UNIT 2

SCALARS & VECTORS

SCALAR QUANTITY

The quantity which require only magnitude to define are called scalar quantity.

Ex-Mass , length, time, volume, area, density, energy, temperature, electric charge etc.

VECTOR QUANTITY

The quantity which require both magnitude & direction to define are called vector quantity.

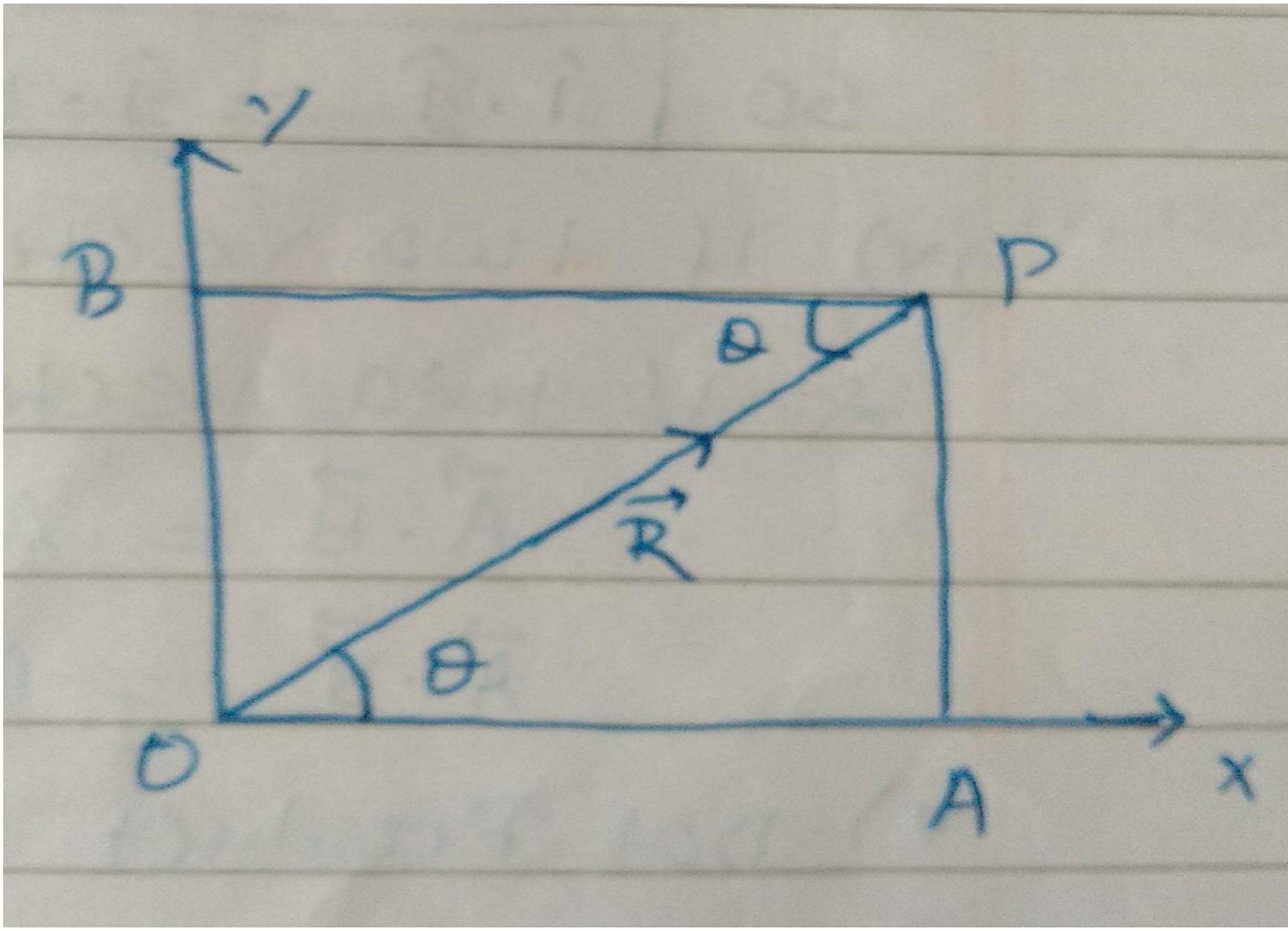
Ex- displacement, velocity, acceleration, momentum, Force, Magnetic moment, electric intensity etc.

