SIMPLE STRESS, STRAIN

CHAPTER – 1

Q.1 Define Poisonous ratio 2006(w) (1-i), 2010(w)(1-a),2012(w)

Ans: If a body is stressed with in its elastic limit, then the lateral strain bears a constant ratio with the linear strain. This constant is known as poisonous ratio (Hooke's law)

Q2. Define Young's modulus of elasticity 2010(w), (1-b) 2006(w) (1-x)

Ans: It can be defined as the ratio of stress by strain of a stressed material

Elasticity = Stress/Strain i.e. $E = \sigma/\in$.

Q3. Srite relation between E, K and G 2010(w)(3), 2006(w)(1-ii)

Ans: Consider a wbe ABCD, A'B'C'D'.

Let the stress acting on faces = σ.

E = young's modulus of elasticity

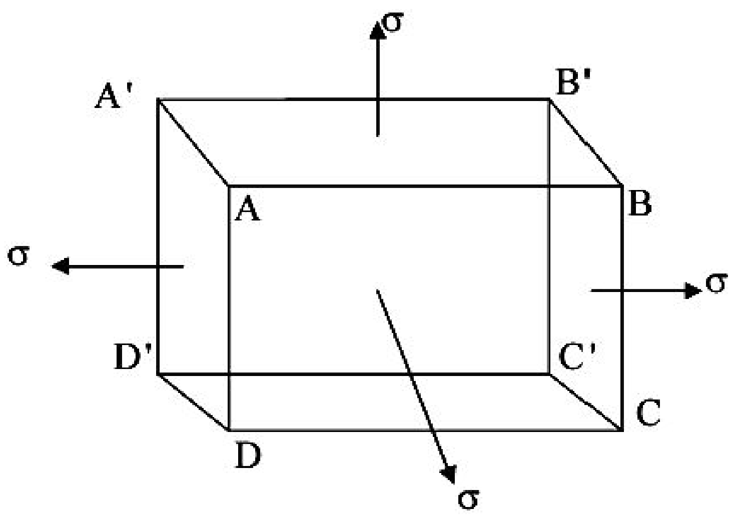
Consider deformation of face AB from ABCD

AB will suffer the following strains

(1) A tensile strain of σ/E.

We know that E = 2C (1+1/m) ----------------- (i)

And Also E = 3K (1−2/m) ---------------------- (ii)



Now

$$E = 2C\left(1+\frac{1}{m}\right) \qquad \Rightarrow \qquad E = 3K\left[\frac{C-E+2C}{C}\right]$$

$$\Rightarrow \frac{E}{2C} = 1+\frac{1}{m} \qquad \Rightarrow \qquad \frac{E}{3K} = \frac{C-E+2C}{C}$$

$$\Rightarrow \frac{1}{m} = \frac{E}{2C} - 1 \qquad \Rightarrow \qquad \frac{E}{3K} = \frac{3C-E}{C}$$

$$\Rightarrow \frac{1}{m} = \frac{E-2C}{2C} \qquad \Rightarrow \qquad \frac{E}{3K} = 3-\frac{E}{C}$$

$$\Rightarrow m = \frac{2C}{E-2C} ----(iii) \qquad \Rightarrow \qquad \frac{E}{3K}+\frac{E}{C} = 3$$

Also

$$E = 3K\left(1-\frac{2}{m}\right) \qquad \Rightarrow \qquad \frac{EC+3KE}{3KC} = 3$$

$$\Rightarrow 3K\left(1-\frac{2}{\frac{2C}{E-2C}}\right) \qquad \Rightarrow \qquad EC+3KE = 9KC$$

$$\Rightarrow E = 3K\left(1-\frac{2(E-2C)}{2C}\right) \qquad \Rightarrow \qquad E = \frac{9KC}{3K+C}$$

$$\Rightarrow E = 3K\left(\frac{2C-2(E-2C}{2C}\right)$$

$$\Rightarrow E = 3K\left[\frac{C-(E-2C)}{C}\right]$$

This is the required Reculion between E,K and G

Q.4. Define strength of material 2007 (w)

Ans: A detailed study of analysis of forces with suitable protective measures for their safe working condition is known as strength of material.

Q.5 Define working stress 2007(w) (1-ii)

Ans: When a body is strained with in elastic limit then some resisting force or restoring force is offered by the body to deformation. This resisting force per unit area of the body is known as working stress.

Problem

A steel rod 25 mm in diameter and 2m long is subjected to an axial pull of 45 KN Find

(i) The intensity of stress

(ii) Strain

(iii) Elongation Take $E = 2 \times 10^5$ N/mm^2 2013(w), 1(c)

Given

D = 25 mm

L = 2 m = 2000 mm

P = 45 KN = 45×10^3 N

$$\text{Area, } A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times (25)^2 = 490.63\text{mm}^2$$

$$E = 2 \times 10^5 \text{ N/mm}^2$$

$$\text{Stress}, \sigma = \frac{P}{A} = \frac{45 \times 10^3}{490.63} = 91.7 \text{ N/mm}^2$$

$$\text{Strain}, \in = \frac{\text{stress}}{E} = \frac{91.7}{2 \times 10^5} = 0.00046$$

$$\text{Elongation}, \delta l = \frac{PL}{AE} = \frac{45 \times 10^3 \times 2000}{490.63 \times 2 \times 10^5}$$

$$= 0.92\text{mm}$$

Problem:

A reinforced concrete circular column 50000 mm^2 cross sectional area carries six rein forcing bars whose total area is 500 mm^2. Find the safe load the column can carry if the concrete is not to be stressed more than 3.5 MPa. Take modular ratio for steel and concrete as 18.

2013(w),3(c)

Area of column (A) = 5000 mm^2

Area of 6 steel bars (A_s) = 500 mm^2

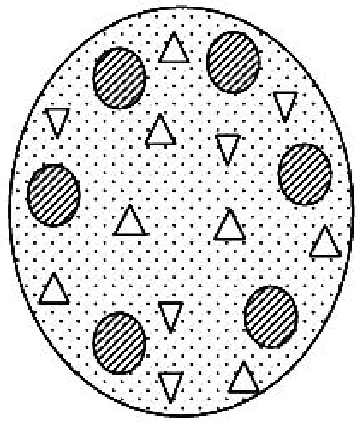
Area of concrete, $A_c = A - A_s = 50{,}000 - 500$

$= 49500$ mm^2

Stress in concrete, $\sigma_c = 3.5$ MPa = 3.5 N/mm^2

Let σ_S = stress in steel

Modular ratio = $E_S/E_C = 18$



$\sigma_S/\sigma_C = E_S/E_C = 18 \Rightarrow \sigma_S = 18\ \sigma_C = 18 \times 3.5 = 63$ N/mm^2

$P = \sigma_C\ ,\ A_c + \sigma_s.A_S$

$= (3.5 \times 49{,}5000) + (63 \times 500) = 173250 + 31500 = 204750$ N = 204.75 KN.

Problem :

A rod of steel is 20 m long at a temperature of 20°C. Find the free expansion of rod when temperature is raised by 65°C. also find the temperature stress when expansion of rod is prevented Take $\alpha = 12\ 10^{-6}/°C$ and $E = 2\ 10^5$ N/mm^2 2013(w), 4(c)

Given :

$L = 20$ m = 20, 000 mm

Rise in temperature, $t = 65^\circ - 20^\circ = 45^\circ C$

$\alpha = 12 \times 10^{-6}/^\circ C$ $E = 2 \times 10^5\ N/mm^2$

Expansion of rod, $l = l\ \alpha\ t$

$= 20{,}000 \times 12 \times 10^{-6} \times 45 = 10.8$ mm

Temperature stress, $= \alpha\ t\ E = 12 \times 10^{-6} \times 45 \times 2 \times 10^5 = 108\ N/mm^2$

Q. State Hooke's law 2014(w)

Ans: When material is loaded within elastic limit, stress is proportional to strain.

Mathematically stress α strain.

i.e. $\frac{\text{stress}}{\text{strain}} = E = \text{constant}$

Where E = young's modulus of elasticity. Define stress and strain 2014(w)

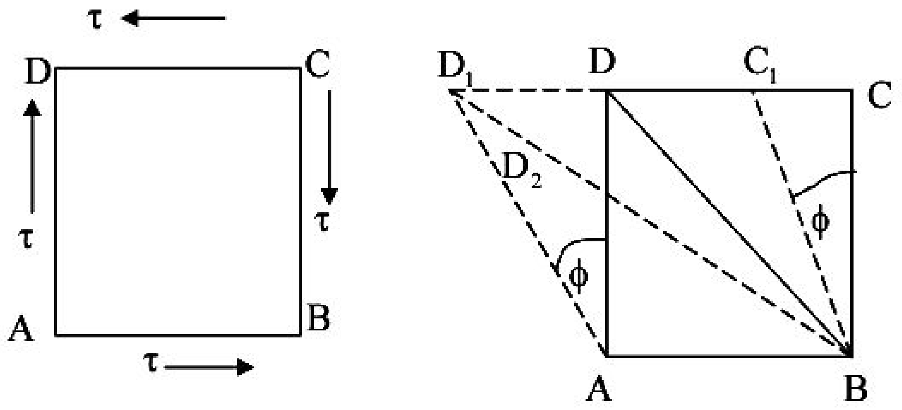
Stress – The restoring force per unit area is known as stress

$$\text{Stress}(\sigma)\frac{\text{Force}}{\text{Area}} = \frac{P}{A}$$

Strain- The deformation per unit length is known as strain.

