15.5.

For maximum power transfer, the condition is

$$R = R_{Th} = \frac{10}{3}\ \Omega.$$

Max. power transferred,

$$P_{max} = \frac{V_{Th}^2}{4R_L} = \frac{V_{Th}^2}{4R} = \frac{60^2}{4(10/3)} = 270\ \text{W}.$$

Example 16: Calculate the value of R which will absorb maximum power from the circuit of Fig. E16.1 Also find the value of maximum power.

Solution: To find the desired answers, we should find V_{Th} and R_{Th} at the load (i.e. R) terminals. For this purpose, we first convert 2A current source in parallel with 15Ω resistance into equivalent voltage source of 2A*15Ω = 30 V in series with 15Ω resistance. The circuit then becomes as shown in Fig. E16.2.

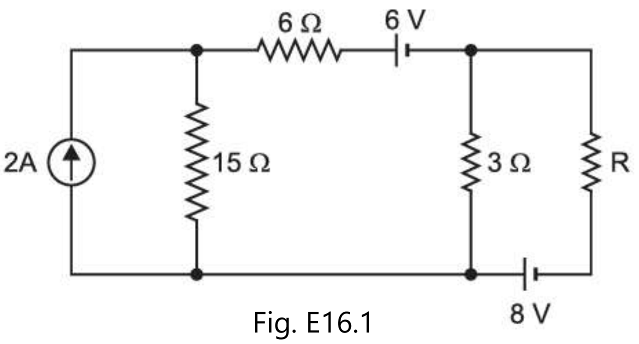


Fig. E16.1

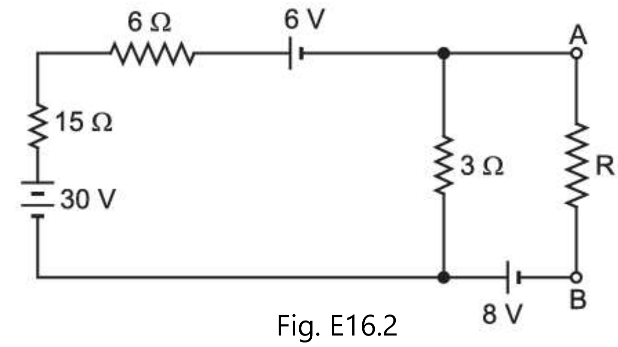


Fig. E16.2

To find V_{Th}, remove R (i.e. load) from the circuit in Fig. E16.2 and the circuit becomes as shown in Fig. E16.3. Then voltage across the open-circuited terminals AB is V_{Th}. Referring to Fig. E16.3,

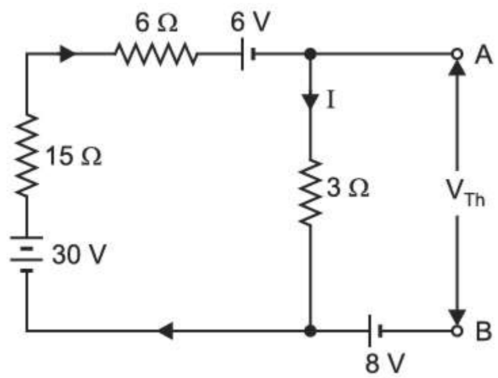


Fig. E16.3

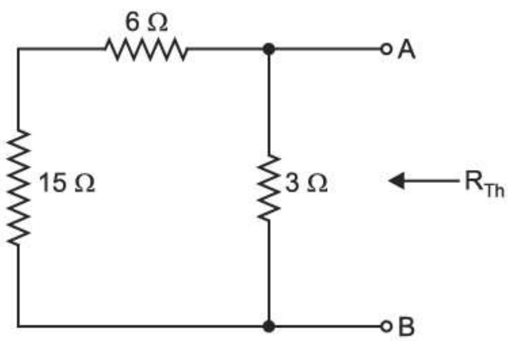


Fig. E16.4

Current in 3Ω resistor,

$$I = \frac{30-6}{15+6+3} = 1\text{ A.}$$

In Fig. E15.3, as we go from point A to point B via 3Ω resistor, we have,

$$V_A - I \times 3 - 8 = V_B$$

Or $\quad V_A - V_B = I \times 3 + 8 = 1 \times 3 + 8 = 11V = V_{Th}$

In order to find R_{Th}, remove R and replace the voltage sources by short in Fig. E15.2. Then circuit becomes as shown in Fig. E15.4 Then resistance at open-circuited terminals AB is R_{Th}.

$$R_{Th} = (15 + 6)\Omega \parallel 3 = \frac{21*3}{21+3} = \frac{21}{8}\ \Omega.$$

For maximum power transfer, the condition is

$$R = R_{Th} = \frac{21}{8}\ \Omega.$$

Max. power transferred,

$$P_{max} = \frac{{V_{Th}}^2}{4R_L} = \frac{{V_{Th}}^2}{4R} = \frac{11^2}{4(21/8)} = 11.524\text{ W.}$$

Example 17: For the circuit shown in Fig. E17.1, find the value of R that will receive maximum power. Determine this power.

Solution: We will use Thevenin's theorem to obtain the results. In order to find V_{Th}, remove the variable load R as shown in Fig. E17.2. Then open-circuited voltage across terminals AB is equal to V_{Th}.

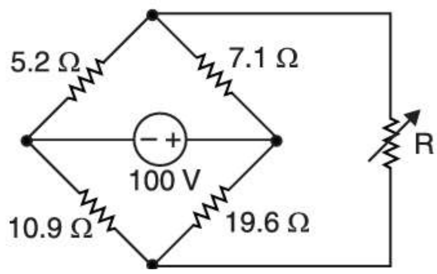


Fig. E17.1

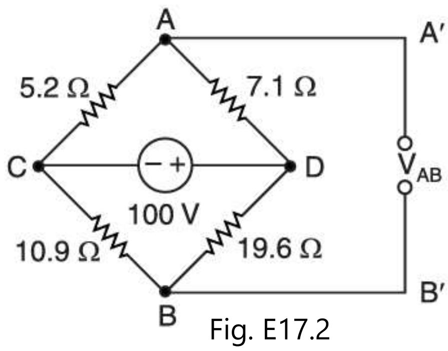


Fig. E17.2

$$\text{Current in branch DAC} = \frac{100}{7.1+5.2} = 8.13\text{ A}$$

$$\text{Current in branch DBC} = \frac{100}{19.6+10.9} = 3.28\text{ A}$$

It is clear from Fig. E17.2 that point A is at higher potential than point B. Applying KVL to the loop A′ACBB′A′, we have,

$$-5{\cdot}2 \times 8{\cdot}13 + 10{\cdot}9 \times 3{\cdot}28 + V_{AB} = 0$$

$$V_{AB} = 6{\cdot}52\text{ V}$$

Now V_{AB}, in Fig. E17.2 is equal to V_{Th} so that $V_{Th} = 6{\cdot}52$ V.

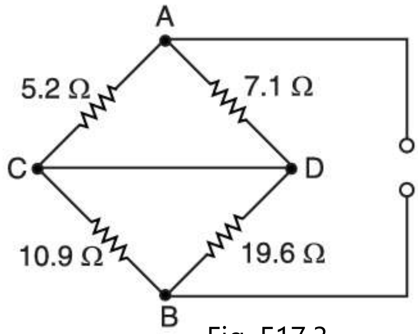


Fig. E17.3

In order to find R_{Th}, replace the 100 V source in Fig. E17.2 by a short. The circuit becomes as shown in Fig. E17.3 The resistance across terminals AB is the Thevenin resistance. Referring to Fig. E17.3,

$$R_{AB} = R_{Th} = (5{\cdot}2 \,||\, 7{\cdot}1) + (10{\cdot}9 \,||\, 19{\cdot}6)$$
$$= 3 + 7 = 10\ \Omega$$

Therefore, for maximum power transfer, $R = R_{Th} = 10\ \Omega$.

$$P_{max} = \frac{{V_{Th}}^2}{4R} = \frac{6.52^2}{4*10} = 1.06\ W.$$

QUESTION BANK

Short Questions With Answer:

Q: State Superposition theorem.

A: It states that in a linear, bilateral network containing more than one energy source, the current which flows at any point is the sum of all the currents which would flow at that point if each source where considered separately and all the other sources replaced for the time being by resistances equal to their internal resistances.

Q: State Thevenin's theorem.

A: It states that any linear, bilateral network having terminals A and B can be replaced by a single source of e.m.f. V_{Th} in series with a single resistance R_{Th}, where the e.m.f. V_{Th} is the voltage obtained across terminals A and B with load, if any removed i.e. it is open-circuited voltage between terminals A and B and the resistance R_{Th} is the resistance of the network measured between terminals A and B with load removed and sources of e.m.f. replaced by their internal resistances. Ideal voltage sources are replaced with short circuits and ideal current sources are replaced with open circuits.

Q: State Norton's theorem.

A: It states that any linear, bilateral network having two terminals A and B can be replaced by a current source I_N in parallel with a resistance R_N where the current source I_N is equal to the current that would flow through AB when A and B are short-circuited and the resistance R_N is the resistance of the network measured between A and B with load removed and the sources of e.m.f. replaced by their internal resistances. Ideal voltage sources are replaced with short circuits and ideal current sources are replaced with open circuits.

Q: State Maximum Power Transfer Theorem.

A: It states that in d.c. circuits, maximum power is transferred from a source to load when the load resistance is made equal to the internal resistance of the source as viewed from the load terminals with load removed and all e.m.f. sources replaced by their internal resistances.

Long Questions:

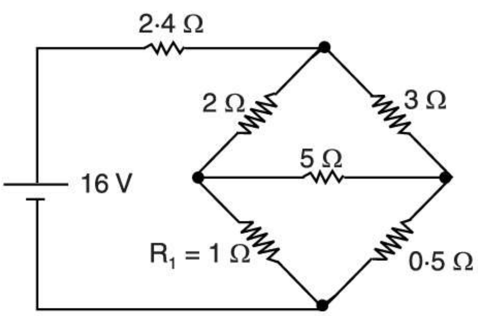


Fig. Q1

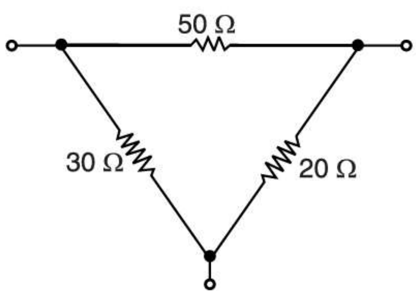


Fig. Q2

Q: Find the total current drawn from the voltage source and the current through R_1 (= 1 Ω) in the circuit shown in Fig. Q1.

Q: Convert the delta network shown in Fig. Q2 into equivalent wye network

Q: Convert the star network shown in Fig. Q3 into equivalent delta network.

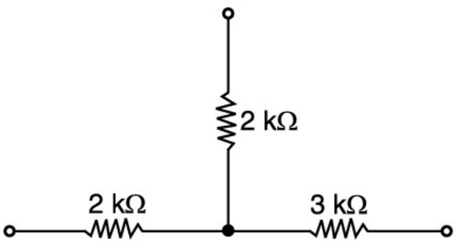


Fig. Q3

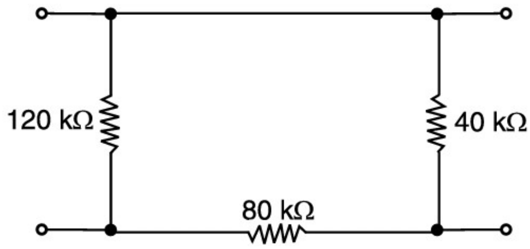


Fig. Q4

Q: Convert the delta network shown in Fig. Q4 into the equivalent wye network.

Q: In the network shown in Fig. Q5, find the resistance between terminals B and C using star/delta transformation.

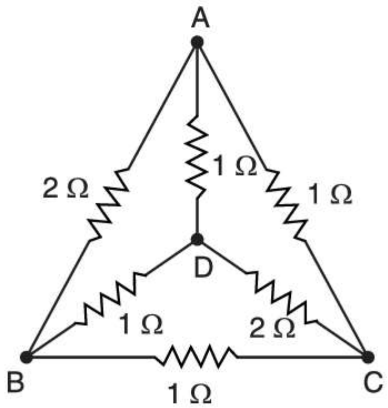


Fig. Q5

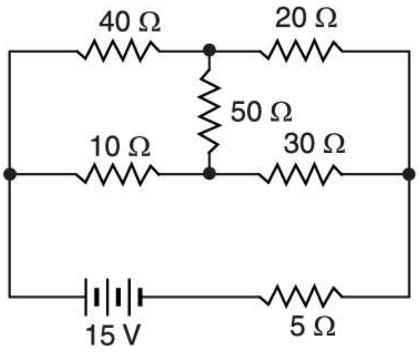


Fig. Q6

Q: In the network shown in Fig. Q6, find the current supplied by the battery using star/delta transformation.

Q: Use the superposition theorem to find the current in R_1 (= 60 Ω) in the circuit shown in figure Q7.

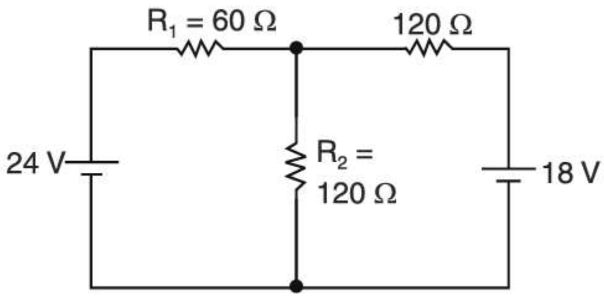


Fig. Q7

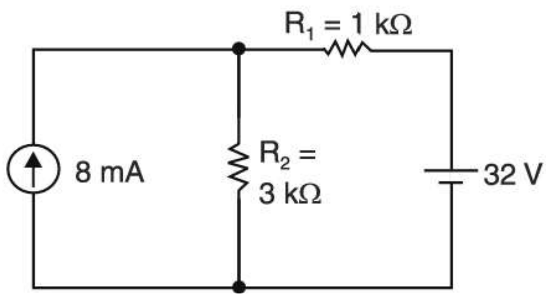


Fig. Q8

Q: Use the superposition theorem to find the current through R_1 (= 1k Ω) in the circuit shown in figure Q8.

Q: Use the superposition theorem to find the current through R_1 (= 10 Ω) in the circuit shown in Figure Q9.

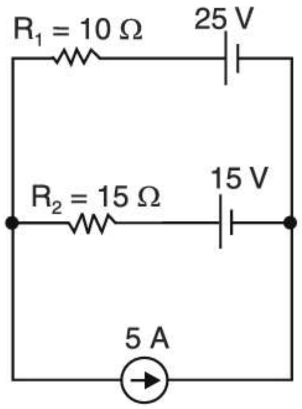


Fig. Q9

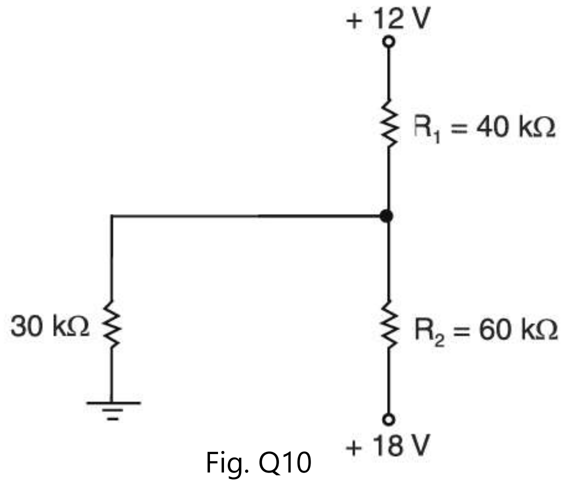


Fig. Q10

Q: Use superposition principle to find the current through resistance R_1 (= 40 KΩ) in the circuit shown in Fig. Q10.

Q: Using superposition principle, find the current through 10 Ω resistor in Fig. Q11.

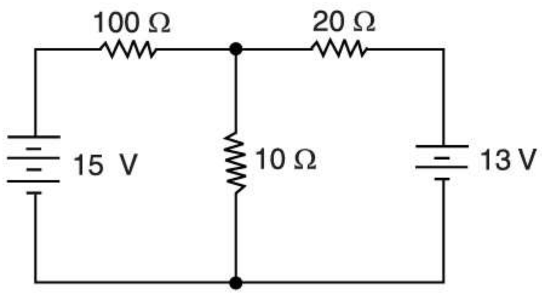


Fig. Q11

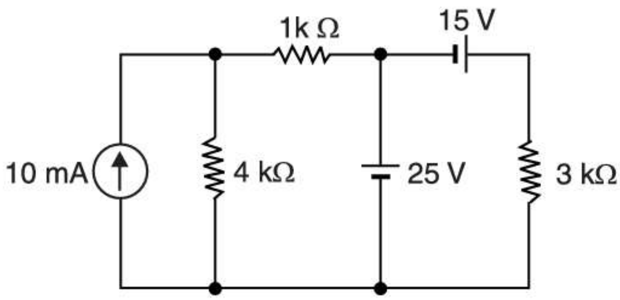


Fig. Q12

Q: Using superposition principle, find the voltage across 4 KΩ resistor in Fig. Q12.

Q: Using Thevenin's theorem, find the current in 10 Ω resistor in the circuit shown in Fig. Q13.

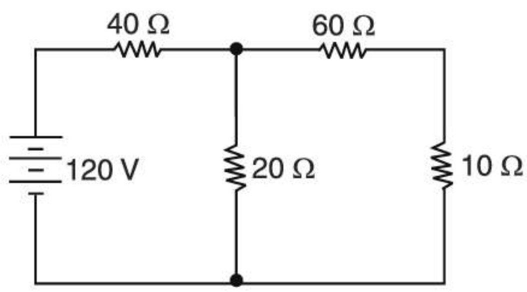


Fig. Q13

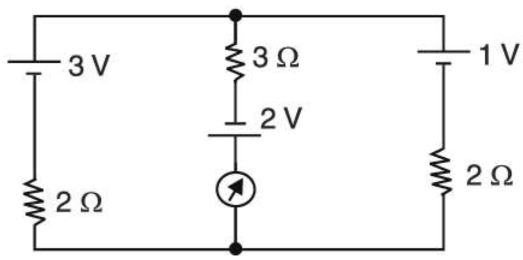


Fig. Q14

Q: Using Thevenin's theorem, find current in the ammeter shown in Fig. Q14.

Q: Using Thevenin's theorem, find p.d. across branch AB of the network shown in Fig. Q15.

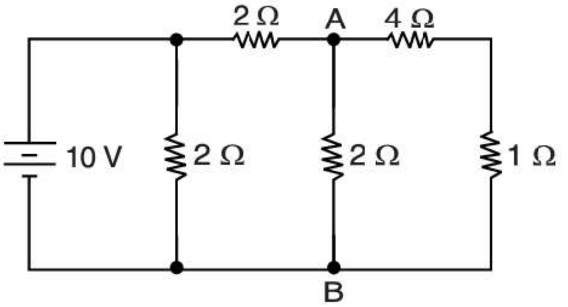


Fig. Q15

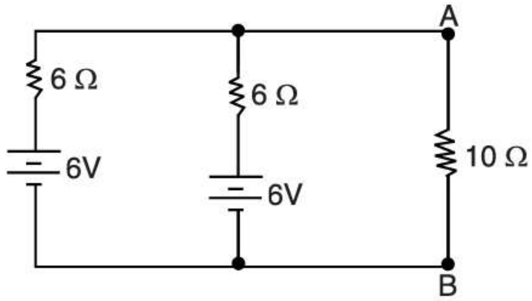


Fig. Q16

Q: Determine Thevenin's equivalent circuit to the left of AB in Fig. Q 16.

Q: A Wheatstone bridge ABCD is arranged as follows: AB = 100 Ω, BC = 99 Ω, CD = 1000 Ω and DA = 1000 Ω. A battery of e.m.f. 10 V and negligible resistance is connected between A and C with A positive. A galvanometer of resistance 100 Ω is connected between B and D. Using Thevenin's theorem, determine the galvanometer current.

Q: Find the Thevenin equivalent circuit of the circuitry, excluding R_1, connected to the terminals x – y in Fig. Q 17.

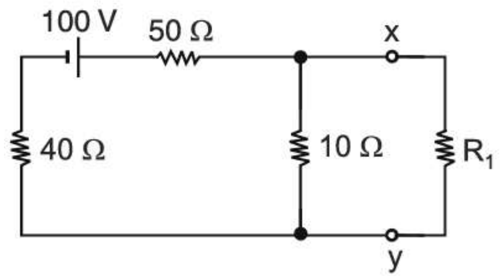


Fig. Q 17

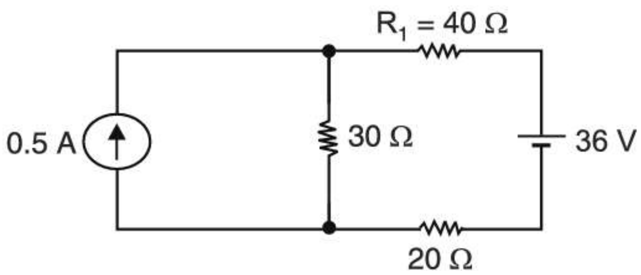


Fig. Q 18

Q: Find the voltage across R_1 in Fig. Q18 by constructing Thevenin equivalent circuit at the R1 terminals. Be sure to indicate the polarity of the voltage.

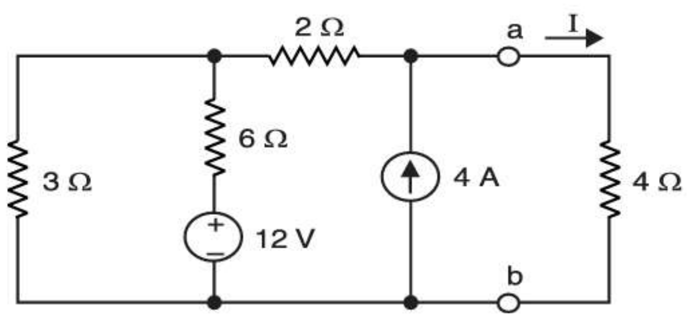


Fig. Q 19

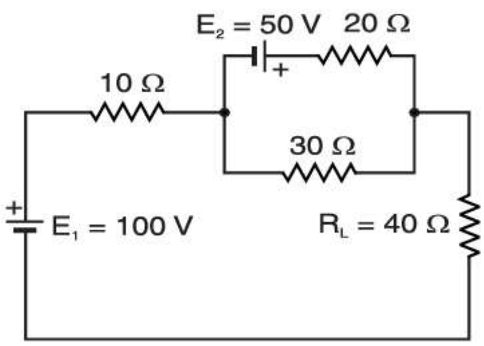


Fig. Q 20

Q: By using Thevenin Theorem, find current I in the circuit shown in Fig. Q 19.

Q: Find Thevenin equivalent circuit in Fig. Q 20.

Q: Using Norton's theorem, find the current in 8 Ω resistor of the network in Fig. Q21.

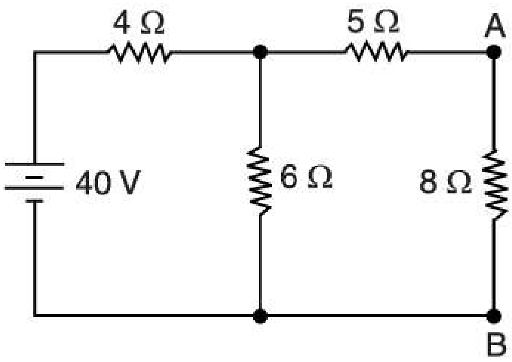


Fig Q21

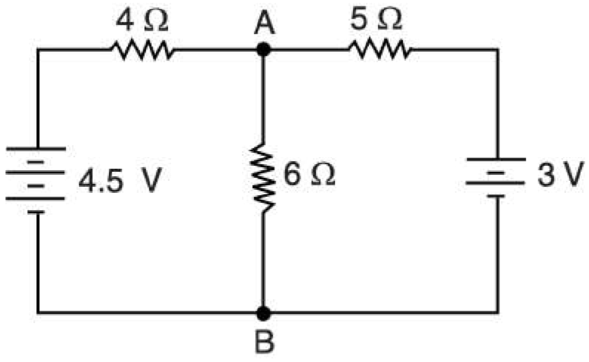


Fig. Q22

Q: Using Norton's theorem, find the current in the branch AB containing 6 Ω resistor of the network shown in Fig. Q22.

Q: Find Norton equivalent circuit to the left of terminals a – b in Fig. Q23.

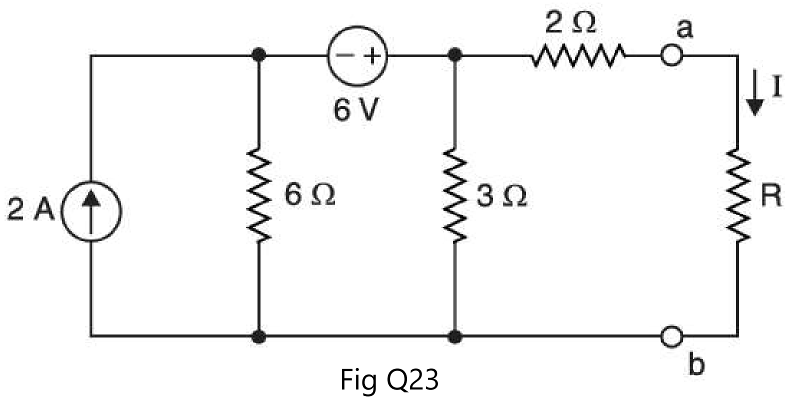


Fig Q23

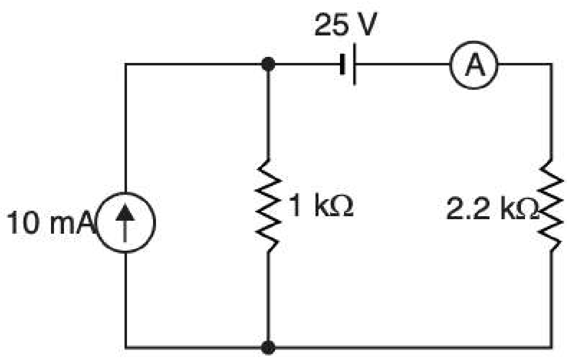


Fig Q24

Q: The ammeter labelled A in Fig. Q24 reads 35 mA. Is the 2·2 kΩ resistor shorted ? Assume that ammeter has zero resistance.

Q: Determine the Norton equivalent circuit and the load current in RL in Fig. Q25. The various circuit values are:

$E = 64$ V, $R_1 = 230$ Ω, $R_2 = 450$ Ω, $R_3 = 260$ Ω, $R_4 = 550$ Ω, $R_5 = 440$ Ω, $R_L = 360$ Ω.

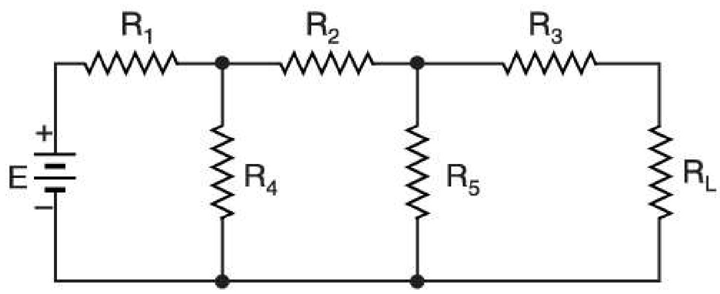


Fig. Q25

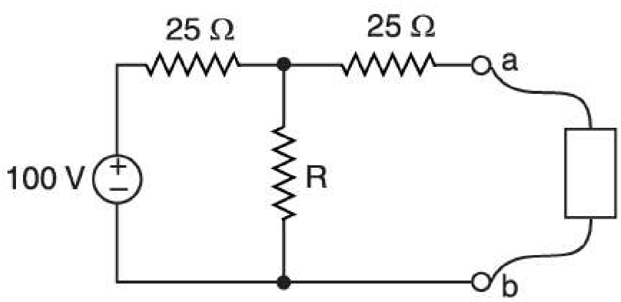


Fig. Q26

Q: In Fig. Q26, replace the network to the left of terminals ab with its Norton equivalent.

Q: Find the value of R_L in Fig. Q27, so that to obtain maximum power in R_L. Also find the maximum power in R_L.

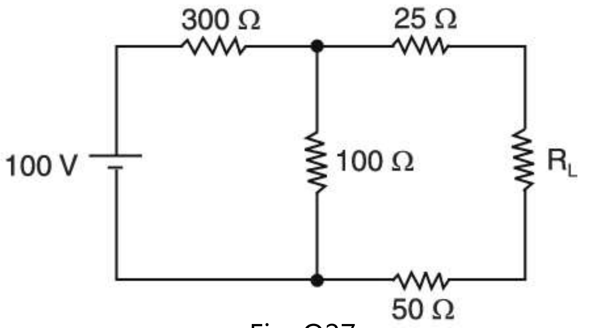


Fig. Q27

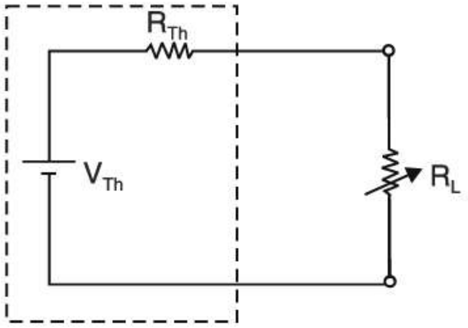


Fig. Q28

Q: What percentage of the maximum possible power is delivered to R_L in Fig. Q28, when $R_L = R_{Th}/2$?

Q: Determine the value of R_L for maximum power transfer in Fig. Q29 and evaluate this power.

