LECTURE NOTES

ON

DESIGN OF MACHINE ELEMENT (TH2)

For

5TH SEM MECHANICAL ENGG

(SCTE&VT SYLLABUS)

Prepared by

ER RAMESH CHANDRA PRADHAN

SR LECT. MECHANICAL

MACHINE DESIGN

WHAT IS A MACHINE?

A machine is the assembly of resistant bodies or links which is used to transmit available energy to do useful work.

MACHINE DESIGN:

- Machine design is defined as the use of scientific principles, engineering techniques and imagination to create a machine or machine element economically.
- Machine Design focuses on the basic principles of following three areas for creation of new machines and improving the existing.
  a) Mechanical behavior of material.
  b) Mechanical parts or machine element.
  c) Life cycle analysis.

CLASIFFICATION OF MACHINE DESIGN

1. **Adaptive design:-**

   Here designers work is concerned with adaptation of existing designs. This type design needs no special knowledge or skill .The designer only makes minor modification in the existing designs of the product.

2. **Development design:-**

   This type of design needs scientific training and design ability in order to modify the exiting design into a new idea by adopting a new material or different method of manufacturing. In this case, though the designer starts from the existing design, but the final product will be quite different from the original product.

3. **New design:-**

   This type of design needs lo of research, technical ability and creative thinking. From this design completely new product will be found.

The designs, depending upon the methods used, may be classified as follows:

- Rational design:-This type of design depends upon mathematical formulae of principal of mechanics.
- Industrial design:-This type of design depends upon the production aspects to manufacture any machine component in the industry.
- System design:-It is the design of any complex mechanical system like a motor car.

- Element design:-It is the design of any element of the mechanical system like piston, crankshaft, connecting rod, etc.
- Computer aided design:-In this of design, creation, modification & analysis of a design is done by using computer system.

DIFFERENT MECHANICAL ENGG. MATERIALS USED IN DESIGN

- Commonly used ferrous alloys are carbon steel, Low-alloy steel, Tool steel, Stainless steel and cast iron.
- Non ferrous alloys are Aluminum alloys, Nickel alloys, Titanium alloys.

Properties of Material:

Properties of material are the characteristics of matter which differentiate one material from the other.

The major properties of material to be studied for selection of material in engineering field are:

1. Physical properties
2. Chemical properties
3. Mechanical properties

Physical Properties:

A materials physical properties denote the physical state of material. Physical properties include

1. Density
2. Specific Heat
3. Thermal Expansion
4. Conductivity
5. Melting Point
6. Porosity
7. Crystal Structure
8. Appearance

Mechanical properties of Materials:

1. Strength
2. Stiffness
3. Elasticity
4. Plasticity
5. Ductility
6. Brittleness
7. Malleability
8. Toughness
9. Hardness
10. Machinability
11. Creep
12. Fatigue

Strength:

- It is the ability of a material to resist the externally applied forces without breaking.
- The internal resistance offered by a part to an externally applied force is called stress.

Stiffness:

It is the ability of materials to resist deformation under the action of load.

Mathematically:

$$\text{Stiffness (K)} = \frac{\text{Load (W)}}{\text{Deflection } (\delta)}$$

Unit: KN / mm or N / mm

This property is considered during selection of material for spring manufacturing.

Elasticity:

It is a property by virtue of which a material regains its original dimension after removal of load. Elasticity is measured by Young's Modulus or Modulus of Elasticity.
Unit – N / mm^2

Plasticity:

- It is the property by virtue of which the material does not regain its original shape after removal of load . I retains its deformed shape permanently.
- This property of the material is necessary for forging & rolling process.

Ductility:

- It is a property by which materials can be drawn into wires with the application of a tensile force.
- Ductile materials have the ability to flow or elongate under load. Example : Cupper , Aluminum , mild steel , nickel, zinc, tin & lead.
- The ductility of a material is commonly measured by means of percentage elongation and percentage reduction in area in a tensile test.

Brittleness:

It is the ability of a material by which it can develop crack under load or it can break suddenly.
Example : Wood , Concrete , Cast iron.

Malleability:

It is the property by virtue of which the material is able to be converted in to thin sheets.
Materials which are more elastic are also more malleable.
Example : Steel , Copper, Al, Brass, Bronze etc.

Toughness:

- It is the property by virtue of which a material is able to resist shock or impact loading.
- Impact loading means applied load fall from a height.
- The amount of energy absorbed per unit volume within elastic limit is know as Resilience.
- In the deign of springs toughness or resilience of material is considered.

Hardness:

- It is the property by which the material is able to resist scratches, marks or wear & tear.
- It also measures the ability of a material to cut another metal.
- Harness is independent of the weight of a material.

Brittle materials are more hard example: Glass, cast iron , concrete.

Machinability:

- It is the property of a material which refers to the ease with which a material can be cut.
- Machinability of a material can be measured by measuring the energy required to remove a unit volume of the material, keeping all machining parameters constant.
- It may be noted that brass can be easily machined than steel.

Creep:

- When a part is subjected to a constant stress at high temperature for a long period of time , it will undergo a slow and permanent deformation called creep.
- This property is considered in designing internal combustion engines, boilers and turbines.
- Super alloys resist creep failure as they can with stand high temperature for a prolonged period without developing stress.

Fatigue:

Fatigue is the property of mater by virtue of which the material fails under stress less than yield stress due to cyclic nature of stress.

Fatigue failure is responsible for 90% of mechanical failure.

Different type of stress:-

When external load is applied on an abject resist distortion due to the applied force for which internal forces developed in the object hose magnitude is equal to externally applied force.

Stress can be defined as the internal resting force acting on unit cross sectional area of the object.

Mathematically,

$$\text{Stress}(\sigma) = \frac{\text{Load}(F)}{\text{c/s area}}$$

Unit is N/mm^2

Before discussing types of stress we will discuss about types of load.

LOAD:

It is defined as any external force acting upon a machine part. The following four types of the load are:-

1. **Dead or stead load:-**

   A load is said to be a dead or stead load, When it does not change in magnitude or direction.

2. **Live or variable load**:-

   A load is said to be a live or variable load, when it changes continually.

3. **Suddenly applied or shock loads:-**

   A load is said to be a suddenly applied or shock load, when it is suddenly applied or removed.

4. **Impact load:-**

   A load is said to be an impact load, when it is applied from certain height with some initial velocity.

Different types of Stresses

- Yield stress:-

  It is defined as the maximum stress at which increase in elongation occurs without increase in load. After yield point on removal of the load the material will not be able to recover its original shape and size. Stress corresponding to yield point is known as yield point stress.

- Ultimate stress:-

  The stress, which attains its maximum value is known as ultimate stress. It I obtained by dividing the largest value of the load reached in a test to the original cross-sectional area of the test piece.

- Working Stress :-

  When designing machine parts, it is desirable to keep the stress lower than the maximum or ultimate stress at which failure of the material takes place. This stress is known as the working stress or design stress. It is also known as safe or allowable stress.

(**Note**: By failure it is no meant actual breaking of the material. Some machine parts are said to fail when they have plastic deformation set in them, and they no more perform their function satisfactorily)

Factor of Safety:

It is defined, as the ratio of the maximum stress to the working stress. Mathematically,

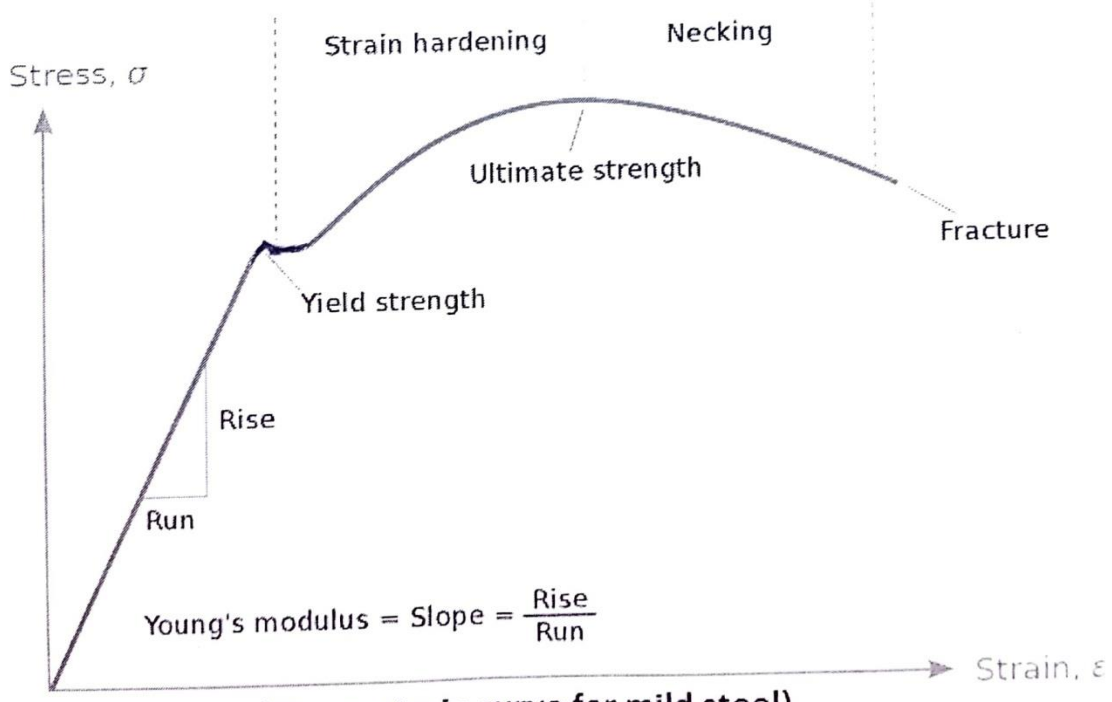
$$\text{Factor of safety} = \frac{\text{(Maximum Stress)}}{\text{(Working or design stress)}}$$

In case of ductile material example mild steel, where the yield point is clearly defined, the factor of safety is based upon the yield point stress.
In such cases,

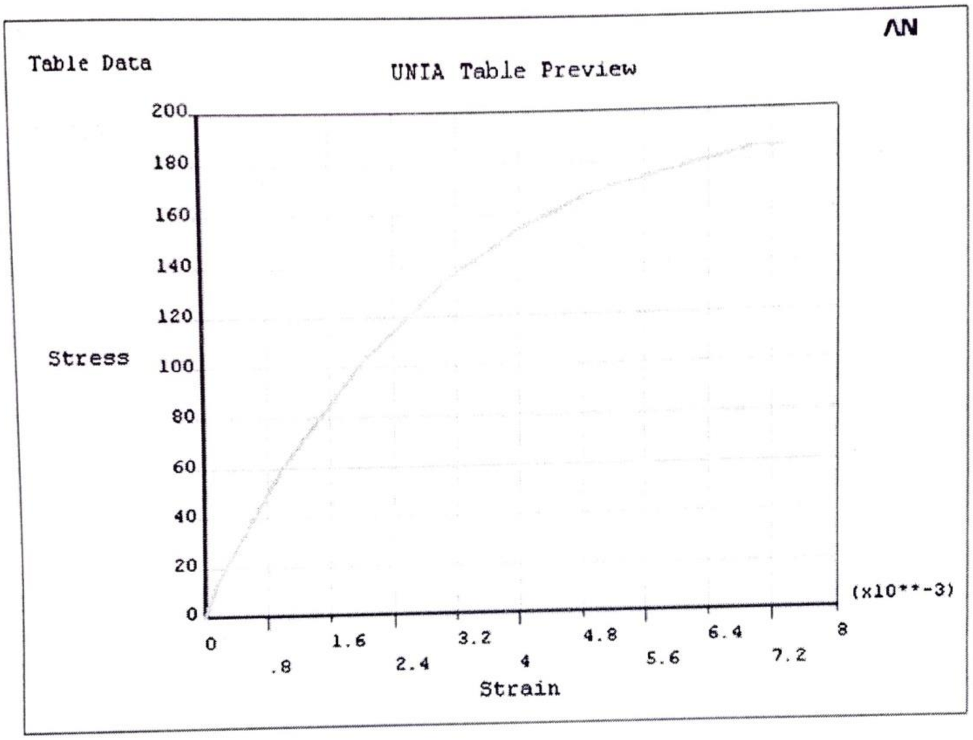
$$\text{Factor of safety} = \frac{\text{(Yield point stress)}}{\text{(Working or design stress)}}$$

In case of brittle materials example cast iron , the yield point is not well defined as for ductile materials.
Therefore , the factor of safety for brittle material is based on ultimate stress.

$$\therefore \text{Factor of safety} = \frac{\text{(Ultimate stress)}}{\text{(Working or design stress)}}$$



(Stress-strain curve for mild steel)



(Stres-strain curve for cast iron)

Mode of Failure to be considered in Machine design

1. **By Elastic deflection:**

- In the transmission system, the shaft which support gears are subjected to load which causes deflection of shaft.
- The maximum force which can be applied on the shaft is limited by the permissible elastic deflection.
- Lateral or torsional rigidity is the criteria for designing such components.
- Most effective method of decreasing the defection of a member is by changing its shape or c/s dimension.

2. **Yielding:**

- Mechanical component made of ductile material loses its engineering usefulness due to a large amount of plastic deformation or yielding of a considerable portion of the member.
- To avoid failure due to yielding working stress for ductile material is always less than they yield stress.

3. **Failure by fracture:**

- Sudden fracture of brittle materials.
- Fracture of cracked or flawed members.
- Progressive fracture due to cyclic load (fatigue) and due to low temperature.

4. **By Fracture:**

- Fracture occurs at much below stress than yield stress if the component has to work under cyclic load.
- Brittle materials cease to function suddenly due to fracture without any plastic deformation.

State the factors governing the design of machine elements:

1. **Strength:**

A machine part shouldn't fail under the effect of forces acting on it. It should have sufficient strength to avoid failure due to fracture or yielding.

2. **Rigidity:**

Machine component should be rigid enough, so that it will not deflect or bend due to forces or moment acting on it. A transmission shaft is designed on the basis of lateral rigidity & torsional rigidity.

3. **Wear resistance :**

A machine component should be wear resistant because wear reduces accuracy of machine tool along with its life cycle. Surface hardening will increase the wear resistance of the machine component.

4. **Safety:**

The shape and dimension of the machine part should ensure safety to the operator of the machine.

5. **Minimum dimension and weight :**

A machine should have minimum possible dimension & weight which will reduce the material cost.

6. **Conformance to the standard:**

Machine part should conform to the national & international standards covering the dimension , profile & material.

7. **Minimum life cycle cost:**

Total cost i.e to be paid for purchasing the parts, operating & maintaining it for its life span should be minimum.

<u>Describe design procedure:</u>

The general procedure to solve a design problem is as follows:

1. **Recognisation of need :**

   First of all the need, aim or purpose for which the machine is to be designed should be recognized.

2. **Mechanism:**

   Select the possible mechanism or group of mechanisms which will give the desired motion.

3. **Analysis of forces:**

   Find the forces acting on each member of the machine and the energy transmitted by each member.

4. **Material selection:**

   Select the material best suited for each member of the machine.

5. **Design of elements:**

   Find the size of each member of the machine by considering the force acting on the member and the permissible stresses for the material used . it should be kept in mind that each member should not deflect or deform more than the permissible limit.

