

**1.A. Define force & write its unit.**

Force is defined as an external agent which changes either state of rest or state of motion of a body.

Unit: SI-Newton (N)

CGS-Dyne

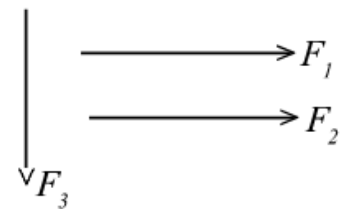
**B. Define coplanar & concurrent force.**

**Co-planer forces :**

*It is defined as lines of action of forces lie on the same plane, known as co-planer force.*

*e.g : The lines of action of forces  $F_1$ ,  $F_2$  and  $F_3$  lie on the same plane.*

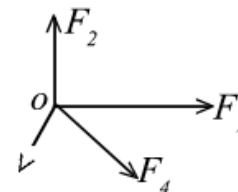
*Hence these forces are called co-planar forces.*



**Concurrent forces :**

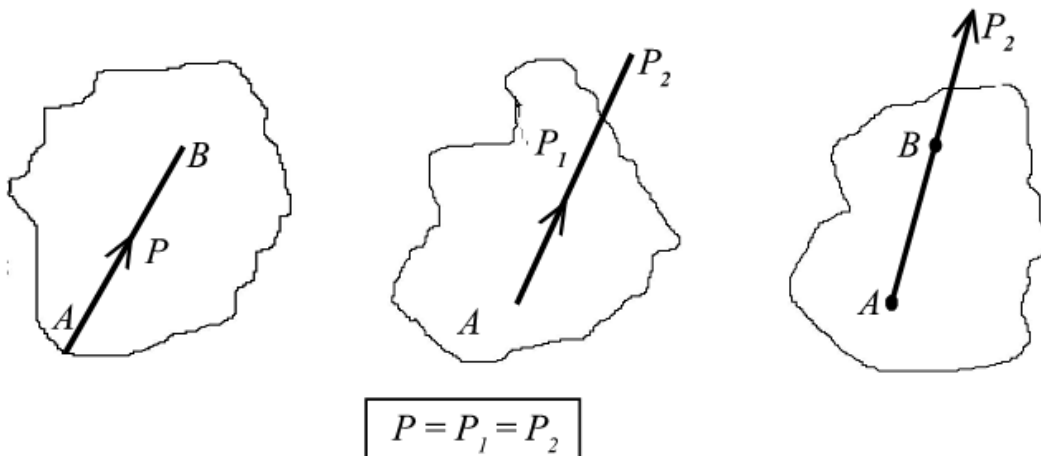
*It is defined as lines of action of forces pass through the common point, known as concurrent forces.*

*e.g. : The lines of action of forces  $F_1$ ,  $F_2$ ,  $F_3$ ,  $F_4$  pass through the common point 'o'. Hence they are called con-current forces.*



**c.State principle of transmissibility.**

*The principle states that "The condition of equilibrium or motion of a rigid body will not be changed if a force acting on a body at a certain point is replaced by a force of same magnitude and same direction but applied at a different point provided the two forces act along the same straight line".*



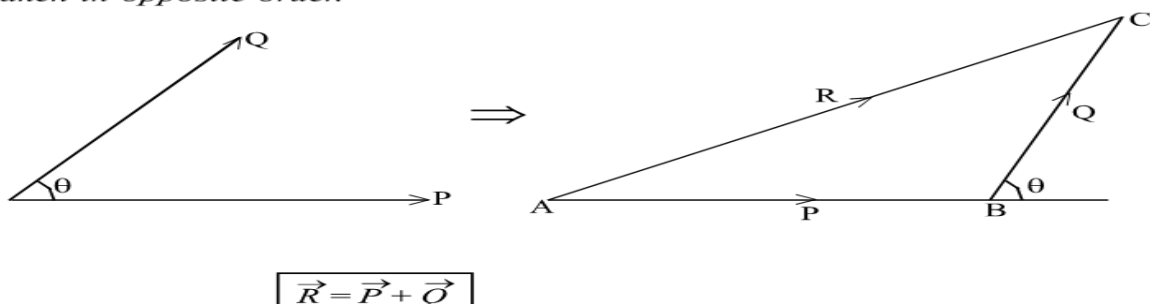
#### D. Define free body diagram & rigid body.