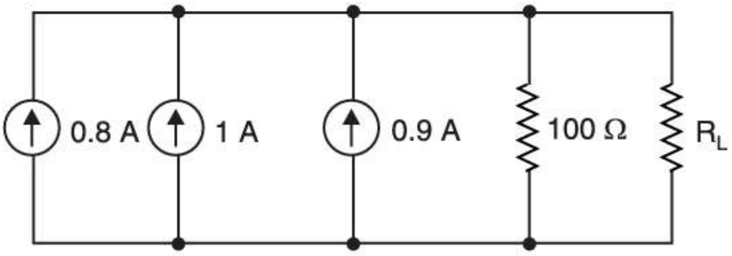


Fig. Q29

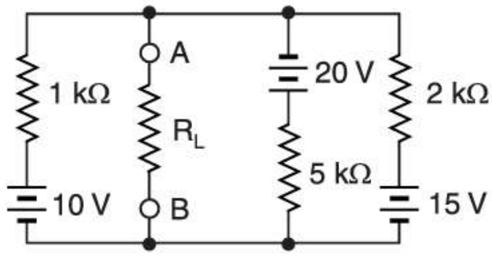


Fig. Q30

Q: What value should R_L be in Fig. Q30 to achieve maximum power transfer to the load?

Q: For the circuit shown in Fig. Q31, find the value of RL for which power transferred is maximum. Also calculate this power.

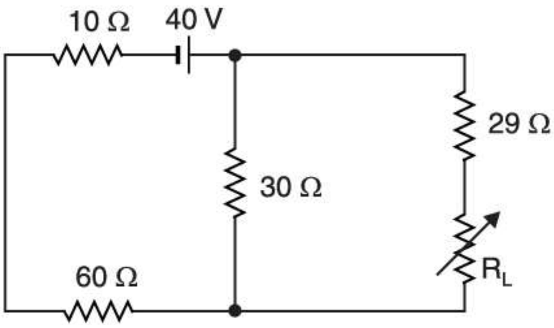


Fig. Q31

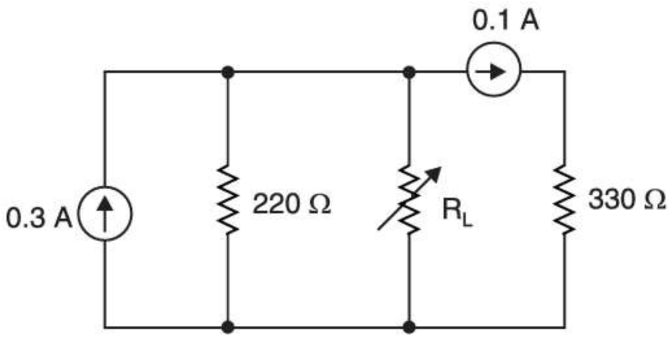


Fig. Q32

Q: Calculate the value of R_L for transference of maximum power in Fig. Q32. Evaluate this power.

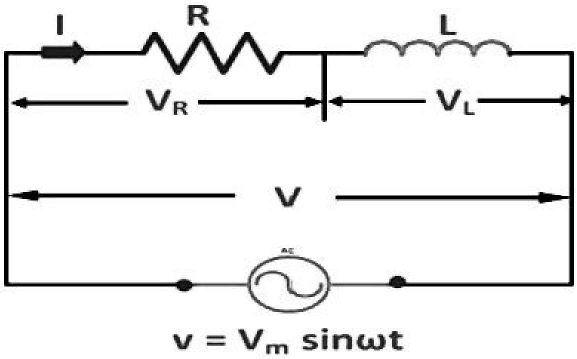
Q: State and prove Maximum Power Transfer Theorem.

CHAPTER 5. AC CIRCUIT AND RESONANCE:

5.1. A.C. through R-L, R-C, R-L-C Circuit:

R-L Series Circuit:

A circuit that contains pure resistance R ohms connected in series with a coil having pure inductance of L (Henry) is known as R- L Series Circuit.



When an AC supply voltage V is applied the current, I flow in the circuit.

I_R and I_L will be the current flowing in the resistor and inductor respectively

Where,

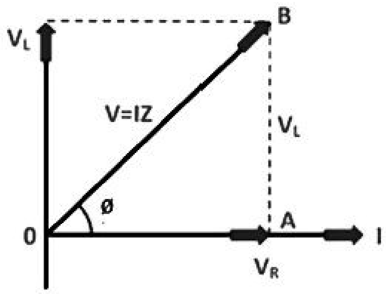
Voltage across the resistor R is $V_R = IR$

Voltage across the inductor L is $V_L = IX_L$

Total voltage of the circuit is V

Phasor Diagram of the R-L Series Circuit:

The phasor diagram of the R-L Series circuit is shown below



Steps to draw the Phasor Diagram of R-L Series Circuit:

The following steps are given below which are followed to draw the phasor diagram step by step.

- Current ***'I'*** is taken as a reference.
- The Voltage drop across the resistance $V_R = I_R$ is drawn in phase with the current I.

- The voltage drop across the inductive reactance $V_L = IX_L$ is drawn ahead of the current I. As the current lags voltage by an angle of 90 degrees in the pure Inductive circuit.
- The vector sum of the two voltages drops V_R and V_L is equal to the applied voltage V.
  Now,

  In right angle triangle OAB

  $V_R = I_R$ and $V_L = IX_L$ where $X_L = 2\pi fL$

  $$V = \sqrt{(V_R)^2 + (V_L)^2} = \sqrt{(IR)^2 + (IX_L)^2}$$

  $$V = I\sqrt{(R)^2 + (X_L)^2}$$

  $$\frac{V}{I} = \sqrt{(R)^2 + (X_L)^2}$$

  $$Z = \sqrt{(R)^2 + (X_L)^2}$$

  Where, Z is the total opposition offered to the flow of alternating current by an R-L Series circuit and is called impedance of the circuit. It is measured in ohms (Ω).

Phase Angle:

In R-L Series Circuit the current lags the voltage by Ø degree angle known as phase angle. It is given by the equation

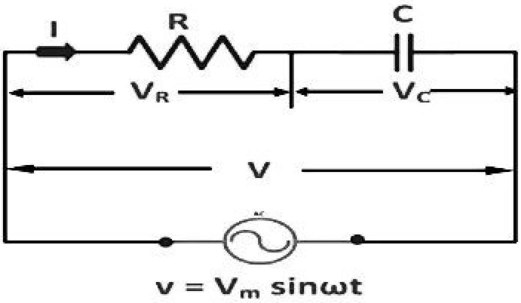
$$\tan\emptyset = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R} \quad or \quad \emptyset = \tan^{-1}\frac{X_L}{R}$$

R-C Series Circuit:

A circuit that contains pure resistance R ohms connected in series with a pure capacitor of capacitance C farads is known as R-C Series Circuit.

A sinusoidal voltage is applied to and current I flows through the resistance (R) and the capacitance (C) of the circuit.

The R-C Series circuit is shown in the figure below



Where,

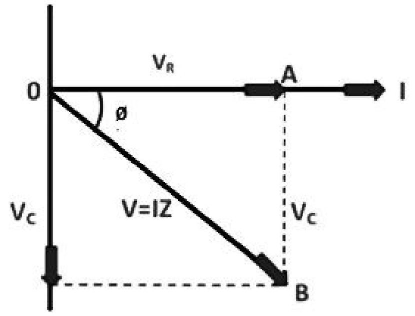
Voltage across the resistance R is $V_R = IR$

Voltage across the capacitor C is $V_C = IX_C$

Total voltage across the R-C Series circuit is V

Phasor Diagram of R-C Series Circuit:

The phasor diagram of the R-C Series circuit is shown below



Steps to draw a Phasor Diagram:

The following steps are used to draw the phasor diagram of RC Series circuit

Take the current I (r.m.s value) as a reference vector

Voltage drop in resistance V_R = IR is taken in phase with the current vector

Voltage drop in capacitive reactance $V_C = IX_C$ is drawn 90 degrees behind the current vector, as current leads voltage by 90 degrees in pure capacitive circuit.

The vector sum of the two voltage drops is equal to the applied voltage V (r.m.s value).

Now, V_R = IR and $V_C = IX_C$ Where, X_C = I/2πfC

In right triangle OAB

V_R = IR and $V_C = IX_C$ where $X_C = \frac{1}{2\pi fC}$

$$V = \sqrt{(V_R)^2 + (V_C)^2} = \sqrt{(IR)^2 + (IX_C)^2}$$

$$V = I\sqrt{(R)^2 + (X_C)^2}$$

$$\frac{V}{I} = \sqrt{(R)^2 + (X_C)^2}$$

$$Z = \sqrt{(R)^2 + (X_C)^2}$$

Where, Z is the total opposition offered to the flow of alternating current by an R-C Series circuit and is called impedance of the circuit. It is measured in ohms (Ω).

Phase Angle:

In R-C Series Circuit the current lags the voltage by Ø degree angle known as phase angle. It is given by the equation

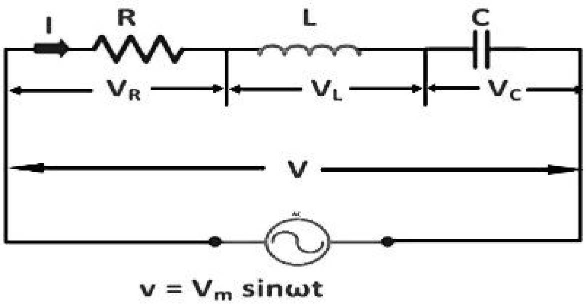
$$\tan\phi = \frac{V_C}{V_R} = \frac{IX_C}{IR} = \frac{X_C}{R} \quad or \quad \phi = \tan^{-1}\frac{X_C}{R}$$

R-L-C Series Circuit:

The RLC Series Circuit is defined as when a pure resistance of R ohms, a pure inductance of L Henry and a pure capacitance of C farads are connected together in series combination with each other.

As all the three elements are connected in series so, the current flowing in each element of the circuit will be same as the total current 'I' flowing in the circuit.

The **R-L-C Circuit** is shown below



In the R-L-C Series Circuit

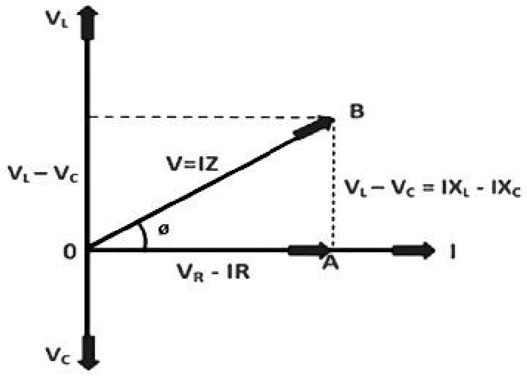
$$X_L = 2\pi f L \quad and \; X_C = \frac{1}{2\pi f C}$$

When the AC voltage is applied through the RLC Series Circuit the resulting current 'I' flows through the circuit, and thus the voltage across each element will be

- V_R = IR that is the voltage across the resistance R and is in phase with the current I.
- V_L = IX_L that is the voltage across the inductance L and it leads the current I by an angle of 90 degrees.
- V_C = IX_C that is the voltage across the capacitor C and it lags the current I by an angle of 90 degrees.

Phasor Diagram of RLC Series Circuit:

The phasor diagram of the RLC Series Circuit when the circuit is acting as an inductive circuit that means (V_L>V_C) is shown below and if (V_L< V_C) the circuit will behave as a capacitive circuit.



Steps to draw the Phasor Diagram of the RLC Series Circuit:

- Take current I as the reference as shown in the figure above
- The voltage across the inductor L that is V_L is drawn leads the current I by a 90 degree angle.
- The voltage across the capacitor c that is V_c is drawn lagging the current I by a 90 degree angle because in capacitive load the current leads the voltage by an angle of 90 degrees.
- The two vectors V_L and V_C are opposite to each other.

$$V = \sqrt{(V_R)^2 + (V_L - V_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$V = I\sqrt{(R)^2 + (X_L - X_C)^2}$$

$$\frac{V}{I} = \sqrt{(R)^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{(R)^2 + (X_L - X_C)^2}$$

It is the total opposition offered to the flow of current by an R-L-C Circuit and is known as Impedance of the circuit.

Phase Angle:

From the phasor diagram, the value of phase angle will be

$$\tan \emptyset = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R} \ or$$

$$\emptyset = \tan^{-1}\frac{X_L - X_C}{R}$$

The Three Cases of R-L-C Series Circuit:

- When $X_L > X_C$, the phase angle ϕ is positive. The circuit behaves as a RL series circuit in which the current lags behind the applied voltage and the power factor is lagging.

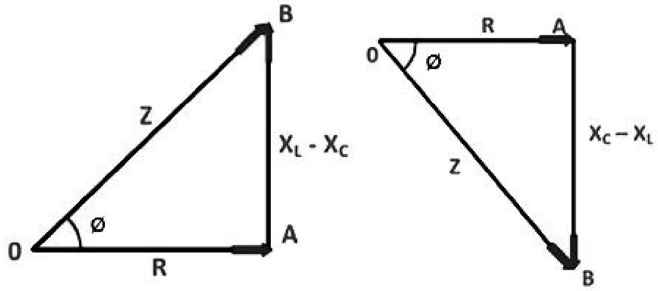
- When $X_L < X_C$, the phase angle ϕ is negative, and the circuit acts as a series RC circuit in which the current leads the voltage by 90 degrees.

- When $X_L = X_C$, the phase angle ϕ is zero, as a result, the circuit behaves like a purely resistive circuit. In this type of circuit, the current and voltage are in phase with each other. The value of power factor is unity.

Impedance Triangle of R-L-C Series Circuit:

When the quantities of the phasor diagram are divided by the common factor I then the right angle triangle is obtained known as impedance triangle.

The impedance triangle of the RLC series circuit, when ($X_L > X_C$) and ($X_C > X_L$) is shown below



If the inductive reactance is greater than the capacitive reactance than the circuit reactance is inductive giving a lagging phase angle.

When the capacitive reactance is greater than the inductive reactance the overall circuit reactance acts as a capacitive and the phase angle will be leading.

From the triangle OAB

$$Z = \sqrt{(R)^2 + (X_L - X_C)^2}$$

$$\cos\emptyset = \frac{R}{Z} = or$$

$$\emptyset = \cos^{-1}\frac{R}{Z}$$

Applications of RLC Series Circuit:

The following are the application of the RLC circuit

- It acts as a variable tuned circuit
- It acts as a low pass, high pass, band pass, band stop filters depending upon the type of frequency.
- The Circuit also works as an oscillator
- Voltage multiplier and pulse discharge circuit

5.2. Solution of problems of AC through R-L, R-C, R-L-C series circuit by complex algebra method:

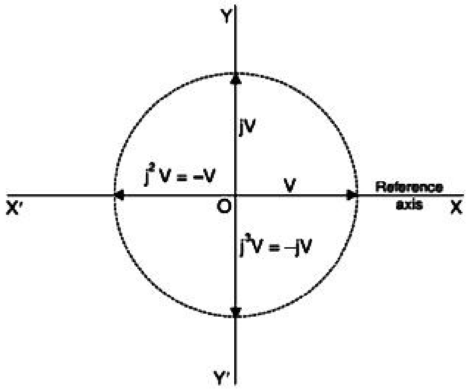
Mathematical representation of phasors, significant of operator "j":

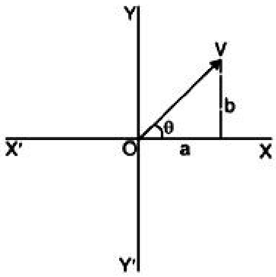
Complex number consists of a "real number" and an "imaginary number" this imaginary number is represented letter " j " known commonly in electrical engineering as the **j-operator**, is used. Thus the letter "j" is placed in front of a real number to signify its imaginary number operation.

Examples of imaginary numbers are: j3, j12, j100 etc.

Then a **complex number** consists of two distinct but very much related parts, a "Real Number" plus an "Imaginary Number".

What is 'J' Operator?





- **j is an operator** which rotates the phasor through 90° in Counter clock wise direction without changing the magnitude of the phasor.
- The operator j occurs with the phasor only when it is along Y-axis Thus when the phasor is along OY-axis, it is jV and when along OY'-axis, it is –jV. The phasor lying along X-axis is not associated with j.
- Since $j = \sqrt{-1}$ and its value cannot be determined, it is called an imaginary number. For this reason, any phasor (or its component) associated with *j* is called the imaginary part.
- *A* phasor (or its component) along *X*-axis is not associated with *j* and is called the real part.

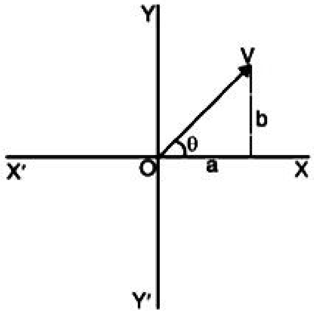
Mathematical Representation of Phasors:

There are four ways of representing a phasor in the mathematical form *viz.* (*i*) Rectangular form (*ii*) Trigonometrically form (*iii*) Polar form and (*iv*) Exponential form.

Rectangular form:

- This method is also known as **symbolic notation.**
- In this method, the phasor is resolved into horizontal and vertical components and is expressed in the complex form as Consider a voltage phasor **V** displaced 90° *CCW (Counter Clock Wise)* from the reference axis (*i.e.* *OX*-axis) as shown in Fig.

- Where "a" is the horizontal or in-phase component of this phasor.
- And "b" is the vertical or quadrature component.
- Therefore, the phasor can be represented in the rectangular form as : **V** = *a* + *j b*



Magnitude of phasor, $V = \sqrt{a^2 + b^2}$

Its angle *w.r.t.* *OX*-axis, $\theta = \tan^{-1}\frac{b}{a}$

Polar form:

It is a usual practice to write the trigonometrical form **V** = *V* (cos θ + *j* sin θ) in what is called polar form as: **V** = *V* ∠ θ

Where *V* is the magnitude of the phasor and θ is its phase angle measured *CCW* from the reference axis *i.e.* *OX*-axis.

A negative angle in the polar form indicates clockwise measurement of the angle from the reference axis.

Hence polar form can be written in general as: **V** = *V* ∠ ± θ

Conversion from One Form to the Other

Rectangular to Polar:

Example: 1 **Find the Polar Form of 3+j4**

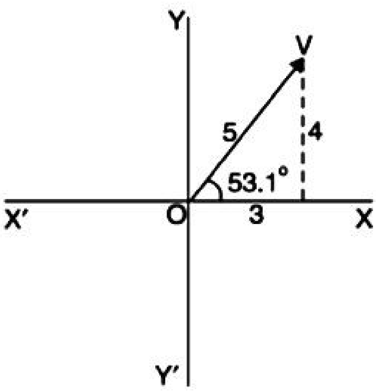
Solution: Where a=3 and b=4

Magnitude of phasor, $V = \sqrt{a^2 + b^2}$

$V = \sqrt{3^2 + 4^2} = 5$

Its angle *w.r.t.* *OX*-axis,

$\theta = \tan^{-1}\frac{b}{a}$



$$\theta = \tan^{-1}\frac{4}{3} = 53.13^0$$

In polar form, $V = 5\angle 53.13^0$

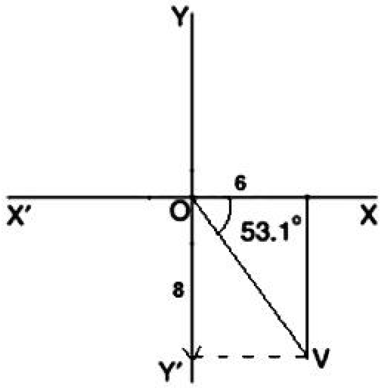
Example: 2 Express the following in polar form (i) 6 – j 8. (ii) –2 + j 5 (iii) –50 – j 75 and (iv) 3 + j 7

Solution: Where a=6 and b= -8

Magnitude of phasor, $V = \sqrt{a^2 + b^2}$

$V = \sqrt{6^2 + (-8)^2} = 10$

Its angle *w.r.t. OX*-axis,



$$\theta = \tan^{-1}\frac{b}{a}$$

$$\theta = \tan^{-1}\frac{(-8)}{6} = -53.13^0$$

In polar form, $V = 10\angle -53.13^0$

Polar to Rectangular:

Example: 1 Find the Rectangular Form of 50 $\angle 36.87^0$

Solution: Magnitude of voltage, $V = 50$

Phase angle, θ = 36.87°

V = V (cos θ + j sin θ)

V= 50 (cos 36.87° + j sin 36.87°)

V=50 (0.8 + j 0.6)

V = (40 + j 30)

The rectangular form is 40 + j 30\

ADDITION, SUBTRACTION, MULTIPLICATION AND DIVISION OF PHASOR QUANTITIES:

Addition and subtraction of Phasor Quantities:

- The rectangular form is best suited for addition and subtraction of phasors.
- If the phasors are given in polar form, they should be first converted into rectangular form.
- Once the phasors are in rectangular form, the real components and the imaginary components can be algebraically added or subtracted.
- The answer (sum or difference) can be left in rectangular form or it can be converted back to polar form if desired.

Addition:

For the addition of phasors in the rectangular form, the Real components are added together and the quadrature or imaginary components are added together.

Let's consider two voltage phasors represented as:

V1 = a1 + j b1

V2 = a2 + j b2

Then resultant, **V = V1 + V2** = (a1 + j b1) + (a2 + j b2)

= (a1 + a2) + j (b1 + b2)

Subtraction:

For the subtraction of phasors in the rectangular form, the Real components are subtracted and the quadrature or imaginary components are subtracted separately.

Let's consider two voltage phasors represented as:

V1 = a1 + j b1

V2 = a2 + j b2

Then resultant, **V = V1 – V2** = (a1 – j b1) + (a2 – j b2)

= (a1 – a2) + j (b1 – b2)

Example: 1

Two vectors are defined as, *A = 4 + j1* and *B = 2 + j3* respectively. Determine the sum and difference of the two vectors in both rectangular (*a +jb*) form.

Solution: Addition: A=4 + j1

B=2+j3

The resultant vector = A+B

= 4 + j1+2+j3

= (4+2) +j (1+3)

= 6+j4

Subtraction: A=4 + j1

B=2+j3

The resultant vector = A–B

= (4 + j1)–(2+j3)

= (4–2) +j (1–3)

= 2+j(–2) or 2–j2

Example: 2 **Determine the resultant voltage of two sinusoidal generators in series whose voltages are V1 = 25⌊15° V and V2 = 15⌊60° V. and show the result in polar form.**

Solution: First convert polar form to rectangular form and then find out the addition.

V1 = 25(cos 15° + j sin 15°) = (24.15 + j 6.47) volts

V2 = 15(cos 60° + j sin 60°) = (7.5 + j 12.99) volts

∴ **V = V1 + V2** = (24.15 + j 6.47) + (7.5 + j 12.99)

= (31.65 + j 19.46) volts

Where a= 31.65 and b=19.46

Magnitude of phasor, $V = \sqrt{31.65^2 + 19.46^2} = 37.15$

$$\theta = \tan^{-1}\frac{b}{a}$$

$$\theta = \tan^{-1}\frac{(19.46)}{31.65} = 31.58^0$$

In Polar form, $V = 37.15\angle 31.58$

Multiplication and Division of Phasors

It is easier to multiply and divide the phasors when they are in polar form than in the rectangular form.

Consider two phasors given by:

$\mathbf{V_1} = a_1 + j\,b_1 = V_1 \angle \theta_1$

$\mathbf{V_2} = a_2 + j\,b_2 = V_2 \angle \theta_2$

Multiplication:

Rectangular form:

$$\mathbf{V_1} \times \mathbf{V_2} = (a_1 + j\,b_1)\,(a_2 + j\,b_2)$$

$$= a_1a_2 + j\,a_1b_2 + j\,a_2b_1 + j^2\,b_1b_2$$

$$= a_1a_2 + j\,a_1b_2 + j\,a_2b_1 - b_1b_2 \qquad (j^2 = -1)$$

$$= (a_1a_2 - b_1b_2) + j\,(a_1b_2 + a_2b_1)$$

Polar Form:

To multiply the phasors that are in polar form, just multiply their magnitudes and algebraically add the phase angles.

If two vectors are in polar form:

Let $V_1 \angle \theta_1$, $V_2 \angle \theta_2$

The resultant is given by $V = (V_1 \angle \theta_1) \times (V_2 \angle \theta_2)$

$V = (V_1 \times V_2) \angle (\theta_1 + \theta_2)$

Division:

Rectangular form:

$$\frac{V_1}{V_2} = \frac{a_1 + jb_1}{a_2 + jb_2}$$

$$= \frac{(a_1 + jb_1)(a_2 - jb_2)}{(a_2 + jb_2)(a_2 - jb_2)}$$

$$= \frac{(a_1 + jb_1)(a_2 - jb_2)}{(a_2 + jb_2)(a_2 - jb_2)}$$

$$= \frac{(a_1 + jb_1)(a_2 - jb_2)}{{a_2}^2 + {b_2}^2}$$

$$= \frac{(a_1a_2 + b_1b_2)}{{a_2}^2 + {b_2}^2} + j\frac{(b_1a_2 - a_1b_2)}{{a_2}^2 + {b_2}^2}$$

Polar Form:

To divide the phasors that are in polar form, divide the magnitudes of phasors and subtract the denominator angle from the numerator angle.

If two vectors are in polar form:

Let $V_1\angle\theta_1$, $V_2\angle\theta_2$

The resultant is given by

$$\frac{V_1}{V_2} = \frac{V_1\angle\theta_1}{V_2\angle\theta_2}$$

$$\frac{V_1}{V_2} = \frac{V_1}{V_2}\angle\theta_1 - \theta_2$$

Example: 3 Two phasors are given in the following form: **A** = (4 + j 3); **B** = (5 + j 6)

Evaluate **A × B** and **A/B** in (i) rectangular form (ii) polar form.

Solution: Given **A** = (4 + j 3); **B** = (5 + j 6)

Rectangular form:

Multiplication:

A × B = (4 + j 3) × (5 + j 6)

= (4×5) + (4× j6) + (j3× 5) + (j^2 3×6)

= 20+j24+j15–18 (j^2 = –1)

= 2+j39

Division:

$$\frac{A}{B} = \frac{4 + j\,3}{5 + j\,6}$$

$$= \frac{(4 + j\,3)(5 - j\,6)}{(5 + j\,6)(5 - j\,6)}$$

$$= \frac{(4 + j\,3)(5 - j\,6)}{5^2 + 6^2}$$

$$= \frac{20 - j24 + j15 - j^2 18}{5^2 - j^2 6^2} = \frac{38 - j9}{61} = \frac{38}{61} - \frac{j9}{61}$$

$$= 0.623 + j\,0.147$$

Polar Form:

Multiplication:

First Convert (4 + *j* 3) and (5 + *j* 6) in to polar form.

A= (4 + *j* 3) = $5\angle 36.86^0$

B= (5 + *j* 6) = *7.81* ∠50.19^0

The resultant is given by = (5∠ 36.86^0) × (7.81∠ 50.19^0)

= (5× 7.81) ∠ (36.86^0+50.19^0)

= 39.05 ∠ 87.05

Division:

$$\frac{A}{B} = \frac{5 \angle 36.86}{7.81 \angle 50.19}$$
$$\frac{A}{B} = \frac{5}{7.81} \angle 36.86 - 50.19 = 0.64 \angle -13.33$$

Exercise:

(1) Perform the following operations and express the final result in polar form:

(i) (8 + j 6) × (–10 – j 7.5) (ii) 5 ∠ 30° + 8 ∠ – 30°

(2) The following three phasors are given:

A = 5 + j 5; B = 50 ∠ 40°; C = 4 + j 0

Perform the following indicated operations: $(i)\,\frac{AB}{C}, (ii)\,\frac{BC}{A}$

(3) Divide (6 + j 7) by (5 + j 3) and express the result in (i) rectangular form (ii) polar form.

PROBLEMS:

Q.1. The current in a circuit is given by (4.5 + j12) A when the applied voltage is (100 + j150) V. Determine (i) the magnitude of impedance and (ii) phase angle

Sol:

$$V = (100 + j50)V = 180.28\angle 56.31^\circ V$$

$$I = (4.5 + j12)A = 12.82\angle 69.44^\circ A$$

(i) $$\therefore\ Z = \frac{V}{I} = \frac{180.28\angle 56.31^\circ}{12.82\angle 69.44^\circ} = 14.06\angle -13.13^\circ \Omega$$

$$\therefore\ Z = 14.06\Omega$$

(ii) $\therefore$ Phase angle, $\Phi = 13.13^\circ\,lead$

Q.2. In an R-L series circuit, $R = 10\Omega$ and $X_L = 8.66\Omega$. If current in the circuit is $(5 - j10)A$, find (i) the applied voltage (ii) power factor and (iii) active power and reactive power.

Sol:

$$Z = (R + jX_L) = (10 + j8.66)\Omega$$

$$= 13.23\angle 40.9^\circ \Omega$$

$$I = (5 - j10)A = 11.18\angle -63.43^\circ A$$

(i) Applied voltage, $V = IZ$

$$= 11.18\angle -63.43^\circ \times 13.23\angle 40.9^\circ$$

$$= 148\angle -22.53^\circ V$$

$$V = 148 volts$$

(ii) Phase angle, $\Phi = 63.43^\circ - 22.53^\circ = 40.9^\circ$

Power factor = $\cos\Phi = \cos 40.9^\circ = 0756 lag$

(iii) Complex **VA, S**=Phase voltage× Conjugate of phasor current

Or, $P + jQ = 148\angle -22.53^\circ \times 11.18\angle 63.43^\circ = 1654.64\angle 40.9^\circ VA$

$$= 1654.64(\cos 40.9^\circ + j\sin 40.9^\circ) = (1250.66 + j1083.36)VA$$

∴ Active power $P = 1250.66W$; Reactive power, $Q = 1083.36VAR$

Q.3. A coil of resistance $R = 12\Omega$ and inductive reactance $(X_L) = 25\Omega$ is connected in series with a capacitive reactance of $X_C = 41\Omega$. The combination is connected to a supply of 230V, 50Hz. Using phasor algebra, find (i) circuit impedance (ii) current and (iii) power consumed.

Sol:

(i) $Z = R + j(X_L - X_C)$

$$= 12 + j(25 - 41) = (12 - j16)\Omega = 20\angle -53.13^\circ \Omega$$

∴ $Z = 20\Omega$

(ii) Taking voltage as the reference phasor,

$V = (230 + j0)V = 230\angle 0^\circ volts$

∴ $$I = \frac{V}{Z} = \frac{230\angle 0^\circ}{20\angle -53.13^\circ} = 11.5\angle 53.13^\circ A$$

∴ $I = 11.5A$

(iii) It is clear that current leads the voltage by 53.13°i.e. $\Phi = 53.13^\circ$

$\therefore$ Power factor $= \cos\Phi = \cos 53.13^\circ = 0.6 lead$

Power consumed, $P = VI\cos\Phi = 230 \times 11.5 \times 0.6 = 1587W$

Q.4. The potential difference measured across a coil is 4.5V, when it carries a direct current of 9A. The same coil when carries an alternating current of 9A at 25Hz, the potential difference is 24V. Find the power and power factor when it is supplied by 50V, 50Hz supply.