6. Modification:

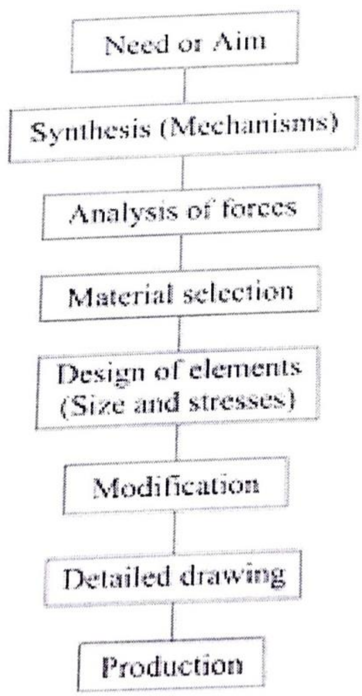
Modify the size of the member to reduce overall cost.

7. Detailed drawing:

Draw the detailed drawing of each component and the assembly of the machine with complete specification for the manufacturing processes.

The detailed drawing along with material of each component.

The flow chart for the general procedure in machine design is shown in fig.



General procedure in Machine Design

Chapter-2

DESIGN OF FASTENING ELEMENTS

Fasteners: It is a Mechanical Joints which is used to become a fixed / attaches to something or holds something in place.

The Fastenings may be classified into the following two groups:

1. The Permanent Fastenings are those fastenings which cannot be disassembled without destroying the connecting components. Examples: Welded joint, Rivet joint.
2. The Temporary or Detachable Fastenings are those fastenings which can be disassembled without destroying the connecting components.

   Examples: 1.Thread Joints
   - a. Bolted Joints
   - b. Screws Joints

   2. Keys
   3. Coupling
   4. Pins Joints
      - a. Cotters Joints
      - b. Knuckle Joints
   5. Pipe Joints

Welded joint:

Welding can be defined as a process of joining metallic parts by heating to a suitable temperature with or without the application of pressure.

Welding is an economical and efficient method for obtaining a permanent joint of metallic parts.

Two distinct application of welding

1. Can be used as a substitute for a riveted joint
2. Welded structure as an alternative method for casting or forging.

Welding advantages over riveting:

Welded joint	Rivet joint
Due to no additional parts except melting of filler rods, welded joints are lighter in weight. Welded steel structures are lighter than the corresponding iron castings by 50% and steel castings by 30%	Requires cover plates, gusset plates, straps, clip angles and large number of rivets which increases the weight
Cost is lesser due to no additional components used	Cost is higher due to usage of additional components listed above
Alterations and additions of the design of welded assemblies can be easily and economically modified	Alterations and additions of design of riveted assemblies are not easier and economically changed
Production time is less	Production time is higher
Welding does not create stress concentration due to lack of drilling holes.	Holes are drilled to accommodate the rivets. The holes reduces the cross-sectional area of the members and result in stress concentration.
Strength of weld is higher. Strength of weld is more than the strength of the plates that are joined together.	Strength of rivets are not high as that of weld joints.
Machine components of certain shapes such as circular steel pipe can easily be constructed by welding.	Machine components of certain shapes such as circular steel pipe , find difficulty in riveting

Disadvantages of welding:

1. The capacity of weld structures to damp vibrations is poor
2. Welding results in a thermal distortion of the parts, there by inducing residual stresses.
3. In many cases, stress-relieving heat treatment is required to relieve residual Stresses.
4. The quality and strength of the welded joint depend upon the skill of the welder. It is difficult to control the quality when a number of welders are involved.
5. The inspection of the welded joint is more specialized and costly compared with the inspection of riveted or cast structures

Eccentrically Loaded Welded Joints

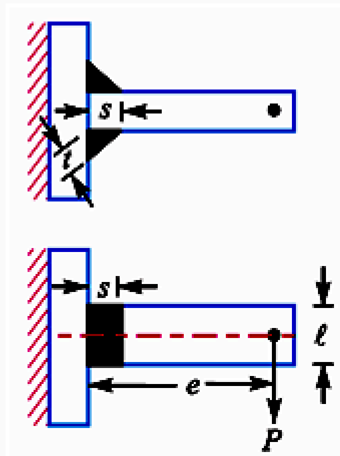


Figure. Bending Stress due to Eccentricity

In many cases the welded joints are eccentrically loaded. Different stresses may get induced depending upon the type of joint and loading. if the stresses are of same nature , those may be vectorially added but for those of different nature, resultant maximum tensile and shear stresses may be calculated. Depending upon the type of joint, eccentricity may lead to bending stress or torsional shear stress in the joint in addition to the direct shear stress induced by applied load.

Eccentricity leading to Bending Stress:

Consider a T-joint subjected to loading as shown in figure.

Let s and l be the size and length of the weld and t be the throat thickness.

Throat area = A = 2. t. l

This applied load may be considered as a load P directly acting on the joint through the CG and a bending moment of magnitude P.e acting on the joint. 1st one will lead to direct shear stress and the 2nd will lead to a bending stress.

Direct Shear Stress,

$$\tau = \frac{P}{A} = \frac{P}{2tl}$$

and Bending Stress,

$$\sigma_b = \frac{My}{I}$$

where y = distance of the point on the weld from the neutral axis

I = Moment of inertia of the weld section

Maximum tensile and shear stress may be calculated as:

$$\sigma_{t_{max.}} = \left(\frac{\sigma_b}{2}\right)^2 + \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + (\tau)^2}$$

and

$$\tau_{max.} = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + (\tau)^2}$$

<u>Eccentricity Leading to Torsional Shear Stress:</u>

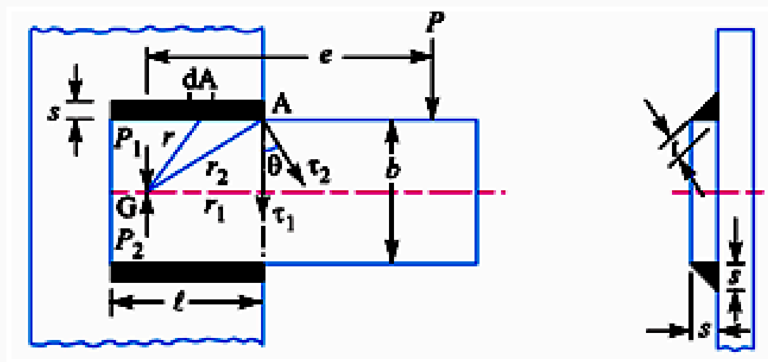


Figure. Shear Stress due to Eccentricity

Let us consider a double parallel fillet weld subjected to an eccentric load P acting at a distance e from the CG of the welds as shown in Figure.

Eccentric force P may be considered as a force P acting on the CG of the joint and a torque equivalent to Pe acting on the joint. The force P through the CG leads to direct shear stress, called primary shear stress and is assumed to be uniformly distributed over the throat area of all welds. The torque Pe causes torsional shear stress called secondary shear stress.

Primary Shear Stress,

$$\tau_1 = \frac{P}{A} = \frac{P}{2tl}$$

And, Secondary Shear Stress,

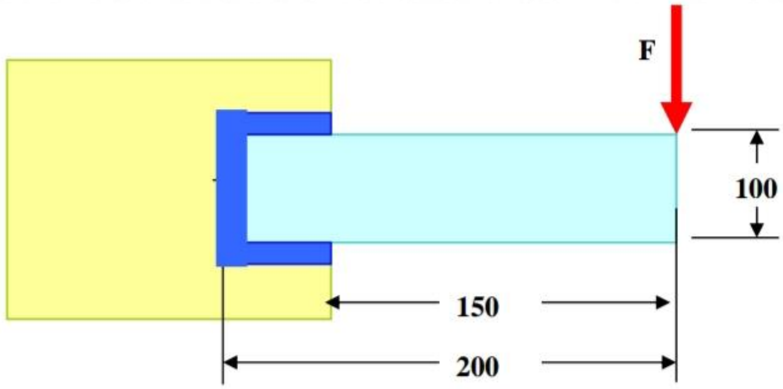
$$\tau_2 = \frac{Mr}{J}$$

where r = distance of the point on the weld from the CG

J = Polar moment of inertia of the weld section

r is calculated from the geometry for the farthest point of the weld from the CG.

Q.1. A rectangular steel plate is welded as a cantilever to a vertical column and supports a single concentrated load of 60 kN as shown in figure below. Determine the weld size if the allowable shear stress in the weld material is 140 MPa.



Ans. The weld is subjected to two shear stresses

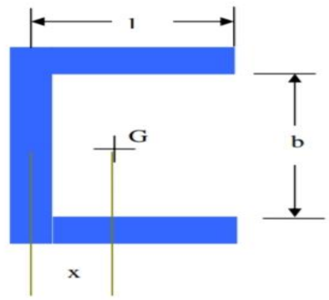
(1) Direct shear of magnitude 60,000/Area of the weld. The area of the throat section is easily found out to be 200 *t* where *t=0.707 h*. Thus direct shear stress is 424/*h* MPa.

(2) The indirect shear stress as a point *r* distance away from the centroid of the throat section has magnitude

$$\tau = \frac{FLr}{J},$$

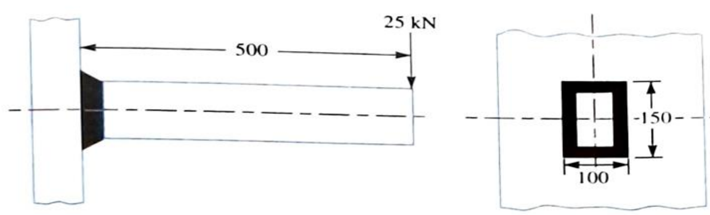
where *J* is the polar moment of area of the throat section and L is the eccentricity of the load. From the geometry of the throat section it may be calculated that the distance of centroid from left end = $x = \frac{l^2}{2l+b} = 12.5$ mm (see figure below) and the polar moment about G is

$$J = \frac{h}{\sqrt{2}}\left[\frac{(b+2l)^3}{12} - \frac{l^2(b+l)^2}{b+2l}\right] = 272530\, h \text{ mm}^4.$$



Thus the indirect shear stress has magnitude $\frac{41.28}{h}r$ MPa. The maximum resultant shear stress depends on both the magnitude and direction of the indirect shear stress. It should be clear that the maximum shear stress appears at the extreme corner of the weld section which is at a distance $\sqrt{(\frac{b}{2})^2 + (l-x)^2} = 62.5$ mm away from the centroid. Noticing that the included angle between the two shear forces as $\cos^{-1}\left(\frac{l-x}{r_{max}}\right) \approx 53.13^0$, the maximum value of the resultant shear stress is found out to be $f_{max} = \frac{2854.62}{h}$ MPa. Since this value should not exceed 140 MPa the minimum weld size must be h= 20.39 mm.

Q.2. A rectangular cross-section bar is welded to a support by means of fillet welds as shown in figure. Determine the size of the welds, if the permissible shear stress in the weld is limited to 75 MPa.



All dimensions in mm

Fig. 10.26

Solution. Given : $P = 25$ kN $= 25 \times 10^3$N ; $\tau_{max} = 75$ MPa $= 75$N/mm^2 ; $l = 100$ mm ; $b = 150$ mm ; $e = 500$ mm

Let s = Size of the weld, and

t = Throat thickness.

The joint, as shown in Fig. 10.26, is subjected to direct shear stress and the bending stress. We know that the throat area for a rectangular fillet weld,

$$A = t\,(2b + 2l) = 0.707\,s\,(2b + 2l)$$

$$= 0.707s\,(2 \times 150 + 2 \times 100) = 353.5\,s \text{ mm}^2 \qquad \ldots (\because t = 0.707s)$$

$\therefore$ Direct shear stress, $\tau = \frac{P}{A} = \frac{25 \times 10^3}{353.5\,s} = \frac{70.72}{s}$ N/mm^2

We know that bending moment,

$$M = P \times e = 25 \times 10^3 \times 500 = 12.5 \times 10^6 \text{ N-mm}$$

From Table 10.7, we find that for a rectangular section, section modulus,

$$Z = t\left(b.l + \frac{b^2}{3}\right) = 0.707\,s\left[150 \times 100 + \frac{(150)^2}{3}\right] = 15\,907.5\,s \text{ mm}^3$$

$\therefore$ Bending stress, $\sigma_b = \frac{M}{Z} = \frac{12.5 \times 10^6}{15\,907.5\,s} = \frac{785.8}{s}$ N/mm^2

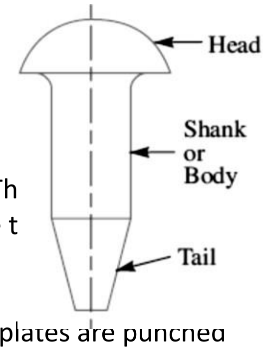
We know that maximum shear stress (τ_{max}),

$$75 = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\,\tau^2} = \frac{1}{2}\sqrt{\left(\frac{785.8}{s}\right)^2 + 4\left(\frac{70.72}{s}\right)^2} = \frac{399.2}{s}$$

$\therefore \qquad s = 399.2 / 75 = 5.32$ mm **Ans.**

Riveted joint:

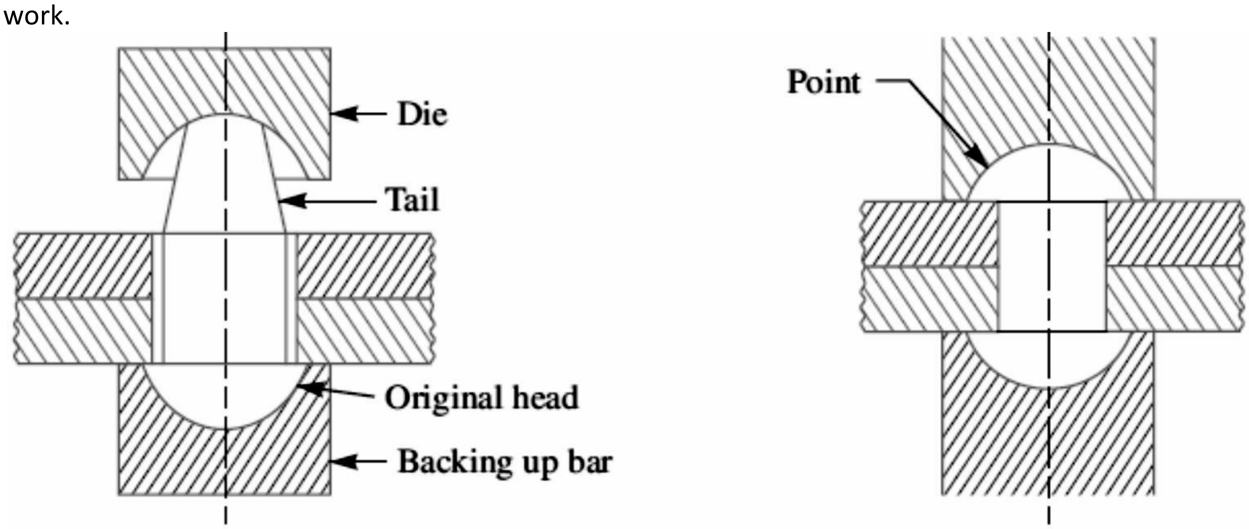
The rivets are used to make permanent fastening between the two or more plates such as in structural work, ship building, bridges, tanks and boiler shells. The riveted joints are widely used for joining light metals. A rivet is a short cylindrical bar with a head integral to it. The cylindrical portion of the rivet is called shank or body and lower portion of shank is known as tail.



Methods of Riveting

The function of rivets in a joint is to make a connection that has strength and tightness. Th necessary to prevent failure of the joint. The tightness is necessary in order to contribute t prevent leakage as in a boiler or in a ship hull (The frame or body of ship).