Strain , $e = \delta l/L$

Q. State relation between modulus of elasticity and modulus of rigidity (c).2014(w)



Consider a cube of length 'l' subjected to a shear stress τ as shown in figure. A little consideration will show that due to these stresses the cube is subjected to some distortion such that the diagonal BD will be elongated and diagonal Ac will be shortened. Let this shear stress (τ) cause shear stress as shown. We see that diagonal BD is distorted to BD.

$$\text{Strain of BD} = \frac{BD_1 - BD}{BD} = \frac{D_1D_2}{BD} = \frac{DD_1COS45}{AD\sqrt{2}}$$

$$= \frac{DD_1}{2AD} = \frac{\phi}{2}$$

We see that the linear strain of diagonals BD is half of shear strain and is tensile in nature. Similarly it can be proved that the linear strain of diagonal AC is also equal to half of shear strain but is compressive in nature, Now this linear strain of diagonal $BD = \frac{\phi}{2} = \frac{\tau}{2C}$ - - - - - -(1)

Where τ = shear stress

C= Modulus of rigidity

Let us now consider this shear stress (τ) acting on the sides AB, CD, CB and AD. Now the effect of this stress is to cause tensile stress an diagonal BD and compressive stress on diagonal AC.

Therefore tensile stress on diagonal BD due to tensile stress on diagonal

$$BD = \frac{\tau}{E} ------(2)$$

Tensile strain on diagonal Bd due to compressive stress on diagonal

$$AC = \frac{1}{m} \times \frac{\tau}{E} ------(3)$$

The combined effect of above two stress on diagonal

$$BD = \frac{\tau}{E} + \frac{1}{m} \times \frac{\tau}{E} = \frac{\tau}{E}\left(1 + \frac{1}{m}\right) = \frac{\tau}{E}\left(\frac{m+1}{m}\right) ------(4)$$

Now equating equations (1) and (2)

$$\frac{\tau}{2C} + \frac{\tau}{E}\left(\frac{m+1}{m}\right) \quad \text{or} \quad C = \frac{mE}{2(m+1)}$$

$$= \frac{2.86 \times 318480}{3(2.86 - 2)} = 353043.7\,N/mm^2$$

Problem:

A composite bar is made up of brass rod of 25 mm diametrer enclosed in a steel tuber of 40 mm external and 35 mm internal diameter. The ends of rod and tube are securely fixed. Find stresses developed is rod and steel tube when the composite bar is subjected to an axial pull of 45 KN. Take E for brass as 80 GPa and E for steel as 200 GPa 2012(w)(2c)

Given:

Diameter of brass road $d_b = 25$ mm

Area of brass rod, $A_b = \pi/4 \times d_b^2 = \pi/4 \times (25)^2 = 490.63\ mm^2$

Area of steel tube, $A_s = \pi/4(40^2 - 35^2) = 294.38\ mm^2$

$P = 45\ KN = 45 \times 10^3\ N$

Let σ_b = stress is brass

σ_S = stress is steel

$E_b = 80$ GPa $= 80 \times 10^3$ N/mm^2

$E_s = 200$ GPa $= 200 \times 10^3$ N/mm^2

$$\frac{\sigma_s}{\sigma_b} = \frac{E_s}{E_b} = \frac{200\times 10^3}{80\times 10^3} = 2.5 \qquad \Rightarrow \sigma_s = 2.5\sigma_b$$

$$P = \sigma_s.A_s + \sigma_b.A_b \quad \Rightarrow 45\times 10^3 = 2.5\sigma_b \times 294.38 + \sigma_b \times 490.63$$

$$\Rightarrow 45\times 10^3 = \sigma_b\left[\left(2.5\times 294.38\right) + 490.63\right]$$

$$\Rightarrow 45\times 10^3 = \sigma_b\left(735.95 + 490.63\right) \qquad \Rightarrow 1226.6\sigma_b = 45\times 10^3$$

$$\Rightarrow \sigma_b = \frac{45\times 10^3}{1226.6} = 36.7\ \mathrm{N/mm^2}$$

$$\sigma_s = 2.5\sigma_b = 2.5\times 3.67 = 91.75\,\mathrm{N/mm^2}$$

Problem

A bar of 20 mm diameter is subjected to a pull of 50 KN. The measured extension over a gauge length of 20 cm is found to be 0.1 mm and change in diameter is 0.0035 mm Evaluate the poisonous ratio, e and is:

2015(w), (1-c)

Diameter of bar, d = 20 mm

Area of bar, $A = \pi/4 \times d^2 = \pi/4 \times (20)^2 = 314$ mm^2

Length of bar, L = 20 cm = 200 mm

Extension of bar, δL=0.1 mm $\qquad$ P = 50 KN = 50×10^3 N

Change in diameter, d = 0.0035 mm

$$\text{Linear strain}, e = \frac{\delta l}{l} = \frac{0.1}{200} 0.0005$$

$$\text{Lateral strain}, = \frac{\delta d}{d} = \frac{0.0035}{20} = 0.000175$$

$$\text{Poisonou's ratio}, \frac{1}{m} = \frac{\text{Lateral strain}}{\text{Linear strain}} = \frac{0.000175}{0.0005} = 0.35$$

or $$m = \frac{1}{0.35} = 2.86$$

$$\text{Stress}, \delta = \frac{P}{A} = \frac{50 \times 10^3}{314} = 159.24 \, N/mm^2$$

$$\text{strain}, e = \frac{\delta L}{L} = \frac{0.1}{200} = 0.0005$$

$$\text{Young's modulus}, E = \frac{\text{Stress}}{\text{Strain}} = \frac{159.24}{0.0005} = 318480 \, N/mm^2$$

$$\text{Bulk modulus}, K = \frac{mE}{3(m-2)}$$

Problem :

A tensile load of 60 KN applied axial on a cylindrical bar of diameter 10 cm. What is the tensile stress on a section perpendicular to the axis of bar 2010(w),2014(w) 1(b)

Load, P = 60 KN = 60 × 10 N

Diameter, d = 10 cm = 0.1 m

Area, $A = \pi/4 \times d^2 = \pi/4 \times (0.1)^2 = 0.00785 \, m^2 = 78550 \, mm^2$

$$\text{Stress}, = \frac{P}{A} = \frac{60 \times 10^3}{7850} N/mm^2 = 7.64 \, N/mm^2$$

Problem:

A material has a Young's modulus 1.3×10^5 N/mm^2 and poisonous ratio of 0.3. Calculate rigidity modulus and bulk modulus 2014(w) 2(b)

Young's modulus, $E = 1.3 \times 10^5\ N/mm^2$

Poisonous ratio, $1/m = 0.3$ or $m = 1/0.3 = 3.33$

$$\text{Bulk modulus, } K = \frac{mE}{3(m-2)}$$

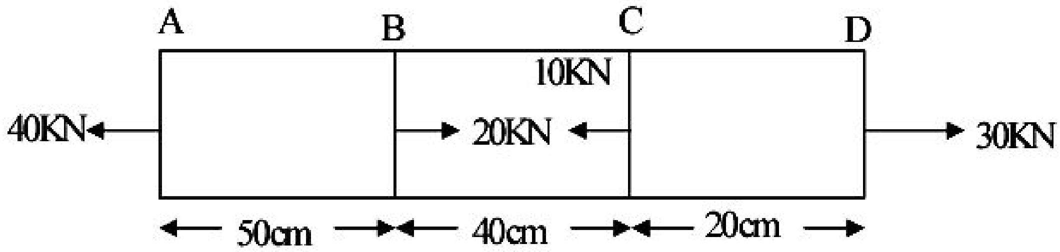
$$= \frac{3.33 \times 1.3 \times 10^5}{3(3.33-2)} = \frac{3.33 \times 1.3 \times 10^5}{3 \times 1.33} = 1.08 \times 10^5\ N/mm^2$$

$$\text{Modulus of rigidity, } C = \frac{mE}{2(m+1)} = \frac{3.33 \times 1.3 \times 10^5}{2(3.33+1)}$$

$$= \frac{3.33 \times 1.3 \times 10^5}{2 \times 4.33} = 0.5 \times 10^5\ N/mm^2$$

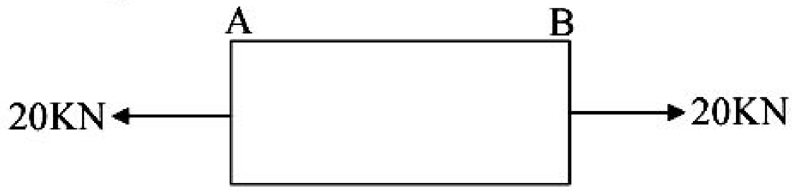
Problem :

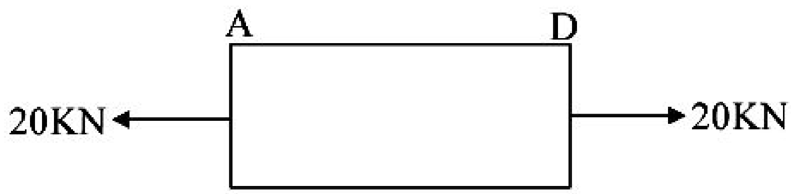
A steel bar 25 mm diameter is loaded as shown in figure. Determine stresses in each part of the total elongation 2014(w)