RESOLUTION OF VECTORS

Resolution of a vector is the process of obtaining the component of vector

Consider the $\vec{R}$= OP in XY plane. Draw a perpendicular from P to X-axis at A and another perpendicular from P to Y-axis at B. θ is the angle made by R with X-axis. In triangle OAP

COS θ = OA /OP => OA = R COS θ equation 1 in triangle OBP



Sin θ = OB/OP => OB = R Sin θ equation 2

Since OA = R_x & OB = R_y

R COS θ = Rx & R Sin θ = Ry

Or R =$\sqrt{(R_x^2+R_y^2)}$

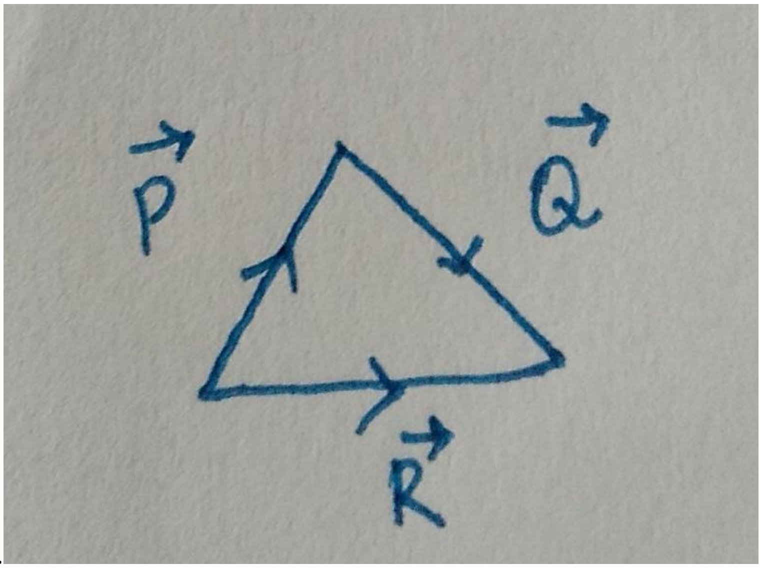
ORTHOGONAL UNIT VECTORS

1f Ax, Ay & Az are 3 rectangular components of $\vec{A}$ along x-axis, y-axis and z-axis respectively then

Ax= î Ax , Ay = ĵ Ay , Az = k̂ Az where î ĵ & k̂ are called orthogonal unit vectors or base vector.