When two plates are to be fastened together by a rivet as shows below, the holes in the plates are punched and reamed or drilled. Punching is the cheapest method and is used for relatively thin plates and in structural work. Since punching injures the material around the hole, therefore drilling is used in most pressure-vessel work.



(*a*) Initial position. (*b*) Final position.

Material of Rivets:

The material of the rivets must be tough and ductile. They are usually made of steel (low carbon steel or nickel steel), brass, aluminum or copper, but when strength and a fluid tight joint is the main consideration, then the steel rivets are used. The rivets for general purposes shall be manufactured from steel conforming to the following Indian Standards:

1. IS: 1148–1982 (Reaffirmed 1992) – Specification for hot rolled rivet bars (up to 40 mm diameter) for structural purposes; or
2. IS: 1149–1982 (Reaffirmed 1992) – Specification for high tensile steel rivet bars for structural purposes.
3. The rivets for boiler work shall be manufactured from material conforming to IS: 1990 – 1973 (Reaffirmed 1992) – Specification for steel rivets and stay bars for boilers.

Manufacture of Rivets

The rivets may be made either by cold heading or by hot forging.

- If rivets are made by the cold heading process, they heat treated so that the stresses set up in the cold heading process are eliminated.
- If they are made by hot forging process, care shall be taken to see that the finished rivets cool gradually.

Note: when the diameter of rivet is 12 mm or less generally cold riveting is adopted.

Types of Rivets:

1. Button Head
2. Counter sunk Head
3. Oval counter Head
4. Pan Head
5. Conical Head

Types of Riveted Joints

1. According to purpose
2. According to position of plates connected
3. According to arrangement of rivets

1. **According to purpose**:

   a) Strong Joints: In these Joints strength is the only criterion. Eg: Beams, Trusses and Machine Joints.

   b) Tight joints: These joints provide strength as well as are leak proof against low pressure. Eg: Reservoir, Containers and tanks.

   c) Strong-Tight Joints: These are the joints applied in boilers and pressure vessels and ensure both strength and leak proofness.

2. **According to position of plates:**

   - **Lap Joint:** A lap joint is that in which one plate overlaps the other and the two plates are then riveted together.

   - **Butt Joint:** A butt joint is that in which the main plates are touching each other and a cover plate (i.e. Strap) is placed either on one side or on both sides of the main plates. The cover plate is then riveted together with the main plates. Butt joints are of the following two types:

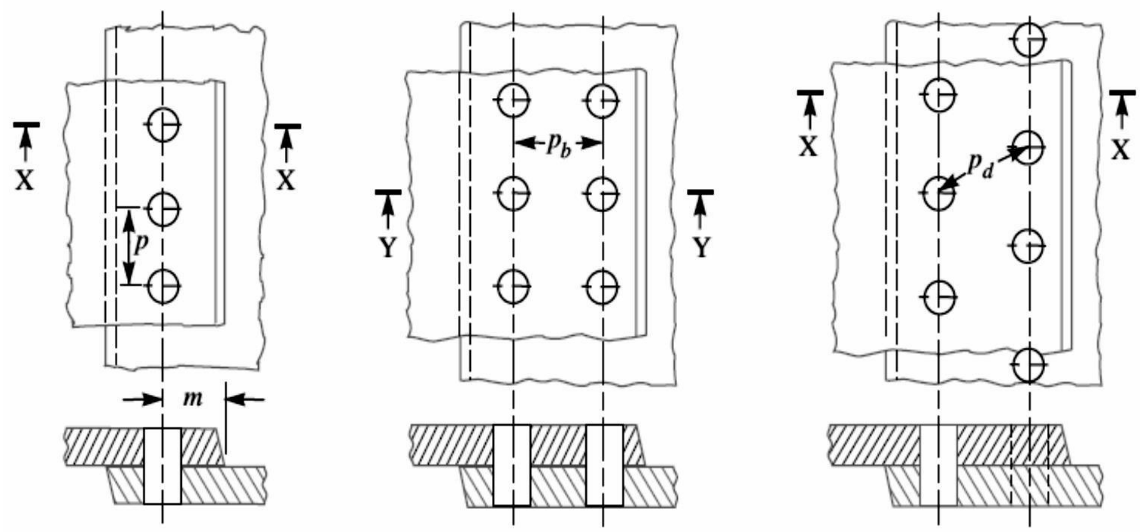
     a. In a single strap butt joint, the edges of the main plates butt against each other and only one cover plate is placed on one side of the main plates and then riveted together.

     b. In a double strap butt joint, the edges of the main plates butt against each other and two cover plates are placed on both sides of the main plates and then riveted together.

3. **According to arrangement of rivets:**

   a. A **single riveted joint** is that in which there is a single row of rivets in a lap joint as shown in Fig. and there is a single row of rivets on each side in a butt joint.

   b. A **double riveted joint** is that in which there are two rows of rivets in a lap joint as shown in Fig. and there are two rows of rivets on each side in a butt joint.

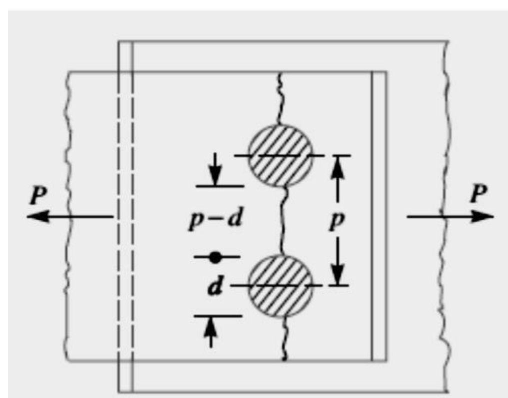


(*a*) Single riveted lap joint. (*b*) Double riveted lap joint (Chain riveting). (*c*) Double riveted lap joint (Zig-zag riveting).

Important terms of Riveted joints:

1. **Pitch (p)**: The Distance between two adjacent rivet holes in a row.
2. **Back pitch (P_b)**: The Distance between two adjacent rows of rivets.
3. **Diagonal pitch(P_d)**: The smallest distance between centers of two rivet holes in adjacent rows of ZIG-Zag riveted joints.
4. **Margin (m)**: It is the distance between center of a rivet hole and nearest edge of the plate.

Modes of Failures of a Riveted Joint



1. **Tearing of the plate at the section weakened by holes**: Due to the tensile stresses in the main plates, the main plate or cover plates may tear off across a row of rivets as shown in Fig. In such cases, we consider only one pitch length of the plate, since every rivet is responsible for that much length of the plate only.

The resistance offered by the plate against tearing is known as tearing resistance or tearing strength or tearing value of the plate.

Let, p = Pitch of the rivets,
d = Diameter of the rivet hole,
t = Thickness of the plate, and
σ_t = Permissible tensile stress for the plate material.

We know that tearing area per pitch length,

$$A_t = (p - d)\, t$$

Tearing resistance or pull required to tear off the plate per pitch length,

$$P_t = A_t . \sigma_t = (p - d)\ t.\ \sigma_t$$

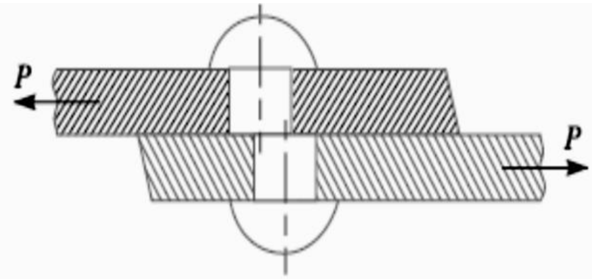
When the tearing resistance (P_t) is greater than the applied load (P) per pitch length, then this type of failure will not occur.

2. Shearing of the rivets:

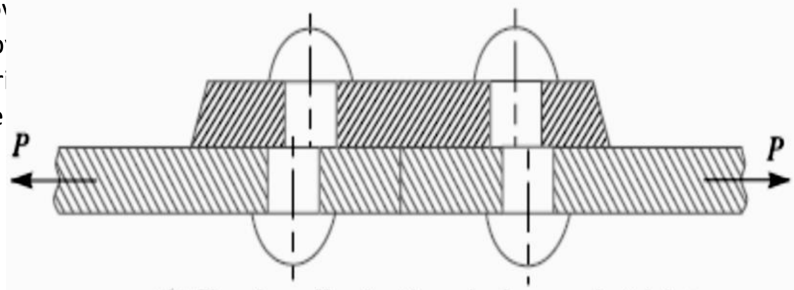
The plates which are connected by the rivets exert tensile stress on the rivets, and if the rivets are unable to resist the stress, they are sheared off as shown in Fig.

It may be noted that the rivets are in single shear in a lap joint and in a single cover butt joint, as shown in Fig.

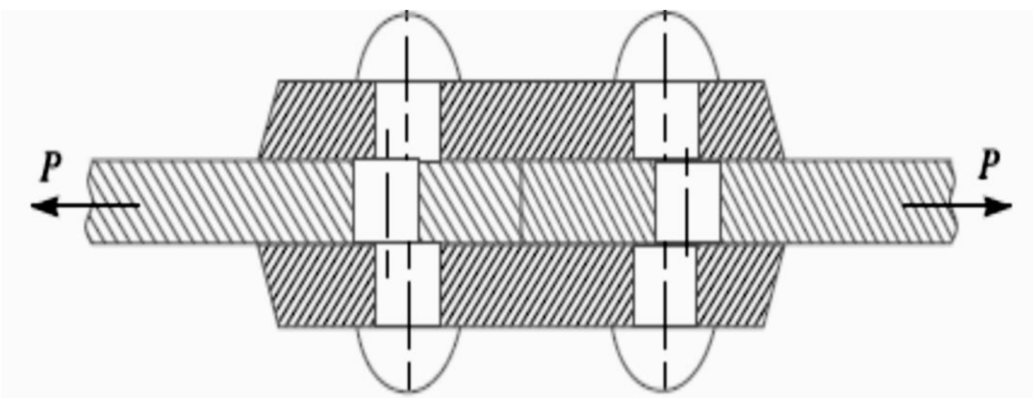
But the rivets are in double shear in a double co butt joint as shown in Fig. The resistance offered b rivet to be sheared off is known as shear resistance or shearing strength or shearing value the rivet.



(a) Shearing off a rivet in a lap joint.



(b) Shearing off a rivet in a single cover butt joint.



Let d = Diameter of the rivet hole,

τ = Safe permissible shear stress for the rivet material,

n = Number of rivets per pitch length.

We know that shearing area,

$$A_s = (\pi/4) \times d^2 \quad \text{... (In single shear)}$$

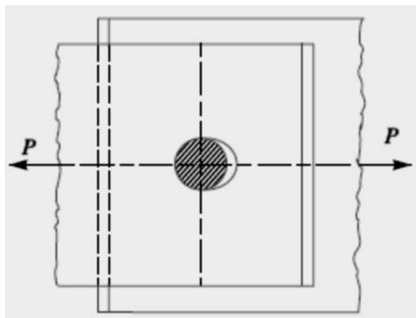
$$= 2 \times (\pi/4) \times d^2 \quad \text{... (Theoretically, in double shear)}$$

Shearing resistance required to shear off the rivet per pitch length,

$$P_s = n \times (\pi/4) \times d^2 \times \tau \quad \text{...(In single shear)}$$
$$= n \times 2 \times (\pi/4) \times d^2 \times \tau \quad \text{...(Theoretically, in double shear)}$$

When the shearing resistance (P_s) is greater than the applied load (P) per pitch length, then this type of failure will occur.

3. **Crushing of the plate or rivets:** Sometimes, the rivets do not actually shear off under the tensile stress, but are crushed as shown in Fig. Due to this, the rivet hole becomes of an oval shape and hence the joint becomes loose. The failure of rivets in such a manner is also known as bearing failure. The area which resists this action is the projected area of the hole or rivet on diametral plane.



The resistance offered by a rivet to be crushed is known as crushing resistance or crushing strength or bearing value of the rivet.

Let d = Diameter of the rivet hole,

t = Thickness of the plate,

σ_c = Safe permissible crushing stress for the rivet or plate material,

and n = Number of rivets per pitch length under crushing.

We know that crushing area per rivet (i.e. projected area per rivet),

$$A_c = d.\ t$$

∴Total crushing area = n. d. t

and crushing resistance or pull required to crush the rivet per pitch length,

$$P_c = n.\ d.\ t.\ \sigma_c$$

When the crushing resistance (P_c) is greater than the applied load (P) per pitch length, then this type of failure will occur.

Note: The number of rivets under shear shall be equal to the number of rivets under crushing.

Unwin's Formula: As a Common Practice for plate thickness greater than 8 mm, the diameter of rivet hole is determined by: $d = 6\sqrt{t}$ (t = thickness of plate)

Strength of a Riveted Joint:

The strength of a joint may be defined as the maximum force, which it can transmit, without causing it to fail.

We have seen that P_t, P_s and P_c are the pulls required to tear off the plate, shearing off the rivet and crushing off the rivet. A little consideration will show that if we go on increasing the pull on a riveted joint, it will fail when the least of these three pulls is reached, because a higher value of the other pulls will never reach since the joint has failed, either by tearing off the plate, shearing off the rivet or crushing off the rivet.

If the joint is continuous as in case of boilers, the strength is calculated per pitch length.

But if the joint is small, the strength is calculated for the whole length of the plate.

Efficiency of a Riveted Joint:

The efficiency of a riveted joint is defined as the ratio of the strength of riveted joint to the strength of the un-riveted or solid plate. We have already discussed that strength of the riveted joint

= **Least of P_t , P_s and P_c**

Strength of the un-riveted or solid plate per pitch length,

$$P = p \cdot t \cdot \sigma_t$$

∴ Efficiency of the riveted joint,

$$\eta = \frac{\text{Least of } P_t, P_S \text{ and } P_C}{p \times t \times \sigma_t}$$

Where, σ_t = Permissible tensile stress of the plate material

p= Pitch of the rivets,

t = Thickness of the plate

Q.1: A double riveted lap joint is made between 15 mm thick plates. The rivet diameter and pitch are 25 mm and 75 mm respectively. If the ultimate stresses are 400 MPa in tension, 320 MPa in shear and 640 MPa in crushing, find the minimum force per pitch which will rupture the joint. If the above joint is subjected to a load such that the factor of safety is 4, find out the actual stresses developed in the plates and the rivets.

Solution. Given : $t = 15$ mm ; $d = 25$ mm ; $p = 75$ mm ; $\sigma_{tu} = 400$ MPa $= 400$ N/mm^2 ; $\tau_u = 320$ MPa $= 320$ N/mm^2 ; $\sigma_{cu} = 640$ MPa $= 640$ N/mm^2

Minimum force per pitch which will rupture the joint

Since the ultimate stresses are given, therefore we shall find the ultimate values of the resistances of the joint. We know that ultimate tearing resistance of the plate per pitch,

$$P_{tu} = (p - d)t \times \sigma_{tu} = (75 - 25)15 \times 400 = 300\ 000 \text{ N}$$

Ultimate shearing resistance of the rivets per pitch,

$$P_{su} = n \times \frac{\pi}{4} \times d^2 \times \tau_u = 2 \times \frac{\pi}{4} (25)^2\, 320 = 314\ 200 \text{ N} \quad ...(\because n = 2)$$

and ultimate crushing resistance of the rivets per pitch,

$$P_{cu} = n \times d \times t \times \sigma_{cu} = 2 \times 25 \times 15 \times 640 = 480\ 000 \text{ N}$$

From above we see that the minimum force per pitch which will rupture the joint is 300 000 N or 300 kN. **Ans.**

Actual stresses produced in the plates and rivets

Since the factor of safety is 4, therefore safe load per pitch length of the joint

$$= 300\ 000/4 = 75\ 000 \text{ N}$$

Let σ_{ta}, τ_a and σ_{ca} be the actual tearing, shearing and crushing stresses produced with a safe load of 75 000 N in tearing, shearing and crushing.