Let dl = total elongation Assuming E for steel = $2 \times 10^5\ N/mm^2$

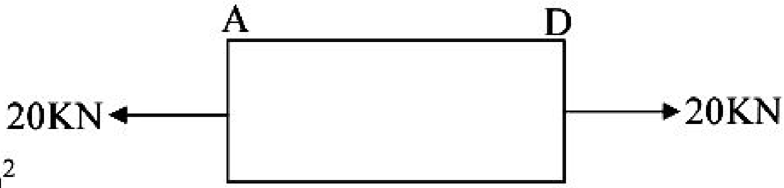
Total elongation





Area of steel bar

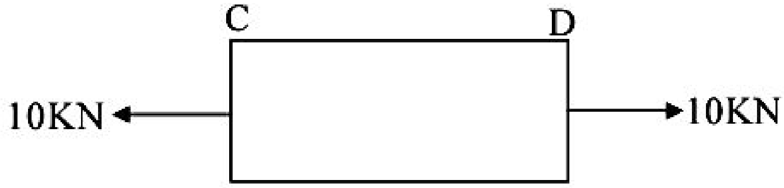
$A = \pi/4 \times (25)^2 = 490.625 \text{ mm}^2$



Total elongation

$$(\delta l) = \frac{P_1L_1}{AE} + \frac{P_2L_2}{AE} + \frac{P_3L_3}{AE}$$

$$= \frac{1}{AE}(P_1L_1 + P_2L_2 + P_3L_3)$$



$$= \frac{1}{490.625 \times 2 \times 10^5}\left[20 \times 10^3 \times 500 + 20 \times 10^3 \times 1100 + 10 \times 10^3 \times 200\right]$$

$$= \frac{1}{490.625 \times 2 \times 10^5} \times 10^3\left[20 \times 500 + 20 \times 1100 + 10 \times 200\right]$$

$$= \frac{1}{490.625 \times 2 \times 100}\left[10,000 + 22,000 + 20,000\right] = 0.35\text{mm}$$

Problem:

A 15 cm dia steel rod passes centrally through a copper tube 50 mm external dia and 40 mm internal dia. The tube is closed at each end by rigid plates of negligible thickness. The nuts are tightened lightly home on the projecting parts of rod. If the temperature of assembly is raised by 60°C, calculate the stresses raised by 60°C, Calculate the stresses developed in steel and copper. Take E for steel and copper as 210 kw/mm^2 and 110 KN/mm^2 respectively. Also α for steel and copper as 12×10^{-6}/°C and 17.5×19^{-6}/°C respectively 2014(w), 7(c)

Given

Diameter of steel rod d_s = 15 cm

Area of steel rod, $A_s = \pi/4 \times d_s^2 = \pi/4 \times (15)^2 \text{ cm}^2 = 176.63 \text{ mm}^2$

Area of copper tube, $A_c = \pi/4\ (50^2 - 40)^2 = 706.5 \text{ mm}^2$

$t = 60°c$

Tension in steel = Compression in copper

$\sigma_S . A_S = \sigma_c . A_C$

$$\Rightarrow \frac{\sigma_s}{\sigma_c} = \frac{A_c}{A_s} = \frac{706.5}{176.63} = 4 \Rightarrow \sigma_s = 4\sigma_c$$

$$E_s + E_c = t(\alpha_c - \alpha_s)$$

$$\Rightarrow \frac{\sigma_s}{E_s} + \frac{\sigma_c}{E_c} = 60\left[17.5\times10^{-6} - 12\times10^{-6}\right]$$

$$\Rightarrow \frac{4\sigma_c}{210\times10^3} + \frac{\sigma_c}{110\times10^3} = 60\times5.5\times10^{-6}$$

$$\Rightarrow \frac{\sigma_c}{10^3}\left[\frac{4}{210} + \frac{1}{110}\right] = 60\times5.5\times10^{-6}$$

$$\Rightarrow \frac{\sigma_c}{10^3}\left[0.019 + 0.00009\right] = 60\times5.5\times10^{-6}$$

$$\Rightarrow \frac{\sigma_c}{10^3}\times0.01 = 60\times5.5\times10^{-6} \Rightarrow \sigma_c = 33\mathrm{N/mm^2}$$

$$\sigma_s = 4\sigma_c = 4\times33 = 132\,\mathrm{N/mm^2}$$

CHAPTER:2

Q.1 **Define temperature stress. 2005(w), 1(j), 2012(w), 2(a) 2014(w)**

Ans: When ever a body is subjects to a change in temperature it undergoes expansion or contraction. But if the deformation of the body is prevented, then the stress which will induced in the body is known as temp. stress.

Q2. Define hoop stress and longitudinal stress.

2012(w), 3(a), 2005(w), 1(c) 2013(w), 5(a), 2014(w)

Ans: Hoop stress: The stress which acts tangentially along the circumference of the shell, this is known as circumferential stress is σ_c

Longitudinal stress: The stress which acts parallel to the longitudinal axis of the shell is known as longitudinal stress σ_c.

Q3. Derive an expression for hoop stress and longitudinal stress for a thin cylinder subjected to an internal pressure 'P' 2012(w) 3(b), 2005(w),2(d), 2006,(2c), 2013, (5b), 2014(w),2015(w), (2b)

Ans: Let l = length of the shell.

P = Intensity of internal pressure

σ_c = circumferential stress.

d = diameter of the circular shell.

t = thickness.

Total pressure along $x - x'$ = Intensity of pressure × Area

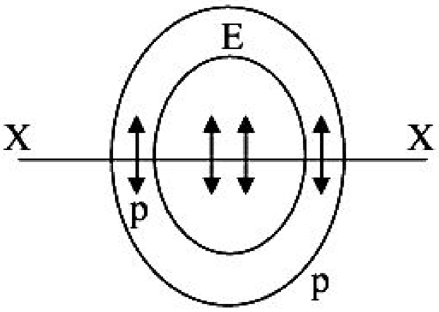
$= P \times d \times l$

Resisting section = 2t l

$$\text{Circumferential stress } \sigma_c = \frac{\text{Total pressure}}{\text{Resisting section}}$$

$$= \sigma_c = \frac{P \times d \times l}{2tl}$$

$$\sigma_c = \frac{Pd}{2t}$$



Longitudinal stress :-

Now total pressure acting along y - y'

= Intensity of pressure × Area.

$= P\,\pi/4\, d^2$

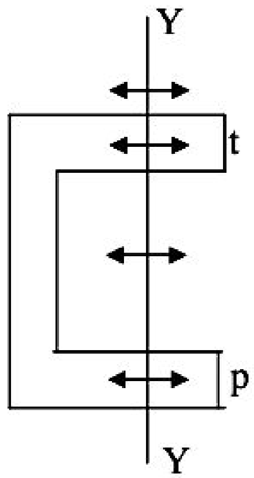
Resisting section = $\pi d \times t$

Longitudinal stress

$$\sigma_L \frac{\text{Total pressure along } yy'}{\text{Resisting section.}}$$

$$= \frac{P \times \pi d^2}{4\pi dt} = \frac{pd}{4t}$$

$$= \sigma_L = \frac{pd}{4t}$$



Q. Find expression for temperature stress for a rise in temperature of $t^{o}C$. when the ends do not yield. Take α co-efficient of expression 'l' as the original length 2014(w), 2015(w) 1(b)

Ans: Consider a body subjected to an increase in temperature.

Let l = original length of body

T = Increase of temperature

α = Co-efficient of linear expansion

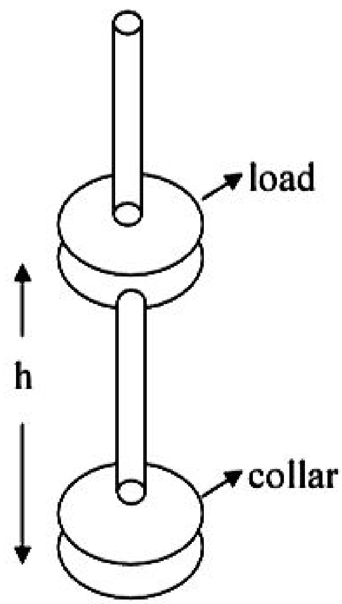
Increase in length due to increase of temperature, $\delta l = l\,\alpha\,t$

When the ends do not yield

Strain, $e = \frac{\delta l}{l} = \frac{l \alpha t}{l} = \alpha t$

Find out stress due to impact loading

Consider a bar subjected to a load applied with impact as shown in figure.



Let p = load applied with impact

A = cross sectional area of bar

E = Modulus of elasticity of bar material

δl = Deformation of bar

σ = stress induced by the application of this load with impact

h = height through which load will fall.

Work done = load × distance = $p(h + \delta l)$ and energy stored, $u = \frac{\sigma^2}{2E} \times Al$

Since energy = workdone

$$\therefore \frac{\sigma^2}{2E} \times Al = p(h + \delta l) = p(h + \frac{\delta}{E} l)$$

$$\therefore \frac{\sigma^2}{2E} \times Al = ph + \frac{p\sigma l}{E}$$

$$\therefore \frac{\sigma^2}{2E} \times Al = \frac{p\sigma l}{E} - ph = 0$$

Multiplying both sides by E/Al

$$\therefore \frac{\sigma^2}{2} - \sigma\left(\frac{p}{A}\right) - \frac{pEh}{Al} = 0$$

This is a quadratic equation

$$\therefore \sigma = \frac{p}{A} \pm \sqrt{\left(\frac{p}{A}\right)^2 + 4 \times \frac{1}{2} \times \frac{pEh}{Al}}$$

$$= \frac{p}{A}\left[1 \pm \sqrt{1 + \frac{2AEh}{pl}}\right]$$

Q. Define strain energy and resistance 2015(w), 2(a)

Strain energy : The amount of energy stored in a body when strained within elastic limit is known as strain energy.