*Free Body Diagram may be drawn for the single body or for a sub system or for whose structure irrespective of whether the system is in equilibrium or not. All the internal as well as external forces must be taken into consideration.*

A body is said to be rigid if it does not undergo deformation whatever force may be applied to the body. In actual practice, there is no body which can be said to be rigid in true sense of terms.

#### E. State triangle law of forces.

**Statement:** "If two forces acting simultaneously on a body, be represented in magnitude and direction by two sides of a triangle taken in order their resultant may be represented in magnitude and direction by the third side of the triangle taken in opposite order."



#### f. State polygon law of forces.

##### POLYGON LAW OF FORCES

It is an extension of Triangle Law of Forces for more than two forces, which states, "If a number of forces acting simultaneously on a particle, be represented in magnitude and direction, by the sides of a polygon taken in order then the resultant of all these forces may be represented, in magnitude and direction, by the closing side of the polygon, taken in opposite order."

##### **Define moment of a force and write its units.**

*Moment of a force about a point may be defined as the turning effect of the force about that point.*

*Moment of the force is expressed as the product of the force and the perpendicular distance of the point, about which the moment is to be found.*

Mathematically,  $\boxed{M = F \times L}$

Where,  $F$  = Force acting on the body

$L$  = Perpendicular distance between the point, about which the moment is to be found out.

**Units:** In SI  $\Rightarrow N - m$  or  $N - mm$

In C.G.S  $\Rightarrow$  dyne - Cm

Clockwise moments are +ve moments

Anticlockwise moments are -ve moments

## 2.a

**Example 1.7.** The following forces act at a point :

- (i) 20 N inclined at  $30^\circ$  towards North of East,
- (ii) 25 N towards North,
- (iii) 30 N towards North West, and
- (iv) 35 N inclined at  $40^\circ$  towards South of West.

Find the magnitude and direction of the resultant force.

**Solution.** The system of given forces is shown in Fig 1.26

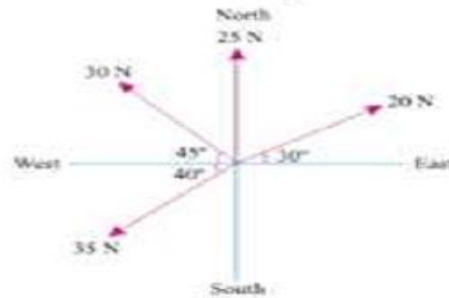


Fig. 1.26

*Magnitude of the resultant force*

Resolving all the forces horizontally *i.e.*, along East-West line,

$$\begin{aligned}\Sigma H &= 20 \cos 30^\circ + 25 \cos 90^\circ + 30 \cos 135^\circ + 35 \cos 220^\circ \text{ N} \\ &= (20 \times 0.866) + (25 \times 0) + 30 (-0.707) + 35 (-0.766) \text{ N} \\ &= -30.7 \text{ N} \dots (i)\end{aligned}$$

and now resolving all the forces vertically *i.e.*, along North-South line,

$$\begin{aligned}\Sigma V &= 20 \sin 30^\circ + 25 \sin 90^\circ + 30 \sin 135^\circ + 35 \sin 220^\circ \text{ N} \\ &= (20 \times 0.5) + (25 \times 1.0) + (30 \times 0.707) + 35 (-0.6428) \text{ N} \\ &= 33.7 \text{ N} \dots (ii)\end{aligned}$$

We know that magnitude of the resultant force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(-30.7)^2 + (33.7)^2} = 45.6 \text{ N Ans.}$$

*Direction of the resultant force*

Let  $\theta$  = Angle, which the resultant force makes with the East.