We know that actual tearing resistance of the plates (P_{ta}),

$$75\ 000 = (p - d)\, t \times \sigma_{ta} = (75 - 25)15 \times \sigma_{ta} = 750\, \sigma_{ta}$$

$$\therefore \quad \sigma_{ta} = 75\ 000 / 750 = 100 \text{ N/mm}^2 = 100 \text{ MPa} \quad \textbf{Ans.}$$

Actual shearing resistance of the rivets (P_{sa}),

$$75\ 000 = n \times \frac{\pi}{4} \times d^2 \times \tau_a = 2 \times \frac{\pi}{4} (25)^2 \tau_a = 982\, \tau_a$$

$$\therefore \quad \tau_a = 75000 / 982 = 76.4 \text{ N/mm}^2 = 76.4 \text{ MPa} \quad \textbf{Ans.}$$

and actual crushing resistance of the rivets (P_{ca}),

$$75\ 000 = n \times d \times t \times \sigma_{ca} = 2 \times 25 \times 15 \times \sigma_{ca} = 750\, \sigma_{ca}$$

$$\therefore \quad \sigma_{ca} = 75000 / 750 = 100 \text{ N/mm}^2 = 100 \text{ MPa} \quad \textbf{Ans.}$$

Q.2: Find the efficiency of the following riveted joints:

1. Single riveted lap joint of 6 mm plates with 20 mm diameter rivets having a pitch of 50 mm. 2. Double riveted lap joint of 6 mm plates with 20 mm diameter rivets having a pitch of 65 mm. Assume Permissible tensile stress in plate = 120 MPa Permissible shearing stress in rivets = 90 MPa Permissible crushing stress in rivets = 180 MPa.

Solution. Given : $t = 6$ mm ; $d = 20$ mm ; $\sigma_t = 120$ MPa $= 120$ N/mm^2 ; $\tau = 90$ MPa $= 90$ N/mm^2 ; $\sigma_c = 180$ MPa $= 180$ N/mm^2

1. *Efficiency of the first joint*

Pitch, $p = 50$ mm ...(Given)

First of all, let us find the tearing resistance of the plate, shearing and crushing resistances of the rivets.

(*i*) *Tearing resistance of the plate*

We know that the tearing resistance of the plate per pitch length,

$$P_t = (p - d)\, t \times \sigma_t = (50 - 20)\, 6 \times 120 = 21\,600 \text{ N}$$

(*ii*) *Shearing resistance of the rivet*

Since the joint is a single riveted lap joint, therefore the strength of one rivet in single shear is taken. We know that shearing resistance of one rivet,

$$P_s = \frac{\pi}{4} \times d^2 \times \tau = \frac{\pi}{4}\,(20)^2\, 90 = 28\,278 \text{ N}$$

(*iii*) *Crushing resistance of the rivet*

Since the joint is a single riveted, therefore strength of one rivet is taken. We know that crushing resistance of one rivet,

$$P_c = d \times t \times \sigma_c = 20 \times 6 \times 180 = 21\,600 \text{ N}$$

∴ Strength of the joint

$$= \text{Least of } P_t, P_s \text{ and } P_c = 21\,600 \text{ N}$$

We know that strength of the unriveted or solid plate,

$$P = p \times t \times \sigma_t = 50 \times 6 \times 120 = 36\,000 \text{ N}$$

∴ Efficiency of the joint,

$$\eta = \frac{\text{Least of } P_t, P_s \text{ and } P_c}{P} = \frac{21\,600}{36\,000} = 0.60 \text{ or } 60\% \quad \textbf{Ans.}$$

2. *Efficiency of the second joint*

Pitch, $p = 65$ mm ...(Given)

(*i*) *Tearing resistance of the plate,*

We know that the tearing resistance of the plate per pitch length,

$$P_t = (p - d)\, t \times \sigma_t = (65 - 20)\, 6 \times 120 = 32\,400 \text{ N}$$

(*ii*) *Shearing resistance of the rivets*

Since the joint is double riveted lap joint, therefore strength of two rivets in single shear is taken. We know that shearing resistance of the rivets,

$$P_s = n \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times \frac{\pi}{4}\,(20)^2\, 90 = 56\,556 \text{ N}$$

(*iii*) *Crushing resistance of the rivet*

Since the joint is double riveted, therefore strength of two rivets is taken. We know that crushing resistance of rivets,

$$P_c = n \times d \times t \times \sigma_c = 2 \times 20 \times 6 \times 180 = 43\,200 \text{ N}$$

∴ Strength of the joint

$$= \text{Least of } P_t, P_s \text{ and } P_c = 32\,400 \text{ N}$$

We know that the strength of the unriveted or solid plate,

$$P = p \times t \times \sigma_t = 65 \times 6 \times 120 = 46\ 800 \text{ N}$$

∴ Efficiency of the joint,

$$\eta = \frac{\text{Least of } P_t, P_s \text{ and } P_c}{P} = \frac{32\ 400}{46\ 800} = 0.692 \text{ or } 69.2\% \quad \textbf{Ans.}$$

Q.3: Design a double riveted lap joint for MS Plates having a thickness 9.5 mm. Calculate the efficiency of the joint. The permissible stresses are: σ_t = 90 MPa, τ_s = 75 MPa, σ_c = 150 MPa.

Design of boiler joints according to IBR

The boiler has a longitudinal joint as well as circumferential joint. The longitudinal joint is used to join the ends of the plate to get the required diameter of a boiler. For this purpose, a butt joint with two cover plates is used. The circumferential joint is used to get the required length of the boiler. For this purpose, a lap joint with one ring overlapping the other alternately is used.
Since a boiler is made up of number of rings, therefore the longitudinal joints are staggered for convenience of connecting rings at places where both longitudinal and circumferential joints occur.

Design of Longitudinal Butt Joint for a Boiler

According to Indian Boiler Regulations (I.B.R), the following procedure should be adopted for the design of longitudinal butt joint for a boiler.

1. Thickness of boiler shell.

First of all, the thickness of the boiler shell is determined by using the thin cylindrical formula, *i.e.*

$$t = \frac{P.D}{2\,\sigma_t \times \eta_l} + 1 \text{ mm as corrosion allowance}$$

Where t = Thickness of the boiler shell,

P = Steam pressure in boiler,

D = Internal diameter of boiler

σ_t = Permissible tensile stress, and

η_l = Efficiency of the longitudinal joint.

The following points may be noted:

(a) The thickness of the boiler shell should not be less than 7 mm.

(b) The efficiency of the joint may be taken from the following table.

Indian Boiler Regulations (I.B.R.) allows a maximum efficiency of 85% for the best joint.

(c) According to I.B.R., the factor of safety should not be less than 4.

2. Diameter of rivets.

After finding out the thickness of the boiler shell (*t*), the diameter of the rivet hole (*d*) may be determined by using Unwin's empirical formula,

i.e. $d = 6\sqrt{t}$, (when *t* is greater than 8 mm)

But if the thickness of plate is less than 8 mm, then the diameter of the rivet hole may be calculated by equating the shearing resistance of the rivets to crushing resistance. In no case, the diameter of rivet hole should not be less than the thickness of the plate, because there will be danger of punch crushing.

3. Pitch of rivets.

The pitch of the rivets is obtained by equating the tearing resistance of the plate to the shearing resistance of the rivets. It may noted that;

(a) The pitch of the rivets should not be less than 2d, which is necessary for the formation of head.

(b) The maximum value of the pitch of rivets for a longitudinal joint of a boiler as per I.B.R. is

$$p_{max} = C \times t + 41.28 \text{ mm}$$

where, t = Thickness of the shell plate in mm,

C = Constant.

- The value of the constant C may be taken from DDB. If the pitch of rivets as obtained by equating the tearing resistance to the shearing resistance is more than p_{max}, then the value of p_{max} is taken.

4. Distance between the rows of rivets.

The distance between the rows of rivets as specified by Indian Boiler Regulations is as follows:

(a) For equal number of rivets in more than one row for lap joint or butt joint, the distance between the rows of rivets (p_b) should not be less than

0.33 p + 0.67 d ……for zig-zig riveting, and

2d …….for chain riveting.

(b) For joints in which the number of rivets in outer rows is **half** the number of rivets in inner rows and if the inner rows are chain riveted, the distance between the outer rows and the next rows should not be less than **0.33 p + 0.67 or 2d**, whichever is greater.

The distance between the rows in which there are full number of rivets shall not be less than 2d.

(c) For joints in which the number of rivets in outer rows is **half** the number of rivets in inner rows and if the inner rows are zig-zig riveted, the distance between the outer rows and the next rows shall not be less than 0.2 p + 1.15 d. The distance between the rows in which there are full number of rivets (zig-zag) shall not be less than **0.165 p + 0.67d**.

Note : In the above discussion, p is the pitch of the rivets in the outer rows.

5. Thickness of butt strap.

According to I.B.R., the thicknesses for butt strap (t_1) are as given below:

(a) The thickness of butt strap, in no case, shall be less than 10 mm.

(b) **t_t = 1.125 t**, for ordinary (chain riveting) single butt strap.

$$t_1 = 1.125\, t \left(\frac{p - d}{p - 2d} \right)$$

For single butt straps, every alternate rivet in outer rows being omitted.

t_1 = 0.625 t, for double butt-straps of equal width having ordinary riveting (chain riveting).

$$t_1 = 0.625\, t \left(\frac{p - d}{p - 2d} \right)$$

For double butt straps of equal width having every alternate rivet in the outer rows being omitted.

(c) For unequal width of butt straps, the thicknesses of butt strap are

t_1 = 0.75 t, for wide strap on the inside, and

t_1 = 0.625 t, for narrow strap on the outside.

6. Margin.

The margin (m) is taken as 1.5 d.

Q.1: Inner diameter of a boiler is 1500 mm and the steam pressure is 2 N/mm^2. Use a proper joint along the length and design it completely. Use following permissible values of stress.

Tension σ_t = 90 MPa, Shear τ = 75 MPa , Crushing σ_c = 150 MPa

Q.2: A cylindrical pressure vessel with a 1.5 m inside diameter is subjected to internal steam pressure of 1.5 MPa. It is made from steel plate by triple-riveted double strap longitudinal butt joint with equal straps. The pitch of the rivets in the outer row is twice of the pitch of the rivets in the inner rows. The rivets are arranged in a zig zag pattern. The efficiency of the riveted joint should be atleast 80 %. The permissible stresses for the plate and rivets in tension, shear and compression are 80, 60 and 120 MPa, respectively. Assume that the rivet in double shear is 1.875 times stronger than in single shear. Design the joint and calculate

1. thickness of the plate,
2. diameter of the rivets,
3. Pitch of the rivets,
4. distance between the rows of rivets,
5. margin
6. thickness of the straps and
7. Efficiency of the joint.

Design of Shafts and Keys

Function of Shafts:-

A shaft is a rotating machine element which is used to transmit power from one place to another. The power is delivered to the shaft by some tangential force and the resultant torque (or twisting moment) set up within the shaft permits the power to be transferred to various machines linked up to the shaft. In order to transfer the power from one shaft to another, the various members such as pulleys, gears etc., are mounted on it. These members along with the forces exerted upon them causes the shaft to bending.

In other words, we may say that a shaft is used for the transmission of torque and bending moment. The various members are mounted on the shaft by means of keys or splines.

Notes:

1. The shafts are usually cylindrical, but may be square or cross-shaped in section. They are solid in cross-section but sometimes hollow shafts are also used.

2. An ***axle,*** though similar in shape to the shaft, is a stationary machine element and is used for the transmission of bending moment only. It simply acts as a support for some rotating body such as hoisting drum, a car wheel or a rope sheave.

3. A ***spindle*** is a short shaft that imparts motion either to a cutting tool (*e.g.* drill press spindles) or to a work piece (*e.g.* lathe spindles).

Material Used for Shafts:-

The material used for shafts should have the following properties:

1. It should have high strength.
2. It should have good machinability.
3. It should have low notch sensitivity factor.
4. It should have good heat treatment properties.
5. It should have high wear resistant properties.

The material used for ordinary shafts is carbon steel of grades 40 C 8, 45 C 8, 50 C 4 and 50 C 12.
The mechanical properties of these grades of carbon steel are given in the following table.

Mechanical properties of steels used for shafts.

Indian standard designation	*Ultimate tensile strength, MPa*	*Yield strength, MPa*
40 C 8	560 - 670	320
45 C 8	610 - 700	350
50 C 4	640 - 760	370
50 C 12	700 Min.	390

When a shaft of high strength is required, then an alloy steel such as nickel, nickel-chromium or chrome-vanadium steel is used.

Types of Shafts:

The following two types of shafts are important from the subject point of view :

1. ***Transmission shafts.*** These shafts transmit power between the source and the machines absorbing power. The counter shafts, line shafts, overhead shafts and all factory shafts are transmission shafts. Since these shafts carry machine parts such as pulleys, gears etc., therefore they are subjected to bending in addition to twisting.

2. ***Machine shafts.*** These shafts form an integral part of the machine itself. The crank shaft is an example of machine shaft.

Standard Sizes of Transmission Shafts:

The standard sizes of transmission shafts are:

25 mm to 60 mm with 5 mm steps;
60 mm to 110 mm with 10 mm steps;
110 mm to 140 mm with 15 mm steps;
and 140 mm to 500 mm with 20 mm steps.
The standard lengths of the shafts are 5 m, 6 m and 7 m.

Design of Shafts

The shafts may be designed on the basis of
1. Strength, and **2.** Rigidity and stiffness.
In designing shafts on the basis of strength, the following cases may be considered:
(a) Shafts subjected to twisting moment or torque only,
(b) Shafts subjected to bending moment only,
(c) Shafts subjected to combined twisting and bending moments, and
(d) Shafts subjected to axial loads in addition to combined torsional and bending loads.

Shafts Subjected to Twisting Moment Only:

When the shaft is subjected to a twisting moment (or torque) only, then the diameter of the shaft may be obtained by using the torsion equation. We know that

$$\frac{T}{J} = \frac{\tau}{r} \qquad \text{...}(i)$$

where T = Twisting moment (or torque) acting upon the shaft,
J = Polar moment of inertia of the shaft about the axis of rotation,
τ = Torsional shear stress, and

r = Distance from neutral axis to the outer most fibre

= $d / 2$; where d is the diameter of the shaft.

We know that for round solid shaft, polar moment of inertia,

$$J = \frac{\pi}{32} \times d^4$$

The equation (*i*) may now be written as

$$\frac{T}{\frac{\pi}{32} \times d^4} = \frac{\tau}{\frac{d}{2}} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \times d^3 \qquad \text{...(ii)}$$

From this equation, we may determine the diameter of round solid shaft (d).

We also know that for hollow shaft, polar moment of inertia,

$$J = \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]$$

where d_o and d_i = Outside and inside diameter of the shaft, and $r = d_o / 2$.

Substituting these values in equation (*i*), we have

$$\frac{T}{\frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]} = \frac{\tau}{\frac{d_o}{2}} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau\left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] \qquad \text{...(iii)}$$

Let k = Ratio of inside diameter and outside diameter of the shaft

$= d_i / d_o$

Now the equation (*iii*) may be written as

$$T = \frac{\pi}{16} \times \tau \times \frac{(d_o)^4}{d_o}\left[1 - \left(\frac{d_i}{d_o}\right)^4\right] = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4) \qquad \text{...(iv)}$$

From the equations (*iii*) or (*iv*), the outside and inside diameter of a hollow shaft may be determined.