Strain energy = work done

Resistance: The strain energy stored in a body when strained within elastic limit is known as resistance.

Problem:

A cylindrical shell 2.5 m long and closed at the ends has an internal diameter of 1.25 m and wall thickness of 20 mm. Calculate the change in dimensions when subjected to an internal pressure of 1.5 MPa. Take E = 200 GPa and 1/m = 0.3 2014(w), 2(c)

Given

L = 2.5 m = 2500 mm.

D = 1.25 m = 1250 mm

T = 20 mm

P = 1.5 Mpa = 1.5 N/mm^2

E = 200 GPa = 200 × 10^3 N/mm^2

1/m = 0.3

Circumferential stress, σ_c = pd/2t N/mm^2

Longitudinal stress σ_l = pd/4t = N/mm^2

$$\text{Change in diameter}, \delta d = \frac{pd^2}{2tE}\left(1 - \frac{1}{2m}\right)$$

$$= \frac{1.5 \times (1250)^2}{2 \times 20 \times 200 \times 10^3}\left(1 - \frac{1}{2} \times 0.3\right) = 0.24\text{mm}$$

$$\text{Change in length}, \delta l = \frac{pdl}{2tE}\left(\frac{1}{2} - \frac{1}{m}\right)$$

$$= \frac{1.5 \times 1250 \times 2500}{2 \times 20 \times 200 \times 10^3}\left(\frac{1}{2} - 0.3\right) = 0.117\text{mm}$$

Problem:

A cylindrical shell 4m long has 1 m internal diameter and 20 mm metal thickness. Calculate the circumferential and longitudinal stress. If the shell is subjected to an internal pressure of 2Mpa. Calculate change in dimension of shell Take E = 200 GPa and poisonous ratio = 0.3

2014(w), 3(c)

l = 4m = 4000 mm

d = 1m = 1000 mm

t = 20 mm

p = 2 MPa = 2 N/mm^2

E = 200 GPa = 200 × 10^3 N/mm^2

1/m = 0.3

Circumferential stress, $\sigma_c = \frac{pd}{2t} = \frac{2\times 1000}{2\times 20} = 50\,N/mm^2$

Longitudinal stress, $\sigma_l = \frac{pd}{4t} = \frac{2\times 1000}{4\times 20} = 25\,N/mm^2$

Change in diameter, $\delta d = \frac{pd^2}{2tE}\left(1 - \frac{1}{2m}\right)$

$= \frac{2\times (1000)^2}{2\times 20\times 200\times 10^3}\left(1 - \frac{1}{2}\times 0.3\right)$

$= 0.2125$ mm

Change in length, $\delta l = \frac{pdl}{2tE}\left(\frac{1}{2} - \frac{1}{m}\right)$

$= \frac{2\times 1000\times 4000}{2\times 20\times 200\times 10^3}\left(\frac{1}{2} - 0.3\right)$

$= 0.2$ mm

Change in volume = ?

$$\text{Hoop strain} \in_c = \frac{pd}{2tE}\left(1-\frac{1}{2m}\right) = \frac{2\times 1000}{2\times 20\times 200\times 10^3}\left(1-\frac{1}{2}\times 0.3\right)$$

$$= 0.00021$$

$$\text{Longitudinal strain}, \in_l = \frac{pd}{2tE}\left(\frac{1}{2}-\frac{1}{2m}\right)$$

$$= \frac{2\times 1000}{2\times 20\times 200\times 10^3}\left(\frac{1}{2}-0.3\right)$$

$$= 0.00005$$

$$\text{Volume of shell, } V = \frac{\pi}{4}\times d^2\times l = \frac{\pi}{4}(1000)^2\times 4000$$

$$= 3.25\times 10\text{mm}$$

$$\frac{\delta V}{V} = 2\in_c + \in_l \qquad \text{or} \qquad \delta V = v\left[2\in_c + \in_l\right] = 3.25\times 10^{-12}\left[2\times 0.00021 + 0.0005\right]$$

$$= 3\times 10^{-15}\text{mm}^3$$

Problem:

A cylindrical vessel closed with plane ends is made of 4 mm thick steel plate. The diameter and length are 250 mm and 750 mm respectively when same is subjected to an internal pressure of 300 N/mm^2. Calculate the following

(i) Longitudinal and hoop stress

(ii) Changes in diameter, length and volume

Assume e = 200 G N/m^2

Poisonous ratio = 0.3 2015(w), 4(c)

Given:

t = 4 mm

d = 250 mm

l = 750 mm

p = 300 N/cm^2 = 3 N/mm^2

E = 200 G N/m^2 = 200 × 10^3 N/mm^2, 1/m = 0.3

$$\text{Circumferential stress, } \sigma_c = \frac{pd}{2t}$$

$$= \frac{3 \times 250}{2 \times 4} = 93.75 \text{ N/mm}^2$$

$$\text{Longitudinal stress, } \sigma_1 = \frac{pd}{4t}$$

$$= \frac{3 \times 250}{4 \times 4} = 46.88 \text{ N/mm}^2$$

$$\text{Change in diameter, } \sigma d = \frac{pd^2}{2tE}\left(1 - \frac{1}{2m}\right)$$

$$= \frac{3 \times 250 \times 750}{2 \times 4 \times 200 \times 10^3} = \left(\frac{1}{2} - 0.3\right) = 0.007\text{mm}$$

Problem:

A cylindrical vessel 2m to 500 mm in diameter with 10 mm plate is subjected to an internal pressure of 3 MPa. Calculate change volume of vessel. Take E = 200 GPa, Poisonous ratio = 0.3 for the vessel material.

2013(w), 5(c)

Given :

$l = 2\text{m} = 2000$ mm

$d = 500$ mm

$p = 3$ MPa $= 3\text{N/mm}^2$

$E = 200$ GPa $= 200 \times 10^3$ N/mm^2

$1/m = 0.3$

$t = 10$ mm

Volume of cylinder, $V = \frac{\pi}{4} \times d^2 \times L$

$$= \frac{\pi}{4} \times (500)^2 \times 2000 = 392500000 mm^3$$

Hoop strain, $\in_c = \frac{pd}{2tE} - \frac{1}{m} \times \frac{pd}{4tE} = \frac{pd}{2tE}\left(1 - \frac{1}{2m}\right)$

$$= \frac{3 \times 500}{2 \times 10 \times 200 \times 10^3}\left(1 - \frac{1}{2} \times 0.3\right) = 0.000319$$

Longitudinal strain, $\in_l = \frac{pd}{4tE} - \frac{1}{m} \times \frac{pd}{2tE}$

$$= \frac{pd}{2tE}\left(\frac{1}{2} - \frac{1}{m}\right) = \frac{3 \times 500}{2 \times 10 \times 200 \times 10^3}\left(\frac{1}{2} - 0.3\right) = 0.000075$$

Volumetric strain, $\frac{\delta V}{v} = 2\in_l + \in_c$

$$\Rightarrow \delta V = V(2\in_l + \in_c)$$

$$= 392500000(2 \times 0.000075 + 0.000319)$$

$$= 184475 mm^3$$

Change in volume $184475 mm^3$

Problem:

A cylindrical shell 3 m long has 1n internal diameter and 15 mm metal thickness. Calculate the circumferential and longitudinal stresses if the shell is subjected to an internal pressure of 1.5 MPa. Also calculate change in dimension of shell. Take E = 200 GPa and poisonous ratio = 0.3 2012(w), 3(c)

Given :

l = 3 m = 3000 mm

d = 1m, = 1000 mm

t = 15 mm

$p = 1.5$ MPa $= 1.5$ N/mm^2

$E = 200$ GPa $= 200 \times 10^3$ N/mm^2

$1/m = 0.3$

$$\text{circumferential stress}, \sigma_c = \frac{pd}{2t} = \frac{1.5 \times 1000}{2 \times 15} = 50\text{N/mm}^2$$

$$\text{longitudinal stress}, \sigma_l = \frac{pd}{4t} = \frac{1.5 \times 1000}{4 \times 15} = 25\text{N/mm}^2$$

$$\text{change in diameter}, \delta_d = \frac{pd^2}{2tE}\left(1 - \frac{1}{2m}\right)$$

$$= \frac{1.5 \times (1000)^2}{2 \times 15 \times 200 \times 10^3}\left(1 - \frac{1}{2} \times 0.3\right)$$

$$= 0.21 \text{ mm}$$

$$\text{Change in length } \delta_l = \frac{pdl}{2tE}\left(\frac{1}{2} - \frac{1}{m}\right)$$

$$= \frac{1.5 \times 1000 \times 3000}{2 \times 15 \times 200 \times 10^3}\left(\frac{1}{2} - 0.3\right) = 0.15\text{mm}$$

Problem :

A cylindrical shell 2.5 m way and closed at the ends has an internal diameter of 1.25 m and wall thickness of 20 mm. Calculate the change in dimension when subjected to an internal pressure is 1.5 MPa. Take E = 200 GPa and 1/m = 0.3 2014(w), 2(c)

Given:

$l = 2.5$ m $= 2500$ mm

$d = 1.25$ m $= 1250$ mm

$t = 20$ mm

$p = 1.5$ MPa $= 1.5$ N/mm^2

$E = 200\ GPa = 200 \times 10^3\ N/mm^2$

$1/m = 0.3$

Change in diameter, $\delta d = ?$

Change in length, $\delta l = ?$

$$\text{Change in diameter}, \delta d = \frac{pd^2}{2tE}\left(1 - \frac{1}{2m}\right)$$

$$= \frac{1.5 \times (1250)^2}{2 \times 20 \times 200 \times 10^3}\left(1 - \frac{1}{2} \times 0.3\right) = 0.25mm$$

$$\text{Change in length } \delta l = \frac{pdl}{2tE}\left(\frac{1}{2} - \frac{1}{m}\right)$$

$$= \frac{1.5 \times 1250 \times 2500}{2 \times 20 \times 200 \times 10^3}\left(\frac{1}{2} - 0.3\right) = 0.12mm$$

CHAPTER:3

PRINCIPAL STRESS AND STRAIN

Q.1. Define principal plane and principal stress ***2006,(1-iii), 2010(1-c)***

Ans: At a point in a strained material, there are three mutually perpendicular plane, which carry only direct stress, no shear stress, is known as principal plane.

