PNS School of Engineering & Technology

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Internal Assessment Examination-2022(1st Semester)

Subject : Th-3-Engineering Mathematics-I

Branch : Civil, Mechanical, Electrical, Comp.Sc & ETC Engineering

Time : $1\frac{1}{2}$ Hours F.M. : 20

1. *Answer the following questions.(any Five)* [2 x 5]

(a) *Express $\sin 1400^0$ in terms of T-ratio of an acute angle.*

(b) *Construct a 2 x 3 matrix having elements $a_{ij} = i + j,\ \forall i, j$*

(c) *If* $A = \begin{bmatrix} 0 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 7 & 6 & 3 \\ 1 & 4 & 5 \end{bmatrix}$ *then*

Find 3A - B

(d) *Find Inverse of the matrix* $\begin{bmatrix} 4 & -2 \\ 3 & 1 \end{bmatrix}$

(e) *Solve :* $\begin{vmatrix} a-1 & 2 \\ 3 & a \end{vmatrix} = 0$

(f) *Find x and y if* $\begin{bmatrix} x+y & 3 \\ 4 & x-y \end{bmatrix} = \begin{bmatrix} 12 & 3 \\ 4 & 6 \end{bmatrix}$

(g) *If* $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix}$ *then find*

M_{11} *and* M_{21} *where* M_{ij} *is the minor of* $a_{ij}\ \forall\ ij$

2. *Answer the following questions.(any Two)* [5 x 2]

(a) *If* $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 1 \end{bmatrix}$ *then verify that*

$$(AB)^T = B^T . A^T$$

(b) *Solve by Cramer's Rule : $4x - y = 9,\ 5x + 2y = 8$*

(c) *Prove that*

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$$

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SOLUTIONS: (1)

1(a) $\sin 1400^\circ$

$= \sin(7\times180+140)$

$= (-1)^7 \sin 140^\circ = -\sin 140^\circ$

$= -\sin(90+50)$

$= -\cos 50^\circ$ (Ans)

1(b) Let the required 2×3 matrix be $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$

$= \begin{bmatrix} 1+1 & 1+2 & 1+3 \\ 2+1 & 2+2 & 2+3 \end{bmatrix}$

$\because a_{ij} = i+j, \forall i,j$

$= \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$ (Ans)

1(c) $A = \begin{bmatrix} 0 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 7 & 6 & 3 \\ 1 & 4 & 5 \end{bmatrix}$

Now

$3A-B = 3\begin{bmatrix} 0 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} - \begin{bmatrix} 7 & 6 & 3 \\ 1 & 4 & 5 \end{bmatrix}$

$= \begin{bmatrix} 0 & 6 & 9 \\ 6 & 3 & 12 \end{bmatrix} - \begin{bmatrix} 7 & 6 & 3 \\ 1 & 4 & 5 \end{bmatrix}$

$= \begin{bmatrix} -7 & 0 & 6 \\ 5 & -1 & 7 \end{bmatrix}$ (Ans)

1(d) Let $A = \begin{bmatrix} 4 & -2 \\ 3 & 1 \end{bmatrix}$

$|A| = \begin{vmatrix} 4 & -2 \\ 3 & 1 \end{vmatrix} = (4)(1)-(3)(-2) = 4+6 = 10 \neq 0$

$\therefore A^{-1}$ exist.

$A^{-1} = \frac{1}{|A|}\begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} 1/10 & 1/5 \\ -3/10 & 2/5 \end{bmatrix}$

(Ans)

1(e) $\begin{vmatrix} a-1 & 2 \\ 3 & a \end{vmatrix} = 0 \Rightarrow a(a-1)-(2)(3)=0$

$\Rightarrow a^2-a-6=0 \Rightarrow a = \dfrac{-(-1)\pm\sqrt{(-1)^2-4.1.(-6)}}{2\times1}$

$\Rightarrow a = \dfrac{1\pm\sqrt{25}}{2} = \dfrac{1\pm5}{2} = \dfrac{1+5}{2}$ or $\dfrac{1-5}{2} = 3$ or -2 (Ans)

1(f) $\begin{bmatrix} x+y & 3 \\ 4 & x-y \end{bmatrix} = \begin{bmatrix} 12 & 3 \\ 4 & 6 \end{bmatrix}$

$\Rightarrow \quad x+y = 12$ —— (1)

and $\quad x-y = 6$ —— (2)

Eq^n(1)+Eq^n(2) $\Rightarrow 2x=18 \Rightarrow x=9$

Eq^n(1)−Eq^n(2) $\Rightarrow 2y=6 \Rightarrow y=3$.