Sol:

Let ***R*** be the D.C. resistance and L be inductance of the coil.

$$R = \frac{4.5}{9} = 0.5\Omega$$

With a.c. current of 25Hz, $Z = \frac{V}{I} = \frac{24}{9} = 2.66\Omega$

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{2.66^2 - 0.5^2} = 2.62\Omega$$

$$X_L = 2\pi f L = 2 \times \pi \times 25 \times L$$

Or, $L = \frac{X_L}{2\pi f} = \frac{2.62}{2 \times \pi \times 25} = 0.0167H$

At 50Hz, $X_L = 2.62 \times 2 = 5.24\Omega$

$$Z = \sqrt{0.5^2 + 5.24^2} = 5.26\Omega$$

$$I = \frac{50}{5.26} = 9.5A$$

$$P = I^2 R = 9.5^2 \times 0.5 = 45 watt.$$

Again, power factor $\cos\Phi = \frac{R}{Z} = \frac{0.5}{5.26} = 0.095 lagging.$

5.3. Solution of problems of AC through R-L, R-C, R-L-C parallel and composite circuits:

PARALLEL CIRCUITS:

In the **Parallel Circuits**, a number of branches are connected in parallel.

Each branch contains a number of components like resistance, inductance and capacitance forming a series circuit.

Each branch of the circuit is analyzed separately as a series circuit and after that, the effects of each branch are combined together.

For circuit calculations, the magnitude and phase angle of current and voltage is taken into consideration.

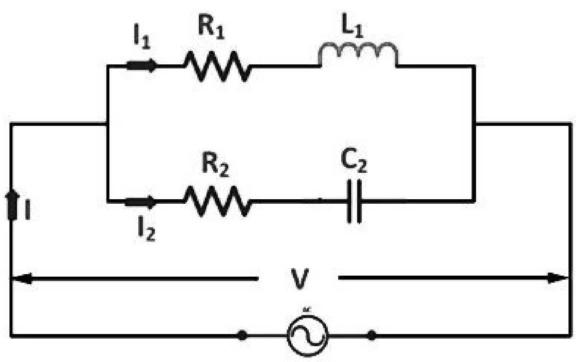
The magnitudes and phase angle voltages and currents are taken into consideration while solving the circuit.

There are mainly three methods of solving the parallel AC circuits. They are as follows

- Phasor Method or Vector Method
- Admittance Method
- Method of Phasor algebra or Symbolic method or J method

➢ **PHASOR METHOD OR VECTOR METHOD**
Steps to Solve Parallel Circuits by Phasor Method

Consider the circuit diagram below to solve the circuit step by step.



☞ **Step 1**

- Draw the circuit diagram as per the given problem. As we are considering the above circuit as an example.
- In this case two branches connected in parallel are taken into consideration.
- One branch contains resistance and inductance in series. The second branch consists of resistance and capacitance in series. The supply voltage is V volts.

☞ **Step 2**

- Find the **impedance** of each branch of the circuit separately, i.e.

$$Z_1 = \sqrt{R_1^2 + X_{L1}^2}$$

Where, $X_{L1} = 2\pi fL_1$

$$Z_2 = \sqrt{R_2^2 + X_{C2}^2}$$

$$where \quad X_{C2} = \frac{1}{2\pi f C_2}$$

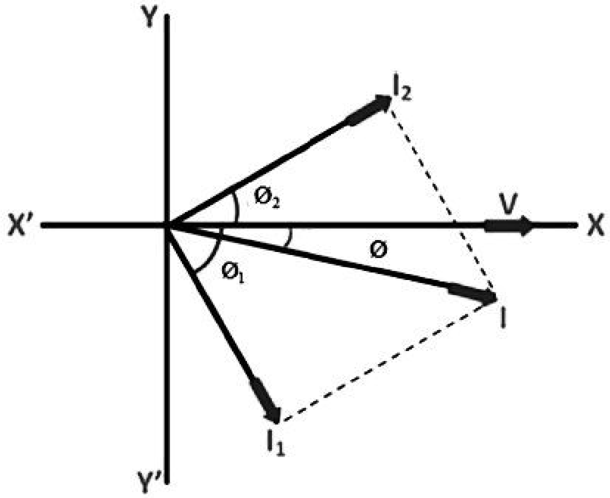
- **Step 3**
  - Determine the magnitude of **current** and **phase angle** with the voltage in each branch.

$$I_1 = \frac{V}{Z_1}, \ \emptyset_1 = \tan^{-1}\frac{X_{L1}}{R_1}$$

$$I_2 = \frac{V}{Z_2}, \ \emptyset_2 = \tan^{-1}\frac{X_{C2}}{R_2}$$

Here, Due to inductive load ϕ_1 is lagging and due to capacitive load ϕ_2 leading.

- **Step 4**
  - Draw the **phasor diagram** taking voltage as the reference. Represent the various branches current on it as shown in the phasor diagram below.



- **Step 5**
  - Now, find the **phasor sum** of the branch currents by the methods of components.

$$I_{XX} = I_1 \cos\emptyset_1 + I_2 \cos\emptyset_2$$

$$I_{YY} = -I_1 \sin\emptyset_1 + I_2 \sin\emptyset_2$$

And therefore, current I will be

$$I = \sqrt{(I_{XX})^2 + (I_{YY})^2}$$

- **Step 6**
  - Find the phase angle ϕ between the total current I and the circuit voltage V.

$$\phi = \tan^{-1}\frac{I_{YY}}{I_{XX}}$$

Here angle ϕ will be lagging as I_{yy} is negative

Power factor of the circuit will be Cos ϕ or

$$\cos\phi = \frac{I_{XX}}{I}\ lagging$$

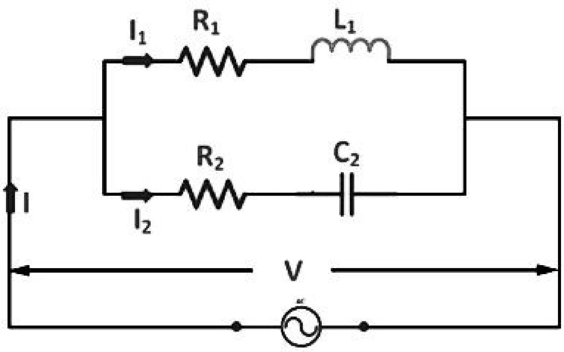
ADMITTANCE METHOD:

- The reciprocal of the impedance of an AC circuit is known as **Admittance** of the circuit. Since impedance is the total opposition offered to the flow of alternating current in an AC circuit.
- Therefore, Admittance is defined as the effective ability of the circuit due to which it allows the alternating current to flow through it.
- It is represented by (Y). The old unit of admittance is mho (℧). Its new unit is Siemens.

$$Y = \frac{1}{Z}$$

Steps for Solving Circuit by Admittance Method

Consider a parallel AC circuit having resistance and capacitance connected in series and resistance and inductance also connected in series as shown in the figure below.



- **Step 1**
  - Draw the circuit as per the given problem.

- **Step 2**

  - Find impedance and phase angle of each branch.

$$Z_1 = \sqrt{R_1^2 + X_{L1}^2},\quad \phi_1 = \tan^{-1}\frac{X_{L1}}{R_1}$$

Where, $X_{L1} = 2\pi f L_1$

$$Z_2 = \sqrt{R_2^2 + X_{C2}^2}, \quad \emptyset_2 = \tan^{-1}\frac{X_{C2}}{R_2}$$

$$where \quad X_{C2} = \frac{1}{2\pi f C_2}$$

Step 3

- Now, find Conductance, Susceptance and Admittance of each branch.

$$g_1 = \frac{R_1}{Z_1^2} \; ; \; b_1 = \frac{X_{L1}}{Z_1^2} \; (negative) \; ; \; Y_1 = \sqrt{g_1^2 + b_1^2}$$

$$g_2 = \frac{R_2}{Z_2^2} \; ; \; b_2 = \frac{X_{C1}}{Z_2^2} \; (Positive) \; ; \; Y_2 = \sqrt{g_2^2 + b_2^2}$$

Step 4

- Find the algebraic sum of conductance and susceptance.
  $G = g_1 + g_2 \; ; \; B = -b_1 + b_2$

Step 5

- Find the total Admittance (Y) of the circuit.
  $Y = \sqrt{G^2 + B^2}$

Step 6

- Find the various branch currents of the circuit.

  $I_1 = VY_1 \; and \; I_2 = VY_2$

Step 7

- Now, find the total current I of the circuit.

  $I = VY$

Step 8

- Find the phase angle of the whole circuit.
  $\emptyset = \tan^{-1}\frac{B}{G}$

  Phase angle will be lagging if B is negative.

Step 9

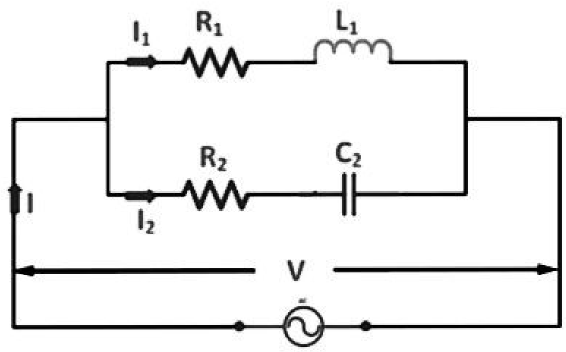
- Now, find the power factor of the circuit.

  $Power\ Factor = \cos\emptyset = \frac{G}{Y}$

METHOD OF PHASOR ALGEBRA OR SYMBOLIC METHOD OR J METHOD

Steps for Solving Circuit by Admittance Method

Consider a parallel AC circuit having resistance and capacitance connected in series and resistance and inductance also connected in series as shown in the figure below.



- **Step 1**
  - Draw the circuit as per the given problem.

- **Step 2**
  - Impedance of the branch 1 is given by $Z_1 = R_1 + jX_{L1}$
    Impedance of the branch 2 is given by $Z_2 = R_2 - jX_{C2}$

- **Step 3**
  - Find the equivalent impedance

$$Z_{equ} = \frac{Z_1 \times Z_2}{Z_1 + Z_2} = \frac{(R_1 + jX_{L1}) \times (R_2 - jX_{C2})}{R_1 + jX_{L1} + R_2 - jX_{C2}}$$

- **Step 4**
  - Find the Total Current 'I'

$$I = \frac{V}{Z_{equ}}$$

- **Step 5**
  - Find the branch Current

$$I_1 = \frac{I \times Z_2}{Z_1 + Z_2} \quad and \quad I_2 = \frac{I \times Z_1}{Z_1 + Z_2}$$

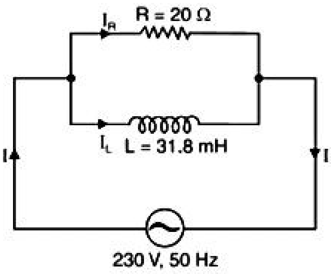
- **Step6**
  - Find the Power by $S = P + jQ \quad S = V\,I^*$
    Where I^* is the complex conjugate of I

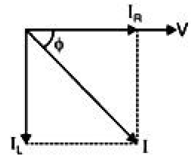
The real part P indicates the Active or real or True Power and Q indicates the Reactive power.

PROBLEMS:

Q.1. A resistance of 20 Ω and a coil of inductance 31·8 mH and negligible resistance are connected in parallel across 230 V, 50 Hz supply. Find (i) the line current (ii) power factor and power consumed by the circuit.

Sol:





$I_R = V / R = 230 / 20 = 11.5A$ in phase with V

$X_L = 2\pi fL = 2\pi \times 50 \times 31.8 \times 10^{-3} = 10\Omega$

$I_L = V / X_L = 230 / 10 = 23A$ lagging V by 90°

The line current I is the phasor sum of I_R and I_L

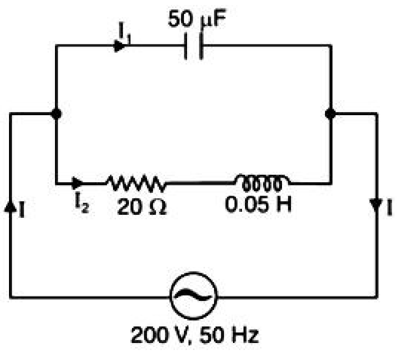
(i) Line current, $I = \sqrt{I_R^2 + I_L^2} = \sqrt{(11.5)^2 + (23)^2} = 25.71A$

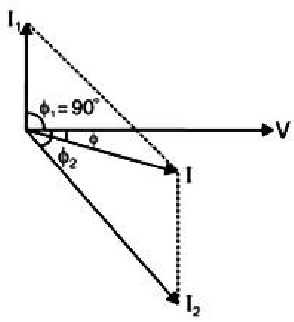
(ii) Power factor, $\cos\phi = I_R / I = 11.5 / 25.71 = 0.447 lag$

Power consumed, $P = VI\cos\phi = 230 \times 25.71 \times 0.447 = 2643 watts$

Q.2. A capacitor of 50μF is connected in parallel with a coil that has a resistance of 20Ω and inductance of 0.05H. If this parallel combination is connected across 200V, 50Hz supply, calculate (i) the line current (ii) power factor and (iii) power consumed.

Sol:





Branch – 1:

$$Z_1 = X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 50 \times 10^{-6}} = 63.7\Omega$$

$$I_1 = V / X_C = 200 / 63.7 = 3.14A$$

The current I_1 leads the applied voltage by $\phi_1 = 90^\circ$ as shown in figure above.

Branch – 2:

$$X_L = 2\pi fL = 2\pi \times 50 \times 0.05 = 15.7\Omega$$

$$Z_2 = \sqrt{R^2 + X_L^2} = \sqrt{(20)^2 + (15.7)^2} = 25.43\Omega$$

$$I_2 = V / Z_2 = 200 / 25.43 = 7.86A$$

Phase angle, $\phi_2 = \tan^{-1}\left(X_L / R \right) = \tan^{-1}(15.7 / 20) = 38.13^\circ$

The current I_2 lags behind the applied voltage by $\phi_2 = 38.13^\circ$

The line current I is the phasor sum of I_1 and I_2.

Resolving the currents into rectangular components, we have,

$$I\cos\phi = I_1\cos\phi_1 + I_2\cos\phi_2$$
$$= 3.14\cos 90^\circ + 7.86\cos 38.13^\circ = 0 + 6.18 = 6.18A$$

$$I\sin\phi = I_1\sin\phi_1 - I_2\sin\phi_2$$
$$= 3.14\sin 90^\circ - 7.86\sin 38.13^\circ = 3.14 - 4.85 = -1.71A$$

(i) ∴ Line current, $I = \sqrt{(I\cos\phi)^2 + (I\sin\phi)^2} = \sqrt{(6.18)^2 + (-1.71)^2} = 6.41A$

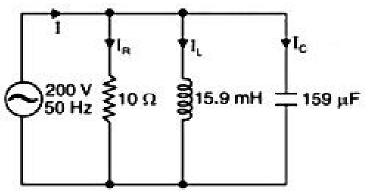
Phase angle, $\phi = \tan^{-1}\left(\frac{-1.71}{6.18}\right) = -15.47^\circ$

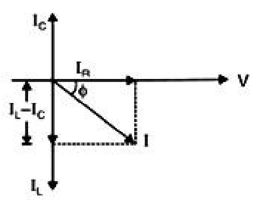
(ii) Power factor $= \cos\phi = \cos(-15.47^\circ) = 0.964 lag$

(iii) Power consumed, $P = VI\cos\phi = 200 \times 6.41 \times 0.964 = 1235.85W$

Q.3. 10 Ω resistor, a 15.9 mH inductor and 159 μF capacitor are connected in parallel to a 200 V, 50 Hz source. Calculate the supply current and power factor.

Sol:





$$X_L = 2\pi fL = 2\pi \times 50 \times 15.9 \times 10^{-3} = 5\Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 159 \times 10^{-6}} = 20\Omega$$

$I_R = V / R = 200/10 = 0A$ ..in phase with V

$I_L = V / X_L = 200/5 = 40A$ ..lags V by 90°

$I_C = V / X_C = 200/20 = 10A$ ..leads V by 90°

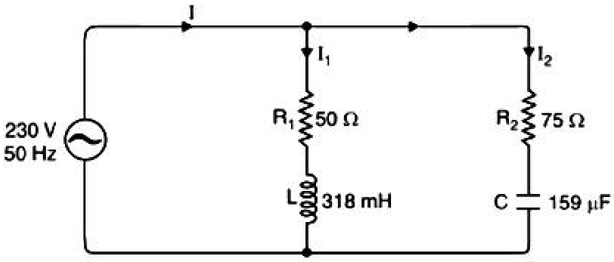
Figure above shows the phasor diagram of the circuit. Here I_L and I_C are 180° out of phase with each other. The supply current I is the phasor sum of I_R and $(I_L - I_C)$.

∴ Supply current, $I = \sqrt{I_R^2 + (I_L - I_C)^2} = \sqrt{20^2 + (40-10)^2} = 36A$

Circuit power factor $= \cos\phi = I_R/I = 20/36 = 0.56 lag$

Q.4. A coil of resistance 50 Ω and inductance 318 mH is connected in parallel with a circuit consisting of a 75 Ω resistor in series with a 159 μF capacitor. The circuit is connected to a 230 V, 50 Hz supply. Determine the supply current and circuit power factor.

Sol:



Now solve the problem by phasor algebra method, we have,

$$V = 230\angle 0^\circ V$$

$$X_L = 2\pi fL = 2\pi \times 50 \times 318 \times 10^{-3} = 100\Omega$$

$$Z_1 = R_1 + jX_L = (50 + j100)\Omega = 112\angle 63.5^\circ \Omega$$

$$I_1 = \frac{V}{Z_1} = \frac{230\angle 0^\circ}{112\angle 63.5^\circ} = 2.05\angle -63.5^\circ A = (0.91 - j1.83)A$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 159 \times 10^{-6}} = 20\Omega$$

$$Z_2 = R_2 - jX_C = (75 - j20)\Omega = 77.6\angle -15^\circ \Omega$$

$$I_2 = \frac{V}{Z_2} = \frac{230\angle 0^\circ}{77.6\angle -15^\circ} = 2.96\angle 15^\circ A = (2.86 + j0.766)A$$

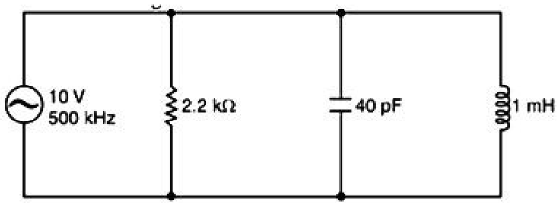
Supply current, $I = I_1 + I_2 = (0.91 - j1.83) + (2.86 + j0.766)$

$$= (3.77 - j1.06)A = 3.92\angle -15.7^\circ A$$

Power factor $= \cos\phi = \cos 15.7^\circ = 0.963 lag$

Q.5. Using admittance method, determine (i) circuit impedance and (ii) circuit current for the circuit shown in Fig. below.

Sol:



(i) $$G = \frac{1}{R} = \frac{1}{2.2 \times 10^3} = 0.455 \times 10^{-3} S = 0.455 mS$$

$$+jB = B_C = \frac{1}{X_C} = 2\pi fC = 2\pi \times 500 \times 10^3 \times 40 \times 10^{-12} = 0.126 mS$$

$$-jB = B_L = \frac{1}{X_L} = \frac{1}{2\pi fL} = \frac{1}{2\pi \times 500 \times 10^3 \times 1 \times 10^{-3}} = 0.318 mS$$

∴ Admittance of the circuit, $Y = G + jB - jB = 0.455 + j0.126 - j0.318 = (0.455 - j0.192) mS$

Now converting it to polar form, we have, $Y = 0.494\angle -22.88^\circ mS$

∴ Circuit impedance, $Z = \frac{1}{Y} = \frac{1}{0.494\angle -22.88^\circ} = 2\angle 22.88^\circ K\Omega$

(ii) Circuit current, $I = VY = 10\angle 0^\circ \times 0.494\angle -22.88^\circ = 5\angle -22.88^\circ mA$

Q.6. Two impedances $Z_1 = (8 + j6)\Omega$ and $Z_2 = (3 - j4)\Omega$ are connected in parallel. If the total current of this combination is 25A, find the power taken by each impedance.

Sol:

$$Z_1 = (8 + j6)\Omega = 10\angle 36.87^\circ \Omega$$

$$Z_2 = (3 - j4)\Omega = 5\angle -53.13^\circ \Omega$$

$$Z_1 + Z_2 = (8 + J6) + (3 - J4) = (11 + J2)\Omega = 11.18\angle 10.3^\circ \Omega$$

$$I_1 = I \frac{Z_2}{Z_1 + Z_2} = 25 \times \frac{5\angle -53.13^\circ}{11.18\angle 10.3^\circ} = 11.18\angle -63.43^\circ A$$

$$I_2 = I\frac{Z_1}{Z_1+Z_2} = 25\times\frac{10\angle 36.87^\circ}{11.18\angle 10.3^\circ} = 22.36\angle 26.57^\circ A$$

Power taken by first branch $= I_1^2 R_1 = (11.18)^2 \times 8 = 1000W$

Power taken by first branch $= I_2^2 R_2 = (22.36)^2 \times 3 = 1500W$

5.4. Power factor & Power triangle:

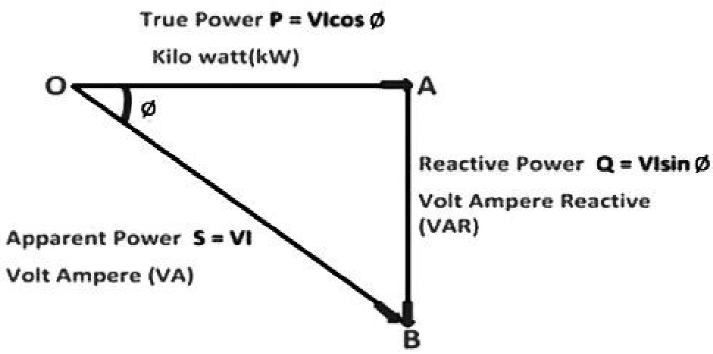
What Is Power Factor?

- It is defined as the cosine of the angle between voltage and current.
- It is dimensionless in nature.
- It is used for both single and three-phase AC circuits. It is also defined as the ratio of true or actual power to the apparent power in the ac systems.
- $\cos\emptyset = \frac{P}{S} = \frac{KW}{KVA}$

POWER TRIANGLE:

Power Triangle is the representation of a right angle triangle showing the relation between active power, reactive power and apparent power.

When each component of the current that is the active component (I cosϕ) or the reactive component (I sinϕ) is multiplied by the voltage V, a power triangle is obtained shown in the figure below:



From the triangle OAB

$S = \sqrt{(P)^2 + (Q)^2}$

$\cos\emptyset = \frac{P}{S} = \frac{KW}{KVA}$

5.5. Deduce expression for active, reactive & apparent power:

ACTIVE, REACTIVE AND APPARENT POWER:

Active Power:

The power which is actually consumed or utilised in an AC Circuit is called True power or Active power or Real power. It is measured in kilowatt (kW) or MW. It is the actual outcomes of the electrical system which runs the electric circuits or load. It is denoted by "P"

Active power P = V I cos ϕ

Reactive Power:

The power which flows back and forth that means it moves in both the directions in the circuit or reacts upon itself, is called Reactive Power. The reactive power is measured in kilo volt-ampere reactive (KVAR) or MVAR. It is denoted by "Q"

Reactive power P_r or Q = V I sin ϕ

Apparent Power:

The product of root mean square (RMS) value of voltage and current is known as Apparent Power. This power is measured in KVA or MVA. It is denoted by "S"

Apparent power S = V x I = VI

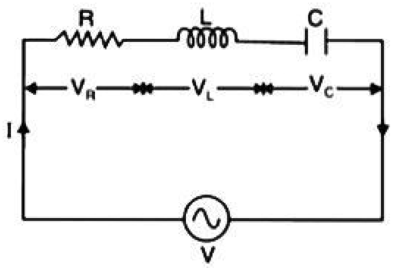
5.6. Derive the resonant frequency of series resonance and parallel resonance circuit:

SERIES RESONANCE:

An a.c. circuit containing reactive elements (L and C) is said to be in **resonance** when the circuit power factor is **unity**.

When the applied voltage and current in an a.c. circuit are in step i.e. phase angle is zero or p.f. is unity, the circuit is said to be in resonance. If this condition exists in a series a.c. circuit, it is called as **series resonance** & if in parallel a.c. circuit, it is called as **parallel resonance**.

Let us take a series R-L-C circuit,



The series R-L-C circuit is said to be at **resonance**, if the circuit power factor is **unity** or **1**.

The impedance of the above circuit is given as,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

So the current, I is given as, $$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Resonance will occur in the R-L-C circuit, when the circuit power factor is unity. This will happen if inductive reactance is equal to the capacitive reactance, i.e. $X_L = X_C$ and this condition arises if we vary the supply frequency, because the inductive & capacitive reactance depends upon the supply frequency (f).

The frequency at which $X_L = X_C$, (i.e. circuit power factor is unity), is called as **resonant frequency** (f_r).

At series resonance, $$X_L = X_C$$

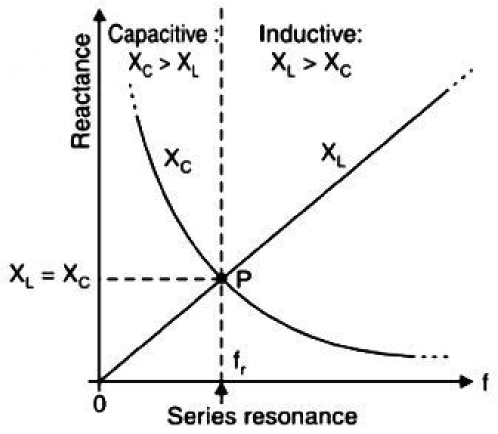
Or, $$2\pi f_r L = \frac{1}{2\pi f_r C}$$

Or, $$f_r = \frac{1}{2\pi\sqrt{LC}} Hz.$$

Effects of series resonance:

- Inductive Reactance = Capacitive Reactance ($X_L = X_C$)
- Resonant Frequency = $f_r = \frac{1}{2\pi\sqrt{LC}} Hz.$
- Circuit Impedance at resonance = $Z_r = Minimum = R$ $(\because X_L = X_C)$
- Circuit current at resonance = $I_r = \frac{V}{Z_r} = \frac{V}{R} = Maximum$
- Circuit power factor = $\cos\phi = 1(unity)$. So circuit behaves as purely resistive circuit.
- Power dissipated in the circuit is maximum
- Since at series resonance the current flowing in the circuit is very large, the voltage drops across L and C are also very large. In fact, these drops are much greater than the applied voltage. However, voltage drop across L–C combination as a whole will be zero because these drops are equal in magnitude but 180° out of phase with each other. So series resonance circuit is called as **voltage magnifying circuit.**
- Series resonance should be avoided in power circuits because the possibility of excessive voltages across the inductive and capacitive elements of the circuit may cause considerable damage. Resonance in power circuits may blow protective fuses, trip circuit breakers or cause damage to other equipment. However, in radio, television and electronic circuits, principles of series resonance are used to increase the signal voltage and current at a desired frequency i.e. at (f_r). .

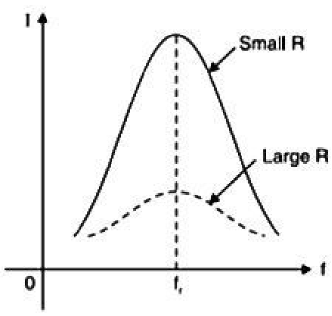
Graphical Representation of different parameters of Series R-L-C circuit:



- Fig. shows the graphical explanation of series resonance
- We know that $X_L = 2\pi fL$, so that $X_L \propto f$. Therefore, graph between X_L and f is a straight line passing through the origin.
- Again $X_C = 1/2\pi fC$, so that $X_C \propto 1/f$. Therefore, graph between X_C and f is a hyperbola.
- Starting at a very low frequency X_C is high and X_L is low and the circuit is predominantly capacitive.
- As the frequency is increased, X_C decreases and X_L increases until a value is reached (point P) where $X_L = X_C$. At this frequency (f_r), the two reactances cancel, making the circuit purely resistive. This condition is **series resonance**.
- As frequency is increased further (i.e. beyond (f_r)), X_L becomes greater than X_C and the circuit is predominantly inductive. Note that at series resonance, the circuit impedance is minimum and is equal to circuit resistance R .

Resonance Curve:

- The curve between current and frequency is known as **resonance curve.**

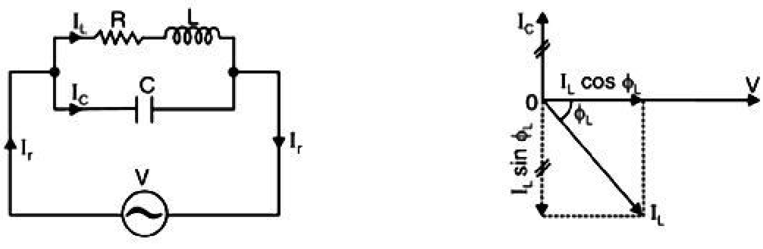


- Fig. above shows the resonance curve of a typical R-L-C series circuit.
- Note that current reaches its maximum value at the resonant frequency (f_r), falling off rapidly on either side at that point.

- It is because if the frequency is below (f_r), $X_L \rangle X_C$ and the net reactance is no longer zero.
- If the frequency is above (f_r), then $X_L \rangle X_C$ and the net reactance is again not zero.
- In both cases, the circuit impedance will be more than the impedance $Z_r (= R)$ at resonance.
- The result is that the magnitude of circuit current decreases rapidly as the frequency changes from the resonant frequency. Note also the effect of resistance in the circuit.
- The smaller the resistance, the greater the current at resonance and sharper the curve. On the other hand, the greater the resistance, the lower the resonant peak and flatter the curve.