It may be noted that

1. The hollow shafts are usually used in marine work. These shafts are stronger per kg of material and they may be forged on a mandrel, thus making the material more homogeneous than would be possible for a solid shaft.

When a hollow shaft is to be made equal in strength to a solid shaft, the twisting moment of both the shafts must be same. In other words, for the same material of both the shafts,

$$T = \frac{\pi}{16} \times \tau\left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] = \frac{\pi}{16} \times \tau \times d^3$$

$$\therefore \quad \frac{(d_o)^4 - (d_i)^4}{d_o} = d^3 \quad \text{or} \quad (d_o)^3\,(1 - k^4) = d^3$$

2. The twisting moment (T) may be obtained by using the following relation :

We know that the power transmitted (in watts) by the shaft,

$$P = \frac{2\pi N \times T}{60} \quad \text{or} \quad T = \frac{P \times 60}{2\pi N}$$

where T = Twisting moment in N-m, and

N = Speed of the shaft in r.p.m.

3. In case of belt drives, the twisting moment (T) is given by

$$T = (T_1 - T_2)\,R$$

where T_1 and T_2 = Tensions in the tight side and slack side of the belt respectively, and

R = Radius of the pulley.

Problem: A line shaft rotating at 200 r.p.m. is to transmit 20 kW. The shaft may be assumed to be made of mild steel with an allowable shear stress of 42 MPa. Determine the diameter of the shaft, neglecting the bending moment on the shaft.

Solution. Given : $N = 200$ r.p.m. ; $P = 20$ kW $= 20 \times 10^3$ W; $\tau = 42$ MPa $= 42$ N/mm^2

Let d = Diameter of the shaft.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ N-m} = 955 \times 10^3 \text{ N-mm}$$

We also know that torque transmitted by the shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\ d^3$$

$\therefore$ $d^3 = 955 \times 10^3 / 8.25 = 115\ 733$ or $d = 48.7$ say 50 mm **Ans.**

Problem: A solid shaft is transmitting 1 MW at 240 r.p.m. Determine the diameter of the shaft if the maximum torque transmitted exceeds the mean torque by 20%. Take the maximum allowable shear stress as 60 MPa.

Solution. Given : $P = 1$ MW $= 1 \times 10^6$ W ; $N = 240$ r.p.m. ; $T_{max} = 1.2\ T_{mean}$; $\tau = 60$ MPa $= 60$ N/mm^2

Let d = Diameter of the shaft.

We know that mean torque transmitted by the shaft,

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{1 \times 10^6 \times 60}{2\pi \times 240} = 39\ 784 \text{ N-m} = 39\ 784 \times 10^3 \text{ N-mm}$$

$\therefore$ Maximum torque transmitted,

$$T_{max} = 1.2\ T_{mean} = 1.2 \times 39\ 784 \times 10^3 = 47\ 741 \times 10^3 \text{ N-mm}$$

We know that maximum torque transmitted (T_{max}),

$$47\ 741 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.78\ d^3$$

$\therefore$ $d^3 = 47\ 741 \times 10^3 / 11.78 = 4053 \times 10^3$

or $d = 159.4$ say 160 mm **Ans.**

Problem: Find the diameter of a solid steel shaft to transmit 20 kW at 200 r.p.m. The ultimate shear stress for the steel may be taken as 360 MPa and a factor of safety as 8.
If a hollow shaft is to be used in place of the solid shaft, find the inside and outside diameter when the ratio of inside to outside diameters is 0.5.

Solution. Given : $P = 20$ kW $= 20 \times 10^3$ W ; $N = 200$ r.p.m. ; $\tau_u = 360$ MPa $= 360$ N/mm^2 ; $F.S. = 8$; $k = d_i / d_o = 0.5$

We know that the allowable shear stress,

$$\tau = \frac{\tau_u}{F.S.} = \frac{360}{8} = 45 \text{ N/mm}^2$$

Diameter of the solid shaft

Let d = Diameter of the solid shaft.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ N-m} = 955 \times 10^3 \text{ N-mm}$$

We also know that torque transmitted by the solid shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 45 \times d^3 = 8.84\, d^3$$

$\therefore$ $d^3 = 955 \times 10^3 / 8.84 = 108\,032$ or $d = 47.6$ say 50 mm **Ans.**

Diameter of hollow shaft

Let d_i = Inside diameter, and

d_o = Outside diameter.

We know that the torque transmitted by the hollow shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau\, (d_o)^3\, (1 - k^4)$$

$$= \frac{\pi}{16} \times 45\, (d_o)^3\, [1 - (0.5)^4] = 8.3\, (d_o)^3$$

$\therefore$ $(d_o)^3 = 955 \times 10^3 / 8.3 = 115\,060$ or $d_o = 48.6$ say 50 mm **Ans.**

and $d_i = 0.5\, d_o = 0.5 \times 50 = 25$ mm **Ans.**

Shafts Subjected to Bending Moment Only

When the shaft is subjected to a bending moment only, then the maximum stress (tensile or compressive) is given by the bending equation. We know that

$$\frac{M}{I} = \frac{\sigma_b}{y} \quad \text{...(i)}$$

where M = Bending moment,

I = Moment of inertia of cross-sectional area of the shaft about the axis of rotation,

σ_b = Bending stress, and

y = Distance from neutral axis to the outer-most fibre.

We know that for a round solid shaft, moment of inertia,

$$I = \frac{\pi}{64} \times d^4 \quad \text{and} \quad y = \frac{d}{2}$$

Substituting these values in equation *(i)*, we have

$$\frac{M}{\frac{\pi}{64} \times d^4} = \frac{\sigma_b}{\frac{d}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b \times d^3$$

From this equation, diameter of the solid shaft (d) may be obtained.

We also know that for a hollow shaft, moment of inertia,

$$I = \frac{\pi}{64}\left[(d_o)^4 - (d_i)^4\right] = \frac{\pi}{64}(d_o)^4(1-k^4) \quad \text{...(where } k = d_i/d_o\text{)}$$

and $y = d_o/2$

Again substituting these values in equation (i), we have

$$\frac{M}{\frac{\pi}{64}(d_o)^4(1-k^4)} = \frac{\sigma_b}{\frac{d_o}{2}} \quad \text{or} \quad M = \frac{\pi}{32}\times\sigma_b\,(d_o)^3(1-k^4)$$

From this equation, the outside diameter of the shaft (d_o) may be obtained.

Note: We have already discussed that the axles are used to transmit bending moment only. Thus, axles are designed on the basis of bending moment only, in the similar way as discussed above.

Problem: A pair of wheels of a railway wagon carries a load of 50 kN on each axle box, acting at a distance of 100 mm outside the wheel base. The gauge of the rails is 1.4 m. Find the diameter of the axle between the wheels, if the stress is not to exceed 100 MPa.

Solution. Given : $W = 50$ kN $= 50 \times 10^3$ N ; $L = 100$ mm ; $x = 1.4$ m ; $\sigma_b = 100$ MPa $= 100$ N/mm^2

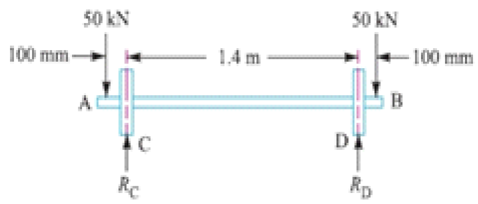


Fig. 14.1

The axle with wheels is shown in Fig. 14.1.

A little consideration will show that the maximum bending moment acts on the wheels at C and D. Therefore maximum bending moment,

$$^*M = W.L = 50 \times 10^3 \times 100 = 5 \times 10^6 \text{ N-mm}$$

Let d = Diameter of the axle.

We know that the maximum bending moment (M),

$$5 \times 10^6 = \frac{\pi}{32}\times\sigma_b\times d^3 = \frac{\pi}{32}\times 100\times d^3 = 9.82\,d^3$$

$\therefore$ $d^3 = 5 \times 10^6 / 9.82 = 0.51 \times 10^6$ or $d = 79.8$ say 80 mm **Ans.**

Shafts Subjected to Combined Twisting Moment and Bending Moment:

When the shaft is subjected to combined twisting moment and bending moment, then the shaft must be designed on the basis of the two moments simultaneously. Various theories have been suggested to account for the elastic failure of the materials when they are subjected to various types of combined stresses. The following two theories are important from the subject point of view :

1. Maximum shear stress theory or Guest's theory. It is used for ductile materials such as mild steel.

2. Maximum normal stress theory or Rankine's theory. It is used for brittle materials such as cast iron.

Let τ = Shear stress induced due to twisting moment, and

σ_b = Bending stress (tensile or compressive) induced due to bending moment.

According to maximum shear stress theory, the maximum shear stress in the shaft,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

Substituting the values of τ and σ_b from Art. 14.9 and Art. 14.10, we have

$$\tau_{max} = \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2} = \frac{16}{\pi d^3}\left[\sqrt{M^2 + T^2}\right]$$

or $$\frac{\pi}{16} \times \tau_{max} \times d^3 = \sqrt{M^2 + T^2} \qquad ...(i)$$

The expression $\sqrt{M^2 + T^2}$ is known as ***equivalent twisting moment*** and is denoted by T_e. The equivalent twisting moment may be defined as that twisting moment, which when acting alone, produces the same shear stress (τ) as the actual twisting moment. By limiting the maximum shear stress (τ_{max}) equal to the allowable shear stress (τ) for the material, the equation (*i*) may be written as

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau \times d^3 \qquad ...(ii)$$

From this expression, diameter of the shaft (d) may be evaluated.

Now according to maximum normal stress theory, the maximum normal stress in the shaft,

$$\sigma_{b(max)} = \frac{1}{2}\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2} \qquad ...(iii)$$

$$= \frac{1}{2} \times \frac{32M}{\pi d^3} + \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2}$$

$$= \frac{32}{\pi d^3}\left[\frac{1}{2}(M + \sqrt{M^2 + T^2})\right]$$

or $$\frac{\pi}{32} \times \sigma_{b\,(max)} \times d^3 = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] \qquad ...(iv)$$

The expression $\frac{1}{2}\left[(M + \sqrt{M^2 + T^2})\right]$ is known as ***equivalent bending moment*** and is denoted by M_e. The equivalent bending moment may be defined as **that moment which when acting alone produces the same tensile or compressive stress (σ_b) as the actual bending moment.** By limiting the maximum normal stress [$\sigma_{b(max)}$] equal to the allowable bending stress (σ_b), then the equation (*iv*) may be written as

$$M_e = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] = \frac{\pi}{32} \times \sigma_b \times d^3 \qquad ...(v)$$

From this expression, diameter of the shaft (d) may be evaluated.

Notes: 1. In case of a hollow shaft, the equations (*ii*) and (*v*) may be written as

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau (d_o)^3 (1 - k^4)$$

and

$$M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \frac{\pi}{32} \times \sigma_b (d_o)^3 (1 - k^4)$$

2. It is suggested that diameter of the shaft may be obtained by using both the theories and the larger of the two values is adopted.

Problem: A solid circular shaft is subjected to a bending moment of 3000 N-m and torque of 10 000 N-m. The shaft is made of 45 C 8 steel having ultimate tensile stress of 700 MPa and a ultimate shear stress of 500 MPa. Assuming a factor of safety as 6, determine the diameter of the shaft.

Solution. Given : M = 3000 N-m = 3 × 10^6 N-mm ; T = 10 000 N-m = 10 × 10^6 N-mm ; σ_{tu} = 700 MPa = 700 N/mm^2 ; τ_u = 500 MPa = 500 N/mm^2

We know that the allowable tensile stress,

$$\sigma_t \text{ or } \sigma_b = \frac{\sigma_{tu}}{F.S.} = \frac{700}{6} = 116.7 \text{ N/mm}^2$$

and allowable shear stress,

$$\tau = \frac{\tau_u}{F.S.} = \frac{500}{6} = 83.3 \text{ N/mm}^2$$

Let d = Diameter of the shaft in mm.

According to maximum shear stress theory, equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(3 \times 10^6)^2 + (10 \times 10^6)^2} = 10.44 \times 10^6 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$10.44 \times 10^6 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 83.3 \times d^3 = 16.36\, d^3$$

$$\therefore \quad d^3 = 10.44 \times 10^6 / 16.36 = 0.636 \times 10^6 \text{ or } d = 86 \text{ mm}$$

According to maximum normal stress theory, equivalent bending moment,

$$M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \frac{1}{2}(M + T_e)$$

$$= \frac{1}{2}(3 \times 10^6 + 10.44 \times 10^6) = 6.72 \times 10^6 \text{ N-mm}$$

We also know that the equivalent bending moment (M_e),

$$6.72 \times 10^6 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 116.7 \times d^3 = 11.46\, d^3$$

$$\therefore \quad d^3 = 6.72 \times 10^6 / 11.46 = 0.586 \times 10^6 \text{ or } d = 83.7 \text{ mm}$$

Taking the larger of the two values, we have

d = 86 say 90 mm **Ans.**

Design of Shafts on the basis of Rigidity:

Sometimes the shafts are to be designed on the basis of rigidity. We shall consider the following two types of rigidity

1. ***Torsional rigidity.*** The torsional rigidity is important in the case of camshaft of an I.C. engine where the timing of the valves would be effected. The permissible amount of twist should not exceed 0.25° per metre length of such shafts. For line shafts or transmission shafts, deflections 2.5 to 3 degree per metre length may be used as limiting value. The widely used deflection for the shafts is limited to 1 degree in a length equal to twenty times the diameter of the shaft.

The torsional deflection may be obtained by using the torsion equation,

$$\frac{T}{J} = \frac{G.\theta}{L} \quad \text{or} \quad \theta = \frac{T.L}{J.G}$$

where θ = Torsional deflection or angle of twist in radians,

T = Twisting moment or torque on the shaft,

J = Polar moment of inertia of the cross-sectional area about the axis of rotation,

$$= \frac{\pi}{32} \times d^4 \quad \text{...(For solid shaft)}$$

$$= \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right] \quad \text{...(For hollow shaft)}$$

G = Modulus of rigidity for the shaft material, and

L = Length of the shaft.

2. ***Lateral rigidity.*** It is important in case of transmission shafting and shafts running at high speed, where small lateral deflection would cause huge out-of-balance forces. The lateral rigidity is also important for maintaining proper bearing clearances and for correct gear teeth alignment. If the shaft is of uniform cross-section, then the lateral deflection of a shaft may be obtained by using the deflection formulae as in Strength of Materials. But when the shaft is of variable cross-section, then the lateral deflection may be determined from the fundamental equation for the elastic curve of a beam, *i.e.*

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

Problem: A steel spindle transmits 4 kW at 800 r.p.m. The angular deflection should not exceed 0.25° per metre of the spindle. If the modulus of rigidity for the material of the spindle is 84GPa, find the diameter of the spindle and the shear stress induced in the spindle.

Solution. Given : $P = 4$ kW = 4000 W ; $N = 800$ r.p.m. ; $\theta = 0.25° = 0.25 \times \frac{\pi}{180} = 0.0044$ rad ; $L = 1$ m = 1000 mm ; $G = 84$ GPa = 84×10^9 N/m² = 84×10^3 N/mm²

Diameter of the spindle

Let d = Diameter of the spindle in mm.