Principal Stress: the magnitude of the direct stress across the principal plane is known as principal stress.

Q2. Derive the principal stresses on a body subjected to two mutually perpendicular direct stresses accompanied with shear stresses

2012(w)1-(b), 2014(w)

Ans: Now let us consider an oblique section inclined with x-x axis an with we are required to find out stresses

Let σ_x = Tensile stress along x-x axis

σ_y = Tensile stress along y-y axis.

ζ = shear stress along x-x axis

θ = Angle which the oblique plane section AB.

First of all consider the equilibrium of the wedge ABC , ABC.

Horizontal force acting on the face AC,

$P_1 = \sigma_x . Ac\ (\rightarrow)$ ……………………. (1)

Vertical force acting on the face AC,

$P_2 = \zeta\ xy\ .\ AC\ (\downarrow)$ ………………… (2)

Similarly, vertical force acting on the face BC,

$P_3 = \sigma_y\ .\ BC\ (\downarrow)$ ……………………. (3)

Horizontal force on the face BC,

$P_4 = \zeta_{xy}\ .\ BC\ (\rightarrow)$ …………………….(4)

Now resolving the force perpendicular to the section AB

$P_n = P_s \sin\theta - P_2 \cos\theta + P_3 \cos\theta - P_4 \sin\theta$

$= \sigma_x\ .\ AC \sin\theta - \zeta_{xy}\ AC\ \cos\theta + \sigma_y\ .\ BC \cos\theta - \zeta_{xy}\ BC \sin\theta.$

Now resolving the force longentically to AB,

$P_1 = P_1 \cos\theta + P_2 \sin\theta - P_3 \sin\theta - P_4 \cos\theta$

$= \sigma_x\ .\ AC \cos\theta + \zeta_{xy}\ AC.\ \text{Sin}\ \theta - \sigma_y\ .\ BC \sin\theta - \zeta_{xy}\ BC \cos\theta.$

We know that normal stress across the section AB, $\sigma_n = p_n/AB$

$$= \frac{\sigma_x\ AC \sin\theta\ -\ \zeta_{xy}\ AC \cos\theta\ +\ \sigma_y\ BC \cos\theta\ -\ \zeta xy\ BC \sin\theta}{AB}$$

$$= \frac{\sigma_x\ AC \sin\theta}{AB} - \frac{\zeta_{xy}\ AC \cos\theta}{AB} + \frac{\sigma_y\ BC \cos\theta}{AB} - \frac{\zeta xy\ BC \sin\theta}{AB}$$

Q3. ***State the relation between maximum shear stress and principal shear stress at a point.*** ***2006(w), 1(iv)***

Ans:
$$=\frac{\frac{\sigma_x AC\sin\theta}{\frac{AC}{\sin\theta}}}{}-\frac{\frac{\zeta_{xy}\ AC\cos\theta}{\frac{AC}{\sin\theta}}}{}+\frac{\frac{\sigma_y\ BC\cos\theta}{\frac{BC}{\cos\theta}}}{}-\frac{\frac{\zeta xy\ BC\sin\theta}{\frac{BC}{\cos\theta}}}{}$$

$$=\sigma_x\ \sin^2\theta-\zeta_{xy}\sin\theta.\cos\theta+\sigma_y\ \cos^2\theta-\zeta xy\sin\theta.\cos\theta$$

$$=\frac{\sigma_x}{2}-(1-\cos 2\theta)+\frac{\sigma_y}{2}(1+\cos 2\theta)-2\zeta_{xy}\sin\theta.\cos\theta$$

$$=\frac{\sigma_x}{2}-\frac{\sigma_x}{2}\cos 2\theta+\frac{\sigma_y}{2}+\frac{\sigma_y}{2}\cos 2\theta-\zeta_{xy}\sin 2\theta$$

$$=\frac{\sigma_x+\sigma_y}{2}-\frac{\sigma_x-\sigma_y}{2}\cos 2\theta-\zeta_{xy}\sin 2\theta.................(5)$$

shear strss i.e. tan gential stress across the sec tion AB.

$$\zeta=\frac{pt}{AB}$$

$$=\frac{\sigma_x AC\ \cos\theta+\zeta_{xy}\ AC\sin\theta-\sigma_y\ BC\sin\theta-\zeta_{xy}\cos\theta}{AB}$$

$$=\frac{\sigma_x AC\ \cos\theta}{AB}+\frac{\zeta_{xy}\ AC\sin\theta}{AB}-\frac{\sigma_y\ BC\sin\theta}{AB}-\frac{\zeta_{xy}\cos\theta}{AB}$$

$$=\frac{\frac{\sigma_x AC\ \cos\theta}{\frac{AC}{\sin\theta}}}{}+\frac{\frac{\zeta_{xy}\ AC\sin\theta}{\frac{AC}{\sin\theta}}}{}-\frac{\frac{\sigma_y\ BC\sin\theta}{\frac{BC}{\sin\theta}}}{}-\frac{\frac{\zeta_{xy}\ BC\cos\theta}{\frac{BC}{\cos\theta}}}{}$$

$$=\sigma_x\sin\theta.\cos\theta+\zeta_{xy}\ \sin^2\theta-\sigma_y\ \sin\theta.\cos\theta-\zeta_{xy}\ \cos^2\theta$$

$$=(\sigma_x-\sigma_y)\sin\theta.\cos\theta+\frac{\zeta_{xy}}{2}(1-\cos 2\theta)-\frac{\zeta_{xy}}{2}(1+\cos 2\theta)$$

$$=\sigma_x-\sigma_y\sin 2\theta+\frac{\zeta_{xy}}{2}-\frac{\zeta_{xy}}{2}\cos 2\theta-\frac{\zeta_{xy}}{2}-\frac{\zeta_{xy}}{2}\cos 2\theta$$

$$=\frac{\sigma_x-\sigma_y}{2}\sin 2\theta-\zeta_{xy}\cos 2\theta.$$

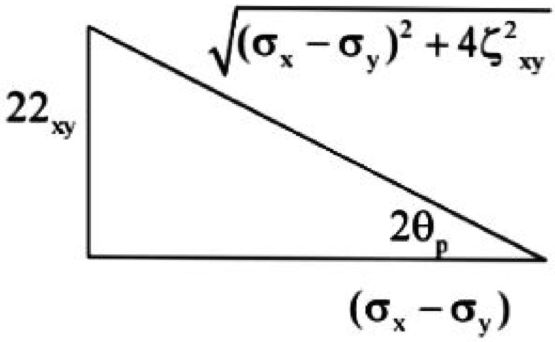
Now the principal stress acting on the principal planes may be found out by equating the on the shear stress to zero. Now let θ_p be the value of the angle for which the shear stress is zero.

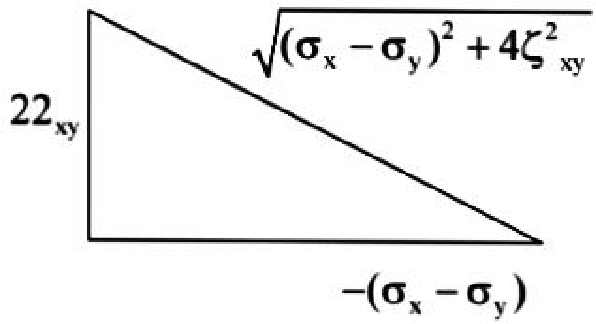
$$\therefore \frac{\sigma_x - \sigma_y}{2}\sin 2\theta_p - \zeta_{xy}\cos 2\theta_p = 0$$

$$\text{or}\ \frac{\sigma_x - \sigma_y}{2}\sin 2\theta_p = \zeta_{xy}\cos 2\theta_p$$

$$\tan 2\theta_p = \frac{2\zeta_{xy}}{\sigma_x - \sigma_y}$$

From the above equation we find that the following two cases satisfy this condition as shown.





Thus we find that there are two principal planes at right angle to each other, their inclination with x-x axis being θ_p and $\theta_p{}^1$.

Now force case-1 we find that

$$\sin 2\theta_{p_1} = \frac{2\zeta_{xy}}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\zeta^2 xy}}$$

$$\cos 2\theta_{p_1} = \frac{(\sigma_x - \sigma_y)}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\zeta^2 xy}}$$

$$\text{similarlyforcase} - 2$$

$$\sin 2\theta_{p_2} = \frac{-2\zeta_{xy}}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\zeta^2 xy}}$$

$$\cos 2\theta_{p_2} = \frac{-(\sigma_x - \sigma_y)}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\zeta^2 xy}}$$

Now the values of principal stress may be found out by substituting the above values of $2\theta_p$ and $2\theta^1_p$.