PARALLEL RESONANCE:

A parallel a.c. circuit containing reactive elements (L and C) is said to be in resonance when the circuit p.f. is unity i.e. reactive component of line current is zero. The frequency at which it occurs is called the resonant frequency (f_r). It is called **parallel resonance** because it concerns a parallel circuit.



(i) (ii)

- The figure – (i) shows a practical parallel circuit consisting of a coil shunted by a capacitor. The figure – (ii) shows the phasor diagram of the circuit.
- The circuit will be in resonance when the reactive component of the line current is zero i.e. $I_C - I_L \sin\phi_L = 0$. It is achieved by varying the supply frequency.
- At parallel resonance, $I_C - I_L \sin\phi_L = 0$

Or, $$I_C = I_L \sin\phi_L$$

Or, $$\frac{V}{X_C} = \frac{V}{Z_L} \times \frac{X_L}{Z_L}$$

Or, $$X_L X_C = Z_L^2$$

Or, $$\frac{\omega_L}{\omega_C} = Z_L^2$$

Or, $$\frac{L}{C} = R^2 + (2\pi f_r L)^2$$

Or, $$(2\pi f_r L)^2 = \frac{L}{C} - R^2$$

Or, $$2\pi f_r L = \sqrt{\frac{L}{C} - R^2}$$

Or, $$f_r = \frac{1}{2\pi L}\sqrt{\frac{L}{C} - R^2}$$

Or, $$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} Hz.$$

If the resistance of the coil is small, then,

$$f_r = \frac{1}{2\pi\sqrt{LC}} Hz.$$

Impedance at Resonance:

- Line current, $I_r = I_L \cos\phi_L$

Or, $$\frac{V}{Z_r} = \frac{V}{Z_L} \times \frac{R}{Z_L}$$

Or, $$\frac{1}{Z_r} = \frac{R}{Z_L^2}$$

Or, $$\frac{1}{Z_r} = \frac{R}{L/C} \qquad \left[\because Z_L^2 = \frac{L}{C}\right]$$

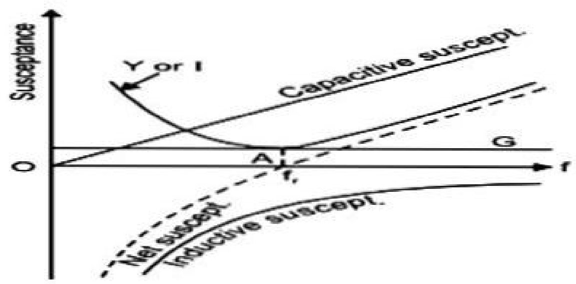
Or, $$Z_r = \frac{L}{CR}$$

At parallel resonance, the impedance is known as dynamic impedance. Its value is very high because ratio L/C is very large at parallel resonance.

Line current at Resonance:

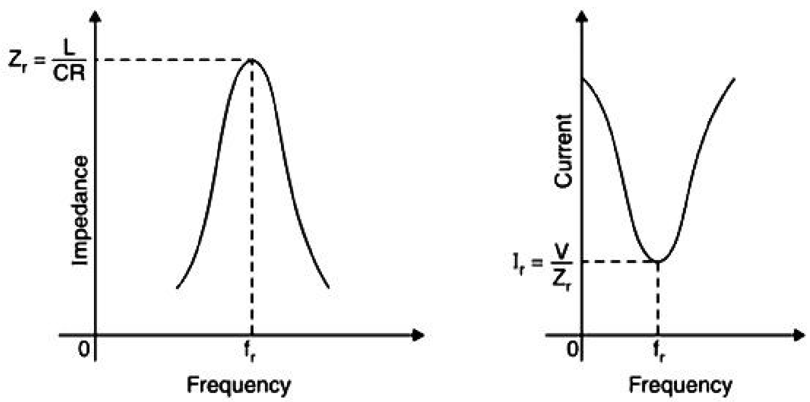
- At resonance, the line current is minimum and is given as; $I_r = \frac{V}{Z_r}$

Graphical Representation of Parallel Resonance:



- The above fig. shows the graph between susceptance (inductive and capacitive) of the two parallel branches and the supply frequency.
- Inductive susceptance, $B_L = -\frac{1}{X_L} = -\frac{1}{2\pi fL}$ or $B_L \propto \frac{1}{f}$. As B_L is inversely proportional to frequency (f), then it is a rectangular hyperbola.
- Capacitive susceptance, $B_C = \frac{1}{X_C} = 2\pi fC$ or $B_C \propto f$. As B_C is directly proportional to frequency (f), then it is a straight line passing through the origin.
- The net susceptance is the difference of the two susceptances and is represented by the dotted hyperbola.
- The circuit admittance is minimum (i.e. circuit impedance is maximum) and equal to the conductance G of the circuit.
- Circuit current is minimum.
- The circuit current is in phase with the supply voltage i.e. circuit p.f. is unity.
- At supply frequency $f \rangle f_r$, the capacitive susceptance of the circuit becomes greater than the inductive susceptance of the circuit. Consequently, the circuit is effectively capacitive and the circuit current leads the supply voltage.
- At supply frequency $f \langle f_r$, inductive susceptance predominates and the circuit becomes effectively inductive.

Impedance-frequency curve & Current-frequency curve:



- The first fig. shows the curve between the impedance and frequency for parallel circuit.
- The frequency below resonance, the capacitive reactance is higher, so more current flows through the coil, so the circuit behaves as inductive and circuit p.f. is lagging.
- The frequency above resonance, the inductive reactance is higher, so more current flows through the capacitor, so the circuit behaves as capacitive and circuit p.f. is leading.
- The second fig. shows the current-frequency curve or resonance curve. The line current is minimum at resonance ($I_r = \frac{V}{Z_r}$). Before and after the resonance frequency, the current is maximum because impedance is minimum.

Comparison of Series and Parallel Resonant circuit:

S. No.	Particular	Series circuit	Parallel circuit
1.	Impedance at resonance	Minimum ($Z_r = R$)	Maximum ($Z_r = L/CR$)
2.	Current at resonance	Maximum ($I_r = V/R$)	Minimum ($I_r = V/Z_r$)
3.	p.f. at resonance	Unity	Unity
4.	Resonant frequency	$f_r = \frac{1}{2\pi\sqrt{LC}}$	$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$
5.	When $f < f_r$	Circuit is capacitive	Circuit is inductive
6.	When $f > f_r$	Circuit is inductive	Circuit is capacitive
7.	Q-factor	X_L/R	X_L/R
8.	It magnifies	Voltage	Current

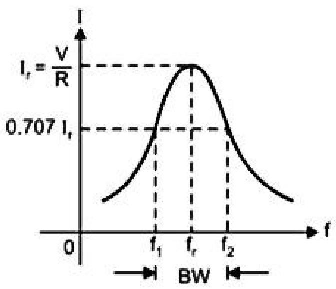
5.7. Define Bandwidth, Selectivity & Q-factor in series circuit:

Quality factor:

- At series resonance, the voltage across L or C is much more than the applied voltage i.e. $V = I_r R$.
- The ratio of $\frac{V_L}{V}$ or $\frac{V_C}{V}$ at resonance is a measure of the quality of a series resonant circuit. It is called the **Q – factor or Quality factor** of the circuit. It is also known as **voltage magnification factor.**
- $Q - factor = \frac{V_L}{V} = \frac{I_r X_L}{I_r R} = \frac{X_L}{R} = \frac{\omega_r L}{R}$ ....(1)
- We know that $f_r = \frac{1}{2\pi\sqrt{LC}}$, so $\omega_r = \frac{1}{\sqrt{LC}}$. Now substituting the value of ω_r in above equation – 1, we get, $Q - factor = \frac{1}{R}\sqrt{\frac{L}{C}}$
- Note that ***Q – factor*** of a series resonant circuit is inversely proportional to the resistance ***(R).*** If ***R*** increases ***Q – factor*** decreases and vice versa.

Bandwidth of a series resonant circuit:

- Bandwidth of a series resonant circuit is the range of frequencies for which the circuit current is equal to or greater than 70.7% of the circuit current at resonance (i.e. I_r).



- Bandwidth, $BW = \Delta f = f_2 - f_1$
- The frequency f_1 (i.e., on the lower side) is called the **lower cut off frequency** and the frequency f_2 (i.e., on the higher side) is called the **upper cut off frequency**.
- $f_1 = f_r - \frac{R}{4\pi L}$ & $f_2 = f_r + \frac{R}{4\pi L}$.
- $f_r = \sqrt{f_1 f_2}$
- $BW = \frac{R}{2\pi L}$, so that, $f_1 = f_r - \frac{BW}{2}$ & $f_2 = f_r + \frac{BW}{2}$

Relation between Bandwidth, resonant frequency & Q – factor is given as $Bandwidth = \frac{f_r}{Q - factor}$.

5.8. Solve Numerical problems:

Ex – 1: A coil of resistance 100 Ω and inductance 100 μH is connected in series with a 100 pF capacitor. The circuit is connected to a 10 V variable frequency source. Calculate (i) the resonant frequency (ii) current at resonance (iii) voltage across L and C at resonance and (iv) Q-factor of the circuit

Sol:

(i) Resonant frequency, $f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{100\times10^{-6}\times100\times10^{-12}}} = 1.59\times10^{6}\,Hz$

(ii) Current at resonance, $I_r = \frac{V}{R} = 10/100 = 0.1A$

(iii) At resonance, $X_L = 2\pi f_r L = 2\pi\times1.59\times10^{6}\times100\times10^{-6} = 1000\Omega$

At resonance, $V_L = I_r X_L = 0.1\times1000 = 100V$

At resonance, $V_C = I_r X_C = 0.1\times1000 = 100V$

(iv) $Q - factor = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{100}\sqrt{\frac{100\times10^{-6}}{100\times10^{-12}}} = 10$

Ex – 2: A choking coil is connected in series with a 20 μF capacitor. With a constant supply voltage of 200 V, it is found that the circuit takes its maximum current of 50 A when the supply frequency is 100 Hz. Calculate (i) resistance and inductance of the choking coil and (ii) voltage across the capacitor. What is the Q-factor of the coil?

Sol: Since current is maximum, the circuit is in resonance i.e.

(i) $I_r = 50A$; $f_r = 100Hz$

$R = \frac{V}{I_r} = \frac{200}{50} = 4\Omega$

$X_C = \frac{1}{2\pi f_r C} = \frac{1}{2\pi\times100\times20\times10^{-6}} = 79.6\Omega$

Now, $X_C = X_L = 79.6\Omega$ (at resonance)

$$\therefore\ L = \frac{X_L}{2\pi f_r} = \frac{79.6}{2\pi \times 100} = 0.127H$$

(ii) $V_C = I_r \times X_C = 50 \times 79.6 = 3980V$

$$Q - factor = \frac{V_C}{V} = \frac{3980}{200} = 19.9$$

Ex – 3: A series RLC circuit has R = 5 Ω, L = 0.2 H and C = 50 µF. The applied voltage is 200 V. Find (i) resonant frequency (ii) Q-factor (iii) bandwidth (iv) upper and lower half-power frequencies (v) current at resonance (vi) current at half-power points (vii) voltage across inductance at resonance.

Sol: $R = 5\Omega; L = 0.2H; C = 50 \times 10^{-6}F;\ V = 200volts$

(i) Resonant frequency, $f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.2 \times 50 \times 10^{-6}}} = 50.33Hz$

(ii) Quality factor, $Q - factor = \frac{\omega_r L}{R} = \frac{2\pi f_r L}{R} = \frac{2\pi \times 50.33 \times 0.2}{5} = 12.65$

(iii) Bandwidth, $BW = \frac{f_r}{Q - factor} = \frac{50.33}{12.65} = 3.98Hz$

(iv) Upper half-power frequency, $f_2 = f_r + \frac{BW}{2} = 50.33 + \frac{3.98}{2} = 52.32Hz$

Lower half-power frequency, $f_1 = f_r - \frac{BW}{2} = 50.33 - \frac{3.98}{2} = 48.34Hz$

(v) Current at resonance, $I_r = \frac{V}{R} = \frac{200}{5} = 40A$

(vi) Current at half-power points $= 0.707 \times I_r = 0.707 \times 40 = 28.28A$

(vii) Voltage across L at resonance $I_r X_L = 40 \times 2\pi \times 50.33 \times 0.2 = 2529.87V$

Ex – 4: The dynamic impedance of a parallel resonant circuit is 500 KΩ. The circuit consists of a 250 pF capacitor in parallel with a coil of resistance 10 Ω. Calculate (i) the coil inductance (ii) the resonant frequency and (iii) the Q-factor of the circuit

Sol:

(i) Dynamic impedance, $Z_r = \frac{L}{CR}$

So inductance of coil, $L = Z_r CR = (500 \times 10^3) \times 250 \times 10^{-12} \times 10 = 1.25mH$

(ii) Resonant frequency, $f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} = \frac{1}{2\pi}\sqrt{\frac{1}{1.25 \times 10^{-3} \times 250 \times 10^{-12}} - \frac{10^2}{(1.25 \times 10^{-3})^2}}$

$$= 284.7KHz$$

(iii) Q-factor of the circuit $= \dfrac{2\pi f_r L}{R} = \dfrac{2\pi \times (284.7\times10^3)\times1.25\times10^{-3}}{10} = 223.6$

EX – 5: An inductor of resistance 10Ω and inductance 100mH is in parallel with a 10mF capacitor. Find (i) the resonant angular frequency (ii) the Q-factor and (iii) the bandwidth.

Sol: $R = 10\Omega; L = 100mH = 100\times10^{-3}H; C = 10nF = 10\times10^{-9}F$

(i) The resonant angular frequency, $\omega_r (= 2\pi f_r)$ is given by;

$$\omega_r = \sqrt{\left(\frac{1}{LC} - \frac{R^2}{L^2}\right)} = \sqrt{\left(\frac{1}{100\times10^{-3}\times10\times10^{-9}} - \frac{10^2}{(100\times10^{-3})^2}\right)} = 3.16\times10^4\, rad/s$$

(ii) Q-factor of the circuit $= \dfrac{\omega_r L}{R} = \dfrac{(3.16\times10^4)\times100\times10^{-3}}{10} = 316$

Bandwidth $= \dfrac{\omega_r}{Q - factor} = \dfrac{3.16\times10^4}{316} = 100 rad/s$

QUESTION BANK

Short Questions With Answer:

Q: Define cycle.

A: One complete set of positive and negative values of alternating quantity is known as cycle.

Q: Define Time Period**.**

A The time taken by an alternating quantity to complete one cycle is called its time period T

Q: Define Frequency.

A: The number of cycles/second is called the frequency of the alternating quantity. Its unit is hertz (Hz).

Q: Define Amplitude.

A: The maximum value, positive or negative, of an alternating quantity is known as its amplitude.

Q: What is Root-Mean-Square (R.M.S.) Value?

A: The r.m.s. value of an alternating current is given by that steady (d.c.) current which when flowing through a given circuit for a given time produces the same heat as produced by the alternating current when flowing through the same circuit for the same time.

Q: What is Average Value?

A: The average value Ia of an alternating current is expressed by that steady current which transfers across any circuit the same charge as is transferred by that alternating current during the same time.

Long Questions:

Q: The potential difference measured across a coil is 4.5 V, when it carries a direct current of 9 A. The same coil when carries an alternating current of 9 A at 25 Hz, the potential difference is 24 V. Find the current, the power and the power factor when it is supplied by 50 V, 50 Hz supply.

Q: An inductive circuit draws 10 A and 1 kW from a 200-V, 50 Hz a.c. supply. Determine : (i) the impedance in cartesian from (a + jb) (ii) the impedance in polar from Z ∠··· (iii) the power factor (iv) the reactive power (v) the apparent power.

Q: A resistance of 20 ohm, inductance of 0.2 H and capacitance of 150 μF are connected in series and are fed by a 230 V, 50 Hz supply. Find XL, XC, Z, Y, p.f., active power and reactive power.

Q: A coil of 0.8 p.f. is connected in series with 110 micro-farad capacitor. Supply frequency is 50 Hz. The potential difference across the coil is found to be equal to that across the capacitor. Calculate the resistance and the inductance of the coil. Calculate the net power factor.

Q: Two impedances consist of (resistance of 15 ohms and series-connected inductance of 0.04 H) and (resistance of 10 ohms, inductance of 0.1 H and a capacitance of 100 μF, all in series) are connectd in series and are connected to a 230 V, 50 Hz a.c. source. Find : (i) Current drawn, (ii) Voltage across each impedance, (iii) Individual and total power factor. Draw the phasor diagram.

Q: A choking coil carries a current of 15 A when supplied from a 50-Hz, 230-V supply. The power in the circuit is measured by a wattmeter and is found to be 1300 watt. Estimate the phase difference between the current and p.d. in the circuit.

Q: Two coils are connected in series. With 2 A d.c. through the circuit, the p.ds. across the coils are 20 and 30 V respectively. With 2 A a.c. at 40 Hz, the p.ds. across the coils are 140 and 100 V respectively. If the two coils in series are connected to a 230-V, 50-Hz supply, calculate (a) the current (b) the power (c) the power factor.

Q: A pure resistance of 50 ohms is in series with a pure capacitance of 100 microfarads. The series combination is connected across 100-V, 50-Hz supply. Find (a) the impedance (b) current (c) power factor (d) phase angle (e) voltage across resistor (f) voltage across capacitor. Draw the vector diagram.

Q: A 240-V, 50-Hz series R-C circuit takes an r.m.s. current of 20 A. The maximum value of the current occurs 1/900 second before the maximum value of the voltage. Calculate (i) the power factor (ii) average power (iii) the parameters of the circuit.

Q: It is desired to operate a 100-W, 120-V electric lamp at its current rating from a 240-V, 50-Hz supply. Give details of the simplest manner in which this could be done using (a) a resistor (b) a capacitor and (c) an indicator having resistance of 10 ••. What power factor would be presented to the supply in each case and which method is the most economical of power.

Q: A voltage e(t) = 100 sin 314 t is applied to series circuit consisting of 10 ohm resistance, 0.0318 henry inductance and a capacitor of 63.6 μF. Calculate (i) expression for i (t) (ii) phase angle between voltage and current (iii) power factor (iv) active power consumed.

Q: Two impedances Z1 and Z2 when connected separately across a 230-V, 50-Hz supply consumed 100 W and 60 W at power factors of 0.5 lagging and 0.6 leading respectively. If these impedances are now connected in series across the same supply, find : (i) total power absorbed and overall p.f. (ii) the value of the impedance to be added in series so as to raise the overall p.f. to unity.

Q: A coil is in series with a 20 μF capacitor across a 230-V, 50-Hz supply. The current taken by the circuit is 8 A and the power consumed is 200 W. Calculate the inductance of the coil if the power factor of the circuit is (i) leading (ii) lagging. Sketch a vector diagram for each condition and calculate the coil power factor in each case.

Q: A coil of resistance 10 Ω and inductance 0.1 H is connected in series with a 150-μF capacitor across a 200-V, 50-Hz supply. Calculate (a) the inductive reactance, (b) the capacitive reactance, (c) the impedance (d) the current, (e) the power factor (f) the voltage across the coil and the capacitor respectively.

Q: A circuit takes a current of 3 A at a power factor of 0.6 lagging when connected to a 115-V, 50-Hz
supply. Another circuit takes a current, of 5 A at a power factor of 0.707 leading when connected to the same supply. If the two circuits are connected in series across a 230-V, 50Hz supply, calculate (a) the current (b) the power consumed and (c) the power factor.

CHAPTER 6. POLYPHASE CIRCUIT:

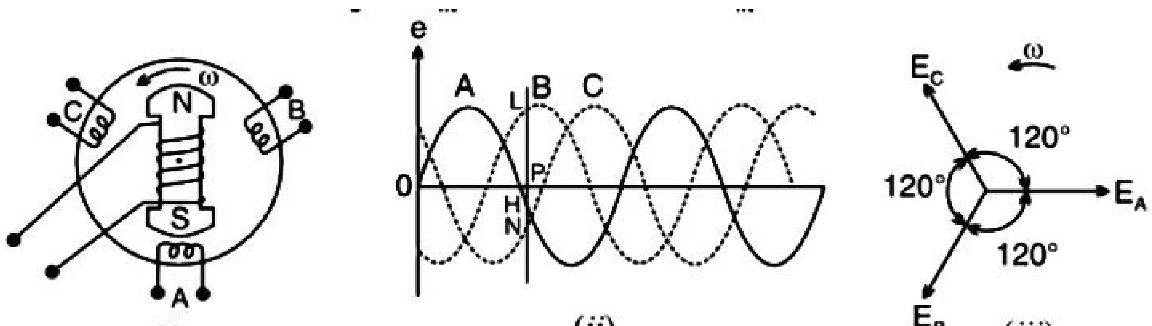
6.1. Concept of Polyphase system and phase sequence:

- Poly means many and phase means winding, so that polyphase means more than one winding. A three phase generator produce three voltages of the same magnitude and frequency but displaced120° electrical from one another.
- Three phase system is by far the most popular because it is the most efficient of all the supply systems.
- In actual 3 – phase alternator, the three windings or coils are stationary and the field rotates.
- The figure – (i) below shows the elementary 3 – phase alternator. The three identical coils A, B and C are symmetrically placed in such a way that e.m.f.s induced in them are displaced 120 electrical degrees from one another.
- Since the coils are identical and are subjected to the same uniform rotating field, the e.m.f.s induced in them will be of the same magnitude and frequency. Figure – (ii) shows the wave diagram of the three e.m.fs whereas figure – (iii) shows the phasor diagram.
- Thus E_A is the r.m.s. value of the e.m.f. induced in coil A. The equations of the three e.m.fs are;

$$e_A = E_m \sin \omega t$$

$$e_B = E_m \sin(\omega t - 120^\circ)$$

$$e_C = E_m \sin(\omega t + 120^\circ)$$



(i) (ii) (iii)

- The sum of the three e.m.fs at every instant is zero.
- Resultant

$$\begin{aligned} &= e_A + e_B + e_C \\ &= E_m[\sin \omega t + \sin(\omega t - 120^\circ) + \sin(\omega t - 240^\circ)] \\ &= E_m[\sin \omega t + 2\sin(\omega t - 180^\circ)\cos 60^\circ] \\ &= E_m[\sin \omega t - 2\sin \omega t \cos 60^\circ] \\ &= 0 \end{aligned}$$

Phase sequence:

The order in which the voltages in the three phases (or coils) of an alternator reach their maximum positive values is called **phase sequence** or **phase order**.

The phase sequence is determined by the direction of rotation of the alternator. Thus in fig – (iii) the phase sequence is ABC and they are 120° out of phase to each other.

6.2. Relation between phase and line quantities in star & delta connection:

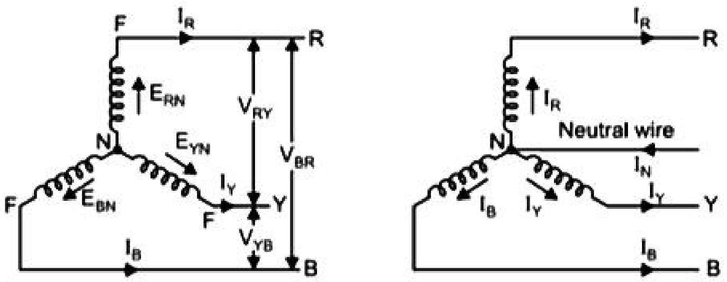
The three windings are interconnected to give rise to two methods of connections viz.

- **(i) Star or Wye (Y) connection**
- **(ii) Mesh or Delta (Δ) connection**

Star or Wye (Y) connection:

In this connection, similar ends (start or finish) of the three windings or phases are connected to a common point and other ends are free ends. The common point is known as **neutral point (N)**. So it is a 3 – phase, 4 – wire system.

The fig. below shows the star connection.

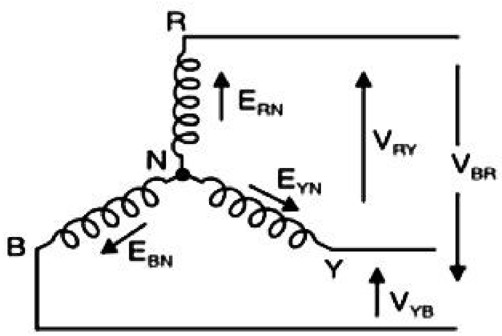


The three phases or windings are named as **R, Y & B.** The voltage between any two lines is called as **line voltage** and the voltage between one phase and neutral is called as **phase voltage.**

Here E_{RN}, E_{YN} & E_{BN} are the phase voltages and V_{RY}, V_{YB} & V_{BR} are the line voltages.

The current flowing in the winding or phase is called as **phase current** and current flowing through the lines is called as the **line current.**

Relation between Line voltage & current with Phase voltage & current in Star connection:



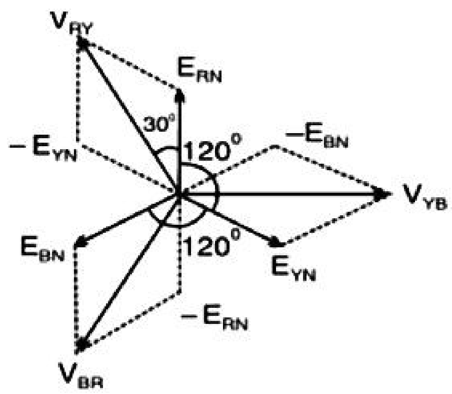


Figure shows a balanced 3 – phase, Y – connected system in which r.m.s values of the e.m.f.s generated are E_{RN}, E_{YN} & E_{BN}. Since the system is balanced, this e.m.f.s will be equal in magnitude (say E_{ph}) but displaced 120° from each other.

From the phasor diagram, **PD** between lines R and Y, $V_{RY} = E_{RN} - E_{YN}$ ***(phasor difference)***

PD between lines Y and B, $V_{YB} = E_{YN} - E_{BN}$ ***(phasor difference)***

PD between lines B and R, $V_{BR} = E_{BN} - E_{RN}$ ***(phasor difference)***

$$\therefore \quad V_{RY} = 2E_{RN} or E_{YN} \cos\left(\theta/2\right) = 2E_{ph}\cos\left(60^\circ/2\right) = 2E_{ph}\cos 30^\circ = 2\times\frac{\sqrt{3}}{2}\times E_{ph} = \sqrt{3}_{Eph}$$

So that, $$E_L = \sqrt{3}E_{ph}$$

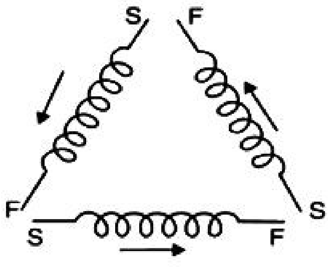
NOTE:

- In star connection, **Line voltage = Phase voltage** ($E_L = \sqrt{3}E_{ph}$).
- As the lines are in series with their respective phases or windings, so **Line current = Phase current** ($I_L = I_{ph}$).
- Line voltages are 30° lead of their respective phase voltages.

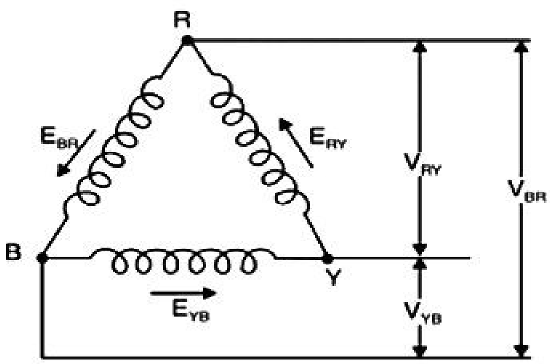
Mesh or Delta (Δ) connection:

In this connection, the finishing end of one winding is connected to starting end of another winding and so on to obtain mesh or delta as shown in figure below. The three lines are taken from three junctions of the mesh or delta and assigned as R, Y & B. This is called 3 – phase, 3 –wire, delta connected system.

In delta connection, there is no neutral exists, therefore, only 3 – phase, 3 – wire system can be found.

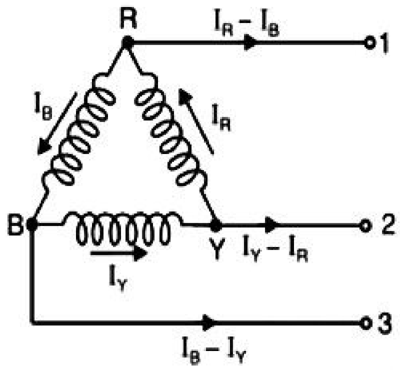


(i)



(ii)

Relation between Line voltage & current with Phase voltage & current in Delta connection:



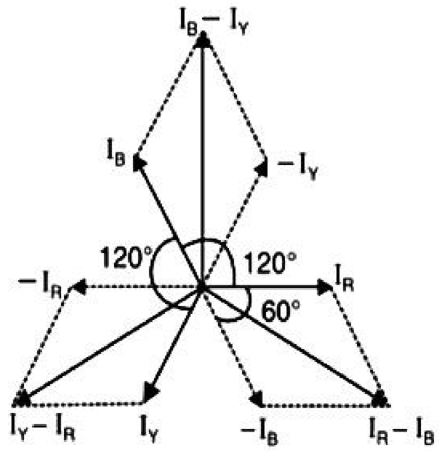


Fig. above shows a balanced 3-phase Δ - connected supply system. It is desired to find the relation between (i) line voltage and phase voltage (ii) line current and phase current.

- As one phase is included between any two lines, so magnitude of voltage between any two lines (i.e. line voltage) is equal to the magnitude of phase voltage. i.e.

  Line voltage (V_L) = Phase voltage (V_{ph})

- Since the system is balanced, the three phase currents I_R, I_Y and I_B are equal in magnitude (say I_{ph}) but displaced 120° from one another as shown in phasor diagram above.
- The current in any line is the phasor difference of the currents in the two phases attached to that line.
- Thus Current in line – 1, $= I_R - I_B$, Current in line – 2, $= I_Y - I_R$, Current in line – 3, $= I_B - I_Y$.
- The current in line – 1 $(I_1) =$ Phasor difference of I_R and I_B
- $\therefore$

$$I_1 = I_L = 2I_{ph}\cos\left(\theta/2\right)$$
$$= 2I_{ph}\cos\left(60^\circ/2\right)$$
$$= 2I_{ph}\cos 30^\circ$$
$$= \sqrt{3}I_{ph}$$

- So the line current, $I_L = \sqrt{3}I_{ph}$
- Line currents are 30° behind the respective phase currents.