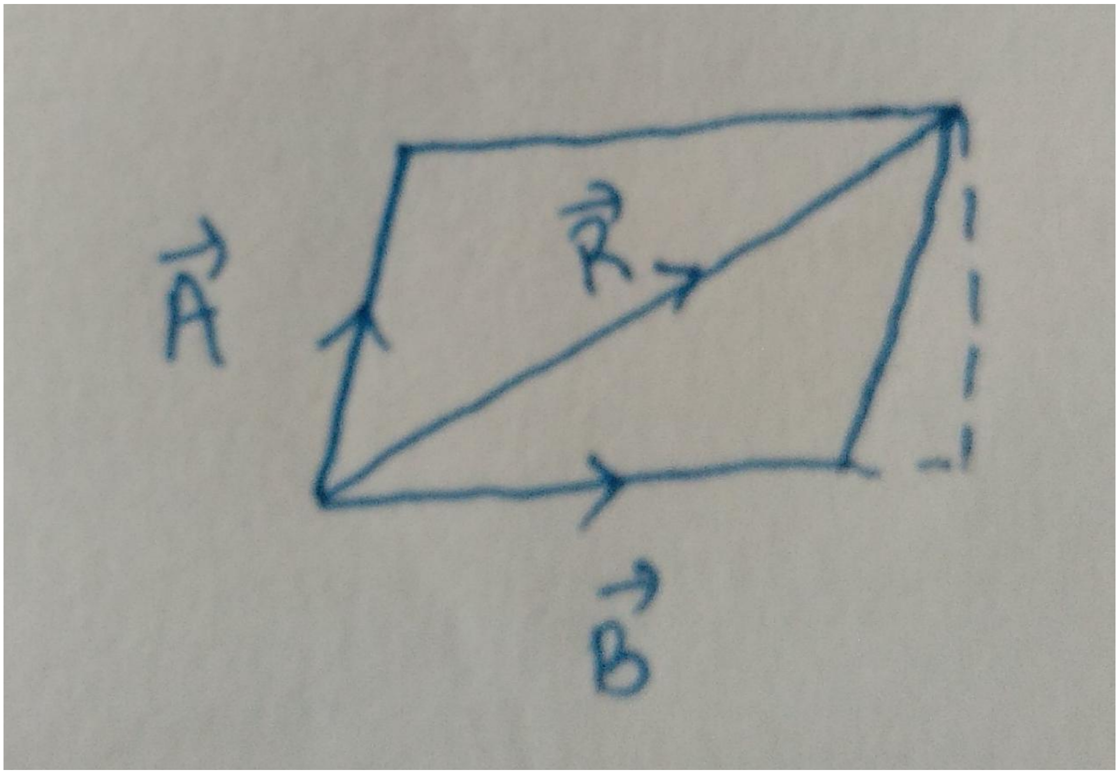
TRIANGLE LAW OF VECTOR ADDITION

It states that if two vectors acting at a point simultaneously be represented in magnitude and direction by the two sides of a triangle taken in order then their resultant vector is represented by the third side of the triangle taken in opposite order.



PARALLELOGRAM LAW OF VECTOR ADDITION

It states that if two vectors acting at a point simultaneously be represented in magnitude and direction by the two adjacent sides of a parallelogram then their resultant vector is represented by the concurrent diagonal of the parallelogram.



Mathematically

$R^2 = A^2 + B^2 + 2AB \text{ Cos } \theta$

DOT PRODUCT OF VECTORS / SCALAR PRODUCT

Dot Product of two vectors is defined as Product of their magnitudes and the cosine of the smaller angle between them.

$\vec{A}.\vec{B}$ = AB COS θ

The result of Dot Product of two vectors is scalar so it is also called as scalar product.

Characteristics

i) Distributive: It obey distributive law

ii) commutative: It obey commutative law.

iii). For two perpendicular vectors dot Product is zero If θ =90° then A·B = 0

So $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{i} \cdot \hat{k} = 0$

(iv) If two vector are parallel then dot product is maximum & if two vector are antiparallel then dot product is Minimum

$\vec{A} \cdot \vec{B}$ = AB COS 0° = AB = Maximum (Fore Parallel)

$\vec{A} \cdot \vec{B}$ = AB COS 180° = -AB = Minimum (Fort Antiparallel)

v) Dot Product of two equal vectons is equal to square of the magnitude of the either vector

$\vec{A} \cdot \vec{A}$ = AA COS 0° = A^2

$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 0$

vi) $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$ & $\vec{B} = B_x \hat{i} + B_y \hat{j} + B_Z \hat{k}$ then $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

CROSS PRODUCT OF VECTORS/VECTOR PRODUCT

Cross Product of two vectors A & B is defined as another vector C whose magnitude is equal to Product of their individual magnitudes and the sine of the smaller angle between them. It is directed along the normal to the plane containing A & B.

Mathematically $\vec{A} \times \vec{B}$ = AB Sin θ $\hat{n}$

where $\hat{h}$ is the unit vector in a direction perpendicular to Plane of $\vec{A}$ & $\vec{B}$ It is also called vector product.

Characteristics

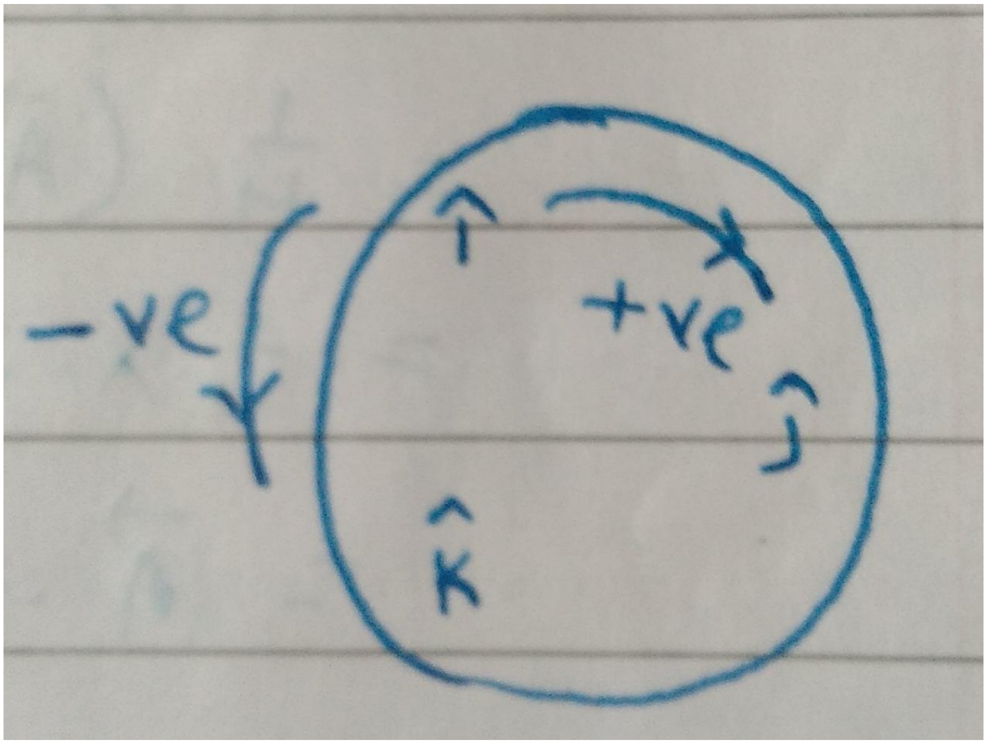
) Distributive: It obey distributive law.