We know that the torque transmitted by the spindle,

$$T = \frac{P \times 60}{2\pi N} = \frac{4000 \times 60}{2\pi \times 800} = 47.74 \text{ N-m} = 47\ 740 \text{ N-mm}$$

We also know that $\frac{T}{J} = \frac{G \times \theta}{L}$ or $J = \frac{T \times l}{G \times \theta}$

or $$\frac{\pi}{32} \times d^4 = \frac{47\ 740 \times 1000}{84 \times 10^3 \times 0.0044} = 129\ 167$$

$\therefore$ $d^4 = 129\ 167 \times 32 / \pi = 1.3 \times 10^6$ or $d = 33.87$ say 35 mm **Ans.**

Shear stress induced in the spindle

Let τ = Shear stress induced in the spindle.

We know that the torque transmitted by the spindle (T),

$$47\ 740 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times \tau\ (35)^3 = 8420\ \tau$$

$\therefore$ $\tau = 47\ 740 / 8420 = 5.67\ \text{N/mm}^2 = 5.67$ MPa **Ans.**

Problem: Compare the weight, strength and stiffness of a hollow shaft of the same external diameter as that of solid shaft. The inside diameter of the hollow shaft being half the external diameter. Both the shafts have the same material and length.

Solution. Given : $d_o = d$; $d_i = d_o / 2$ or $k = d_i / d_o = 1/2 = 0.5$

Comparison of weight

We know that weight of a hollow shaft,

$$W_H = \text{Cross-sectional area} \times \text{Length} \times \text{Density}$$

$$= \frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right] \times \text{Length} \times \text{Density} \quad ...(i)$$

and weight of the solid shaft,

$$W_S = \frac{\pi}{4} \times d^2 \times \text{Length} \times \text{Density} \quad ...(ii)$$

Since both the shafts have the same material and length, therefore by dividing equation (*i*) by equation (*ii*), we get

$$\frac{W_H}{W_S} = \frac{(d_o)^2 - (d_i)^2}{d^2} = \frac{(d_o)^2 - (d_i)^2}{(d_o)^2} \quad ...(\because d = d_o)$$

$$= 1 - \frac{(d_i)^2}{(d_o)_2} = 1 - k^2 = 1 - (0.5)^2 = 0.75 \textbf{ Ans.}$$

Comparison of strength

We know that strength of the hollow shaft,

$$T_H = \frac{\pi}{16} \times \tau\ (d_o)^3\ (1 - k^4) \quad ...(iii)$$

and strength of the solid shaft,

$$T_S = \frac{\pi}{16} \times \tau \times d^3 \quad ...(iv)$$

Dividing equation (*iii*) by equation (*iv*), we get

$$\frac{T_H}{T_S} = \frac{(d_o)^3\ (1 - k^4)}{d^3} = \frac{(d_o)^3\ (1 - k^4)}{(d_o)^3} = 1 - k^4 \quad ...(\because d = d_o)$$

$$= 1 - (0.5)^4 = 0.9375 \textbf{ Ans.}$$

Comparison of stiffness

We know that stiffness

$$= \frac{T}{\theta} = \frac{G \times J}{L}$$

∴ Stiffness of a hollow shaft,

$$S_H = \frac{G}{L} \times \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right] \quad ...(v)$$

and stiffness of a solid shaft,

$$S_S = \frac{G}{L} \times \frac{\pi}{32} \times d^4 \quad ...(vi)$$

Dividing equation (*v*) by equation (*vi*), we get

$$\frac{S_H}{S_S} = \frac{(d_o)^4 - (d_i)^4}{d^4} = \frac{(d_o)^4 - (d_i)^4}{(d_o)^4} = 1 - \frac{(d_i)^4}{(d_o)^4} \quad ...(\because d = d_o)$$

$$= 1 - k^4 = 1 - (0.5)^4 = 0.9375$$ **Ans.**

Design of Keys

Function of Keys:-

A key is a piece of mild steel inserted between the shaft and hub or boss of the pulley to connect these together in order to prevent relative motion between them. It is always inserted parallel to the axis of the shaft. Keys are used as temporary fastenings and are subjected to considerable crushing and shearing stresses. A keyway is a slot or recess in a shaft and hub of the pulley to accommodate a key.

Types of Keys:-

The keys are classified into the following types:

1. Sunk keys, **2.** Saddle keys, **3.** Tangent keys, **4.** Round keys and **5.** Splines.

Sunk Keys

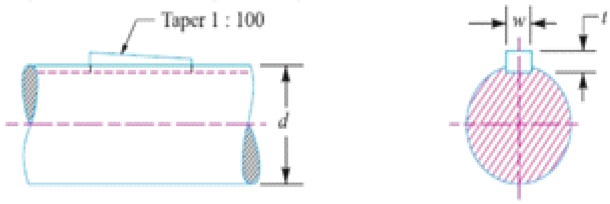
The sunk keys are provided half in the keyway of the shaft and half in the keyway of the hub or boss of the pulley. The sunk keys are of the following types :

1. ***Rectangular sunk key.*** A rectangular sunk key is shown in Fig. 13.1. The usual proportions of this key are :

Width of key, $w = d / 4$; and thickness of key, $t = 2w / 3 = d / 6$

where d = Diameter of the shaft or diameter of the hole in the hub.

The key has taper 1 in 100 on the top side only.



2. *Square sunk key.* The only difference between a rectangular sunk key and a square sunk key is that its width and thickness are equal, *i.e.*

$$w = t = d / 4$$

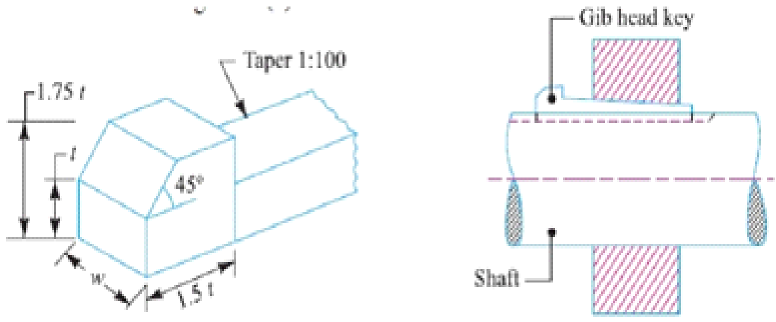
3. *Parallel sunk key.* The parallel sunk keys may be of rectangular or square section uniform in width and thickness throughout. It may be noted that a parallel key is a taperless and is used where the pulley, gear or other mating piece is required to slide along the shaft.

4. *Gib-head key.* It is a rectangular sunk key with a head at one end known as ***gib head.*** It is usually provided to facilitate the removal of key. A gib head key is shown in Fig. 1(*a*) and its use in shown in Fig. 1(*b*).

The usual proportions of the gib head key are :

Width, $w = d / 4$;

and thickness at large end, $t = 2w / 3 = d / 6$

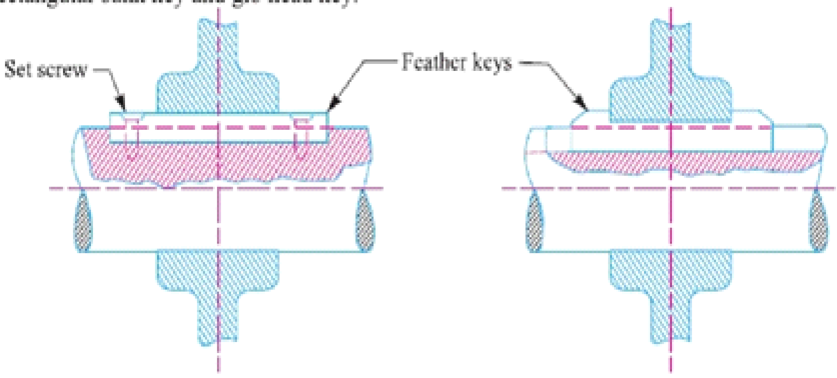


(a) (b)

Gib-head key

5. *Feather key.* A key attached to one member of a pair and which permits relative axial-movement is known as ***feather key.*** It is a special type of parallel key which transmits a turning moment and also permits axial movement. It is fastened either to the shaft or hub, the key being a sliding fit in the key way of the moving piece.

The feather key may be screwed to the shaft as shown in Fig. 2 (*a*) or it may have double gib heads as shown in Fig..2 (*b*). The various proportions of a feather key are same as that of rectangular sunk key and gib head key.



(*a*) (*b*)

6. *Woodruff key.* The woodruff key is an easily adjustable key. It is a piece from a cylindrical disc having segmental cross-section in front view as shown in Fig. 13.4. A woodruff key is capable of tilting in a recess milled out in the shaft by a cutter having the same curvature as the disc from which the key is made. This key is largely used in machine tool and automobile construction.

The main advantages of a woodruff key are as follows :

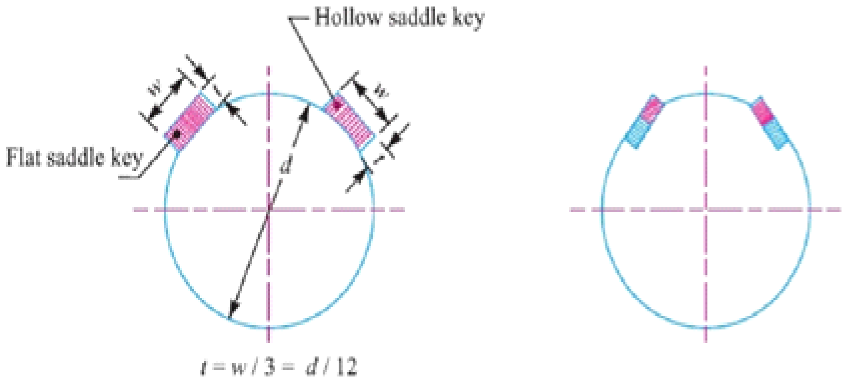
1. It accommodates itself to any taper in the hub or boss of the mating piece.

2. It is useful on tapering shaft ends. Its extra depth in the shaft *prevents any tendency to turn over in its keyway.

The disadvantages are :

1. The depth of the keyway weakens the shaft.

2. It can not be used as a feather.

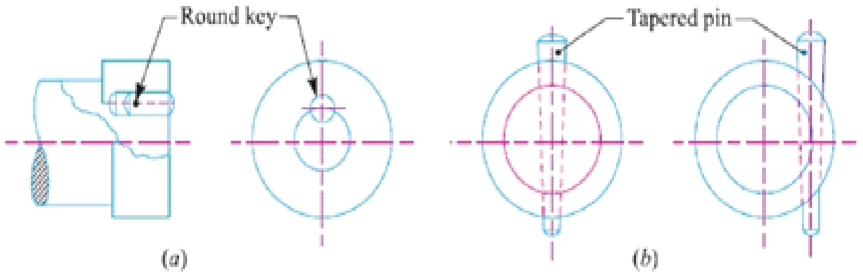


Saddle key. Tangent key.

Round Keys:-

The round keys, as shown in Fig. 5(*a*), are circular in section and fit into holes drilled partly in the shaft and partly in the hub. They have the advantage that their keyways may be drilled and reamed after the mating parts have been assembled. Round keys are usually considered to be most appropriate for low power drives.

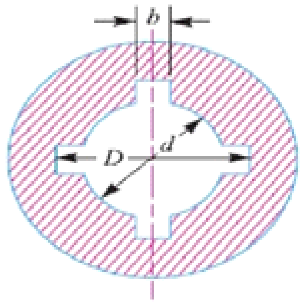
Sometimes the tapered pin, as shown in Fig. 5 (*b*), is held in place by the friction between the pin and the reamed tapered holes.



(*a*) (*b*)

Splines:-

Sometimes, keys are made integral with the shaft which fits in the keyways broached in the hub. Such shafts are known as ***splined shafts*** as shown in Fig. 6. These shafts usually have four, six, ten or sixteen splines. The splined shafts are relatively stronger than shafts having a Single keyway.
The splined shafts are used when the force to be transmitted is large in proportion to the size of the shaft as in automobile transmission and sliding gear transmissions. By using splined shafts, we obtain axial movement as well as positive drive is obtained.



$D = 1.25\ d$ and $b = 0.25\ D$

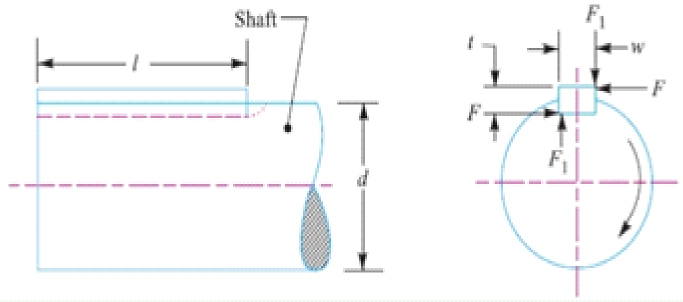
Forces acting on a Sunk Key:-

When a key is used in transmitting torque from a shaft to a rotor or hub, the following two types of forces act on the key :

1. Forces (F_1) due to fit of the key in its keyway, as in a tight fitting straight key or in a tapered key driven in place. These forces produce compressive stresses in the key which are difficult to determine in magnitude.

2. Forces (F) due to the torque transmitted by the shaft. These forces produce shearing and Compressive (or crushing) stresses in the key.

The distribution of the forces along the length of the key is not uniform because the forces are concentrated near the torque-input end. The non-uniformity of distribution is caused by the twisting of the shaft within the hub.
The forces acting on a key for a clockwise torque being transmitted from a shaft to a hub are shown in Fig. 7
In designing a key, forces due to fit of the key are neglected and it is assumed that the distribution of forces along the length of key is uniform.

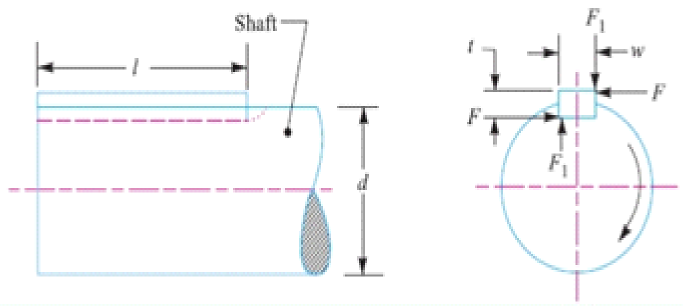


Strength of a Sunk Key:-

A key connecting the shaft and hub is shown in Fig.

Let T = Torque transmitted by the shaft,
F = Tangential force acting at the circumference of the shaft
d = Diameter of shaft,
l = Length of key,

w = Width of key.
t = Thickness of key, and



τ and σ_c = Shear and crushing stresses for the material of key.

A little consideration will show that due to the power transmitted by the shaft, the key may fail due to shearing or crushing.

Considering shearing of the key, the tangential shearing force acting at the circumference of the shaft,

$$F = \text{Area resisting shearing} \times \text{Shear stress} = l \times w \times \tau$$

∴ Torque transmitted by the shaft,

$$T = F \times \frac{d}{2} = l \times w \times \tau \times \frac{d}{2} \quad ...(i)$$

Considering crushing of the key, the tangential crushing force acting at the circumference of the shaft,

$$F = \text{Area resisting crushing} \times \text{Crushing stress} = l \times \frac{t}{2} \times \sigma_c$$

∴ Torque transmitted by the shaft,

$$T = F \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \quad ...(ii)$$

The key is equally strong in shearing and crushing, if

$$l \times w \times \tau \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \quad \text{...[Equating equations (i) and (ii)]}$$

or

$$\frac{w}{t} = \frac{\sigma_c}{2\tau} \quad ...(iii)$$

The permissible crushing stress for the usual key material is at least twice the permissible shearing stress. Therefore from equation ***(iii)***, we have $w = t$. In other words, a square key is equally strong in shearing and crushing.