6.3. Power equation in 3 – phase balanced circuit:

In star connection:

The total power in a star circuit is the sum of powers in the three phases. For a balanced load, the power in each phase is same.

So that, Total power, $P = 3 \times$ power in each phase $= 3 \times V_{ph} \times I_{ph} \times \cos\phi$ (if phase value is taken)

Again in star connection, $V_L = \sqrt{3}V_{ph}$ & $I_L = I_{ph}$. So that, Total power, $P = 3 \times \frac{V_L}{\sqrt{3}} \times I_L \times \cos\phi$

Or, $$P = \sqrt{3}V_L I_L \cos\phi \text{ (KW)}$$

Also, Reactive power is given as $$Q = \sqrt{3}V_L I_L \sin\phi \text{ (KVAR)}$$

$\therefore$ Total power or apparent power $(S) = \sqrt{P^2 + Q^2} = \sqrt{3}V_L I_L$

In delta connection:

The total power in a star circuit is the sum of powers in the three phases. For a balanced load, the power in each phase is same.

So that, Total power, $P = 3 \times$ power in each phase $= 3 \times V_{ph} \times I_{ph} \times \cos\phi$ (if phase value is taken)

Again in star connection, $V_L = V_{ph}$ & $I_L = \sqrt{3}I_{ph}$. So that, Total power, $P = 3 \times V_L \times \frac{I_L}{\sqrt{3}} \times \cos\phi$

Or, $$P = \sqrt{3}V_L I_L \cos\phi \text{ (KW)}$$

Also, Reactive power is given as $$Q = \sqrt{3}V_L I_L \sin\phi \text{ (KVAR)}$$

$\therefore$ Total power or apparent power $(S) = \sqrt{P^2 + Q^2} = \sqrt{3}V_L I_L$

6.4. Numerical:

Ex – 1: Phase voltages of a star-connected alternator are $E_R = 231 \angle 0°$ V; $E_Y = 231 \angle -120°$ V and $E_B = 231 \angle +120°$ V. What is the phase sequence of the system? Compute the line voltages E_{RY} and E_{YB}.

Sol:

The phase voltage $E_B = 231 \angle +120°$ V can be written as $E_B = 231 \angle -240°$ V.

Therefore, the three phase voltages are:

$E_R = 231 \angle 0°$ V; $E_Y = 231 \angle -120°$ V; $E_B = 231 \angle -240°$ V

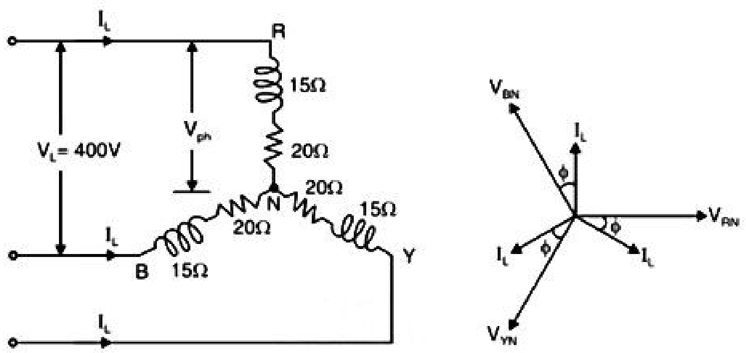
It is clear that E_R is the reference voltage. Now E_Y lags behind E_R by 120° while E_B lags behind E_R by 240°.

Therefore, the phase sequence is RYB.

Further, the 3-phase system is balanced so that the magnitudes of line voltages E_{RY} and E_{YB} are:

$$E_{RY} = E_{YB} = \sqrt{3} \times 231 = 400V.$$

Ex – 2: Three coils, each having a resistance of 20 Ω and an inductive reactance of 15 Ω, are connected in star to a 400 V, 3-phase, 50 Hz supply. Calculate (i) the line current (ii) power factor and (iii) power supplied.



Data given: $V_L = 400V, R_{ph} = 20\Omega, X_{Lph} = 15\Omega,\ f = 50Hz$

Sol: Phase voltage $(V_{ph}) = \dfrac{V_L}{\sqrt{3}} = \dfrac{400}{\sqrt{3}} = 231\,\text{V}$

Impedance/phase, $Z_{ph} = \sqrt{R_{ph}^2 + X_{Lph}^2} = \sqrt{20^2 + 15^2} = 25\Omega$

Phase current, $I_{ph} = \dfrac{V_{ph}}{I_{ph}} = \dfrac{231}{25} = 9.24A$

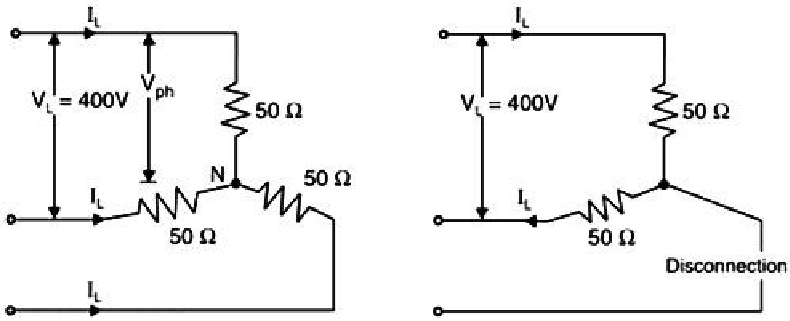
∴ **Line current,** $I_L = I_{ph} = 9.24A$

∴ **Power factor,** $\frac{R_{ph}}{Z_{ph}} = \frac{20}{25} = 0.8 lag$

∴ **Power supplied,** $P = \sqrt{3}V_L I_L \cos\phi = \sqrt{3} \times 400 \times 9.24 \times 0.8 = 5121W$

Or, $P = 3I_{ph}^2 R_{ph} = 3 \times (9.24)^2 \times 20 = 5121W$

Ex – 3: Three 50 ohm resistors are connected in star across 400 V, 3-phase supply. (i) Find phase current, line current and power taken from the mains. (ii) What would be the above values if one of the resistors were disconnected ?



Data given: $V_L = 400V, R_{ph} = 50\Omega$

Sol: Phase voltage $(V_{ph}) = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231$V; $R_{ph} = 50\Omega$; $\cos\phi = 1$

(i) When the three resistors are star-connected:

Phase current, $I_{ph} = \frac{V_{ph}}{R_{ph}} = 231/50 = 4.62A$

Line current, $I_L = I_{ph} = 4.62A$

∴ **Power taken,** $P = \sqrt{3}V_L I_L \cos\phi = \sqrt{3} \times 400 \times 4.62 \times 1 = 3200W$

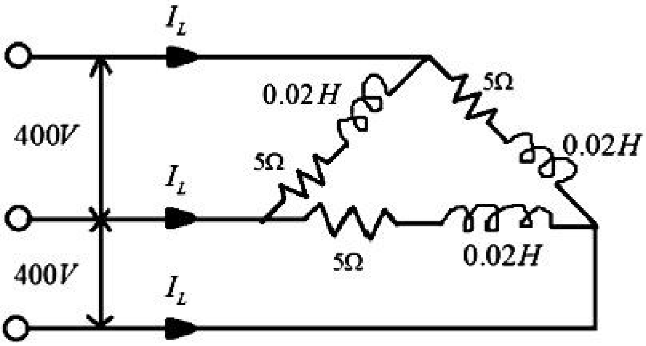
(ii) When one of the resistors is disconnected:

When one of the resistor is disconnected, the remaining two resistors are connected in series across the line voltage. So the circuit behaves as a single phase circuit.

∴ $I_{ph} = I_L = \frac{400}{50+50} 4A$

∴ Power taken, $P = V_L I_L \cos\phi = 400 \times 4 \times 1 = 1600W$

Ex – 4: Three similar coils each having a resistance of 5Ω and an inductance of 0.02H are connected in delta to a 440V, 3-phase, 50Hz supply. Calculate the line current and total power absorbed.



Data given: $V_L = 440V, R_{ph} = 5\Omega, L_{ph} = 0.02H$, $f = 50Hz$

Sol: Reactance of the coil/phase, $X_L = 2\pi fL = 2\pi \times 50 \times 0.02 = 6.28\Omega$

Impedance/phase, $Z_{ph} = \sqrt{R_{ph}^2 + X_{Lph}^2} = \sqrt{5^2 + 6.28^2} = 8.05\Omega$

Power factor, $\frac{R_{ph}}{Z_{ph}} = \frac{5}{8.05} = 0.622 lag$

Phase voltage $(V_{ph}) = V_L = 440V$ V

∴ Phase current, $I_{ph} = {}^{V_{ph}}/_{Z_{ph}} = {}^{440}/_{8.05} = 54.6A$

∴ **Line current,** $I_L = \sqrt{3} I_{ph} = \sqrt{3} \times 54.6 = 94.8A$

∴ **Power absorbed,** $P = \sqrt{3} V_L I_L \cos\phi = \sqrt{3} \times 440 \times 94.8 \times 0.622 = 45000W$

Ex – 5: A 3-phase, 400 V, 50 Hz a.c. supply is feeding a 3-phase delta-connected load with each phase having a resistance of 25 Ω, an inductance of 0.15 H and a capacitor of 120 μF in series. Find line current, volt-amp, active power and reactive volt-amp.

Data given: $V_L = 400V, R_{ph} = 25\Omega, L_{ph} = 0.15H, C_{ph} = 120\mu F$, $f = 50Hz$

Sol: Reactance of the coil/phase, $X_L = 2\pi fL = 2\pi \times 50 \times 0.15 = 47.1\Omega$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 120 \times 10^{-6}} = 26.54\Omega$$

Net reactance/phase, $X = X_L - X_C = 47.1 - 26.54 = 20.56\Omega$

Impedance/phase, $Z_{ph} = \sqrt{R^2 + X^2} = \sqrt{(25)^2 + (20.56)^2} = 32.37\Omega$

Power factor, $\cos\phi = \frac{R}{Z_{ph}} = 25/32.37 = 0.772 lag$

Phase current, $I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{400}{32.37} = 12.36A$

$\therefore$ **Line current,** $I_L = \sqrt{3}I_{ph} = \sqrt{3} \times 12.36 = 21.4A$

$\therefore$ **Total active power,** $P = \sqrt{3}V_L I_L \cos\phi = \sqrt{3} \times 400 \times 21.4 \times 0.772 = 11446W$

$\therefore$ **Total apparent power,** $S = P / \cos\phi = 11446 / 0.772 = 14830VA$

$\therefore$ **Total reactive power,** $Q = \sqrt{S^2 - P^2} = \sqrt{(14830)^2 - (11446)^2} = 9430VAR$

6.5. Measurement of 3 – phase power by two wattmeter method:

In **two-wattmeter method** for the measurement of 3-phase power, the current coils of the two wattmeters are connected in any two lines and the potential coil of each joined to the third line as shown in Fig. (Star connected load) below.

We now prove from first principles that the algebraic sum of the readings of the two wattmeters gives the total power drawn by the 3-phase load (Y or Δ, balanced or unbalanced). Also the power factor of the load can be determined from the wattmeter readings.

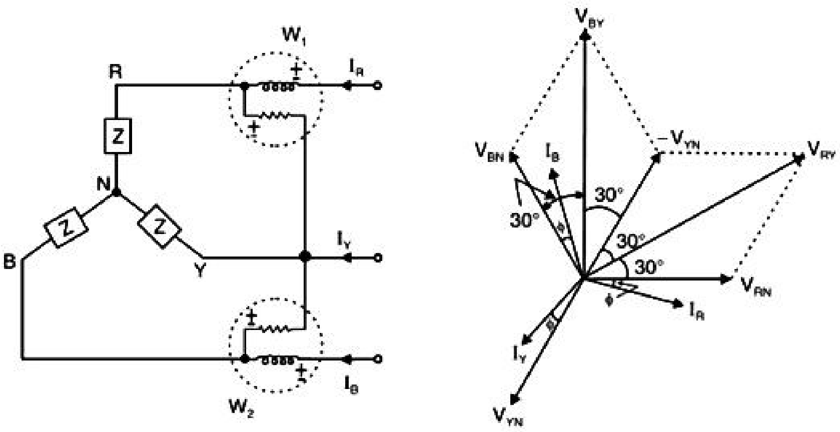


Fig. shows a balanced Y-connected load; the p.f. angle of load impedance being ϕ lag.

Let V_{RN}, V_{YN} and V_{BN} be the r.m.s. values of the three load phase voltages (phase sequence being RYB) and I_R, I_Y and I_B the r.m.s. values of line currents.

These currents will lag behind their respective phase voltages by ϕ as shown in the phasor diagram.

Current through current coil of $W_1 = I_R$

P.D. across potential coil of W_1, $V_{RY} = V_{RN} - V_{YN}$ ...phasor difference

The angle between V_{RY} and I_R is $(30°+\phi)$

∴ So $$W_1 = V_{RY} I_R \cos(30^\circ + \phi)$$

Current through current coil of $W_2 = I_B$

P.D. across potential coil of W_1, $V_{BY} = V_{BN} - V_{YN}$ ...phasor difference

The angle between V_{RY} and I_R is $(30°- \phi)$

∴ So $$W_2 = V_{BY} I_B \cos(30^\circ - \phi)$$

Since the load is balanced, $V_{RY} = V_{BY} = V_L$

And $$I_R = I_B = I_L$$

∴ So $$W_1 = V_L I_L \cos(30^\circ + \phi)$$

∴ So $W_2 = V_L I_L \cos(30^\circ - \phi)$

∴ **Total power,** $W_1 + W_2 = V_L I_L [\cos(30^\circ + \phi) + \cos(30^\circ - \phi)]$

$$= V_L I_L [(\cos 30^\circ \cos\phi - \sin 30^\circ \sin\phi) + (\cos 30^\circ \cos\phi + \sin 30^\circ \sin\phi)]$$

$$= V_L I_L (2\cos 30^\circ \cos\phi)$$

$$= \sqrt{3} V_L I_L \cos\phi \text{ (Total power in the 3 – phase load)}$$

Hence the algebraic sum of the two wattmeter readings gives the total power consumed in the 3 – phase load.

Power factor:

$$W_2 = V_L I_L \cos(30^\circ - \phi)$$

$$W_1 = V_L I_L \cos(30^\circ + \phi)$$

∴ $W_2 + W_1 = \sqrt{3} V_L I_L \cos\phi$

Now taking the difference between two wattmeter readings, we get,

$$W_2 - W_1 = \sqrt{3} V_L I_L \sin\phi$$

∴ $\tan\phi = \sqrt{3}\dfrac{W_2 - W_1}{W_2 + W_1}$ (∴ For lagging P.F.)

∴ $\tan\phi = \sqrt{3}\dfrac{W_1 - W_2}{W_2 + W_1}$ (∴ For leading P.F.)

From $\tan\phi$, we get the ϕ and hence the load power $\cos\phi$ can be calculated.

Effect of load power factor on Wattmeter readings:

ϕ	0°	60°	More than 60°	90°
cos ϕ	1	0.5	< 0.5	0
W_2	positive	positive	positive	positive
W_1	positive	0	negative	negative
Conclusion	$W_1 = W_2$ Total power $= W_1 + W_2$	$W_1 = 0$ Total power $= W_2$	Total power $= W_2 - W_1$	$W_2 = -W_1$ Total power $= 0$

6.6. Solve Numerical Problems:

EX – 1: Two-wattmeter method is used to measure the power taken by a 3-phase induction motor on no load. The wattmeter readings are 375 W and –50 W. Calculate (i) power factor of the motor at no load (ii) phase difference of voltage and current in two wattmeters (iii) reactive power taken by the load.

Data given:

Higher-reading wattmeter, **W_2 = 375 W**; Lower-reading wattmeter, **W_1 = – 50 W**

Solution:

(i) $$\tan\phi = \sqrt{3}\,\frac{W_2 - W_1}{W_2 + W_1} = \sqrt{3} \times \frac{375-(-50)}{375+(-50)} = \sqrt{3} \times \frac{425}{325} = 2.265$$

$\therefore$ $$\phi = \tan^{-1}(2.265) = 66.18^\circ$$

$\therefore$ No load P. F. = $\cos\phi = \cos(66.18^\circ) = 0.404 lag$

(ii) Phase angle in wattmeter, $W_2 = 30^\circ - \phi = 30^\circ - 66.18^\circ = -36.18^\circ$

Phase angle in wattmeter, $W_1 = 30^\circ + \phi = 30^\circ + 66.18^\circ = 96.18^\circ$

(iii) Reactive power $\sqrt{3}(W_2 - W_1) = \sqrt{3}(375+50) = 736.12 VAR$

EX – 2: A 3-phase motor load has a p.f. of 0.397 lagging. Two wattmeters connected to measure power show the input as 30 KW. Find the reading on each wattmeter.

Solution:

Let W_2 = Higher-reading wattmeter, **W_1 =** Lower-reading wattmeter,

$\therefore$ $$W_2 + W_1 = 30 KW$$

Power factor angle, $\phi = \cos^{-1}(0.397) = 66.6^\circ$

∴ $$\tan\phi = \sqrt{3}\frac{W_2 - W_1}{W_2 + W_1} = 2.311$$

Or, $$W_2 - W_1 = 40KW$$

Now solving the equations, we get, $W_2 = 35KW$; $W_1 = -5KW$

EX – 3: Two wattmeters are used to measure power in a 3-phase balanced load. The wattmeter readings are 8.2 kW and 7.5 kW. Calculate (i) total power (ii) power factor and (iii) total reactive power. **(ANS: 15.7KW, 0.997, 1.21KVAR)**

EX – 4: A balanced 3-phase load takes 10 kW at a p.f. of 0.9 lagging. Calculate the readings on each of the two wattmeters connected to read the input power. **(ANS: 6398W, 3602W)**

EX – 5: A 440 V, 3-phase induction motor has an output of 20.7 kW at a p.f. of 0.82 and efficiency 85%. Calculate the readings on each of the two wattmeters connected to measure input. **(ANS: 12.35KW, 5.25KW)**

QUESTION BANK

Short Questions With Answer:

Q: What do you mean by a three-phase balanced load?

A: Any three-phase load will be balanced when the loads (impedances) connected in three phases are the same in magnitude as well as in phase.

Q: What is a three-phase unbalanced load?

A: Any three-phase load will be unbalanced if impedances in one or more phases differ from the impedance(s) of the remaining phase(s).

Q: Why is an unbalanced load not normally used on a 3-phase 3-wire system? Are any line/ phase voltages equal in such situations?

A: Normally, the unbalanced load is not employed on 3-phase, 3-wire ungrounded star-connected systems because unbalanced loading may cause different voltage drops in as well lines. Consequently, the three-phase voltages are different (or unbalanced) in magnitude as well as in-phase and it is quite possible that one phase voltage may exceed the line voltage. Such a condition is undesirable since some loads may operate inefficiently due to lowering of the voltage and the other equipment may get damaged due to over-voltage.

Q: Differentiate between balanced and unbalanced three-phase supply and balanced and unbalanced three-phase load.

A: 3-phase supply will be balanced when line-to-line voltages are equal in magnitude and displaced in phase by 120 electrical degrees with respect to each other. On the other hand, a 3-placed supply will be unbalanced when either of the three-phase voltages is unequal in magnitude or the phase angle between these phase voltages is not equal to 120°.

A 3-phase load circuit is said to be balanced when the loads (impedances) connected in the different phases are the same in magnitude as well as in phase. On the other hand, any three-phase load in which the impedances in one or more phases differ from the impedances of other phases is called the unbalanced three-phase load.

Q: What do you mean by the phase sequence of three-phase supply?

A: The phase sequence is meant the order in which the currents or voltages in different phases attain their maximum values one after the other.

Q: What is phase sequence in a 3-phase system? How is the given phase sequence reversed?

A: The phase sequence is the order or sequence in which the currents or voltages in different phases attain their maximum values one after the other. The given phase sequence can be reversed by interchanging any two terminals of the supply.

Q: Differentiate between star and delta connections.

A: In a star-connected system similar (either start or finish) terminals of the three phases are connected together to provide star or neutral points while in a delta-connected system start terminal of one phase is connected to the finished terminal of the second phase and the start terminal of the second phase is connected to finish terminal of the third phase and so on to provide a closed circuit in a 3-phase system.

Long Questions:

Q: A balanced star-connected load of (8 + j6) Ω per phase is connected to a balanced 3-phase 400-V supply. Find the line current, power factor, power and total volt-amperes.

Q: 3 Three equal star-connected inductors take 8 kW at a power factor 0.8 when connected across a 460 V, 3-phase, 3-phase, 3-wire supply. Find the circuit constants of the load per phase

Q: A resistance of 20 ohm, inductance of 0.2 H and capacitance of 150 μF are connected in series and are fed by a 230 V, 50 Hz supply. Find XL, XC, Z, Y, p.f., active power and reactive power.

Q: A three pahse 400-V, 50 Hz, a.c. supply is feeding a three phase deltaconnected load with each phase having a resistance of 25 ohms, an inductance of 0.15 H, and a capacitor of 120 microfarads in series. Determine the line current, volt-amp, active power and reactive volt-amp.

Q: A star-connected alternator supplies a delta connected load. The impedance of the load branch is (8 + j6) ohm/phase. The line voltage is 230 V. Determine (a) current in the load branch, (b) power consumed by the load, (c) power factor of load, (d) reactive power of the load.

Q: A Δ -connected balanced 3-phase load is supplied from a 3-phase, 400-V supply. The line current is 20 A and the power taken by the load is 10,000 W. Find (i) impedance in each branch (ii) the line current, power factor and power consumed if the same load is connected in star.

Q: Three identical impedances are connected in delta to a 3 φ supply of 400 V. The line current is 35 A and the total power taken from the supply is 15 kW. Calculate the resistance and reactance values of each impedance.

Q: The load connected to a 3-phase supply comprises three similar coils connected in star. The line currents are 25 A and the kVA and kW inputs are 20 and 11 respectively. Find the line and phase voltages, the kVAR input and the resistance and reactance of each coil. If the coils are now connected in delta to the same three-phase supply, calculate the line currents and the power taken.

Q: Each phase of a delta-connected load comprises a resistor of 50 Ω and capacitor of 50 μ F in series. Calculate (a) the line and phase currents (b) the total power and (c) the kilovoltamperes when the load is connected to a 440-V, 3-phase, 50-Hz supply.

Q: Three similar choke coils are connected in star to a 3-phase supply. If the line currents are 15 A, the total power consumed is 11 kW and the volt-ampere input is 15 kVA, find the line and phase voltages, the VAR input and the reactance and resistance of each coil.

Q: The load in each branch of a delta-connected balanced 3-φ circuit consists of an inductance of 0.0318 H in series with a resistance of 10 Ω. The line voltage is 400 V at 50 Hz. Calculate (i) the line current and (ii) the total power in the circuit.

Q: A 3-phase load consists of three similar inductive coils, each of resistance 50 Ω and inductance 0.3 H. The supply is 415 V, 50 Hz, Calculate (a) the line current (b) the power factor and (c) the total power when the load is (i) star-connected and (ii) delta-connected.

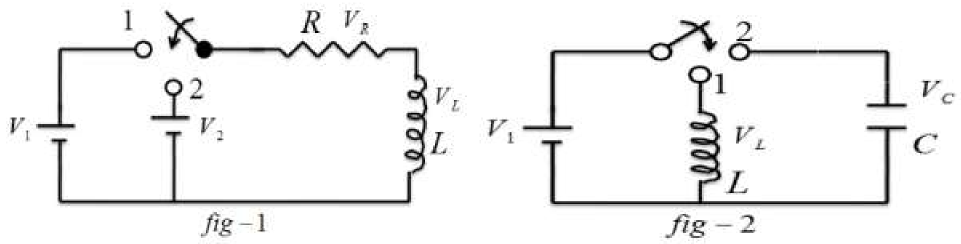
Q: Phase voltage and current of a star-connected inductive load is 150 V and 25 A. Power factor of load is 0.707 (lag). Assuming that the system is 3-wire and power is measured using two wattmeters, find the readings of wattmeters.

Q: In a balanced 3-phase 400-V circuit, the line current is 115.5 A. When power is measured by two wattmeter method, one meter reads 40 kW and the other zero. What is the power factor of the load? If the power factor were unity and the line current the same, what would be the reading of each wattmeter?

Q: The input power to a three-phase motor was measured by two wattmeter method. The readings were 10.4 KW and – 3.4 KW and the voltage was 400 V. Calculate (a) the power factor (b) the line current.

CHAPTER 7. TRANSIENTS:

7.1. Steady state & transient state response:



If in a circuit, the excitation will change or the circuit element will change, then the response (i.e. voltage or current across element) also changes or varies. This variation in the circuit is known as **Transient**.

- The time taken to exit the transient is called as **transient time**.
- In storing elements i.e. inductor or capacitor the transient will only occur.
- Transient analysis is not valid for purely resistive circuit.
- **In a** system, when certain input changes, it takes a while for the output to stabilize and reach its final **state**. This interim phase is called **transient** phase. The final **state** is the **steady state** and system will stay there indefinitely until some input changes again.

Disadvantages of Transient:

(i) Electronic devices are operated at lower frequency or may be damaged.
(ii) Motor will run at high temperature leads to vibration, noise and excessive heat.
(iii) Due to high temperature the motor winding will be damaged.

Notes:

- Inductor stores the energy in the form of current and stores it in its magnetic field. The stored energy is given as $E = \frac{1}{2}Li^2$
- Capacitor stores the energy in the form of voltage and stores it in its electric field. The stored energy is given as $E = \frac{1}{2}Cv^2$
- Inductor and Capacitor does not dissipate any energy, if there is no dissipating element i.e. the resistance.

Standard Charging equation:

The standard charging equation is given by,

$$y(t) = y_0\left(1 - e^{\frac{-t}{\lambda}}\right)$$

Where λ = time constant in second

t = Time in second

At $t = 0$, $$y(t) = y_0\left(1 - e^{\frac{-0}{\lambda}}\right) = y_0\left(1 - e^0\right) = y_0(1-1) = 0$$

When, $$t = \lambda, y(t) = y_0\left(1 - e^{-1}\right) = 0.632 y_0$$

$$t = 2\lambda, y(t) = y_0\left(1 - e^{-2}\right) = 0.865 y_0$$

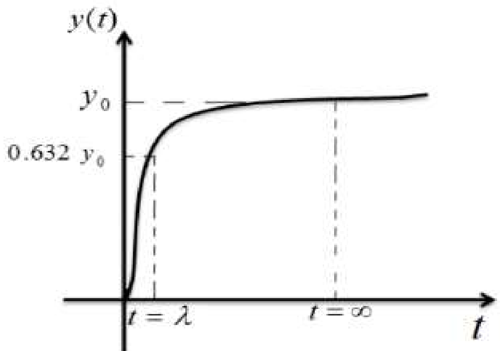
$$t = 3\lambda, y(t) = y_0\left(1 - e^{-3}\right) = 0.951 y_0$$

$$t = 4\lambda, y(t) = y_0\left(1 - e^{-4}\right) = 0.865 y_0$$

$$t = 5\lambda, y(t) = y_0\left(1 - e^{-5}\right) = 0.993 y_0$$

$$t = \infty, y(t) = y_0\left(1 - e^{-\infty}\right) = y_0$$

Now plotting the curve between t & $y(t)$,we have,



[Standard charging curve]

Time constant: It is the time taken by the response to achieve 63.2% of its normal value or maximum value or standard value.

Settling time: Settling time is defined as the time taken by the response to reach nearby its maximum value.

Standard Discharging equation:

The standard discharging equation is given by,

$$y(t) = y_0 e^{\frac{-t}{\lambda}}$$

Where λ = time constant in second

t = Time in second

At $t = 0$, $$y(t) = y_0 e^{\frac{-0}{\lambda}} = y_0 e^0 = y_0(1) = y_0$$

When, $t=\lambda\,,\ y(t)=y_0e^{\frac{-\lambda}{\lambda}}=y_0e^{-1}=0.368y_0$

$$t=2\lambda\,,\ y(t)=y_0e^{\frac{-2\lambda}{\lambda}}=y_0e^{-2}=0.135y_0$$

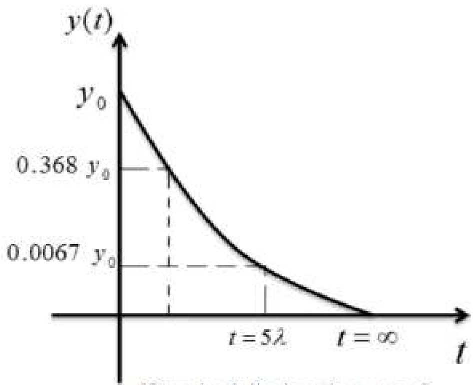
$$t=3\lambda\,,\ y(t)=y_0e^{\frac{-3\lambda}{\lambda}}=y_0e^{-3}=0.049y_0$$

$$t=4\lambda\,,\ y(t)=y_0e^{\frac{-4\lambda}{\lambda}}=y_0e^{-4}=0.018y_0$$

$$t=5\lambda\,,\ y(t)=y_0e^{\frac{-5\lambda}{\lambda}}=y_0e^{-5}=0.0067y_0$$

$$t=\infty,\ y(t)=y_0e^{-\infty}=0$$

Now plotting the curve between t & $y(t)$,we have,



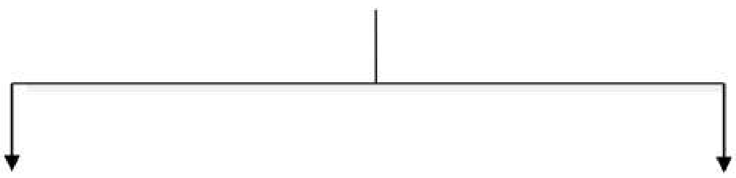
[Standard discharging curve]

Time constant: Time constant is defined as the time taken by the response to reach 36.8% of its normal value or maximum value..

Settling time: Settling time is defined as the time taken by the response to reach nearby its minimum value or zero value.