$\vec{A}$ x ($\vec{B} + \vec{C}$ +........) = $\vec{A}$ X $\vec{B}$ +$\vec{A}$ x $\vec{C}$ + ...

ii) Anti commutative $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$ but $\vec{A} \times B = -(\vec{B} \times \vec{A})$

iii) For two perpendicular vectors cross product is maximum $\vec{A}$ X $\vec{B}$ = AB Sin90° $\hat{n}$ = AB $\hat{n}$

$\hat{i} \times \hat{j} = \hat{k}$ $\hat{j} \times \hat{i} = -\hat{k}$



$\hat{j} \times \hat{k} = \hat{i}$ $\hat{k} \times \hat{j} = -\hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$ $\hat{i} \times \hat{k} = -\hat{j}$

iv) If two vectors are parallel on antiparallel $\vec{A} \times \vec{B} = 0\,\hat{n}$ = a null vector As Sin 0° =0 & Sin 180° =0

v) CROSS Product of two equal vector is null vector. $\vec{A} \times \vec{A}$ = a null vector. As Sin 0° = 0 & Sin 180° = o

vi) $\vec{A} = Ax\,\hat{i} + Ay\,\hat{j} + Az\,\hat{k}$ & $\vec{B} = Bx\,\hat{i} + By\,\hat{j} + B_Z\,\hat{k}$ then $\vec{A} \times \vec{B}$

$$\begin{matrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ Ax & Ay & Az \\ Bx & By & BZ \end{matrix}$$

or $\vec{A} \times \vec{B} = \hat{i}\,(AyBz - AzBy) - \hat{j}\,(AxBz - AzBx) + \hat{k}\,(AxBy - AyBz)$

vii) Magnitude of cross product of two vector is equal to area of Parallelogram formed with the two vectors as the two sides.

NEUMERICALS

1) What is the condition that A. B = $\vec{A}$ x $\vec{B}$

Let A.B = $\vec{A}$ X $\vec{B}$

=> AB Cos θ = AB Sin θ

=> Cos θ = Sin θ

It is possible only when 0 = 45° as Cos 45° = Sin 45°

2) If $\vec{A}$ = î - 2 ĵ - 5k̂ & $\vec{B}$ = 2 î + ĵ - 4k̂ find A X B

$$\vec{A} \text{ X } \vec{B} = \begin{matrix} i & j & k \\ 1 & -2 & -5 \\ 2 & 1 & -4 \end{matrix}$$

Or $\vec{A}$ X $\vec{B}$ = î (8 + 5) - ĵ (-4 +10) + k̂ (1 + 4)

$\vec{A}$ X $\vec{B}$ = 13 î - 6 ĵ + 5 k̂

4) A Particle gets displaced through 13 î - 6 ĵ + 5 k̂ due to a force 13 î - 6 ĵ + 5 k̂. The displacement and force are measure in MKS units. Find the Work done a force.

(Ans 20 joule)

5) Resolve a vector 5N which makes an angle 45° with X axis

(Ans $\frac{5}{\sqrt{2}}$ & $\frac{5}{\sqrt{2}}$)

TWO MARK QUESTIONS

1 Define scalar quantity and give two examples .

2 Define vector quantity and give two examples .

3 Define orthogonal vector .

4 Define Dot product of vectors.

5 Define cross product of vectors.

6 Write any two properties of Dot product.

7 Write any two properties of Cross product.

8 State Triangle law of vector addition.

9 State Parallelogram law of vector addition.

FIVE MARK QUESTIONS

1 Define Dot product of vectors and write its properties.

2 Define Dot product of vectors and write its properties.

UNIT-3

KINEMATICS

REST

A body is said to be in rest if the position of the body doesn't changes with respect to time.