In order to find the length of the key to transmit full power of the shaft, the shearing strength of the key is equal to the torsional shear strength of the shaft.

We know that the shearing strength of key,

$$T = l \times w \times \tau \times \frac{d}{2} \quad \text{...(iv)}$$

and torsional shear strength of the shaft,

$$T = \frac{\pi}{16} \times \tau_1 \times d^3 \quad \text{...(v)}$$

...(Taking τ_1 = Shear stress for the shaft material)

From equations *(iv)* and *(v)*, we have

$$l \times w \times \tau \times \frac{d}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$\therefore \quad l = \frac{\pi}{8} \times \frac{\tau_1 d^2}{w \times \tau} = \frac{\pi d}{2} \times \frac{\tau_1}{\tau} = 1.571\, d \times \frac{\tau_1}{\tau} \quad \text{... (Taking } w = d/4) \quad \text{...(vi)}$$

When the key material is same as that of the shaft, then $\tau = \tau_1$.

$$\therefore \quad l = 1.571\, d \quad \text{... [From equation (vi)]}$$

Problem: Design the rectangular key for a shaft of 50 mm diameter. The shearing and crushing stresses for the key material are 42 MPa and 70 MPa.

Solution. Given : d = 50 mm ; τ = 42 MPa = 42 N/mm^2 ; σ_c = 70 MPa = 70 N/mm^2

The rectangular key is designed as discussed below:

From Table 13.1, we find that for a shaft of 50 mm diameter,

Width of key, w = 16 mm **Ans.**

and thickness of key, t = 10 mm **Ans.**

The length of key is obtained by considering the key in shearing and crushing.

Let l = Length of key.

Considering shearing of the key. We know that shearing strength (or torque transmitted) of the key,

$$T = l \times w \times \tau \times \frac{d}{2} = l \times 16 \times 42 \times \frac{50}{2} = 16\,800\, l \text{ N-mm} \quad \text{...(i)}$$

and torsional shearing strength (or torque transmitted) of the shaft,

$$T = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42\,(50)^3 = 1.03 \times 10^6 \text{ N-mm} \quad \text{...(ii)}$$

From equations *(i)* and *(ii)*, we have

$$l = 1.03 \times 10^6 / 16\,800 = 61.31 \text{ mm}$$

Now considering crushing of the key. We know that shearing strength (or torque transmitted) of the key,

$$T = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} = l \times \frac{10}{2} \times 70 \times \frac{50}{2} = 8750\, l \text{ N-mm} \quad \text{...(iii)}$$

From equations *(ii)* and *(iii)*, we have

$$l = 1.03 \times 10^6 / 8750 = 117.7 \text{ mm}$$

Taking larger of the two values, we have length of key,

l = 117.7 say 120 mm **Ans.**

Problem: A 45 mm diameter shaft is made of steel with a yield strength of 400 MPa. A parallel key of size 14 mm wide and 9 mm thick made of steel with a yield strength of 340 MPa is to be used. Find the required length of key, if the shaft is loaded to transmit the maximum permissible torque. Use maximum shear stress theory and assume a factor of safety of 2.

Solution. Given : d = 45 mm ; σ_{yt} for shaft = 400 MPa = 400 N/mm^2 ; w = 14 mm ; t = 9 mm ; σ_{yt} for key = 340 MPa = 340 N/mm^2; $F.S.$ = 2

Let l = Length of key.

According to maximum shear stress theory (See Art. 5.10), the maximum shear stress for the shaft,

$$\tau_{max} = \frac{\sigma_{yt}}{2 \times F.S.} = \frac{400}{2 \times 2} = 100 \text{ N/mm}^2$$

and maximum shear stress for the key,

$$\tau_k = \frac{\sigma_{yt}}{2 \times F.S.} = \frac{340}{2 \times 2} = 85 \text{ N/mm}^2$$

We know that the maximum torque transmitted by the shaft and key,

$$T = \frac{\pi}{16} \times \tau_{max} \times d^3 = \frac{\pi}{16} \times 100\,(45)^3 = 1.8 \times 10^6 \text{ N-mm}$$

First of all, let us consider the failure of key due to shearing. We know that the maximum torque transmitted (T),

$$1.8 \times 10^6 = l \times w \times \tau_k \times \frac{d}{2} = l \times 14 \times 85 \times \frac{45}{2} = 26\,775\, l$$

$\therefore$ $l = 1.8 \times 10^6 / 26\,775 = 67.2$ mm

Now considering the failure of key due to crushing. We know that the maximum torque transmitted by the shaft and key (T),

$$1.8 \times 10^6 = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = l \times \frac{9}{2} \times \frac{340}{2} \times \frac{45}{2} = 17\,213\, l$$

$$\ldots\left(\text{Taking } \sigma_{ck} = \frac{\sigma_{yt}}{F.S.}\right)$$

$\therefore$ $l = 1.8 \times 10^6 / 17\,213 = 104.6$ mm

Taking the larger of the two values, we have

l = 104.6 say 105 mm **Ans.**

Effect of Keyways:-

A little consideration will show that the keyway cut into the shaft reduces the load carrying capacity of the shaft. This is due to the stress concentration near the corners of the keyway and reduction in the cross-sectional area of the shaft. It other words, the torsional strength of the shaft is reduced. The following relation for the weakening effect of the keyway is based on the experimental results by H.F. Moore.

$$e = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{h}{d}\right)$$

where e = Shaft strength factor. It is the ratio of the strength of the shaft with keyway to the strength of the same shaft without keyway,

w = Width of keyway,

d = Diameter of shaft, and

$$h = \text{Depth of keyway} = \frac{\text{Thickness of key } (t)}{2}$$

It is usually assumed that the strength of the keyed shaft is 75% of the solid shaft, which is somewhat higher than the value obtained by the above relation.

In case the keyway is too long and the key is of sliding type, then the angle of twist is increased in the ratio k_θ as given by the following relation :

$$k_\theta = 1 + 0.4\left(\frac{w}{d}\right) + 0.7\left(\frac{h}{d}\right)$$

where k_θ = Reduction factor for angular twist.

Problem: A 15 kW, 960 r.p.m. motor has a mild steel shaft of 40 mm diameter and the extension being 75 mm. The permissible shear and crushing stresses for the mild steel key are 56 MPa and 112 MPa. Design the keyway in the motor shaft extension. Check the shear strength of the key against the normal strength of the shaft.

Solution. Given : $P = 15$ kW $= 15 \times 10^3$ W ; $N = 960$ r.p.m. ; $d = 40$ mm ; $l = 75$ mm ; $\tau = 56$ MPa $= 56$ N/mm^2 ; $\sigma_c = 112$ MPa $= 112$ N/mm^2

We know that the torque transmitted by the motor,

$$T = \frac{P \times 60}{2\pi N} = \frac{15 \times 10^3 \times 60}{2\pi \times 960} = 149 \text{ N-m} = 149 \times 10^3 \text{ N-mm}$$

Let w = Width of keyway or key.

Considering the key in shearing. We know that the torque transmitted (T),

$$149 \times 10^3 = l \times w \times \tau \times \frac{d}{2} = 75 \times w \times 56 \times \frac{40}{2} = 84 \times 10^3\, w$$

$\therefore$ $w = 149 \times 10^3 / 84 \times 10^3 = 1.8$ mm

This width of keyway is too small. The width of keyway should be at least $d/4$.

$\therefore$ $w = \dfrac{d}{4} = \dfrac{40}{4} = 10$ mm **Ans.**

Since $\sigma_c = 2\tau$, therefore a square key of $w = 10$ mm and $t = 10$ mm is adopted.

According to H.F. Moore, the shaft strength factor,

$$e = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{h}{d}\right) = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{t}{2d}\right) \quad \ldots (\because h = t/2)$$

$$= 1 - 0.2\left(\frac{10}{20}\right) - \left(\frac{10}{2 \times 40}\right) = 0.8125$$

$\therefore$ Strength of the shaft with keyway,

$$= \frac{\pi}{16} \times \tau \times d^3 \times e = \frac{\pi}{16} \times 56\,(40)^3\; 0.8125 = 571\,844 \text{ N}$$

and shear strength of the key

$$= l \times w \times \tau \times \frac{d}{2} = 75 \times 10 \times 56 \times \frac{40}{2} = 840\,000 \text{ N}$$

$$\therefore \frac{\text{Shear strength of the key}}{\text{Normal strength of the shaft}} = \frac{840\,000}{571\,844} = 1.47 \;\textbf{Ans.}$$

DESIGN OF MACHINE ELEMENTS

CHAPTER-4

Design of Coupling

Prepared by :

Er. Ramesh Chandra Pradhan

Lecturer in

MECHANICAL DEPARTMENT

PNS School of Engineering & Technology

Nishamani Vihar, Marshaghai, Kendrapara

Design of Coupling

Shaft Coupling:-

Shafts are usually available up to 7 metres length due to inconvenience in transport. In order to have a greater length, it becomes necessary to join two or more pieces of the shaft by means of a coupling.
Shaft couplings are used in machinery for several purposes, the most common of which are the Following:

1. To provide for the connection of shafts of units that are manufactured separately such as a motor and generator and to provide for disconnection for repairs or alternations.
2. To provide for misalignment of the shafts or to introduce mechanical flexibility.
3. To reduce the transmission of shock loads from one shaft to another.
4. To introduce protection against overloads.
5. It should have no projecting parts

Couplings

Note : A coupling is termed as a device used to make permanent or semi-permanent connection where as a clutch permits rapid connection or disconnection at the will of the operator.

Requirements of a Good Shaft Coupling:-

A good shaft coupling should have the following requirements:

1. It should be easy to connect or disconnect.
2. It should transmit the full power from one shaft to the other shaft without losses.
3. It should hold the shafts in perfect alignment.
4. It should reduce the transmission of shock loads from one shaft to another shaft.
5. It should have no projecting parts.

Types of Shafts Couplings:-

Shaft couplings are divided into two main groups as follows :

1. ***Rigid coupling.*** It is used to connect two shafts which are perfectly aligned. Following types of rigid coupling are important from the subject point of view:

(*a*) Sleeve or muff coupling.
(*b*) Clamp or split-muff or compression coupling, and
(*c*) Flange coupling.

2. ***Flexible coupling.*** It is used to connect two shafts having both lateral and angular misalignment. Following types of flexible coupling are important from the subject point of view :
(*a*) Bushed pin type coupling,

(*b*) Universal coupling, and
(*c*) Oldham coupling.

Sleeve or Muff-coupling:-

It is the simplest type of rigid coupling, made of cast iron. It consists of a hollow cylinder whose inner diameter is the same as that of the shaft. It is fitted over the ends of the two shafts by means of a gib head key, as shown in Fig. 13.10. The power is transmitted from one shaft to the other shaft by means of a key and a sleeve. It is, therefore, necessary that all the elements must be strong enough to transmit the torque. The usual proportions of a cast iron sleeve coupling are as follows :

Outer diameter of the sleeve, $D = 2d + 13$ mm

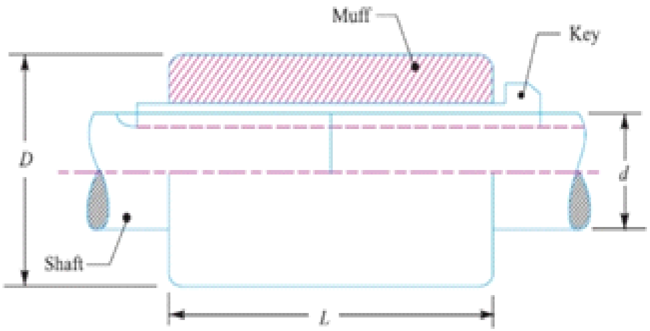
and length of the sleeve, $L = 3.5\,d$

where d is the diameter of the shaft.

In designing a sleeve or muff-coupling, the following procedure may be adopted.

1. *Design for sleeve*

The sleeve is designed by considering it as a hollow shaft.



Let T = Torque to be transmitted by the coupling, and

τ_c = Permissible shear stress for the material of the sleeve which is cast rion. The safe value of shear stress for cast iron may be taken as 14 MPa.

We know that torque transmitted by a hollow section,

$$T = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D}\right) = \frac{\pi}{16} \times \tau_c \times D^3\,(1 - k^4) \qquad \text{... } (\because k = d/D)$$

From this expression, the induced shear stress in the sleeve may be checked.

2. *Design for key*

The key for the coupling may be designed in the similar way as discussed in Art. 13.9. The width and thickness of the coupling key is obtained from the proportions.

The length of the coupling key is atleast equal to the length of the sleeve (*i.e.* 3.5 *d*). The coupling key is usually made into two parts so that the length of the key in each shaft,

$$l = \frac{L}{2} = \frac{3.5\,d}{2}$$

After fixing the length of key in each shaft, the induced shearing and crushing stresses may be checked. We know that torque transmitted,

$$T = l \times w \times \tau \times \frac{d}{2} \qquad \text{... (Considering shearing of the key)}$$

$$= l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \qquad \text{... (Considering crushing of the key)}$$

Note: The depth of the keyway in each of the shafts to be connected should be exactly the same and the diameters should also be same. If these conditions are not satisfied, then the key will be bedded on one shaft while in the other it will be loose. In order to prevent this, the key is made in two parts which may be driven from the same end for each shaft or they may be driven from opposite ends.

Example 13.4. *Design and make a neat dimensioned sketch of a muff coupling which is used to connect two steel shafts transmitting 40 kW at 350 r.p.m. The material for the shafts and key is plain carbon steel for which allowable shear and crushing stresses may be taken as 40 MPa and 80 MPa respectively. The material for the muff is cast iron for which the allowable shear stress may be assumed as 15 MPa.*

Solution. Given : P = 40 kW = 40 × 10^3 W; N = 350 r.p.m.; τ_s = 40 MPa = 40 N/mm²; σ_{cs} = 80 MPa = 80 N/mm²; τ_c = 15 MPa = 15 N/mm²

The muff coupling is shown in Fig. 13.10. It is designed as discussed below :

A type of muff couplings.

Note : This picture is given as additional information and is not a direct example of the current chapter.

1. *Design for shaft*

Let d = Diameter of the shaft.

We know that the torque transmitted by the shaft, key and muff,

$$T = \frac{P \times 60}{2\,\pi N} = \frac{40 \times 10^3 \times 60}{2\,\pi \times 350} = 1100 \text{ N-m}$$

$$= 1100 \times 10^3 \text{ N-mm}$$

We also know that the torque transmitted (T),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$\therefore$ d^3 = 1100 × 10^3/7.86 = 140 × 10^3 or d = 52 say 55 mm **Ans.**

2. *Design for sleeve*

We know that outer diameter of the muff,

$$D = 2d + 13 \text{ mm} = 2 \times 55 + 13 = 123 \text{ say } 125 \text{ mm} \textbf{ Ans.}$$

and length of the muff,

$$L = 3.5\, d = 3.5 \times 55 = 192.5 \text{ say } 195 \text{ mm} \textbf{ Ans.}$$

Let us now check the induced shear stress in the muff. Let τ_c be the induced shear stress in the muff which is made of cast iron. Since the muff is considered to be a hollow shaft, therefore the torque transmitted (T),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D}\right) = \frac{\pi}{16} \times \tau_c \left[\frac{(125)^4 - (55)^4}{125}\right]$$

$$= 370 \times 10^3\, \tau_c$$

$\therefore$ τ_c = 1100 × 10^3/370 × 10^3 = 2.97 N/mm²

Since the induced shear stress in the muff (cast iron) is less than the permissible shear stress of 15 N/mm², therefore the design of muff is safe.