7.2. Response to R-L, R-C & RLC circuit under DC condition:

Transient response is of two types:

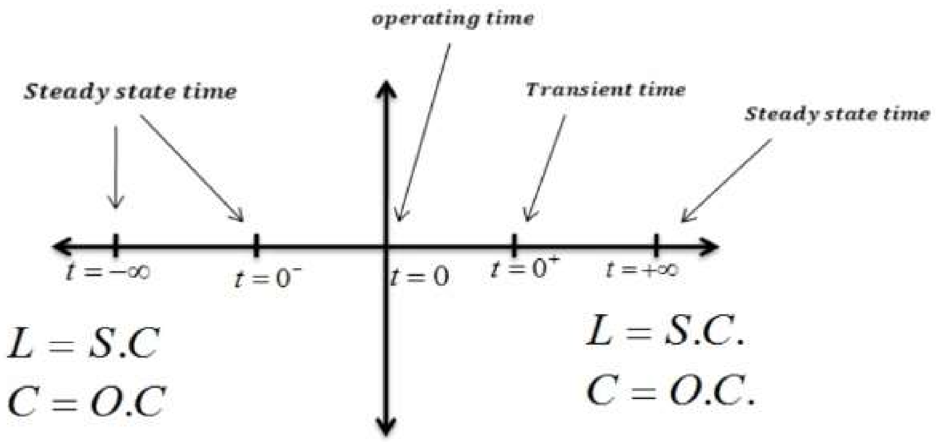


Forced response
[Charging response]

without forced response
or
Source free response

[Discharging response]

NOTE:



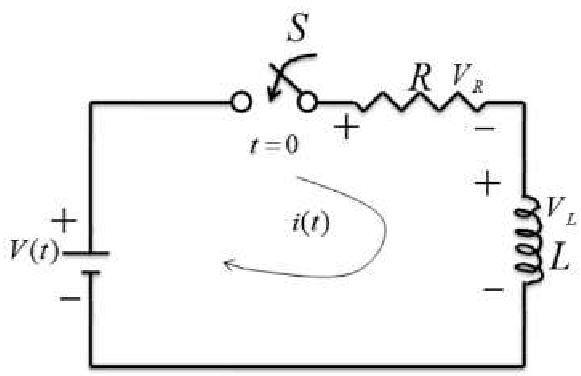
$t = 0 \rightarrow$ It is the operating time when switch is just close or open.

$t = 0^{-} \rightarrow$ It is the time just before the operating time i.e. steady state time.

$t = 0^{+} \rightarrow$ It is the time just after the operating time i.e. transient time.

$t = \infty \rightarrow$ It is the operating time or normal time or steady state time.

- **Charging response in R-L circuit (forced response):**



We know that, $i = \frac{v}{R}, V_L = L\frac{di}{dt}, i_C = C\frac{dv}{dt}$

In the circuit above applying KVL, we have,

$$V_R + V_L = V$$

$$= i(t)R + L\frac{di(t)}{dt} = v(t)$$

$$= Ri(t) + L\frac{di(t)}{dt} = v(t)$$

Now divide L in both the sides, we get,

$$\frac{di(t)}{dt} + \frac{R}{L}i(t) = \frac{v(t)}{L} \text{ .... (1).}$$

The eq. (1) is a linear differential equation in 1st order. $\frac{di}{dt} + Pi = Q$

The solution of the linear differential equation is given as,

$$C.F. + P.I.$$

$C.F =$

$$\frac{di(t)}{dt} + \frac{R}{L}i(t) = 0$$

Or, $\frac{di(t)}{dt} = -\frac{R}{L}i(t)$

Now integrating both the side, we get,

Or, $\int \frac{di}{i} = -\frac{R}{L}\int dt$

$\log i = -\frac{R}{L}t + \mathrm{k}$

Or, $e^{\log i} = Ce^{-\frac{R}{L}t}$

$i(t) = Ce^{-\frac{R}{L}t}$

$P.I =$

Put $t = \infty$

$i(t) = \frac{V}{R}$

The solution of the linear differential equation is given as,

$$C.F. + P.I.$$

$i(t) = C.F + P.I$

$= Ce^{-\frac{R}{L}t} + \frac{V}{R}$

Put $t = 0$, as it is the starting point

$$0 = Ce^{-0} + \frac{V}{R}$$

$$C = -\frac{V}{R}$$

Now the current $i(t)$ is given as,

$$i(t) = \frac{V}{R} - \frac{V}{R}e^{-\frac{t}{\lambda}}$$

$$= \frac{V}{R}\left(1 - e^{-\frac{t}{\lambda}}\right)$$

$\therefore$ $$i(t) = I_0\left(1 - e^{-\frac{t}{\lambda}}\right)$$ → **charging equation of current.**

Where λ = time constant and is given by,

$$\lambda = \frac{L_{eq}}{R_{eq}}$$

Voltage across ***R*** is given as,

$\therefore$ $$V_R = i \times R = \frac{V}{R}\left(1 - e^{-\frac{t}{\lambda}}\right)R$$

$\therefore$ $$V_R = V\left(1 - e^{-\frac{t}{\lambda}}\right)$$

Voltage across ***L*** is given as,

$\therefore$ $$V_L = L\frac{di(t)}{dt}$$

$$= L\frac{d}{dt}\left[I_0\left(1 - e^{-\frac{t}{\lambda}}\right)\right]$$

$$= L\frac{d}{dt}\left[I_0\left(1-e^{-\frac{t}{\lambda}}\right)\right]$$

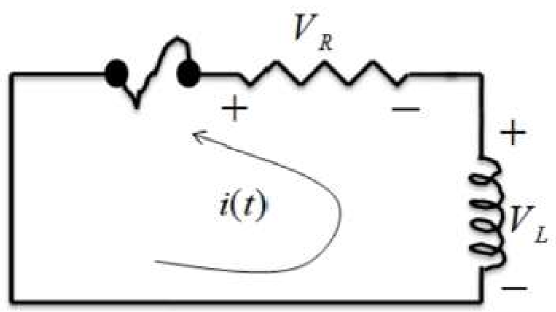
$$= LI_0\frac{d}{dt}\left[1-e^{-\frac{t}{\lambda}}\right]$$

$$= LI_0\left[0-\left(-\frac{t}{\lambda}\right)e^{-\frac{t}{\lambda}}\right]$$

$$= LI_0\left[0+\frac{R}{L}te^{-\frac{t}{\lambda}}\right]$$

$$\therefore \quad V_L = Ve^{-\frac{t}{\lambda}}$$

- **Discharging response in R-L circuit (source free response):**



Now the inductor starts discharging through the resistance and by applying **KVL** we have,

$$V_R + V_L = 0$$

Or, $$i(t)R + L\frac{di(t)}{dt} = 0$$

Now divide ***L*** in both the sides, we get,

$$\frac{di(t)}{dt} + \frac{R}{L}i(t) = 0 \text{ .... (1).}$$

The eq. (1) is a linear differential equation in 1st order. $\frac{di}{dt} + Pi = Q$

The solution of the linear differential equation is given as,

$$C.F. + P.I.$$

$C.F =$

$$\frac{di(t)}{dt} + \frac{R}{L}i(t) = 0$$

Or, $\frac{di(t)}{dt} = -\frac{R}{L}i(t)$

Now integrating both the side, we get,

Or, $\int \frac{di}{i} = -\frac{R}{L}\int i(t)$

$\log i = \frac{R}{L}t + \mathrm{k}$

Or, $e^{\log i} = Ce^{-\frac{R}{L}t}$

$i(t) = Ce^{-\frac{R}{L}t}$

$P.I =$

If $Q = 0$, $P.I = 0$

The solution of the linear differential equation is given as,

$$C.F. + P.I.$$

$i(t) = C.F + P.I$

$= Ce^{-\frac{R}{L}t}$

Applying initial condition at $t = 0$,

$\frac{V}{R} = Ce^{-0}$

$C = \frac{V}{R}$

Now the current $i(t)$ is given as,

$i(t) = \frac{V}{R}e^{-\frac{t}{\lambda}}$

$\therefore \quad i(t) = I_0 e^{-\frac{t}{\lambda}}$ **→ discharging equation of current.**

Where λ = time constant and is given by,

$$\lambda = \frac{L_{eq}}{R_{eq}}$$

Voltage across ***R*** is given as,

$$\therefore \qquad V_R = i(t) \times R = I_0 e^{-\frac{t}{\lambda}} \times R$$

$$\therefore \qquad V_R = V_0 e^{-\frac{t}{\lambda}}$$

Voltage across ***L*** is given as,

$$\therefore \qquad V_L = L \frac{di(t)}{dt}$$

$$= L \frac{d}{dt}\left[I_0 e^{-\frac{t}{\lambda}} \right]$$

$$= L \frac{d}{dt}\left[I_0 e^{-\frac{R}{L}t} \right]$$

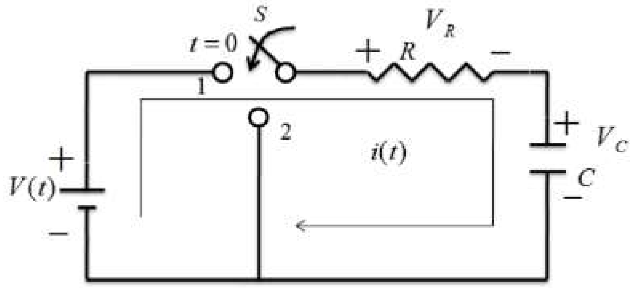
$$= L I_0 \left[\left(-\frac{R}{L} \right) e^{-\frac{R}{L}t} \right]$$

$$= -I_0 \times \mathrm{Re}^{-\frac{R}{L}t}$$

$$\therefore \qquad V_L = -V_0 e^{-\frac{t}{\lambda}}$$

NOTE: When the inductor is discharging, the voltage across inductor (V_L) is always negative.

❖ Charging response in R-C circuit (forced response):



When the switch (S) in position 1 at $t = 0$,

By applying ***KVL,*** We get,

$$V = V_R + V_C$$

Or, $IR + V_C = V$

Or, $C\frac{dV_C}{dt}R + V_C = V$ $\qquad \because i_C = C\frac{dv}{dt}$

Or, $dv = \frac{1}{C}\int i_C dt$

Or, $V_C = \frac{1}{C}\int i_C dt$

Or, $\frac{dV_C}{dt} + \frac{V_C}{RC} = \frac{V}{RC}$ .... (1) $\qquad$ ($\because$ Dividing R on both the sides).

The above equation (1) is a first order differential equation and in the form of $\frac{dv}{dt} + pv = Q$.

Now the solution of the above eq. (1) is given by,

$$C.F. + P.I.$$

$C.F =$

$$\frac{dV_C}{dt} + \frac{V_C}{RC} = 0$$

$$\frac{dV_C}{dt} = -\frac{V_C}{RC}$$

Or, $$\frac{dV_C}{V_C} = -\frac{1}{RC}dt$$

Now integrating both the sides, we have, Or, $$\int \frac{dV_C}{V_C} = -\frac{1}{RC}\int dt$$

Or, $$\log V_C = -\frac{t}{RC} + \mathrm{k}$$

Or, $$V_C = Ce^{-\frac{t}{RC}}$$

$P.I =$

At $t = \infty, V_C = V$

The net solution is

$$C.F. + P.I.$$

$$V_C = V + Ce^{-\frac{t}{RC}}$$

Now, putting the initial condition at $t = 0, V_C = 0$

$$0 = V + Ce^{-0}$$

Or, $$0 = V + C$$

Or, $$C = -V$$

Now putting the value of C in above equation, we get,

$$V_C(t) = V + (-V)e^{-\frac{t}{RC}}$$

Or, $$V_C(t) = V - Ve^{-\frac{t}{RC}}$$

Again, $$\lambda = RC$$

So, $$V_C(t) = V_0\left(1 - e^{-\frac{t}{\lambda}}\right)$$

Where $\lambda =$ time constant and is given by,

$\lambda = R_{eq} \times C_{eq}$

Now net current or capacitor current is given as,

$$i_C(t) = C\frac{dV_C}{dt}$$

$$= C\frac{d}{dt}\left[V_0\left(1-e^{-\frac{t}{\lambda}}\right)\right]$$

$$= CV_0\frac{d}{dt}\left[\left(1-e^{-\frac{t}{\lambda}}\right)\right]$$

$$= CV_0\left[0-e^{-\frac{t}{RC}}\times\left(-\frac{1}{RC}\right)\right]$$

$$= \frac{V_0}{R}e^{-\frac{t}{RC}}$$

$\therefore$ $$i(t) = I_0 e^{-\frac{t}{\lambda}} Amp.$$

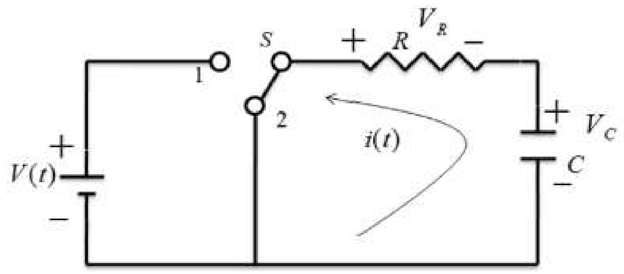
$\therefore$ Now voltage across resistor (V_R) is

$$V_R = i(t)\times R$$

$$= I_0 e^{-\frac{t}{\lambda}}\times R$$

$\therefore$ $$V_R = V_0\times e^{-\frac{t}{\lambda}}$$

❖ **Discharging response in R-C circuit (source free response):**



When the switch moves from (1) to (2), the capacitor starts discharging.

Now by applying ***KVL***, We get,

$$V_R + V_C = 0$$

Or, $$i(t)R + V_C = 0$$

Or, $$C\frac{dV_C(t)}{dt} \times R + V_C = 0 \qquad \because \quad i(t) = C\frac{dV_C(t)}{dt}$$

Now dividing ***RC*** on both the sides, we have,

Or, $$\frac{dV_C(t)}{dt} + \frac{1}{RC}V_C = 0 \text{ .... (1).}$$

The above equation (1) is in the form of first order differential equation i.e. $\frac{dv}{dt} + pv = Q$

∴ Now the solution of the above eq. (1) is given by,

$$C.F. + P.I.$$

$C.F =$

$$\frac{dV_C(t)}{dt} + \frac{V_C}{RC} = 0$$

Or, $$\frac{dV_C(t)}{dt} = -\frac{1}{RC}V_C(t)$$

Now integrating both the sides,

Or, $$\int \frac{dV_C(t)}{V_C(t)} = -\frac{1}{RC}\int dt$$

Or, $$\log V_C(t) = -\frac{t}{RC} + \text{k}$$

Or, $V_C(t) = Ce^{-\frac{t}{RC}}$

$P.I =$

At $Q = 0, P.I = 0$

Now the solution is given as,

$$C.F. + P.I.$$

$$V_C(t) = Ce^{-\frac{t}{RC}} + 0$$

Now putting the initial condition at $t = 0$

$$V = Ce^{-0}$$

Or, $C = V$

$\therefore$ $$V_C(t) = V_0 e^{-\frac{t}{RC}} volt$$

Now $i(t) = C\frac{dV_C(t)}{dt}$

$$= C\frac{d}{dt}\left(V_0 e^{-\frac{t}{RC}}\right)$$

$$= CV_0\frac{d}{dt}\left(e^{-\frac{t}{RC}}\right)$$

$$= CV_0\left(-\frac{1}{RC}\right) \times e^{-\frac{t}{RC}}$$

$$= -\frac{V_0}{R}e^{-\frac{t}{RC}}$$

$\therefore$ $$i(t) = -I_0 e^{-\frac{t}{\lambda}} Amp$$

Where $\lambda =$ time constant and is given by,

$\lambda = R_{eq} \times C_{eq}$

$\therefore$ Voltage across resistance $V_R(t)$ is given as,

$$V_R(t) = i(t) \times R$$

$$= -I_0 e^{-\frac{t}{\lambda}} \times R$$

$$V_R(t) = -V_0 \times e^{-\frac{t}{\lambda}} volt$$

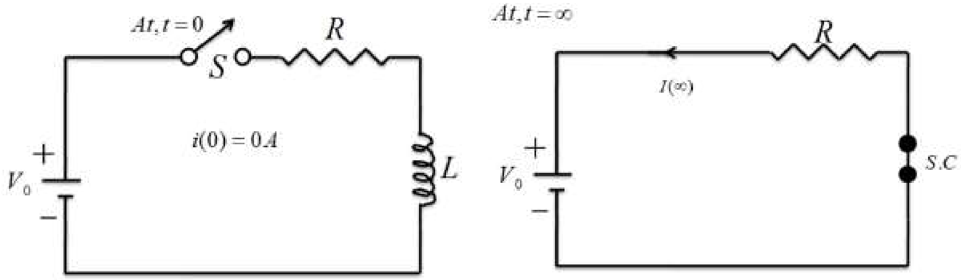
NOTE:

1) $i(t) = I\left(\substack{Final\ value \\ \infty}\right) + \left[I\left(\substack{initial\ value \\ 0}\right) - I\left(\substack{Final\ value \\ \infty}\right)\right] \times e^{-\frac{t}{\tau}} \rightarrow for\ inductor$

2) $V_C(t) = V_C\left(\substack{Final\ value \\ \infty}\right) + V_C\left[\left(\substack{initial\ value \\ 0}\right) - V_C\left(\substack{Final\ value \\ \infty}\right)\right] \times e^{-\frac{t}{\tau}} \rightarrow for\ capacitor$

R – L Circuit:

Case-1: (charging)



At $t = 0, i(0) = 0Amp$

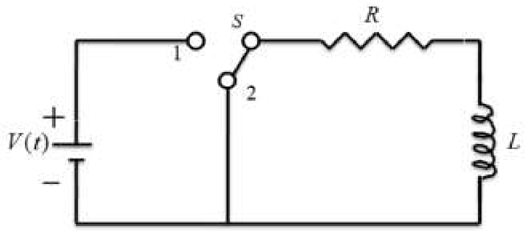
At $t = \infty, i(\infty) = \frac{V_0}{R} = I_0 Amp$

$$i(t) = I_0 + (0 - I_0) \times e^{-\frac{t}{\lambda}}$$

Or, $$i(t) = I_0 - I_0 \times e^{-\frac{t}{\lambda}}$$

Or, $$i(t) = I_0\left(1 - e^{-\frac{t}{\lambda}}\right)$$

Case-2: (Discharging)



At $t = 0, i(0) = I_0$

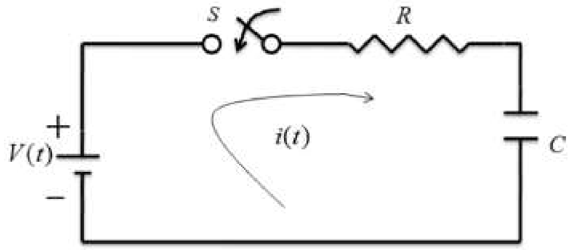
At $t = \infty, i(\infty) = 0$

$$i(t) = 0 + (I_0 - 0) \times e^{-\frac{t}{\lambda}}$$

Or, $$i(t) = I_0 \times e^{-\frac{t}{\lambda}}$$

<u>$R - C$ Circuit:</u>

Case-1: (charging)



At, $t = 0, V_C(t) = V_C(0) = 0$

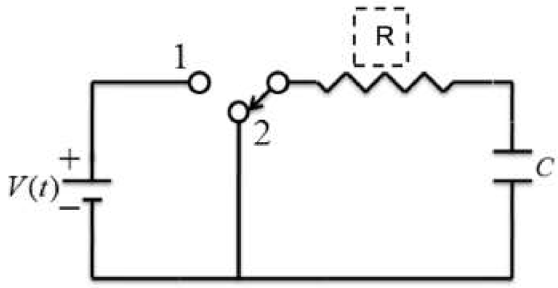
At, $t = \infty, V_C(t) = V_C(\infty) = V_0$

$$V_C(t) = V_0 + [0 - V_0] \times e^{-\frac{t}{\lambda}}$$

$$= V_0 - V_0 \times e^{-\frac{t}{\lambda}}$$

$$V_C(t) = V_0\left(1 - e^{-\frac{t}{\lambda}}\right) volt$$

Case-2: (Discharging)



At, $t = 0, V_C(t) = V_C(0) = V_0$

At, $t = \infty, V_C(t) = V_C(\infty) = 0$

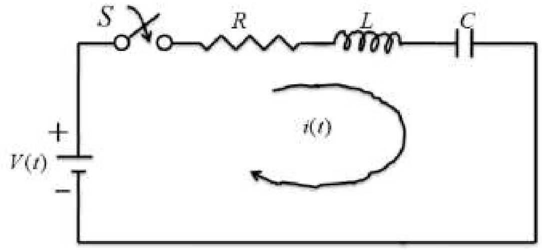
$$V_C(t) = 0 + [V_0 - 0] \times e^{-\frac{t}{\lambda}}$$

$$= V_0 \times e^{-\frac{t}{\lambda}}$$

$$V_C(t) = V_0 \times e^{-\frac{t}{\lambda}} volt$$

❖ DC response of an R-L-C circuit:

Consider a circuit consisting of a resistance, inductance and capacitance as shown in fig. The capacitor and inductor in the circuit initially uncharged and are in series with the resistor. When the switch S is closed at $t = 0$, we can find the complete solution for the current. Application of Kirchhoff's voltage law to the circuit results the following differential equation.



Applying **KVL** to the above circuit, we have, $V_R + V_L + V_C = V$

$$V = Ri + L\frac{di}{dt} + \frac{1}{C}\int idt \text{ ....} (1)$$

By differentiating the above equation w.r.t. **t**, we get,

$$0 = R\frac{di}{dt} + L\frac{d^2t}{dt^2} + \frac{1}{C}i$$

Or, $$\frac{d^2t}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0 \text{ .... } (2)$$ (Dividing ***L*** in all)

The above equation-(2) is a second order differential equation with only the complementary function. The particular solution for the above equation is zero. The characteristics equation for this type of differential equation is

$$\left(D^2 + \frac{R}{L}D + \frac{1}{LC}\right) \times i = 0 \text{ .... } (3) \qquad \left(\because \frac{d}{dt}\right) = D$$

Now the roots of the equations-(3) are D_1 & D_2

$$D_1, D_2 = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

Let $\alpha = \frac{R}{2L}$ & $\beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

So $D_1 = -\alpha + \beta$ and $D_2 = -\alpha - \beta$

Where $\omega = \frac{1}{\sqrt{LC}}$ **→Natural frequency,** $\alpha = \frac{R}{2L}$ **→ Damping coefficient and**

$$\frac{\alpha}{\omega} = \xi = \frac{R}{2}\sqrt{\frac{C}{L}} \rightarrow$$ **Damping factor**

Case-1: If $\left(\frac{R}{2L}\right)^2 \rangle \frac{1}{LC} \rightarrow$ over damped response (roots are negative, real and unequal).

So $D_1 = -\alpha + \beta$ and $D_2 = -\alpha - \beta$

The solution of the above equation is

$$i = C_1 e^{-D_1 t} + C_2 e^{-D_2 t}$$

Case-2: If $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC} \rightarrow$ critically damped response (roots are negative, real and equal).

So $D_1 = D_2 = -\alpha$

The solution of the above equation is

$$i = C_1 e^{-\alpha t} + C_2 e^{-\alpha t}$$

Case-3: If $\left(\frac{R}{2L}\right)^2 \langle \frac{1}{LC} \rightarrow$ under damped response (roots will be complex conjugate).

So $D_1 = -\alpha + j\beta$ and $D_2 = -\alpha - j\beta$

The solution of the above equation is

$$i = e^{-\alpha t}\left[C_1 \cos \beta t + C_2 \sin \beta t\right]$$

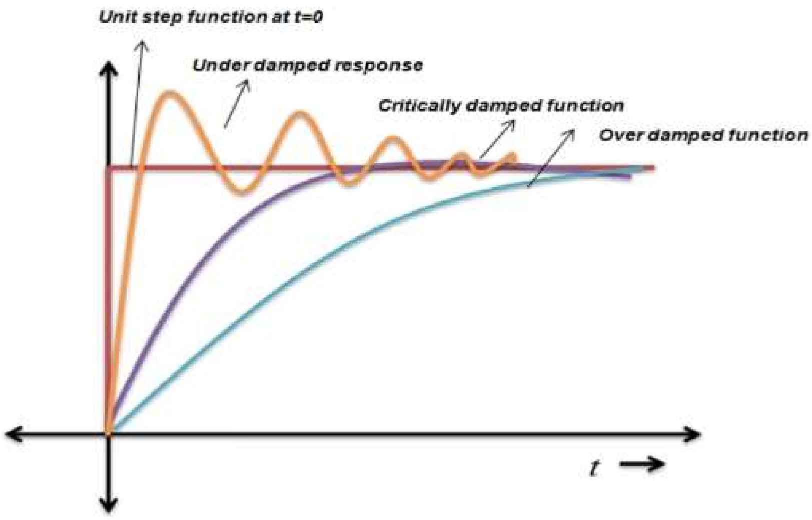
Case-4: If $R = 0 \rightarrow$ only oscillation response (roots will be imaginary).

So $D_1 = j\sqrt{\frac{1}{LC}} = j\omega$ and $D_2 = -j\omega$

The solution of the above equation is

$$i = \left[C_1 \cos \omega t + C_2 \sin \omega t\right]$$

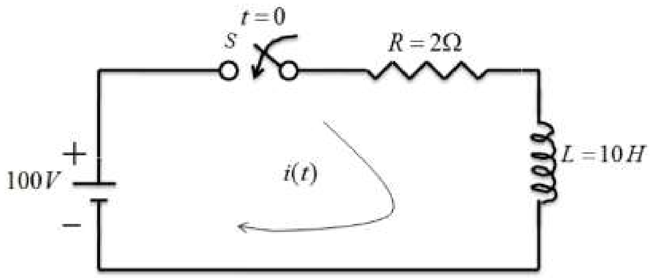
Now the responses of above four cases are shown in fig. below.



7.3. Solve numerical problems:

Examples:

Q.1. Find the current in a series ***R-L*** circuit having ***R=2Ω & L=10H,*** while a dc voltage of ***100V*** is applied. What is the value of this current after 5sec of switching **ON**?



Ans:

We know that, the charging current equation is given as,

$$i(t) = I_0\left(1 - e^{-\frac{t}{\lambda}}\right)$$

λ = time constant and is given as,

$$\lambda = \frac{L_{eq}}{R_{eq}} = \frac{10}{2} = 5\sec$$

Steady state current $I_0 = I_{S.S} = \frac{V_0}{R} = \frac{100}{2} = 50A$

$$\therefore \qquad i(t) = I_0\left(1 - e^{-\frac{t}{\lambda}}\right)$$

$$= 50\left(1 - e^{-\frac{5}{5}}\right)$$

$$= 50\left(1 - e^{-1}\right)$$

$$= 50(1 - 0.3678)$$

$$= 31.61A$$

$$\therefore \qquad i(t) = 31.61A$$

Q.2. A dc voltage of ***100V*** is applied to a coil of ***R=10Ω & L=20H.*** What is the value of current 0.2 sec after switching ***ON*** and the time taken for the current to reach one half of its final value?

Ans:

We know that, the charging current equation is given as,

$$i(t) = I_0\left(1 - e^{-\frac{t}{\lambda}}\right)$$

λ = time constant and is given as,

$$\lambda = \frac{L_{eq}}{R_{eq}} = \frac{20}{10} = 2\sec$$

Steady state current $I_0 = I_{S.S} = \frac{V_0}{R} = \frac{100}{10} = 10A$

$$\therefore \qquad i(t) = I_0\left(1 - e^{-\frac{t}{\lambda}}\right)$$

$$= 10\left(1 - e^{-\frac{0.2}{2}}\right)$$

$$= 10\left(1 - e^{-0.1}\right)$$

$$= 10(1 - 0.9048)$$

$$= 0.952A$$

$\therefore \quad i(t) = 0.952A$

Again time taken ***(t)*** to reach half of I_0 i.e. $\frac{I_0}{2} = \frac{10}{2} = 5A$ is given as,

$$5 = 10\left(1 - e^{-\frac{t}{0.2}}\right)$$

Or, $\quad 1 - e^{-\frac{t}{0.2}} = \frac{5}{10}$

Or, $\quad 1 - 0.5 = e^{-\frac{t}{0.2}}$

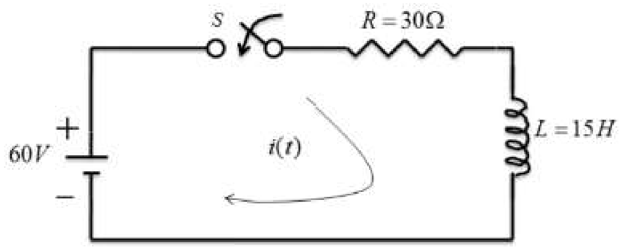
Or, $\quad \ln\left(e^{-\frac{t}{0.2}}\right) = \ln(0.5)$

Or, $\quad -\frac{t}{0.2} = -0.6931$

Or, $\quad t = 0.2 \times 0.6931 = 0.1386$

$\therefore \quad t = 0.1386\,\text{sec}$

Q.3. A series ***R-L*** circuit with ***R=30Ω & L=15H*** has a constant voltage ***V=60V*** applied at ***t=0*** as shown in fig. below. Determine the current ***i,*** the voltage across resistor and across inductor.



Ans:

We know that, the charging current equation is given as,

$$i(t) = I_0\left(1 - e^{-\frac{t}{\lambda}}\right)$$

λ = time constant and is given as,

$$\lambda = \frac{L_{eq}}{R_{eq}} = \frac{15}{30} = 0.5\sec$$

Steady state current $I_0 = I_{S.S} = \frac{V_0}{R} = \frac{60}{30} = 2A$

$\therefore$ $$i(t) = I_0\left(1 - e^{-\frac{t}{\lambda}}\right)$$

$$= 2\left(1 - e^{-\frac{t}{0.5}}\right)$$

$$= 2\left(1 - e^{-2t}\right)A$$

$\therefore$ $$i(t) = 2\left(1 - e^{-2t}\right)Amp.$$

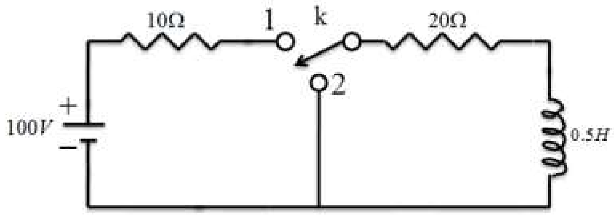
$\therefore$ Voltage across the resistor $(V_R) =$

$$V_R = i(t) \times R = 2\left(1 - e^{-2t}\right) \times 30 = 60 \times \left(1 - e^{-2t}\right)volt$$

$\therefore$ Voltage across the inductor $(V_L) =$

$$V_L = V_0 \times e^{-2t} = 60 \times e^{-2t}volt$$

Q.4. In the circuit below, the switch 'k' is kept first in at position-1 & steady state condition is reached. At ***t=0,*** the switch is moved to position-2. Find the current in both the cases.