MOTION

A body IS said to be in motion if the position of the body Changes with respect to time.

DISPLACEMENT

The shortest distance between two position is called displacement. It is a vector quantity. It is denoted by $\vec{S}$.

Its Si & CGS units are Met & cm. Dimensional formula of displacement is L^1

VELOCITY

The time rate of Change of displacement is called velocity . It is a vector quantity. It is denoted by $\vec{V}$

Mathematically $\vec{V} = \vec{S}$/t

In SI & CGS system the unit of velocity is met/sec & cm / Sec . Its dimension formula is L^1T^{-1}

ACCELERATION

The time rate of change of velocity is called acceleration It is a vector quantity. It is denoted by $\vec{a}$

Its SI & CGS units are met / sec^2 & cm /sec^2.

Mathematically $\vec{a}=\vec{v}/t$

The dimensional formula of acceleration is $[L^1T^{-2}]$

MOMENTUM

The amount of motion contained in a body is called momentum. It in denoted by P. It is a vector quantity.

Mathematically P=MV

where M = Mays of the body.

V = velocity of the body.

Its S I & C G S units are gm / sec^2 and met / sec^2

Its dimensional formula is $[M^1L^1T^{-1}]$

FORCE

The time rate of Change of momentum is called force. It is a vector quantity. It is denoted by $\vec{F}$

Mathematically $\vec{F} = \frac{dP}{dt}$

Or $\vec{F} = \frac{d}{dt}(M\vec{V})$

O $\vec{F} = M\frac{d}{dt}\vec{V} = M\vec{a}$

In SI system 1f M=1 kg $\vec{a}$ = 1 m/sec² then F = 1 kg x 1 m/sec² = I newton

In CGS system if M=1 gm & a = 1cm/ sec^2 then F = 1gm XI cm / sec² = 1 dyne

1 newton = 10^5 dyne

Its dimensional formula is [M'L'T^{-2}]

EQUATIONS OF MOTION IN A STRAIGHT LINE

The equations of motion are as follows.

(1) $\vec{V} = \vec{U} + \vec{a}t$ (2) $\vec{S} = \vec{U}t + \frac{1}{2}\vec{a}t^2$ (3) $V^2 - U^2 = 2\vec{a}\vec{S}$

where $\vec{V}$= Final velocity of body

$\vec{U}$ = Intial velocity of body

$\vec{a}$ = acceleration of body

$\vec{S}$ = displacement of body

t = time of observation,

UNIFORM CIRCULAR MOTION ANGULAR DISPLACEMENT, ANGULAR VELOCITY & ANGULAR ACCELERATION.

If a particle is moving in such a way that its distance from a fixed Point is and always constant then the

Particle is said to be in circular motion. The path of the particle is called a circle. The fixed distance is the radius of the circle.

If a body moves in fixed circular orbit with same speed then it is said to be in uniform circular motion. If a body is in Uniform Circular motion then the angle through which the body get displaced is called ANGULAR DISPLACEMENT.

If $\Delta\theta$ = Angular displacement , $\Delta s =$ Linear displacement and r = radius of circle.

Then $\Delta\theta = \frac{\Delta s}{r}$

In vector form $\overrightarrow{\Delta\theta} = \frac{\overrightarrow{\Delta s}}{\vec{r}}$

The time rate of Change of angular displacement is called angular velocity. It is denoted by ω.

$\omega = \frac{\overrightarrow{\Delta\theta}}{\Delta t}$

Or $\omega = \frac{\Delta s}{\Delta t\, r} = \frac{\vec{v}}{r}$

Or $\vec{v} = \vec{\omega} \times \vec{r}$

Relation between Angular velocity and linear velocity.

The time rate of change of angular velocity is called ANGULAR ACCELERATION. It is denoted by α.

Mathematically α = dω /dt = (dv / dt) × (1 / r)

Or a = $\alpha\, r$

Relation between linear acceleration of angular acceleration.

where a = Linear acceleration.

In vector form $\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \overrightarrow{v}$

PROJECTILE MOTION

When a particle is thrown from the surface of earth under the action of gravity then it will travel in a curved path. The particle is then called the projectile and the motion of the Particle is called projectile motion.

Ex-i) A ball thrown into space.

ii) A bullet fired from a gun. iii) A bomb dropped from a tower.