Maximum principal stress.

$$\sigma_{p_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x + \sigma_y}{2}\cos 2\theta - \zeta_{xy}\sin 2\theta$$

$$= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x + \sigma_y}{2} \times \frac{\sigma_x + \sigma_y}{\sqrt{(\sigma_x + \sigma_y)^2 + 4\zeta^2 xy}} + \zeta_{xy} \times \frac{2\zeta_{xy}}{\sqrt{(\sigma_x + \sigma_y)^2 + 4\zeta^2 xy}}.$$

$$= \frac{\sigma_x + \sigma_y}{2} + \frac{\sqrt{(\sigma_x + \sigma_y)^2 + 4\zeta^2 xy}}{2}$$

$$= \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \zeta^2_{xy}}$$

Minimum principal stress

$$\sigma_{p_2} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x + \sigma_y}{2} \times \frac{\zeta_{xy}}{\sqrt{(\sigma_x - \sigma_y)}}$$

$$+\zeta_{xy} \times \frac{-2\zeta_{xy}}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\zeta_{xy}}}$$

$$= \frac{\sigma_x + \sigma_y}{2} + \frac{-(\sigma_x + \sigma_y)^2 - 2\zeta^2_{xy}}{2\sqrt{(\sigma_x + \sigma_y)^2 + 4\zeta^2_{xy}}}$$

$$= \frac{\sigma_x + \sigma_y}{2} - \frac{(\sigma_x + \sigma_y)^2 + 4\zeta^2_{xy}}{2\sqrt{(\sigma_x + \sigma_y)^2 + 4\zeta^2_{xy}}}$$

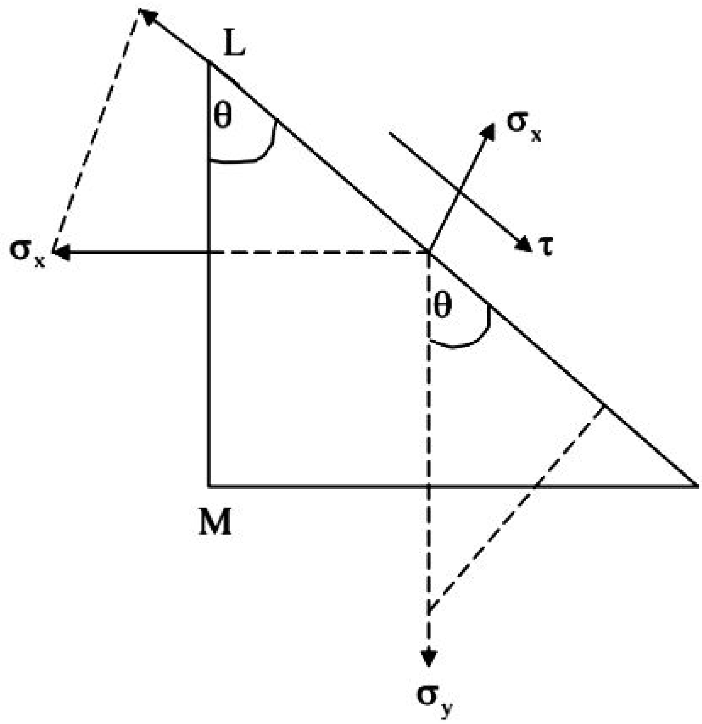
$$= \frac{\sigma_x + \sigma_y}{2} - \frac{\sqrt{(\sigma_x + \sigma_y)^2 + 4\zeta^2_{xy}}}{2}$$

$$= \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \zeta^2_{xy}}$$

Problem

Derive an expression for stresses in two mutually perpendicular directions stresses. 2014(w), 3(b)

Consider that direct stresses σ_x and σ_y act across the faces LM and MN and that the block has unit depth perpendicular to LMN. Let the stresses τ and σ_n act on the same plane at an angle 'θ' to LM.



Resolving normal to LN.

$$\sigma_n \times LN = \sigma_x \times LM\cos\theta + \sigma_y MN\sin\theta$$

$$\sigma_n = \sigma_x \times \frac{LM}{MN}\cos\theta + \sigma_y.\frac{MN}{LN}\sin\theta$$

$$= \sigma_x.\cos^2\theta + \sigma_y.\sin^2\theta = \frac{\sigma_x}{2}\times 2\cos^2\theta + \frac{\sigma_y}{2}\times 2\sin^2\theta$$

$$= \frac{\sigma_x}{2}\left(1-\sin^2\theta+\cos^2\theta\right) + \frac{\sigma_y}{2}\left(1-\cos^2\theta+\sin^2\theta\right)$$

$$= \frac{\sigma_x+\sigma_y}{2} + \sigma_x\left[\frac{\cos^2\theta-\sin^2\theta}{2}\right] - \sigma_y\left[\frac{\cos^2\theta-\sin^2\theta}{2}\right]$$

$$= \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x+\sigma_y}{2}\cos 2\theta$$

when $\upsilon = 0$ $\quad \sigma_n = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2} = \sigma_x$

when $\upsilon = \frac{\pi}{2}$ $\quad \sigma_n = \frac{\sigma_x+\sigma_y}{2} - \frac{\sigma_x-\sigma_y}{2} = \sigma_y$

Resolving parallel to LN

$$\tau \times LN = \sigma_x \times LM \sin\theta - \sigma_y \times MN \cos\theta$$

$$\tau = \sigma_x \frac{LM}{LN}\sin\theta - \sigma_y \frac{MN}{LN}\cos\theta$$

$$= \sigma_x.\cos\theta.\sin\theta - \sigma_y.\sin\theta.\cos\theta = (\sigma_x - \sigma_y)\sin\theta$$

$$= \frac{\sigma_x - \sigma_y}{2}\sin 2\theta$$

The maximum value of τ occurs when

$$2\theta = \frac{\pi}{2} \quad \text{or} \quad \theta = \frac{\pi}{4}$$

$$\tau_{max} = \frac{\sigma_x - \sigma_y}{2}$$

Resultant stress, $\sigma_R = \sqrt{\sigma_n^2 + \tau^2}$

Problem:

The stresses at a point in a component are 100 MPa (tensile) and 50 MPa (compressive). Determine the magnitude of normal and shear stresses on a plane inclined at an angle of 25 with the tensile stress angle of 25° with the tensile stress. Also determine the direction of resultant stress and magnitude of maximum intensity of shear stress 2012(w), 1(c).

Given:

$$\sigma_x = 100\text{MPa} = 100\,\text{N/mm}^2$$

$$\sigma_y = -50\text{MPa} = -50\,\text{N/mm}^2$$

$$\vartheta = 25^\circ$$

$$\text{Normal stress}, \sigma_n = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta$$

$$= \frac{100 + (-50)}{2} - \frac{100 - (-50)}{2} \times \cos 2 \times 25^\circ$$

$$= \frac{100 - 50}{2} - \frac{100 + 50}{2} \times \cos 50^\circ$$

$$= \frac{50}{2} - \frac{150}{2} \times \cos 50^\circ = 25 - 75\cos 50^0 = -23.23\text{N/mm}^2$$

$$\text{Shear stress}, \tau = \frac{\sigma_x - \sigma_y}{2} \times \sin 2\theta$$

$$= \frac{100-(-50)}{2} \times \sin 2 \times 25^\circ$$

$$= \frac{100+50}{2} \times \sin 50^\circ = 75 \sin 50^0 = 57.45 N/mm^2$$

Direction of Resultant stress

$$\tan\theta = \frac{\tau}{\sigma_n} = \frac{57.45}{-23.23} = -2.47$$

$$\Rightarrow \theta = \tan^{-1}(-2.47) = -68^\circ$$

Magnitude of maximum shear stress

$$T_{max} = \pm\frac{\sigma_x - \sigma_y}{2} = \pm\frac{100-(-50)}{2}$$

$$= \pm\frac{100+50}{2} N/mm^2 = \pm 75 N/mm^2$$

Problem:

A point in a strained material is subjected to a stress as shown below. Calculate principal stress ii) Maximum shear stress and also the plane along which and also the plane along which it acts. 2014(w),3(c)

Given :

$\sigma_x = 50\ MN/m^2$

$\sigma_y = 100\ MN/m^2$

$\tau = 25\ MN/m^2$

$$\text{Major principal stress, } \sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

$$\frac{50+100}{2} + \sqrt{\left(\frac{50-100}{2}\right)^2 + (25)^2} = 110.35 MN/m^2$$

$$\text{Minor principal stress}, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

$$= \frac{50+100}{2} - \sqrt{\left(\frac{50-100}{2}\right)^2 + (25)^2} = 39.65 MN/m^2$$

$$\text{Max. shear stress, } \tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

$$= \sqrt{\left(\frac{50-100}{2}\right)^2 + (25)^2} = 35.35\,\text{MN/m}^2$$

Angle made by principal planes.

$$\tan 2\vartheta_p = \frac{2\tau}{\sigma_x - \sigma_y} = \frac{2\times 25}{50-100} = -1$$

or $\quad 2\vartheta_p = \tan^{-1}(-1) = 135^\circ$

or $\quad \vartheta_p = 67.5^\circ \quad$ or $\quad 157.5^\circ$

Q. Write short notes on Mohr's circle 2014(w)

Ans: We have already discussed analytical method for determination of various stresses across a section. Another method known as graphical method is used for determination of stresses. This is done by drawing a Mohr's circle of stresses.