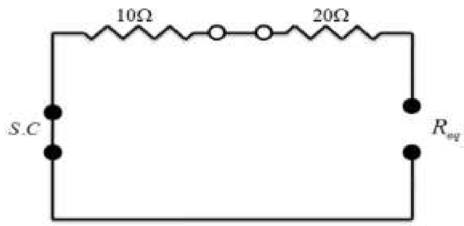


Ans: When ***'k'*** is in **position-1**, the charging current equation is given as,

$$i(t) = I_0\left(1 - e^{-\frac{t}{\lambda}}\right)$$

λ = time constant and is given as,

$$\lambda = \frac{L_{eq}}{R_{eq}} = \frac{0.5}{30} = \frac{1}{60}\sec$$



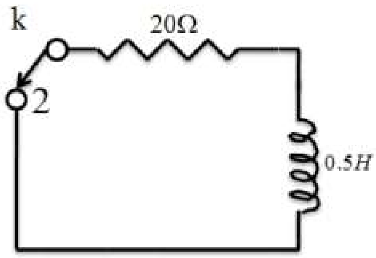
Steady state current $I_0 = I_{S.S} = \frac{V_0}{R_{eq}} = \frac{100}{30} = 3.33A$

$$\therefore \quad i(t) = I_0\left(1 - e^{-\frac{t}{\lambda}}\right)$$

$$= 3.33\left(1 - e^{-60t}\right)A$$

$$\therefore \quad i(t) = 3.33\left(1 - e^{-60t}\right)Amp.$$

When ***'k'*** is in **position-2** at **t=0**, the current starts discharging and the discharging current equation is given as,



λ = time constant and is given as,

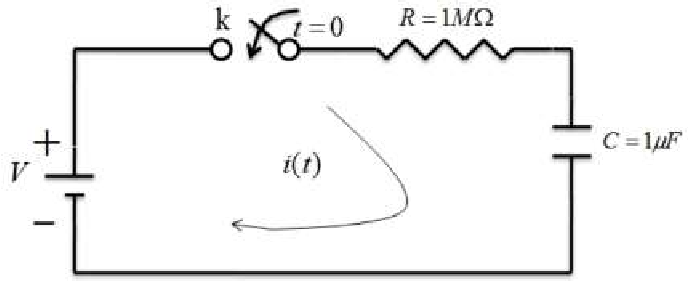
$$\lambda = \frac{L_{eq}}{R_{eq}} = \frac{0.5}{20} = \frac{1}{40}\sec$$

$$i(t) = I_0 \times e^{-\frac{t}{\lambda}}$$

$$= 3.33 \times e^{-40t}\,Amp.$$

$$\therefore \quad i(t) = 3.33 \times e^{-40t}\,Amp.$$

Q.5. Calculate the time taken by a capacitor of $1\mu F$ and is series with a $1M\Omega$ resistance to be charging up to 80% of its final value?



Ans: As we know that $i = \frac{dq}{dt}$ = rate of change of charge. So from this relation, $q \propto i$

The charging current of capacitor is given as,

$$i(t) = I_0\left(1 - e^{-\frac{t}{\lambda}}\right)$$

Similarly, $$q(t) = Q_0\left(1 - e^{-\frac{t}{\lambda}}\right)$$

Time constant (λ) is given as $\lambda = R_{eq} \times C_{eq} = 1\times 10^6 \times 10 \times 10^{-6} = 1\,\text{sec}.$

So, $$0.8Q_0 = Q_0\left(1 - e^{-\frac{t}{\lambda}}\right)$$

$$= 1 - e^{-\frac{t}{1}} = \frac{0.8Q_0}{Q_0}$$

Or, $$1 - e^{-t} = 0.8$$

Or, $$e^{-t} = 1 - 0.8$$

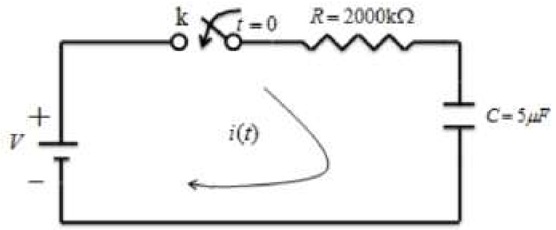
Or, $$e^{-t} = 0.2$$

Or, $$\ln\left(e^{-t}\right) = \ln 0.2$$

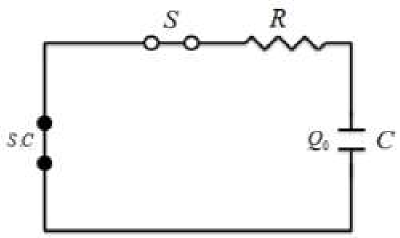
Or, $$-t = -1.609$$

Or, $$t = 1.609\,\text{sec}.$$

Q.6. A dc constant voltage source feeds a resistance of $2000\text{k}\Omega$ in series with a $5\mu F$ capacitor. Find the time taken by the capacitor, when the charge retained will decay to 50% of the initial value, the voltage source being short circuited?



Ans:



$i = \frac{dq}{dt}$ = rate change of charge, so $q \propto i$

The charging current of capacitor is given as,

$$i(t) = I_0 \times e^{-\frac{t}{\lambda}}$$

$$q(t) = Q_0 \times e^{-\frac{t}{\lambda}}$$

Again, $\lambda = R_{eq} \times C_{eq} = 2000 \times 10^3 \times 5 \times 10^{-6} = 10\text{sec}.$

Now the discharging of capacitor is given as,

$$q(t) = Q_0 \times e^{-\frac{t}{\lambda}}$$

Or, $$0.5Q_0 = Q_0 \times e^{-\frac{t}{10}}$$

Or, $$e^{-\frac{t}{10}} = \frac{0.5Q_0}{Q_0}$$

Or, $$e^{-\frac{t}{10}} = 0.5$$

Or, $\ln\left(e^{-\frac{t}{10}}\right) = \ln 0.5$

Or, $-\frac{t}{10} = -0.6931$

Or, $t = 6.93\,\text{sec}$

QUESTION BANK

Short Questions With Answer:

Q: Define response.

A: The current flowing through or voltage across branches in the circuit is called response.

Q: Define transient response.

A: The voltage or current are changed from one transient state to another transient state is called transient response.

Q: What is transient?

A: The state (or condition) of the circuit from the transient of switching to attainment of steady state is called transient state or simply transient.

Q: Why transient occurs in electric circuits?

A: The inductance will not allow the sudden change in current and the capacitance will not allow sudden change in voltage. Hence inductive and capacitive circuits (or in general reactive circuits0transient occurs during switching operation.

Q: Define time constant of RL circuit.

A: The time constant of RL circuit is defined as the ratio of inductance and resistance of the circuit. Time constant $\tau = L/R$

Q: Define time constant of RC circuit.

A: The time constant of RC circuit is defined as the product of capacitance and resistance of the circuit. Time constant $\tau = \mathrm{RC}$

Q: What is damping ratio?

A: The ratio of resistance of the circuit and resistance for critical damping is called damping ratio.

Q: What is critical damping?

A: The critical damping is the condition of the circuit at which the oscillations in the response are just eliminated. This is possible by increasing the value of resistance in the circuit.

Q: What is critical resistance?

A: The critical resistance is the value of the resistance of the circuit to achieve critical damping.

Long Questions:

Q: A balanced star-connected load of (8 + j6) Ω per phase is connected to a balanced 3-phase 400-V supply. Find the line current, power factor, power and total volt-amperes.

Q: 3 Three equal star-connected inductors take 8 kW at a power factor 0.8 when connected across a 460 V, 3-phase, 3-phase, 3-wire supply. Find the circuit constants of the load per phase.

Q: A resistance of 20 ohm, inductance of 0.2 H and capacitance of 150 μF are connected in series and are fed by a 230 V, 50 Hz supply. Find XL, XC, Z, Y, p.f., active power and reactive power.

Q: A three pahse 400-V, 50 Hz, a.c. supply is feeding a three phase deltaconnected load with each phase having a resistance of 25 ohms, an inductance of 0.15 H, and a capacitor of 120 microfarads in series. Determine the line current, volt-amp, active power and reactive volt-amp.

Q: A star-connected alternator supplies a delta connected load. The impedance of the load branch is (8 + j6) ohm/phase. The line voltage is 230 V. Determine (a) current in the load branch, (b) power consumed by the load, (c) power factor of load, (d) reactive power of the load.

Q: A Δ -connected balanced 3-phase load is supplied from a 3-phase, 400-V supply. The line current is 20 A and the power taken by the load is 10,000 W. Find (i) impedance in each branch (ii) the line current, power factor and power consumed if the same load is connected in star.

Q: Three identical impedances are connected in delta to a 3 φ supply of 400 V. The line current is 35 A and the total power taken from the supply is 15 kW. Calculate the resistance and reactance values of each impedance.

Q: The load connected to a 3-phase supply comprises three similar coils connected in star. The line currents are 25 A and the kVA and kW inputs are 20 and 11 respectively. Find the line and phase voltages, the kVAR input and the resistance and reactance of each coil. If the coils are now connected in delta to the same three-phase supply, calculate the line currents and the power taken.

Q: Each phase of a delta-connected load comprises a resistor of 50 Ω and capacitor of 50 μ F in series. Calculate (a) the line and phase currents (b) the total power and (c) the kilovoltamperes when the load is connected to a 440-V, 3-phase, 50-Hz supply.

Q: Three similar choke coils are connected in star to a 3-phase supply. If the line currents are 15 A, the total power consumed is 11 kW and the volt-ampere input is 15 kVA, find the line and phase voltages, the VAR input and the reactance and resistance of each coil.

Q: The load in each branch of a delta-connected balanced 3-φ circuit consists of an inductance of 0.0318 H in series with a resistance of 10 Ω. The line voltage is 400 V at 50 Hz. Calculate (i) the line current and (ii) the total power in the circuit.

Q: A 3-phase load consists of three similar inductive coils, each of resistance 50 Ω and inductance 0.3 H. The supply is 415 V, 50 Hz, Calculate (a) the line current (b) the power factor and (c) the total power when the load is (i) star-connected and (ii) delta-connected.

Q: Phase voltage and current of a star-connected inductive load is 150 V and 25 A. Power factor of load is 0.707 (lag). Assuming that the system is 3-wire and power is measured using two wattmeters, find the readings of wattmeters.

Q: In a balanced 3-phase 400-V circuit, the line current is 115.5 A. When power is measured by two wattmeter method, one meter reads 40 kW and the other zero. What is the power factor of the load? If the power factor were unity and the line current the same, what would be the reading of each wattmeter?

CHAPTER 8. TWO PORT NETWORK:

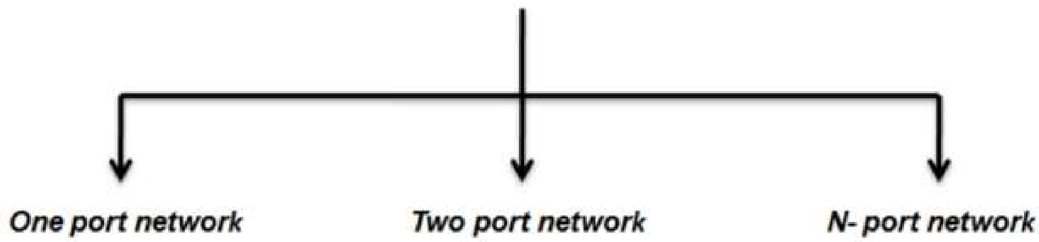
Network:

When a no. of impedances are connected together to form a system that consists of sets of interconnected system to do specific on assigned function is called as a ***network***.

Or,

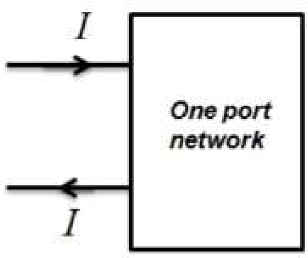
The electrical N/W is a combination of numerous electric elements that are resistance ***(R),*** inductance ***(L)*** & capacitor ***(C).***

Port of a Network:



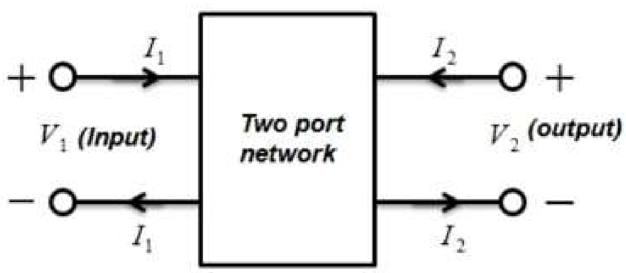
One port network:

Any active or passive network having only two terminals can be represented by a **one port network**.



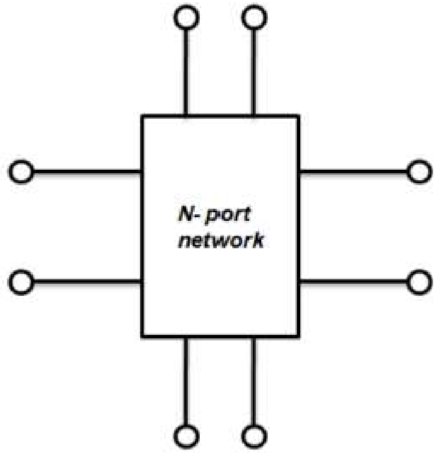
Two port network:

Any active or passive network having two pairs of terminals (or 4- terminals) can be represented as **two port network**.



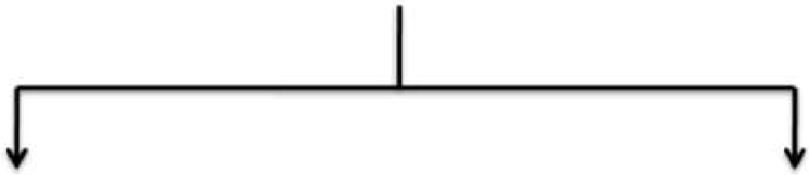
N- Port network:

If the network representations contains ***n-pair*** of terminal can be represented by an **n-port network**.



Network configuration:

Depending on the configuration of impedance a network can be specified as follows-



T- section

(When a network section looks lie a 'T', it is known as T- section).

π -Section

(When a network section loos lie a, π it is known as π – section)

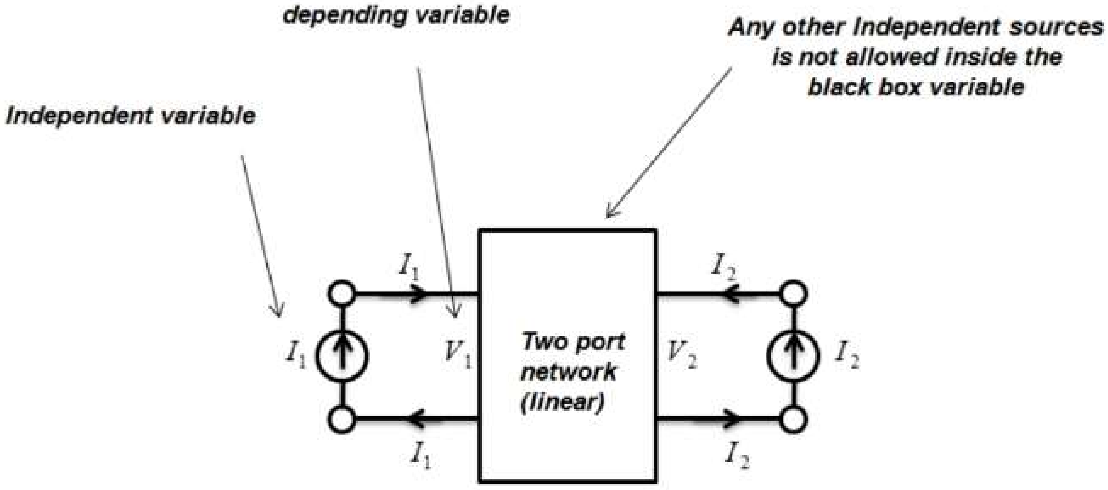
Parameter Representation:

NOTE:

- Which type of source we are applied to the black box, depending upon them, the black box having corresponding parameters.
- The combination of sources applied are as follows:

    a) Current source at input & current source at output $\rightarrow$ **Z-parameter network**.
    b) Voltage source at input & current source at output $\rightarrow$ **Inverse h-parameter network.**
    c) Current source at input & voltage source at output $\rightarrow$ **H-parameter network.**
    d) Voltage source at input & voltage source at output $\rightarrow$ **Y-parameter network.**

8.1. Z-parameter or open circuit parameter or Impedance parameter:



Now applying the super position principles:

$$V_1 = f(I_1)\Big|_{I_2=0},\ V_2 = f(I_1)\Big|_{I_2=0}$$

$$V_1 = f(I_2)\Big|_{I_1=0},\ V_2 = f(I_2)\Big|_{I_1=0}$$

$\therefore \quad V_1 = f(I_1) + f(I_2)$

$\therefore \quad V_2 = f(I_1) + f(I_2)$

$\therefore \quad V_1 = Z_{11}I_1 + Z_{12}I_2$ .... (1).

$\therefore \quad V_2 = Z_{21}I_1 + Z_{22}I_2$ .... (2).

If the output is **open circuit** i.e. $I_2 = 0$,

$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}$$

$$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0}$$

If the input is **open circuit** i.e. $I_1 = 0$,

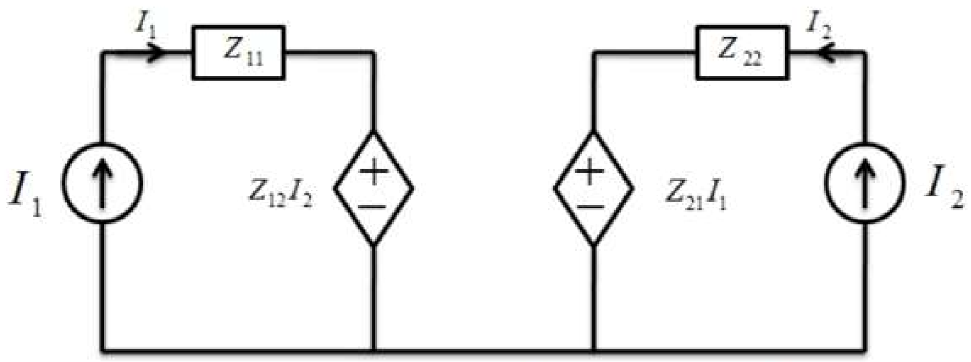
$$Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$$

$$Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$$

Where, $Z_{11} = Input\ driving\ impedance,\ Z_{12} = Input\ transfer\ impedance.$

$Z_{21} = Output\ transfer\ impedance, Z_{22} = Output\ driving\ impedance$

These Z_{11}, Z_{12}, Z_{21} & Z_{22} are the open circuited parameter or impedance parameters or **Z-parameters.**

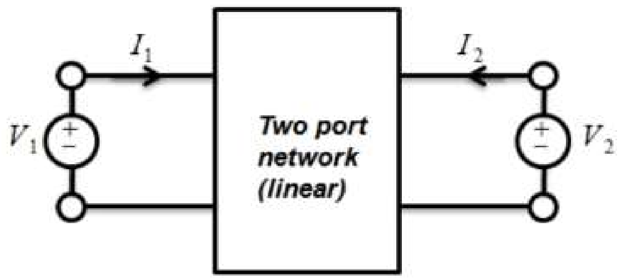


[Internal equivalent circuit of Z-parameter network]

REMEMBER:

If $Z_{11} = Z_{22}$, then the network is **Symmetrical**. If $Z_{12} = Z_{21}$, then network is ***Reciprocal***.

8.2. Y-parameter or short circuit parameter or Admittance parameter:



Now applying the super position principles:

$$I_1 = f(V_1)\big|_{V_2=0}\ ,\ I_2 = f(V_1)\big|_{V_2=0}$$

$$I_1 = f(V_2)\big|_{V_1=0}\ ,\ I_2 = f(V_2)\big|_{V_1=0}$$

$\therefore$ $$I_1 = f(V_1) + f(V_2)$$

$\therefore \qquad I_2 = f(V_1) + f(V_2)$

$\therefore \qquad I_1 = Y_{11}V_1 + Y_{12}V_2$ .... (1).

$\therefore \qquad I_2 = Y_{21}V_1 + Y_{22}V_2$ .... (2).

If the output is **short circuited** i.e. $V_2 = 0$,

$$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}$$

$$Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$$

If the input is **short circuited** i.e. $V_1 = 0$,

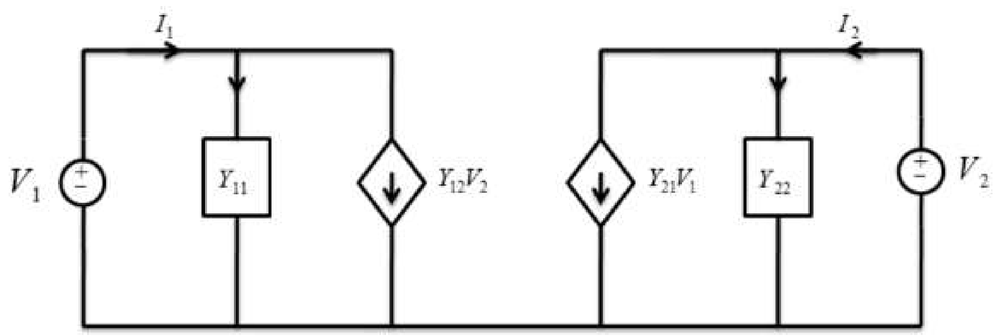
$$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$$

$$Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$$

Where, $Y_{11} = Input\ driving\ admittance,\ y_{12} = Input\ transfer\ admittance.$

$Y_{21} = Output\ transfer\ admittance, Y_{22} = Output\ drivingadmittance$

These Y_{11}, Y_{12}, Y_{21} & Y_{22} are the short circuited parameters or admittance parameters or **Y-parameters.**

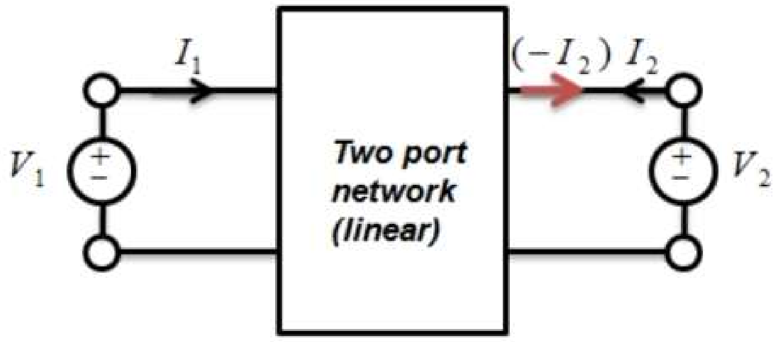


[Internal equivalent circuit of Y-parameter network]

REMEMBER:

If $Y_{11} = Y_{22}$, then the network is **Symmetrical** and if $Y_{12} = Y_{21}$, then the network is **Reciprocal**.

8.3. ABCD-parameter or transmission parameter:



Now applying the super position principles:

$\therefore \quad V_1 = f(V_2) + f(-I_2)$

$\therefore \quad I_1 = f(V_2) + f(-I_2)$

$\therefore \quad V_1 = AV_2 + B(-I_2)$ .... (1).

$\therefore \quad I_1 = CV_2 + D(-I_2)$ .... (2).

If $I_2 = 0$,

$$A = \frac{V_1}{V_2}\Big|_{I_2=0}$$

$$C = \frac{I_1}{V_2}\Big|_{I_2=0}$$

If $V_2 = 0$,

$$B = -\frac{V_1}{I_2}\Big|_{V_2=0}$$

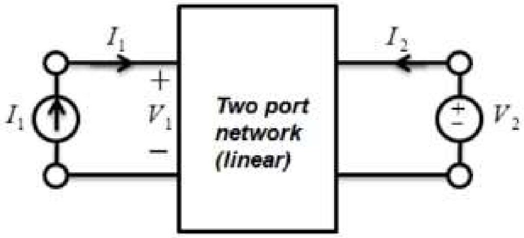
$$D = -\frac{I_1}{I_2}\Big|_{V_2=0}$$

These $A, B, C \,\&\, D$ are the transmission parameters or **ABCD-parameters.**

REMEMBER:

If $\Delta T = 1$, then the network is **Reciprocal** and if $A = D$, then the network is **Symmetrical**.

8.4. Hybrid-parameter or h- parameter:



Now applying the super position principles:

$\therefore \qquad V_1 = f(I_1) + f(V_2)$

$\therefore \qquad I_2 = f(I_1) + f(V_2)$

$\therefore \qquad V_1 = h_{11}I_1 + h_{12}V_2$ .... (1).

$\therefore \qquad I_2 = h_{21}I_1 + h_{22}V_2$ .... (2).

If the output is **short circuited** i.e. $V_2 = 0$

$$h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0}$$

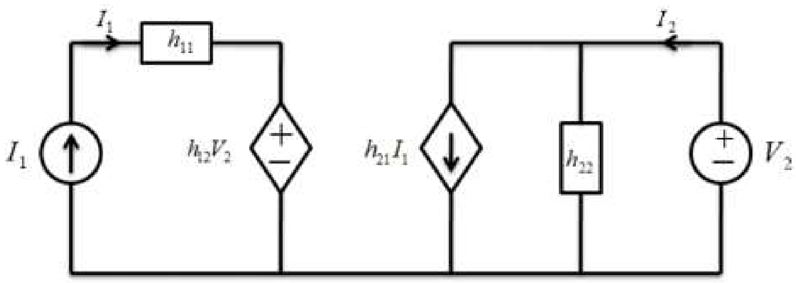
$$h_{21} = \frac{I_2}{I_1}\Big|_{V_2=0}$$

If the input is **open circuited** i.e. $I_1 = 0$

$$h_{12} = \frac{V_1}{V_2}\Big|_{I_1=0}$$

$$h_{22} = \frac{I_2}{V_2}\Big|_{I_1=0}$$

These h_{11}, h_{12}, h_{21} & h_{22} are the Hybrid parameters or **h-parameters.**



[Internal equivalent circuit of h-parameter network]

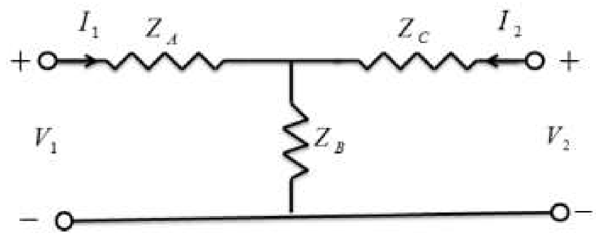
REMEMBER:

If $h_{12} = -h_{21}$, then the network is **Reciprocal** and if $\Delta h = 1$, then the network is **Symmetrical**.