ANGLE OF PROJECTION, TRAJECTORY, MAXIMUM HEIGHT, TIME OF FLIGHT & HORIZONTAL RANGE

consider a Particle thrown with a velocity at an angle with horizontal. The projectile rises to a a height and comes back to C on the level of Projection.

Let $\vec{u}$ - velocity of projection.

θ Angle of projection.

$\vec{v}$ velocity of Projectile at position A.

(x,y) co-ordinate of A

T - Time at which the projectile is at A.

g - Acceleration due to gravity.

T→ Time taken to reach from O to B = Time of ascent = Time of descent

v_x, v_y- component of velocity of projectile along x-axis & y-axis respectively.

Now $\vec{u}$ is broken into u$\cos\theta$ along x-axis and u$\sin\theta$ along y axis

We know $\vec{v} = \vec{u}+\vec{a}t$

then $v_x = u\cos\theta + 0t = u\cos\theta$ equation 1

$v_y = u\sin\theta - gt$ equation 2

$v^2 = v_x^2 + v_y^2$

Again $\vec{s} = \vec{u}t + \frac{1}{2}$ a +²

Or x = u$\cos\theta \cdot$ t + $\frac{1}{2}$ x0x+²

⇨ x = ucosθ t

or t = x/ ucosθ equation 3

simllarly y = usinθ t - $\frac{1}{2}$g +²

using the value of equation -3 in above equation we can write

$$y = usin\theta \times \frac{x}{ucos\theta} - \frac{gx2}{ucos\theta \times ucos\theta}$$

or $y = tan\theta.\ x + \left(\frac{g}{2\ ucos\theta\ ucos\theta}\right) x^2$

or $y = ax + bx^2$

This is the equation of Parabola.

It is known as equation of trajectory.

where a = tanθ & $b = \frac{-g}{2\ ucos\theta\ ucos\theta}$

1) TIME OF FLIGHT

It is the time taken by the Projectile to remain above the horizontal plane through the point of Projection. It is denoted by 2T

At Position B Vy=0, t = T

using the value in equation 2 we get

0 = usinθ -gT

or $T = \frac{usin\theta}{g}$ Time of ascent.

Time of flight $2T = \frac{2usin\theta}{g}$ equation 6

2) MAXIMUM HEIGHT

It is the highest distance travelled by the Projectile in vertical direction through the point of projection. It is denoted by H.

We know y = usinθ t - g +² /2

At Position B y = y_{max} = H and t = T = usinθ/g

Using the value of y & t in above equ" we get

H = (usinθ usinθ /g) - g u^2 $sin^2\theta$ /$2g^2$

=) H = $u^2 sin^2\theta$ / 2g equation 7

Max Height

3) HORIZONTAL RANGE

It is the maximum distance travelled. by the Projectile in horizontal direction through the point of Projection. It is denoted by R

we know x = ucosθt

At Position C x= xmax=R , t= 2T = 2usinθ/g

using the value of x & t in above equation we get

R = ucosθ. 2usinθ / g

R = u^2 sin2θ / g Horizontal Range

CONDITION FOR MAXIMUM HORIZONTAL RANGE

we know R = U^2sin2θ/ g

It means horizontal range depends upon velocity of projection & angle of projection. Fore a fixed value of u , R depends only on value of θ. Range will be maximum if sin2θ is maximum

sin2θ = 1 = sin 90°

- ⇨ $\theta = 45°$
- ⇨ $R_{max} = u^2/g$

Maximum horizontal Range

TWO MARK QUESTIONS

1 Define displacement. Write its SI unit and dimensional formula .

2 Define velocity. Write its SI unit and dimensional formula ..

3 Define acceleratoin. Write its SI unit and dimensional formula .

4 Define force. Write its SI unit and dimensional formula .

5 Define momentum. Write its SI unit and dimensional formula ..

6 Write equations of motion along a straight line.

7 Define angular displacement. Write its SI unit and dimensional formula .

8 Define angular velocity. Write its SI unit and dimensional formula.

9 Define angular acceleration. Write its SI unit and dimensional formula.

10 Define Time of flight.

11 Define maximum height

12 Define horizontal range.

13 What is the condition of maximum horizontal range ?

14 Write the relation between linear and angular velocity.

FIVE MARK QUESTIONS

1 State and explain Time of flight.

2 State and explain horizontal range.

3 State and explain maximum height.

10 MARK QUESTIONS

1 Obtain the equation of trajectory of a projectile fired at an angle θ with the horizontal and find the value of maximum height time of flight and horizontal range.