We know that,

$$\tan \theta = \Sigma V / \Sigma H = 33.7 / -30.7 = -1.098 \text{ or } \theta = 47.7^\circ$$

Since  $\Sigma H$  is negative and  $\Sigma V$  is positive, therefore resultant lies between  $90^\circ$  and  $180^\circ$ . Thus actual angle of the resultant =  $180^\circ - 47.7^\circ = 132.3^\circ$  Ans.

## 2.b.i.

*Two forces are acting at an angle of  $120^\circ$ . The greater force is 40N and the resultant is acting at  $90^\circ$  to the smaller force. Find the magnitude of the smaller force ?*

Here,  $\theta = 120^\circ$

$$\alpha = 90^\circ$$

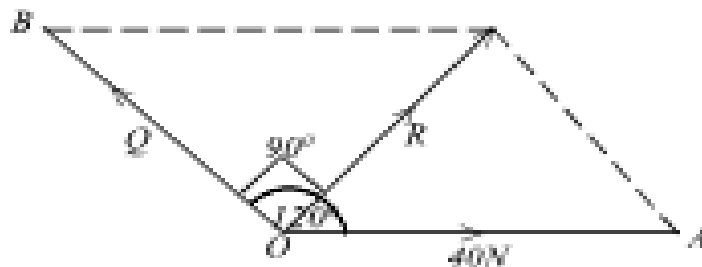
$$\alpha = 120^\circ - 90^\circ = 30^\circ$$

$$P = 40\text{N}$$

$$Q = ?$$

$$\therefore \tan \alpha = \frac{Q \sin \theta}{P + Q \cos \theta}$$

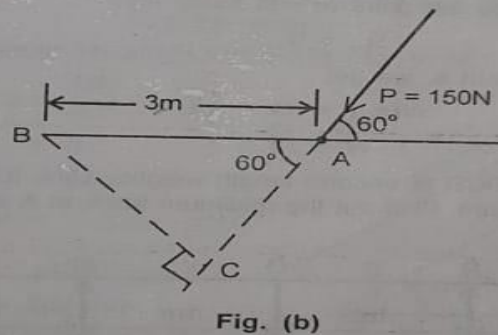
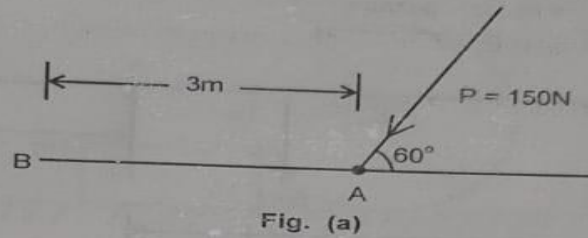
$$\Rightarrow \tan 30^\circ = \frac{Q \sin 120^\circ}{40 + Q \cos 120^\circ} \Rightarrow \boxed{Q = 20\text{N}}$$



## 2.b.ii.

**EXAMPLE – 1 :** A force of 150 N is acting at a point A as shown in the figure. Find out the moment of this force about a point B.

**Solution :**



Moment of force P about point B

$$\begin{aligned}
 &= \text{Force} \times \text{perpendicular distance between point B and line of action of force} \\
 &= P \times BC \\
 &= P \times AB \sin 60^\circ \\
 &= 150 \times 3 \times \sin 60^\circ \\
 &= 150 \times 3 \times 0.866 \\
 &= 389.71 \text{ N-m (clockwise)}
 \end{aligned}$$

## 2.c State and proof Varignon's theorem.

### VARIGNON'S THEOREM

Varignon's theorem states that the algebraic sum of the moment, two forces about any point in their plane is equal to the moment of the resultant about the same point.