8.5. Interrelation between different parameters:

- **Z in terms of Y-parameter & Y in terms of Z-parameter:**

  - **Z in terms of Y-parameter:**



We know that, the equation for **Z-parameter** is given as,

$$\therefore \quad V_1 = Z_{11}I_1 + Z_{12}I_2 \text{ .... (1).}$$

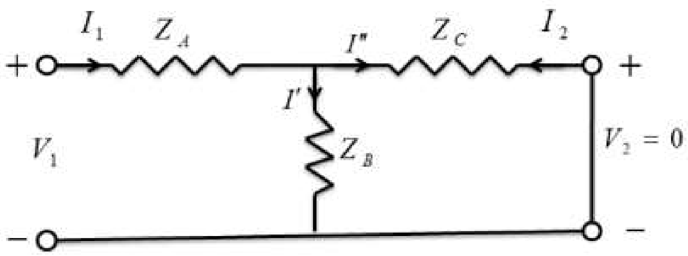
$$\therefore \quad V_2 = Z_{21}I_1 + Z_{22}I_2 \text{ .... (2).}$$

The equation for **Y-parameter** is given as,

$$\therefore \quad I_1 = Y_{11}V_1 + Y_{12}V_2 \text{ .... (3).}$$

$$\therefore \quad I_2 = Y_{21}V_1 + Y_{22}V_2 \text{ .... (4).}$$

Case: 1 When, $V_2 = 0 \;\; Y_{11} = \frac{I_1}{V_1}$ & $Y_{21} = \frac{I_2}{V_1}$



$$Z_{eq} = (Z_B \| Z_C) + Z_A = \frac{Z_B Z_C}{Z_B + Z_C} + Z_A$$

$$\therefore \quad I_1 = \frac{V_1}{\frac{Z_B Z_C}{Z_B + Z_C} + Z_A} = \frac{V_1}{\frac{Z_A(Z_B + Z_C) + Z_B Z_C}{Z_B + Z_C}}$$

Or, $$I_1 = \frac{V_1(Z_B + Z_C)}{Z_A Z_B + Z_A Z_C + Z_B Z_C}$$

Or, $$\frac{I_1}{V_1} = \frac{(Z_B + Z_C)}{Z_A Z_B + Z_A Z_C + Z_B Z_C} = \frac{Z_{22}}{\Delta Z}$$

∴ $$Y_{11} = \frac{Z_{22}}{\Delta Z}$$

$\underline{I'' = -I_2,}$

$$I'' = \frac{I_1 \times Z_B}{Z_B + Z_C} = -I_2$$

Or, $$\frac{I_2}{V_1} = \frac{-\frac{I_1 Z_B}{Z_B + Z_C}}{V_1} = -\frac{V_1(Z_B + Z_C)Z_B}{V_1(Z_A Z_B + Z_A Z_C + Z_B Z_C) \times (Z_B + Z_C)}$$

Or, $$\frac{I_2}{V_1} = -\frac{Z_{21}}{\Delta Z}$$

Or, $$Y_{21} = -\frac{Z_{21}}{\Delta Z}$$

Similarly, $$Y_{22} = \frac{Z_{11}}{\Delta Z} \ \& \ Y_{12} = -\frac{Z_{12}}{\Delta Z}$$

- **Y in terms of Z-parameter:**

$$Z_{11} = \frac{Y_{22}}{\Delta Y}, Z_{12} = -\frac{Y_{12}}{\Delta Y}$$

$$Z_{21} = -\frac{Y_{21}}{\Delta Y}, Z_{22} = \frac{Y_{11}}{\Delta Y}$$

- **Z in terms of ABCD-parameter & ABCD in terms of Z-parameter:**

- **Z in terms of ABCD-parameter:**

$$A = \frac{Z_{11}}{Z_{21}}, B = \frac{\Delta Z}{Z_{21}}, C = \frac{1}{Z_{21}}, D = \frac{Z_{22}}{Z_{21}}$$

➢ **ABCD in terms of Z-parameter:**

$$Z_{11} = \frac{A}{C}, Z_{12} = \frac{AD - BC}{C}, Z_{21} = \frac{1}{C}, Z_{22} = \frac{D}{C}$$

❖ **ABCD in terms of Y-parameter & Y in terms of ABCD-parameter:**

➢ **ABCD in terms of Y-parameter:**

$$Y_{11} = \frac{D}{B}, Y_{12} = \frac{-AD - BC}{B}, Y_{21} = -\frac{1}{B}, Y_{22} = \frac{A}{B}$$

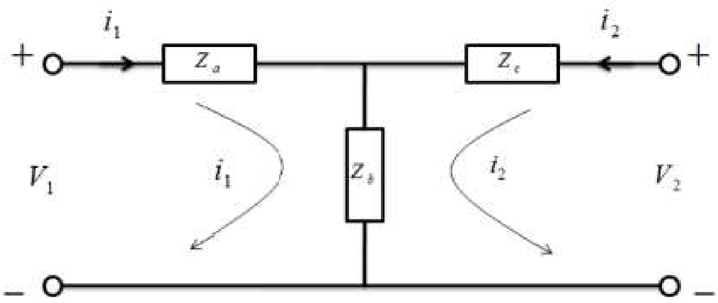
➢ **Y in terms of ABCD-parameter:**

$$A = -\frac{Y_{22}}{Y_{21}}, B = -\frac{1}{Y_{21}}, C = \frac{Y_{11}}{Y_{21}}, D = -\frac{Y_{11}Y_{12} - Y_{12}Y_{21}}{Y_{21}}$$

8.6. T **&** π **representation:**

Parameters of standard circuit:

1) $T-$**Network:**



By applying KVL: at loop-1

$$-V_1 + i_1 Z_a + (i_1 + i_2) Z_b = 0$$

Or, $V_1 = i_1 Z_a + i_1 Z_b + i_2 Z_b$

$= (Z_a + Z_b) \times i_1 + Z_b i_2$ .... (1)

Again at loop-2,

$$V_2 = i_2 Z_c + (i_1 + i_2) Z_b$$
$$= i_2 Z_c + i_1 Z_b + i_2 Z_b$$
$$= (Z_b + Z_c) \times i_2 + i_1 Z_b \text{ .... (2)}$$

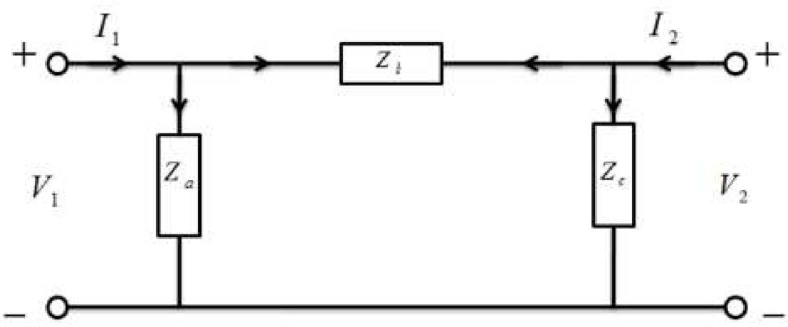
In matrix form:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_a + Z_b & Z_b \\ Z_b & Z_b + Z_c \end{bmatrix} \times \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$= \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \times \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\therefore \quad Z_{11} = Z_a + Z_b$$
$$Z_{12} = Z_{21} = Z_b$$
$$Z_{22} = Z_b + Z_c$$

2) $\pi -$ Network:



By applying KVL, we get,

$$I_1 = \frac{V_1}{Z_a} + \frac{V_1 - V_2}{Z_b}$$

$$I_2 = \frac{V_2}{Z_c} + \frac{V_2 - V_1}{Z_b}$$

Or, $$I_1 = \left[\frac{1}{Z_a} + \frac{1}{Z_b}\right] \times V_1 + \left[-\frac{1}{Z_b}\right] \times V_2 \text{ .... (1)}$$

Or, $I_2 = \left[\frac{1}{Z_b} + \frac{1}{Z_c}\right] \times V_2 + \left[-\frac{1}{Z_b}\right] \times V_1$ .... (2)

In matrix form:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{Z_a} + \frac{1}{Z_b} & -\frac{1}{Z_b} \\ -\frac{1}{Z_b} & \frac{1}{Z_b} + \frac{1}{Z_c} \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$= \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

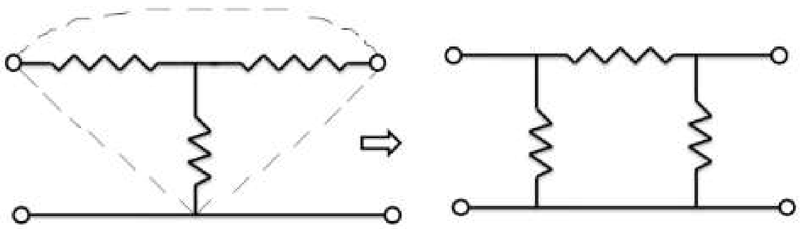
$$Y_{11} = \frac{1}{Z_a} + \frac{1}{Z_b}$$

$$Y_{12} = Y_{21} = -\frac{1}{Z_b}$$

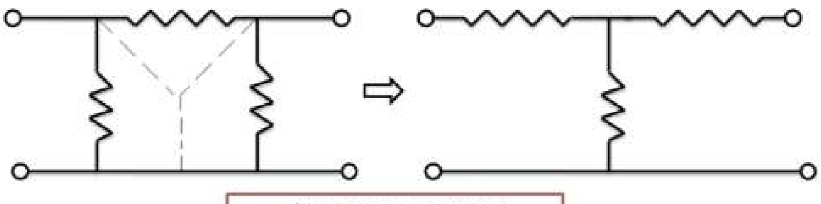
$$Y_{22} = \frac{1}{Z_b} + \frac{1}{Z_c}$$

NOTES:

- $[Z] = [Y]^{-1} |Y| \neq 0$
- $[Y] = [Z]^{-1} |Z| \neq 0$
- Or you can apply $Y \rightarrow \Delta$ or $\Delta \rightarrow Y$ conversion.



Y to Δ conversion

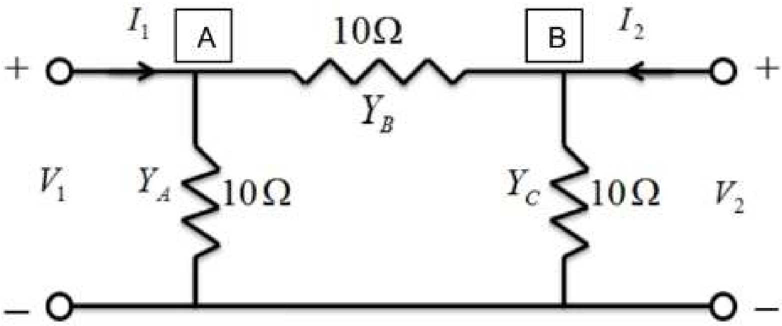


Δ to Y conversion

8.7. Solve numerical problems:

Examples:

Q.1. Find the Y-parameters of the network?



Solution:

As it is a π network, so we can directly find the Y-parameter.

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad \text{.................................... (1)}$$

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad \text{.................................... (2)}$$

Y-parameter $= \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Y_A + Y_B & -Y_B \\ -Y_B & Y_B + Y_C \end{bmatrix}$

$$= \begin{bmatrix} \frac{1}{10} + \frac{1}{10} & -\frac{1}{10} \\ -\frac{1}{10} & \frac{1}{10} + \frac{1}{10} \end{bmatrix} = \begin{bmatrix} 0.2 & -0.1 \\ -0.1 & 0.2 \end{bmatrix} \text{ (Ans)}$$

Or,

At node-A, **by KCL,**

$$I_1 = \frac{V_1}{10} + \frac{V_1 - V_2}{10} = \frac{V_1}{10} + \frac{V_1}{10} - \frac{V_1}{10}$$

$$= V_1\left(\frac{1}{10}+\frac{1}{10}\right)-\left(\frac{1}{10}\right)V_2$$

Or, $I_1 = 0.2V_1 - 0.1V_2$ .... (3)

At node-B, **by KCL,**

$$I_2 = \frac{V_2}{10}+\frac{V_2-V_1}{10} = \frac{V_2}{10}+\frac{V_2}{10}-\frac{V_1}{10}$$

$$= V_2\left(\frac{1}{10}+\frac{1}{10}\right)-\left(\frac{1}{10}\right)V_1$$

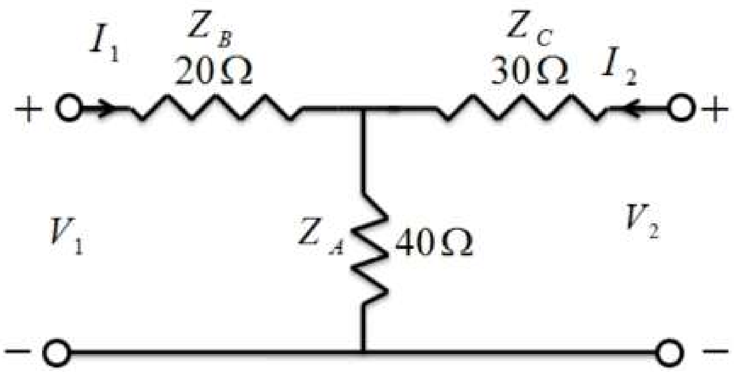
Or, $I_2 = -0.1V_1 + 0.2V_2$ .... (4)

Now comparing eq. (3) & (4), we get,

$$Y_{11} = 0.2, Y_{12} = -0.1$$

$$Y_{21} = -0.1, Y_{22} = 0.2$$

Q.2. Determine the Z-parameters of the network?



Solution:

As it is a ***T*** network, so we can directly find the Z-parameter.

We know that,

$V_1 = Z_{11}I_1 + Z_{12}I_2$ .... (1)

$V_2 = Z_{21}I_1 + Z_{22}I_2$ .... (2)

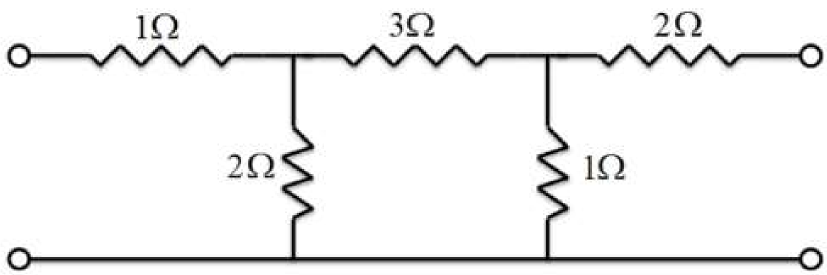
$$Z_{11} = Z_A + Z_B = 20 + 40 = 60\Omega$$

$$Z_{12} = Z_{21} = Z_B = 40 = 40\Omega$$

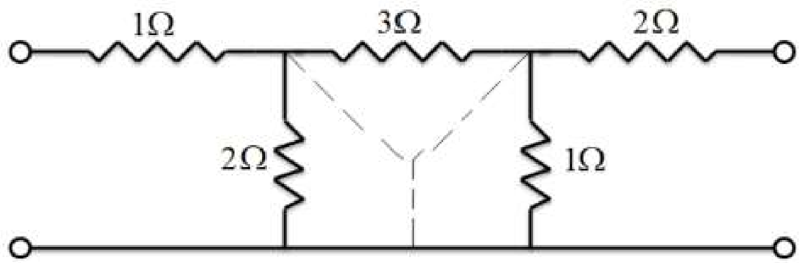
$$Z_{22} = Z_B + Z_C = 40 + 30 = 70\Omega$$

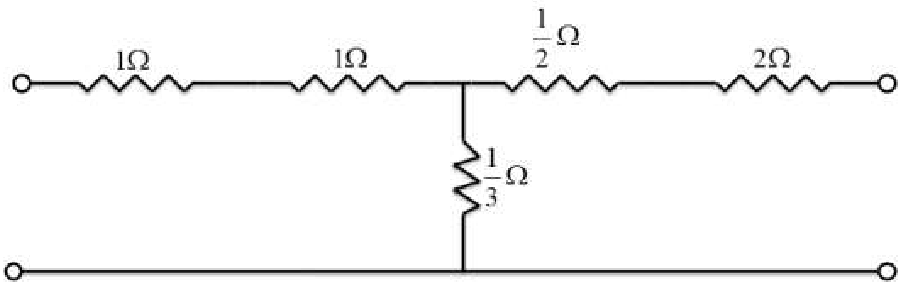
$$\therefore \qquad Z - parameter = \begin{bmatrix} 60 & 40 \\ 40 & 70 \end{bmatrix}$$

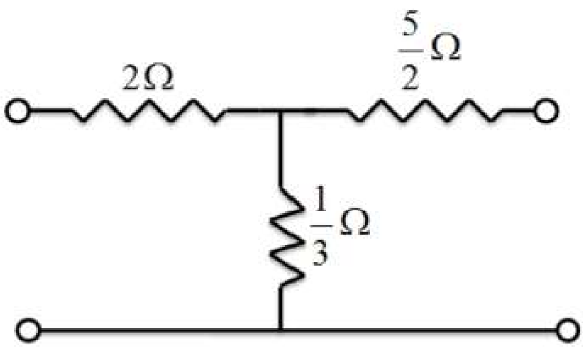
Q.3. Determine the Z-parameters of the network?



Solution: The circuit can be converted to a T-network, so it is easy to find the Z-parameter.







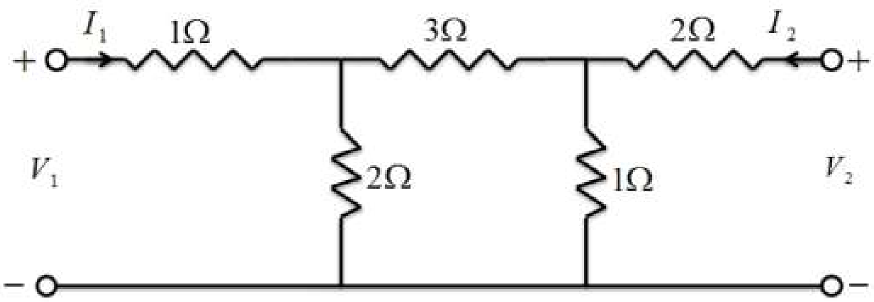
$\therefore \quad Z_{11} = Z_A + Z_B = 2 + \frac{1}{3} = \frac{6+1}{3} = \frac{7}{3}\Omega$

$$Z_{12} = Z_{21} = Z_B = \frac{1}{3}\Omega$$

$$Z_{22} = Z_B + Z_C = \frac{1}{3} + \frac{5}{2} = \frac{2+15}{6} = \frac{17}{6}\Omega$$

$\therefore \quad Z - parameter = \begin{bmatrix} \frac{7}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{17}{6} \end{bmatrix}$

Or,



Case-1: Take $I_2 = 0$

We know that,

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \text{ .... (1)}$$

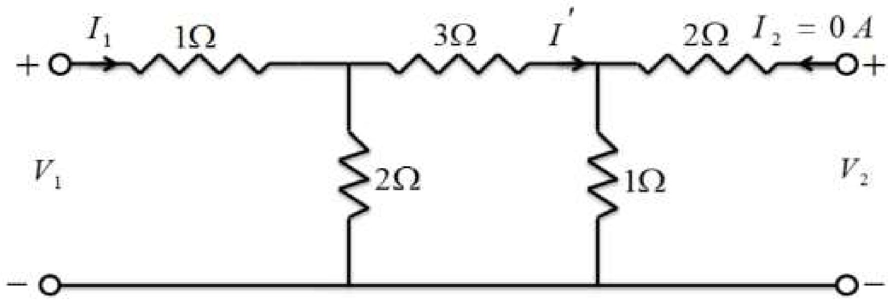
$$V_2 = Z_{21}I_1 + Z_{22}I_2 \text{ .... (2)}$$

$$Z_{11} = \frac{V_1}{I_1} = R_{eq.}$$

$$\therefore \; R_{eq.} = (4\|2) + 1 = \frac{8}{6} + 1 = \frac{8+6}{6} = \frac{14}{6} = \frac{7}{3}\Omega$$

$$\therefore \; Z_{11} = \frac{7}{3}\Omega$$

Again, $\quad Z_{21} = \frac{V_2}{I_1}$



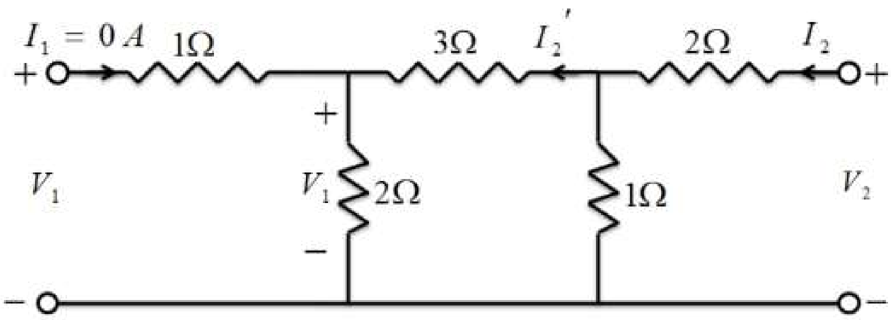
$$I' = \frac{I_1 \times 2}{2+4} = \frac{I_1}{3}$$

$\therefore$ $$V_2 = I' \times 1 = \frac{I_1}{3} \times 1 = \frac{I_1}{3}$$

Or, $$\frac{V_2}{I_1} = \frac{1}{3}$$

Or, $$Z_{21} = \frac{1}{3}\Omega$$

Case-2: Take $I_1 = 0$



$$Z_{22} = \frac{V_2}{I_2} = R_{eq.}$$

$$\therefore\ R_{eq.} = (5\|1) + 2 = \frac{5}{6} + 2 = \frac{5+12}{6} = \frac{17}{6}\Omega$$

$$\therefore\ Z_{22} = \frac{17}{6}\Omega$$

Again, $$Z_{12} = \frac{V_1}{I_2}$$

$$I_2' = \frac{I_2 \times 1}{5+1} = \frac{I_2}{6}$$

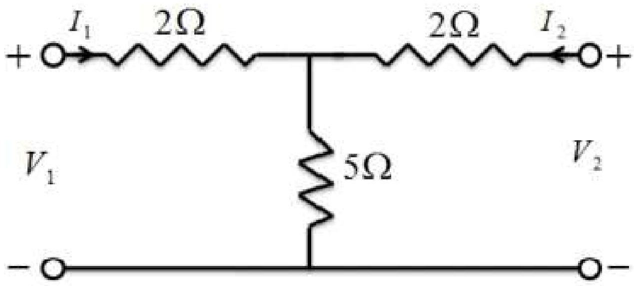
$\therefore \quad V_1 = I_2' \times 2 = \frac{I_2}{6} \times 2 = \frac{I_2}{3}$

Or, $\quad \frac{V_1}{I_2} = \frac{1}{3}$

Or, $\quad Z_{12} = \frac{1}{3}\Omega$

$\therefore \quad Z - parameter = \begin{bmatrix} \frac{7}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{17}{6} \end{bmatrix}$

Q.4. Determine the ABCD-parameters of the network?



Solution:

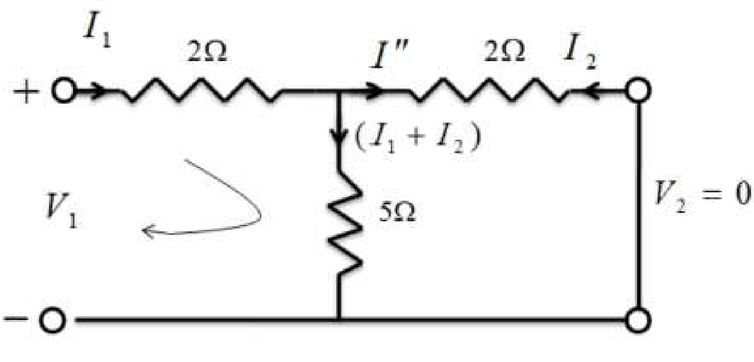
We know that, for **ABCD-parameter**,

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

Case-1: $V_2 = 0$

$$B = -\frac{V_1}{I_2} \qquad D = -\frac{I_1}{I_2}$$



Applying KVL,

$$V_1 = 7I_1 + 5I_2 \text{ .... (1).}$$

Now applying current division rule,

$$I'' = \frac{I_1 \times 5}{5+2} = -I_2$$

Or, $$V_1 = \left(-\frac{7I_2}{5} \times 7\right) + 5I_2$$

$$= \left(-\frac{49I_2}{5}\right) + 5I_2$$

Or, $$V_1 = \frac{-49I_2 + 25I_2}{5}$$

Or, $$5V_1 = -24I_2$$

Or, $$\frac{V_1}{I_2} = -\frac{24}{5}$$

$\therefore$ $$B = -\frac{V_1}{I_2} = -\left(-\frac{24}{5}\right) = \frac{24}{5} = 4.8\Omega$$

$\therefore$ $$B = 4.8\Omega$$

$$D = -\frac{I_1}{I_2}$$

$$I'' = -I_2 = \frac{5I_1}{7}$$

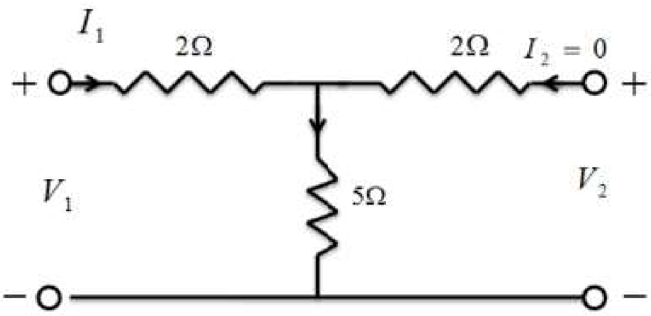
Or, $$\frac{I_1}{I_2} = -\frac{7}{5}$$

$\therefore$ $$D = -\frac{I_1}{I_2} = -\left(-\frac{7}{5}\right) = \frac{7}{5} = 1.4$$

$$D = 1.4$$

Case-2: $I_2 = 0$

$$A = \frac{V_1}{V_2} \quad C = \frac{I_1}{V_2}$$



$$I_1 = \frac{V_1}{7}, V_2 = I_1 \times 5 = \frac{V_1}{7} \times 5 = \frac{5V_1}{7}$$

Or, $\frac{V_1}{V_2} = \frac{7}{5}$

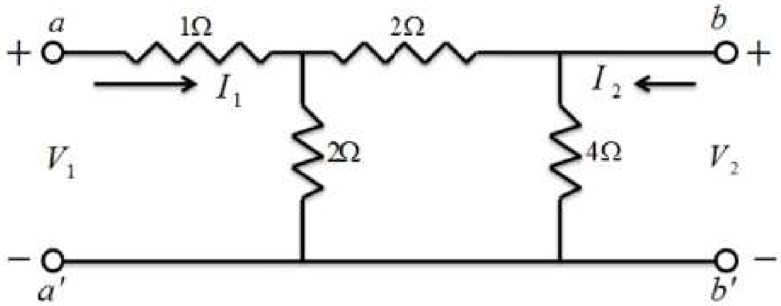
∴ $A = \frac{V_1}{V_2} = \frac{7}{5}$

Again, $V_2 = 5I_1$

Or, $\frac{I_1}{V_2} = \frac{1}{5}$

∴ $C = \frac{I_1}{V_2} = \frac{1}{5} mho$

Q.5. Determine the h-parameters of the network?



Solution:

We know that,

If the output is **short circuited** i.e. $V_2 = 0$

$$h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0}$$

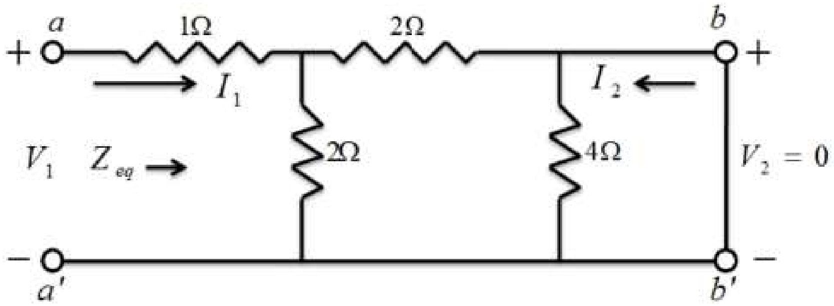
$$h_{21} = \frac{I_2}{I_1}\Big|_{V_2=0}$$

If the input is **short circuited** i.e. $I_1 = 0$

$$h_{12} = \frac{V_1}{V_2}\Big|_{I_1=0}$$

$$h_{22} = \frac{I_2}{V_2}\Big|_{I_1=0}$$

If $b-b'$ is short-circuited, $V_2 = 0$ and the network looks as shown in fig. below.



$$h_{21} = \frac{I_2}{I_1}\Big|_{V_2=0}; V_1 = I_1 Z_{eq}$$

Z_{eq} = The equivalent impedance as viewed from port $a-a'$ is $(2\Omega \| 2\Omega) + 1\Omega = 2\Omega$

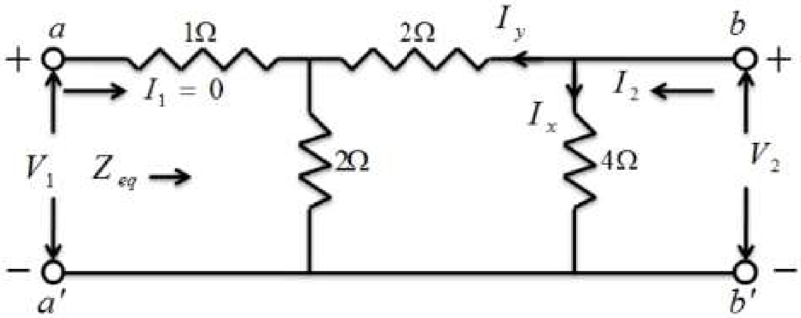
So, $V_1 = 2I_1$

$$h_{11} = \frac{V_1}{I_1} = 2\Omega$$

$$h_{21} = \frac{I_2}{I_1}\Big|_{V_2=0}$$

So, $-I_2 = \frac{I_1}{2}$, Or, $h_{21} = \frac{I_2}{I_1} = -\frac{1}{2}$

If $a-a'$ is open-circuited, $I_1 = 0$ and the network looks as shown in fig. below.



$$h_{12} = \frac{V_1}{V_2}\Big|_{I_1=0} and V_1 = 2I_y; I_y = \frac{I_2}{2}$$

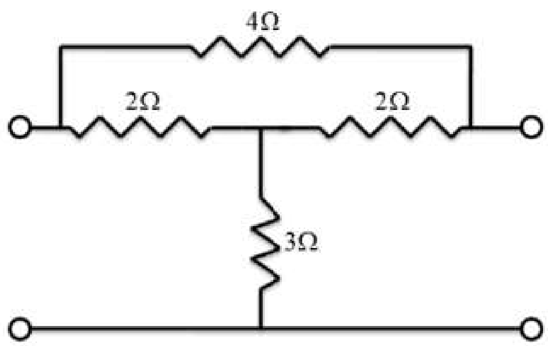
$$V_2 = 4I_x; I_x = \frac{I_2}{2}$$

$$h_{12} = \frac{V_1}{V_2}\Big|_{I_1=0} = \frac{1}{2}, h_{22} = \frac{I_2}{V_2}\Big|_{I_1=0} = \frac{1}{2}$$

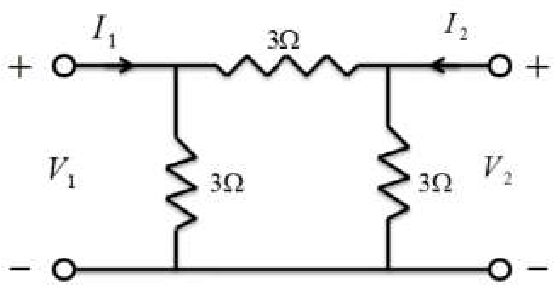
$$\therefore \qquad h - parameter = \begin{bmatrix} 2 & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

QUESTION BANK

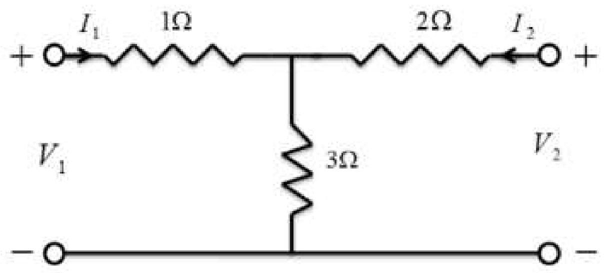
Q.1. For the network shown finds the Z-parameter?



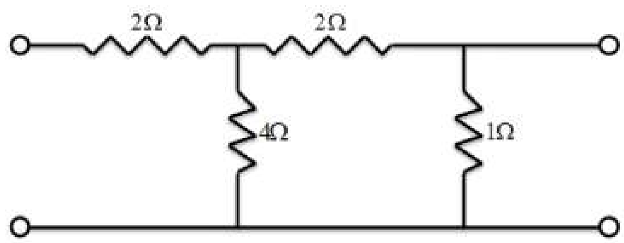
Q.2. For the network shown finds the Y-parameter and Z-parameter?



Q.3. For the network shown finds the ABCD-parameter or transmission parameter?



Q.4. For the network shown finds the h-parameter?



Q.5. For the network shown finds the h-parameter?