$\therefore \quad x=9, \; y=3$ (Ans)

1(g) $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix}$

$M_{11} = \begin{vmatrix} 0 & 2 \\ 1 & 1 \end{vmatrix} = (0)(1)-(2)(1) = 0-2 = -2$

$M_{21} = \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} = (2)(1)-(3)(1) = 2-3 = -1$

$\therefore \quad M_{11} = -2, \; M_{21} = -1$ (Ans)

2(a) $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \end{bmatrix}$ $\quad B = \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 1 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ -2 & 7 \end{bmatrix}$$

$$\therefore (AB)^T = \begin{bmatrix} 2 & -2 \\ 5 & 7 \end{bmatrix} \text{ ——— } (1)$$

Now $A^T = \begin{bmatrix} 1 & 3 \\ 2 & -2 \\ 3 & 1 \end{bmatrix}$, $B^T = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 1 \end{bmatrix}$

$$B^T \cdot A^T = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & -2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 5 & 7 \end{bmatrix} \text{ ——— } (2)$$

From (1) and (2) $(AB)^T = B^T \cdot A^T$ $\square$.

(b) Given system is $\left.\begin{array}{l} 4x - y = 9 \\ 5x + 2y = 8 \end{array}\right\}$ ——— (1)

Determinant of system

$$\Delta = \begin{vmatrix} 4 & -1 \\ 5 & 2 \end{vmatrix} = (4)(2) - (5)(-1) = 13 \neq 0$$

Now $\Delta x = \begin{vmatrix} 9 & -1 \\ 8 & 2 \end{vmatrix} = (9)(2) - (8)(-1) = 26$

$\Delta y = \begin{vmatrix} 4 & 9 \\ 5 & 8 \end{vmatrix} = (4)(8) - (5)(9) = 32 - 45 = -13$

$x = \frac{\Delta x}{\Delta} = \frac{26}{13} = 2$, $y = \frac{\Delta y}{\Delta} = \frac{-13}{13} = -1$

$\therefore x = 2, y = -1$ (Ans)

2(c)

$$LHS = \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix}$$

$$= \begin{vmatrix} a-b & b-c & c \\ a^2-b^2 & b^2-c^2 & c^2 \\ bc-ca & ca-ab & ab \end{vmatrix} \qquad \begin{matrix} C_1 \to C_1 - C_2 \\ C_2 \to C_2 - C_3 \end{matrix}$$

$$= \begin{vmatrix} a-b & b-c & c \\ (a+b)(a-b) & (b+c)(b-c) & c^2 \\ -c(a-b) & -a(b-c) & ab \end{vmatrix}$$

$$= (a-b)(b-c) \begin{vmatrix} 1 & 1 & c \\ a+b & b+c & c^2 \\ -c & -a & ab \end{vmatrix}$$

(Taking $a-b$, $b-c$ from C_1 and C_2 respectively)

$$= (a-b)(b-c) \begin{vmatrix} 0 & 1 & c \\ a-c & b+c & c^2 \\ a-c & -a & ab \end{vmatrix} \qquad (C_1 \to C_1 - C_2)$$

$$= (a-b)(b-c)(c-a) \begin{vmatrix} 0 & 1 & c \\ -1 & b+c & c^2 \\ -1 & -a & ab \end{vmatrix} \qquad (\text{Taking } c-a \text{ from } C_1)$$

$$= (a-b)(b-c)(c-a) \begin{vmatrix} 0 & 1 & c \\ 0 & a+b+c & c^2-ab \\ -1 & -a & ab \end{vmatrix} \qquad (R_2 \to R_2 - R_3)$$

$$= (a-b)(b-c)(c-a)(-1) \begin{vmatrix} 1 & c \\ a+b+c & c^2-ab \end{vmatrix}$$

(Expanding w.r.t C_1)

(5)

$= (a-b)(b-c)(c-a)(-1)\{c^2 - ab - ac - bc - c^2\}$

$= (a-b)(b-c)(c-a)(ab+bc+ca) = RHS$ □.