**Proof.**

**Case (i)** When the forces are concurrent

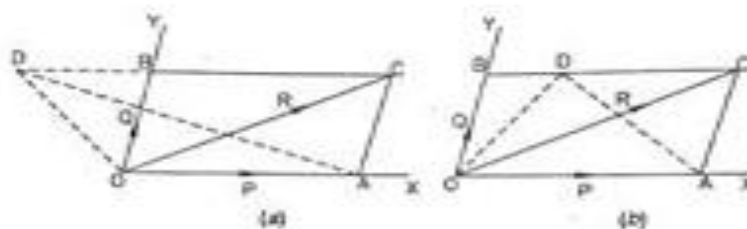


Fig 1.44

Let P and Q be any two forces acting at a point O along lines OX and OY respectively and let D be any point in their plane as shown in Fig 1.44.

Line DC is drawn parallel to OX to meet OY at B. Let in some suitable scale, line OB represent the force Q in magnitude and direction and let in the same scale, OA represent the force P in magnitude and direction.

With OA and OB as the adjacent sides, parallelogram OACB is completed and OC is joined. Let R be the resultant of forces P and Q. Then, according to the "Theorem of parallelogram of forces", R is represented in magnitude and direction by the diagonal OC of the parallelogram OACB.

The point D is joined with points O and A. The moments of P, Q and R about D are given by 2 x area of  $\Delta AOD$ , 2 x area of  $\Delta OBD$  and 2 x area of  $\Delta OCD$  respectively.

With reference to Fig1.44(a), the point D is outside the  $\angle AOB$  and the moments of P, Q and R about D are all anti-clockwise and hence these moments are treated as +ve.

Now, the algebraic sum of the moments of P and Q about

$$\begin{aligned}
 D &= 2\Delta AOD + 2\Delta OBD \\
 &= 2(\Delta AOD + \Delta OBD) \\
 &= 2(\Delta AOC + \Delta OBD) \text{ [See note below]} \\
 &= 2(\Delta OBC + \Delta OBD) \\
 &= 2\Delta OCD = \text{Moment of R about D.}
 \end{aligned}$$

[Note. As AOC and AOD are on the same base and have the same altitude,  $\Delta AOC = \Delta OBD$ .

Again, As AOC and OBC have equal bases and equal altitudes,  $\Delta AOC = \Delta OBC$ ].

With reference to Fig 1.44 (b), the point D is within the  $\angle AOB$  and the moments of P, Q and R about D are respectively anti-clockwise, clockwise and anti-clockwise.

Now, the algebraic sum of the forces P and Q about

$$\begin{aligned}
 D &= 2\Delta AOD - 2\Delta OBD = 2(\Delta AOD - \Delta OBD) = 2(\Delta AOC - \Delta OBD) = 2(\Delta OBC - \Delta OBD) \\
 &= 2\Delta OCD = \text{Moment of R about D}
 \end{aligned}$$

**Case (ii) :** When the forces are parallel



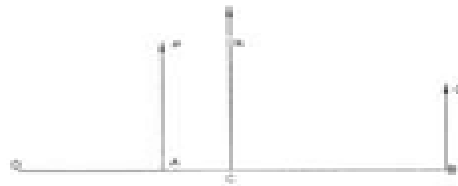


Fig 1.45

Let P and Q be any two like parallel forces (i.e. the parallel forces whose lines of action are parallel and which act in the same sense) and O be any point in their plane.

Let R be the resultant of P and Q.

Then,  $R = P + Q$

From O, line OACB is drawn perpendicular to the lines of action of forces P, Q and R intersecting them at A, B and C respectively as shown in Fig 1.45.

Now, algebraic sum of the moments of P and Q about O

$$= P \times OA + Q \times OB$$

$$= P \times (OC - AC) + Q \times (OC + BC)$$

$$= P \times OC - P \times AC + Q \times OC + Q \times BC.$$

$$\text{But } P \times AC = Q \times BC$$

Algebraic sum of the moments of P and Q about O

$$= P \times OC + Q \times OC$$

$$= (P + Q) \times OC = R \times OC = \text{Moment of R about O.}$$

In case of unlike parallel forces also it can be proved that the algebraic sum of the moments of two unlike parallel forces (i.e. the forces whose lines of action are parallel but which act in reverse senses) about any point in their plane is equal to the moment of their resultant about the same point.