The construction of Mohr's circle of stresses as well as determination of normal, shear and resultant stresses is very easier than the analytical method. More over there is a little chances of committing error in this method.

The angle is taken with reference to x-x axis. All the angles traced in anticlockwise direction to x-x axis are taken as negative where those in clockwise direction as positive. The value of angle 'υ' until and unless mentioned is taken as positive and drawn clock wise.

The measurement above x-x axis and to right of y-y axis is taken positive where as those below x-x axis and to left of y-y axis is taken negative.

Thus we find that there are two principal planes at right angle to each other, their inclination with x-x axis being θ_p and θ_p^1.

Now force case-1 we find that

$$\sin 2\theta_{p_1} = \frac{2\zeta_{xy}}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\zeta^2 xy}}$$

$$\cos 2\theta_{p_1} = \frac{(\sigma_x - \sigma_y)}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\zeta^2 xy}}$$

similarlyforcase – 2

$$\sin 2\theta_{p_2} = \frac{-2\zeta_{xy}}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\zeta^2 xy}}$$

$$\cos 2\theta_{p_2} = \frac{-(\sigma_x - \sigma_y)}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\zeta^2 xy}}$$

Now the values of principal stress may be found out by substituting the above values of $2\theta_p$ and $2\theta^1_p$.

Maximum principal stress.

$$\sigma_{p_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x + \sigma_y}{2}\cos 2\theta - \zeta_{xy}\sin 2\theta$$

$$= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x + \sigma_y}{2} \times \frac{\sigma_x + \sigma_y}{\sqrt{(\sigma_x + \sigma_y)^2 + 4\zeta^2 xy}} + \zeta_{xy} \times \frac{2\zeta_{xy}}{\sqrt{(\sigma_x + \sigma_y)^2 + 4\zeta^2 xy}}.$$

$$= \frac{\sigma_x + \sigma_y}{2} + \frac{\sqrt{(\sigma_x + \sigma_y)^2 + 4\zeta^2 xy}}{2}$$

$$= \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \zeta^2{}_{xy}}$$

$$\sigma_{p_2} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x + \sigma_y}{2} \times \frac{\zeta_{xy}}{\sqrt{(\sigma_x - \sigma_y)}}$$

$$+\zeta_{xy} \times \frac{-2\zeta_{xy}}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\zeta_{xy}}}$$

$$= \frac{\sigma_x + \sigma_y}{2} + \frac{-(\sigma_x + \sigma_y)^2 - 2\zeta^2{}_{xy}}{2\sqrt{(\sigma_x + \sigma_y)^2 + 4\zeta^2{}_{xy}}}$$

$$= \frac{\sigma_x + \sigma_y}{2} - \frac{(\sigma_x + \sigma_y)^2 + 4\zeta^2{}_{xy}}{2\sqrt{(\sigma_x + \sigma_y)^2 + 4\zeta^2{}_{xy}}}$$

$$= \frac{\sigma_x + \sigma_y}{2} - \frac{\sqrt{(\sigma_x + \sigma_y)^2 + 4\zeta^2{}_{xy}}}{2}$$

$$= \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \zeta^2{}_{xy}}$$

Problem:

A plane stress at a point is defined as $\sigma_x = 20$ MPa, $\sigma_y = 40$ MPa and z_{xy} =10 Mpa where the symbols have their usual meaning. Find thd principal stresses at the point and angles between principal planes. 2015(w),2(c)

Given :

$\sigma_x = 20\text{ MPa} = 20\text{ N/mm}^2$

$\sigma_y = 40\text{ MPa} = 40\text{ N/mm}^2$

$\tau = 10\text{ MPa} = 10\text{ N/mm}^2$

Manor principal stress, $\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$

$$\frac{20+40}{2} + \sqrt{\left(\frac{20-40}{2}\right)^2 + (10)^2} = \frac{60}{2} + \sqrt{\left(\frac{-20}{2}\right)^2 + (10)^2}$$

$$= 30 + 14.14 = 44.14\text{N/mm}^2$$

Minor Principal stress $\sigma_2 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$

$$= \frac{20+40}{2} - \sqrt{\left(\frac{20-40}{2}\right)^2 + (10)^2} = \frac{60}{2} - \sqrt{\left(\frac{-20}{2}\right)^2 + (10)^2}$$

$$= 30 - 14.14 = 15.86 \text{N/mm}^2$$

Angle made by principal planes

$$\tan 2\vartheta_p = \frac{2\tau}{\sigma_x - \sigma_y} \Rightarrow 2\vartheta_p = \tan^{-1}\left(\frac{2\tau}{\sigma_x - \sigma_y}\right)$$

$$= \tan^{-1}\left(\frac{2 \times 10}{20-40}\right) = \tan^{-1}(-1) = 135^\circ$$

$$\Rightarrow \vartheta_p = 67.5^\circ \quad \text{or} \quad 157.5^\circ$$

Problem : The principal stress at a point a bar are 200 N/mm^2 (tensile) and 100 N compressive. Determine the resultant stress in magnitude and direction on a plane inclined at 60° to the axis of major principal stress. Find maximum intensity of shear stress in material at this point 2013(w)2c

Given :

$\sigma_x = 200$ N/mm^2

$\sigma_y = -100$ N/mm^2

$\theta = 60^\circ$

Normal stress,

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta$$

$$= \frac{200 + (-100)}{2} - \frac{200 - (-100)}{2}\cos(2 \times 60^\circ)$$

$$= \frac{200 - 100}{2} - \frac{200 + 100}{2}\cos 120^\circ = 50 - 150\cos 120^\circ = 125 \text{N/mm}^2$$

$$\text{shear stress}\,(\tau) = \frac{\sigma_x - \sigma_y}{2} \times \sin 2\theta$$

$$= \frac{200-(-100)}{2} \times \sin 2\times 60^{o} = 150 \sin 120^{o} = 129.9\,N/mm^2$$

$$\text{Resultant stress}, \sigma_R = \sqrt{\sigma_n^2 + \tau^2}$$

$$= \sqrt{(125)^2 + (129.9)^2} = 180.27\,N/mm^2$$

Maximum intensity of shear stress

$$\tau_{max} = \pm \frac{\sigma_x - \sigma_y}{2} = \pm \frac{200-(100)}{2} = \pm 150\,N/mm^2$$

CH-4

Bending Moment & Shear Force

Short Questions & Answers

Q.1. What is Beam ?

Ans. Anymember of structure or machine whose one dimension is very large as compared to the other dimension and which can take lateral force in axial plane is called Beam or Beamis a structural member which is subjected to transverse loading.

Q.2. Write the types of Beam ?

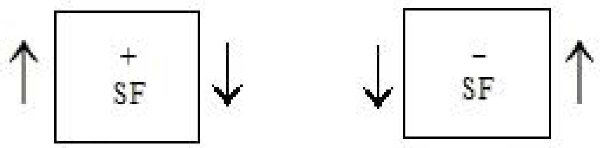
Ans. Beam are of 5 types, such as
(a) Cantilever Beam
(b) Simply supported Beam
(c) Overhang Beam
(d) Fixed Beam or Buitt-in-Beam.
(e) Continous Beam

Q.3. Writes the types of Load.

Ans. Load are of 3 types, such as :
(a) Concentrated or pointload
(b) Uniformly distributed load
(c) Uniformy Varying Load.

Q.4. Whatis shear force ?

Ans. The algebraic sum of all the vertical forces either left or right of the section of beam is known as shear force. It is denoted by S.F.



When the resultant of the forces to the left is upward or to the right is downward, the SF is +ve.

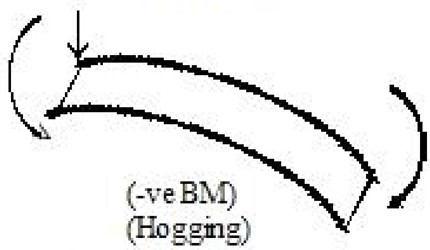
When the resultant of the forces to the left is downward or to the right is upward, the SF is -ve.

Q.5. What is Bending Moment ?

Ans. The algebraic sum of moments of all the vertical forces acting either left or right of the section of beam, is known as Bending Moment. It is denoted by BM.

(+ve BM)
(Sagging)

The moment on the left section is clockwise and on the right portion anti-clockwise, known as +ve BM or Sagging.



(-ve BM)
(Hogging)

The moment on the left section is anti-clockwise and on the right portion is clockwise, known as -ve BM or Hogging.

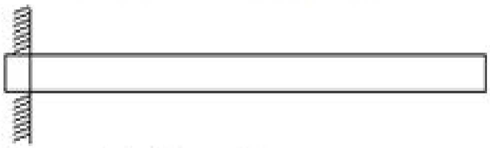
Q.6. What is SFD and BMD ?

Ans. A SFD (Shear Force Disgram) is one which shows the variation of the shear force along with length of the beam, called SFD.

A BMD (Bending Moment diagram) is one which shows the variation of the bending moment along the length of beam, called BMD.

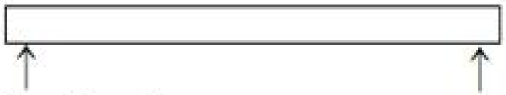
Q.7. What is cantilever Beam ?

Ans. A beam which is fixed at one end and free at other end is known as Cantilever Beam..



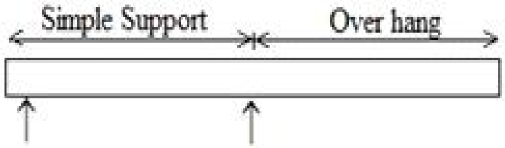
Q.8. What is simply suppotted Beam ?

Ans. A beam supported or resting freely on the support at its both ends, is known as simply supported Beam.



Q.9. What is Overhang Beam ?

Ans. If the end portion of the beam is extended beyond the support, such beam is known as Overhang Beam.



Q.10. Define point of contraflexure or point of inflection ?

Ans. It is point, where the bending moment is zero after changing its sign from positive to negative or vice versa.

Q.11. Define Maximum Bending Moment ?

Ans. It is the point where the shear force is zero, after changing its sign from positive to negative or vice versa.

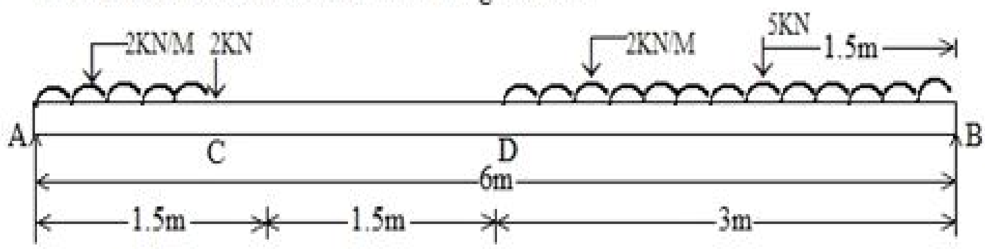
Long Questions :

Q.1 Draw SFD and BMD for contilever beam carrying U.d.l.

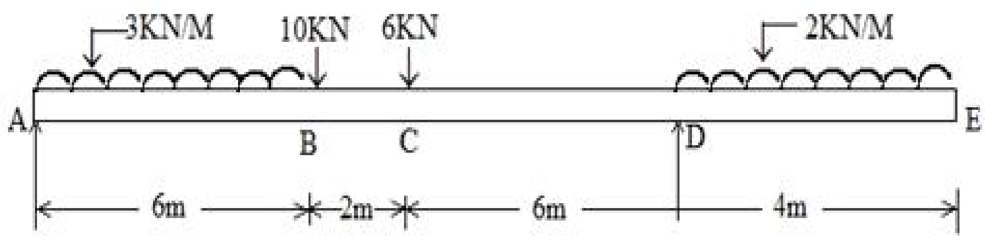
Q.2. Draw SFD and BMD for simple supported beam carrying U.d.l.

Q.3. Draw S.F.D and BMD for overhang beam carrying U.d.l.

Q.4. Draw S.F.D and B.M.D as shown in fig. below :



Q.5. Draw S.F.D and B.M.D as shown in fig. below and determine point of contraflexure.



CH-5

Theory of Simple Bending

Short Questions & Answers

Q.1. Define Bending Stress.

Ans. When some external load acts on a beam, the shear force and bending moments are set up at all section of the beam. Due to the shear force and bending moment, the beam undergoes certain deformation. The material of the beam will offer resistance or stresses against these deformation. These stress are known as Bending Stresses.

Q.2. Define Pure Bending or Simple Bending.

Ans. If a length of a beam is subjected to Constant Bending Moment and no shear force, then the stresses will be set up in that length of the beam due to BM only and that length of the beam is said to be in Pure Bending or Simple Bending. The stresses setup in that length of beam and known as Bending Stresses.

Q.3. What is Neutral Axis ?

Ans. The neutral axis of any transverse section of beam is defined as the line of intersection of the neutral layer with the transverse section. It is written as N.A.

Q.4. What is Moment of Relistance ?

Ans. Due to pure Bending, the layers above the N.A are subjected to compressive stresses, where as the layers below the N.A are subjected to tensie stresses. Due to these stresses, the forces will e acting on the layer. These forces will have moment about the N.A. The total moment of these forces about the N.A for a section is known as Moment of Resistance.

Q.5. What is Section Modules :

Ans. It is defined as the ratio of moment of inertia of a section about the neutral axis to the distance of the outermost layer from the natural axis. It is denoted by 'Z'.

$$\boxed{Z = \frac{I}{y_{max}}}$$

Where, I = M.O.I about neutral axis.

y_{max} = Distance of the outermost layer from the neutral axis.

Long Questions :

Q.1. Write the Assumptions of theory of simple Bending.

Q.2. Derive the relation. $\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}$ (Bending Equation of Bending Formula)

CH-6

Cmbined Direct & Bending Stresses

Short Questions & Answers

Q.1. Define Column & Strut.

Ans. If the member of the stracture is vertical and both of its end are fixed rigidly while subjected to axial compressive load, the member is known as Column.

Ex. : A vertical piller between the roof and floor.

If the member of the structure is not vertical and one or both of its ends are hinged or Pin joined, the bar known as Strut.

Ex. : Connecting rods, Piston rods etc.

Q.2. Define Slenderness Ratio.

Ans. The ratio of the actual length of a column to the least radius of gyration of the column is known as Slenderness Ratio.

Mathematically, Slenderness Ratio = $\frac{\text{Actual Length}}{\text{Least radious of gyration}} = \frac{l}{K}$

Q.3. Define Buckling load or Crippling load.

Ans. If the length of the column is comparision to its lateral dimensions is very larg. Such columns do not fail by crushing alone but also by bending known as buckling or cripping and load at which the column column just buckle is known as Buckling Load or Crippling Load

Q.4. Write the formula for combined direct & Bending Stresses.

Ans. $\sigma_{max} = \sigma_a + \sigma_b = \frac{P}{A} + \frac{6.P.e}{A\text{-}b}$

or $\sigma_{max} = \frac{P}{A}$]

& $\sigma_{min} = \frac{P}{A}$]

P = Direct Load

A = Area of cass-sea

e = eccentricity

b = Width

Q.5. Write the formule of Crippling Stress.

Ans. Crippling Stress = $\frac{\text{Crippling Load}}{A} = \frac{P}{A} = \frac{\pi^2 E \times A}{A \times \left(\frac{Le}{K}\right)^2} = \frac{\pi^2 E}{\left(\frac{Le}{K}\right)^2}$

Q.5. Write the end condition for long column.

Ans. The following four types of end conditions of the column are :

Both the ends of the column are hinged or pinned

One end is fixed and other end is free

Both the ends of column are fixed.

One end is fixed and other is pinned.

Long Questions :

Q.1. Write the Assumptions of Euler's Column Theory.

Q.2. Expression for Crippling load. When the end of column is fixed and the other end is Hinged or Pinned.

Q.3. Expression for Crippling Load. When one end of the column is fixed & other end is free.

Q.4. Expression for Crippling Laod. When both the end of the column are fixed.

Q.5. Expression for Crippling Load. When both ends of the column are hinged or pinned.

CH-7
Torsion
Short Questions & Answers

Q.1. What is Torsion ?

Ans. A shaft is said to be in torsion, when equal and opposite torques are applied at the two ends of the shaft. The torque is equal to the product of the force applied and radius of the shift. Due to the application of the torques at the two ends, the shaft is subjected to a twisting moment. This causes the shear stresses and shear strains in the material shaft.

Q.2. What is Pure Torsion ?

Ans. The circular shaft is said to be in the state of pure torsion, when the circular shaft is subjected to torque only without being acted uponby any bending moment.

Q.3. Define Polar Modulus.

Ans. It is defined as the ratio of polar moment of inertix to the radius of the shift. It is also called torsional section modules.

It is densted by Zp

$\boxed{\therefore Zp = \frac{J}{R}}$ For Solid Shaft, $J = \frac{\pi}{32} d^4$

For Hllow Shaft, $J = \frac{\pi}{32} (do^4 - de^4)$

Q.4. What is Strength of Shaft end Torsional Rigidity ?

Ans. ✷The strength of shaft means the maximum torque or maximum power the shaft can transmit.

✷The torsional rigidity or stiffiness of the shaft is defined as the product of modulus Rigidity and Polar moment of inertia of the shaft.

Torsional Rigidity = C x J C = Modulus of Rigidity

J = Polar moment of inertia.

Torsional rigidity is also defined as the torque required to produce a twist of one radian per unit length of the shaft.

Q.5. Write the Torsion Equetion or Torsion Formula.

Ans. The equation is $\frac{T}{J} = \frac{\tau}{r} = \frac{C.\theta}{l}$

where, T = Torque or Twisting Moment in N-M or N-mm

J = Polar moment of inertia in m^4 or mm^4.

τ = Shear Stress in N/m^2 or N/mm^2

r = radius of shaft in m or mm

C = Modulus of Rigidity in N/m^2 or N/mm^2

θ = Angle turned by the shaft in radian

l = Length of shaft in m or mm.

<u>Long Questions.</u>

Q.1. Write the assumption of Torsion equation.

Q.2. Derive the relation $\frac{T}{J} = \frac{\tau}{r} = \frac{C.\theta}{l}$ i.e. Torsion equation or torsion formula.

Q.3. Derive the maximum torque transmitted by a solid circular shaft.

Q.4. Defive the maximum torque transmitted by a Hollow circular shaft.

Q.5. Comparison between solid shaft and Hollow shaft.