3. *Design for key*

From Table 13.1, we find that for a shaft of 55 mm diameter,

Width of key, w = 18 mm **Ans.**

Since the crushing stress for the key material is twice the shearing stress, therefore a square key may be used.

$\therefore$ Thickness of key, $t = w$ = 18 mm **Ans.**

We know that length of key in each shaft,

$$l = L / 2 = 195 / 2 = 97.5 \text{ mm} \textbf{ Ans.}$$

Let us now check the induced shear and crushing stresses in the key. First of all, let us consider shearing of the key. We know that torque transmitted (T),

$$1100 \times 10^3 = l \times w \times \tau_s \times \frac{d}{2} = 97.5 \times 18 \times \tau_s \times \frac{55}{2} = 48.2 \times 10^3 \tau_s$$

$$\therefore \quad \tau_s = 1100 \times 10^3 / 48.2 \times 10^3 = 22.8 \text{ N/mm}^2$$

Now considering crushing of the key. We know that torque transmitted (T),

$$1100 \times 10^3 = l \times \frac{t}{2} \times \sigma_{cs} \times \frac{d}{2} = 97.5 \times \frac{18}{2} \times \sigma_{cs} \times \frac{55}{2} = 24.1 \times 10^3 \sigma_{cs}$$

$$\therefore \quad \sigma_{cs} = 1100 \times 10^3 / 24.1 \times 10^3 = 45.6 \text{ N/mm}^2$$

Since the induced shear and crushing stressesare less than the permissible stresses, therefore the design of key is safe.

Clamp or Compression Coupling:-

It is also known as ***split muff coupling.*** In this case, the muff or sleeve is made into two halves and are bolted together as shown in Fig. 13.11. The halves of the muff are made of cast iron. The shaft ends are made to a butt each other and a single key is fitted directly in the keyways of both the shafts. One-half of the muff is fixed from below and the other half is placed from above. Both the halves are held together by means of mild steel studs or bolts and nuts. The number of bolts may be two, four or six. The nuts are recessed into the bodies of the muff castings. This coupling may be used for heavy duty and moderate speeds. The advantage of this coupling is that the position of the shafts need not be changed for assembling or disassembling of the coupling. The usual proportions of the muff for the clamp or compression coupling are:

Diameter of the muff or sleeve, $D = 2d + 13$ mm

Length of the muff or sleeve, $L = 3.5\,d$

where d = Diameter of the shaft.

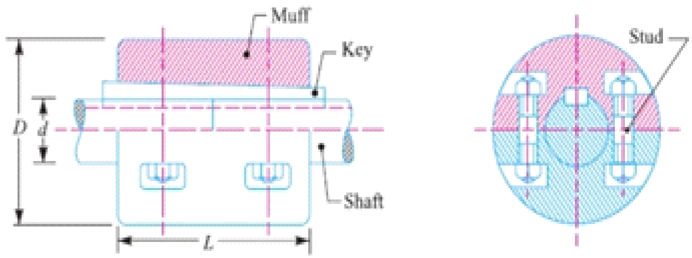


Fig. 13.11. Clamp or compression coupling.

In the clamp or compression coupling, the power is transmitted from one shaft to the other by means of key and the friction between the muff and shaft. In designing this type of coupling, the following procedure may be adopted.

1. *Design of muff and key*

The muff and key are designed in the similar way as discussed in muff coupling (Art. 13.14).

2. *Design of clamping bolts*

Let T = Torque transmited by the shaft,

d = Diameter of shaft,

d_b = Root or effective diameter of bolt,

n = Number of bolts,

σ_t = Permissible tensile stress for bolt material,
μ = Coefficient of friction between the muff and shaft, and
L = Length of muff.

We know that the force exerted by each bolt

$$= \frac{\pi}{4}(d_b)^2 \sigma_t$$

$\therefore$ Force exerted by the bolts on each side of the shaft

$$= \frac{\pi}{4}(d_b)^2 \sigma_t \times \frac{n}{2}$$

Let p be the pressure on the shaft and the muff surface due to the force, then for uniform pressure distribution over the surface,

$$p = \frac{\text{Force}}{\text{Projected area}} = \frac{\frac{\pi}{4}(d_b)^2 \sigma_t \times \frac{n}{2}}{\frac{1}{2} L \times d}$$

$\therefore$ Frictional force between each shaft and muff,

$$F = \mu \times \text{pressure} \times \text{area} = \mu \times p \times \frac{1}{2} \times \pi d \times L$$

$$= \mu \times \frac{\frac{\pi}{4}(d_b)^2 \sigma_t \times \frac{n}{2}}{\frac{1}{2} L \times d} \times \frac{1}{2}\pi d \times L$$

$$= \mu \times \frac{\pi}{4}(d_b)^2 \sigma_t \times \frac{n}{2} \times \pi = \mu \times \frac{\pi^2}{8}(d_b)^2 \sigma_t \times n$$

and the torque that can be transmitted by the coupling,

$$T = F \times \frac{d}{2} = \mu \times \frac{\pi^2}{8}(d_b)^2 \sigma_t \times n \times \frac{d}{2} = \frac{\pi^2}{16} \times \mu\,(d_b)^2 \sigma_t \times n \times d$$

From this relation, the root diameter of the bolt (d_b) may be evaluated.

Note: The value of μ may be taken as 0.3.

Example 13.5. *Design a clamp coupling to transmit 30 kW at 100 r.p.m. The allowable shear stress for the shaft and key is 40 MPa and the number of bolts connecting the two halves are six. The permissible tensile stress for the bolts is 70 MPa. The coefficient of friction between the muff and the shaft surface may be taken as 0.3.*

Solution. Given : P = 30 kW = 30 × 10³ W ; N = 100 r.p.m. ; τ = 40 MPa = 40 N/mm² ; n = 6 ; σ_t = 70 MPa = 70 N/mm² ; μ = 0.3

1. *Design for shaft*

Let d = Diameter of shaft.

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{30 \times 10^3 \times 60}{2\pi \times 100} = 2865 \text{ N-m} = 2865 \times 10^3 \text{ N-mm}$$

We also know that the torque transmitted by the shaft (T),

$$2865 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$\therefore$ $d^3 = 2865 \times 10^3 / 7.86 = 365 \times 10^3$ or $d = 71.4$ say 75 mm **Ans.**

2. *Design for muff*

We know that diameter of muff,

$$D = 2d + 13 \text{ mm} = 2 \times 75 + 13 = 163 \text{ say } 165 \text{ mm } \textbf{Ans.}$$

and total length of the muff,

$$L = 3.5\, d = 3.5 \times 75 = 262.5 \text{ mm } \textbf{Ans.}$$

3. *Design for key*

The width and thickness of the key for a shaft diameter of 75 mm (from Table 13.1) are as follows :

Width of key, w = 22 mm **Ans.**

Thickness of key, t = 14 mm **Ans.**

and length of key = Total length of muff = 262.5 mm **Ans.**

4. *Design for bolts*

Let d_b = Root or core diameter of bolt.

We know that the torque transmitted (T),

$$2865 \times 10^3 = \frac{\pi^2}{16} \times \mu (d_b)^2 \sigma_t \times n \times d = \frac{\pi^2}{16} \times 0.3\, (d_b)^2\ 70 \times 6 \times 75 = 5830 (d_b)^2$$

$$\therefore \quad (d_b)^2 = 2865 \times 10^3 / 5830 = 492 \quad \text{or} \quad d_b = 22.2 \text{ mm}$$

From Table 11.1, we find that the standard core diameter of the bolt for coarse series is 23.32 mm and the nominal diameter of the bolt is 27 mm (M 27). **Ans.**

13.16 Flange Coupling

A flange coupling usually applies to a coupling having two separate cast iron flanges. Each flange is mounted on the shaft end and keyed to it. The faces are turned up at right angle to the axis of the shaft. One of the flange has a projected portion and the other flange has a corresponding recess.

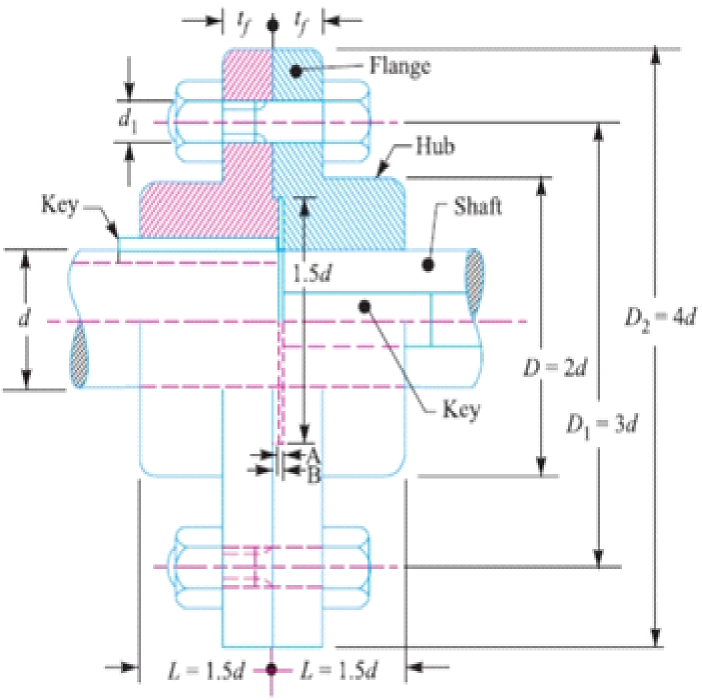


Fig. 13.12. Unprotected type flange coupling.

This helps to bring the shafts into line and to maintain alignment. The two flanges are coupled together by means of bolts and nuts. The flange coupling is adopted to heavy loads and hence it is used on large shafting.

The flange couplings are of the following three types :

1. *Unprotected type flange coupling*:-

In an unprotected type flange coupling, as shown in Fig. 13.12, each shaft is keyed to the boss of a flange with a counter sunk key and the flanges are coupled together by means of bolts. Generally, three, four or six bolts are used. The keys are staggered at right angle along the circumference of the shafts in order to divide the weakening effect caused by keyways. The usual proportions for an unprotected type cast iron flange couplings, as shown in Fig. 13.12, are as follows : If d is the diameter of the shaft or inner diameter of the hub, then Outside diameter of hub,

$$D = 2\,d$$

Length of hub, $L = 1.5\,d$

Pitch circle diameter of bolts,

$$D_1 = 3d$$

Outside diameter of flange,

$$D_2 = D_1 + (D_1 - D) = 2\,D_1 - D = 4\,d$$

Thickness of flange, $t_f = 0.5\,d$

Number of bolts = 3, for d upto 40 mm

= 4, for d upto 100 mm

= 6, for d upto 180 mm

2. *Protected type flange coupling*. In a protected type flange coupling, as shown in Fig. 13.13, the protruding bolts and nuts are protected by flanges on the two halves of the coupling, in order to avoid danger to the workman.

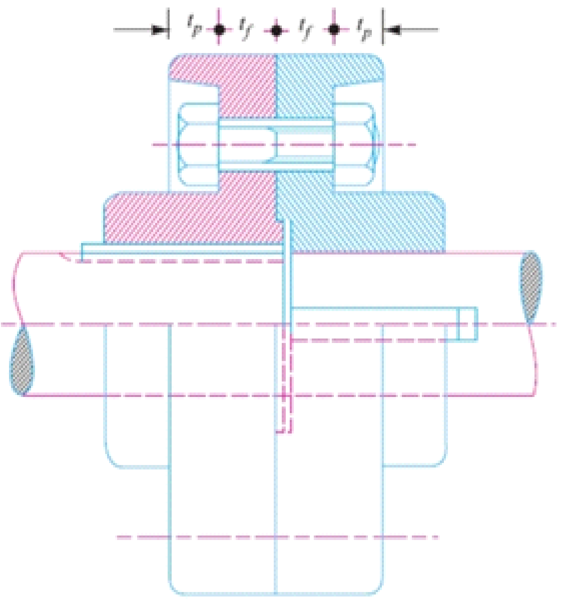


Fig. 13.13. Protective type flange coupling.

The thickness of the protective circumferential flange (t_p) is taken as 0.25 d. The other proportions of the coupling are same as for unprotected type flange coupling.

3. *Marine type flange coupling*. In a marine type flange coupling, the flanges are forged integral with the shafts as shown in Fig. 13.14. The flanges are held together by means of tapered headless bolts, numbering from four to twelve depending upon the diameter of shaft.

The number of bolts may be choosen from the following table.

Table 13.2. Number of bolts for marine type flange coupling. (According to IS : 3653 – 1966 (Reaffirmed 1990))

Shaft diameter (mm)	35 to 55	56 to 150	151 to 230	231 to 390	Above 390
No. of bolts	4	6	8	10	12

The other proportions for the marine type flange coupling are taken as follows :

Thickness of flange $= d / 3$

Taper of bolt = 1 in 20 to 1 in 40

Pitch circle diameter of bolts, $D_1 = 1.6\, d$

Outside diameter of flange, $D_2 = 2.2\, d$

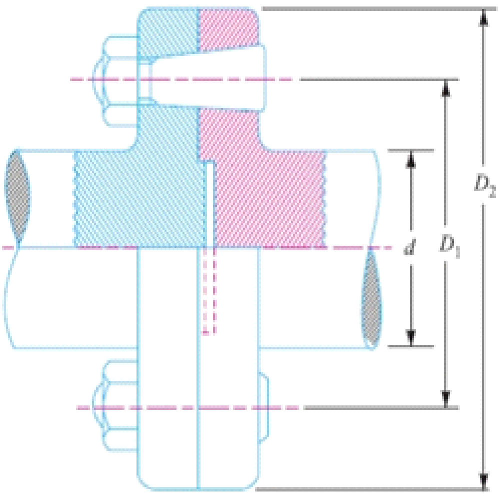


Fig. 13.14. Marine type flange coupling.

Design of Flange Coupling:-

Consider a flange coupling as shown in Fig. 13.12 and Fig. 13.13.

Let d = Diameter of shaft or inner diameter of hub,

D = Outer diameter of hub,

d_1 = Nominal or outside diameter of bolt,

D_1 = Diameter of bolt circle,

n = Number of bolts,

t_f = Thickness of flange,

τ_s, τ_b and τ_k = Allowable shear stress for shaft, bolt and key material respectively

τ_c = Allowable shear stress for the flange material *i.e.* cast iron,

σ_{cb}, and σ_{ck} = Allowable crushing stress for bolt and key material respectively.

The flange coupling is designed as discussed below :

1. *Design for hub*

The hub is designed by considering it as a hollow shaft, transmitting the same torque (T) as that of a solid shaft.

$$\therefore \quad T = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D} \right)$$

The outer diameter of hub is usually taken as twice the diameter of shaft. Therefore from the above relation, the induced shearing stress in the hub may be checked.

The length of hub (L) is taken as 1.5 d.

2. *Design for key*

The key is designed with usual proportions and then checked for shearing and crushing stresses.

The material of key is usually the same as that of shaft. The length of key is taken equal to the length of hub.

3. *Design for flange*

The flange at the junction of the hub is under shear while transmitting the torque. Therefore, the troque transmitted,

T = Circumference of hub × Thickness of flange × Shear stress of flange × Radius of hub

$$= \pi D \times t_f \times \tau_c \times \frac{D}{2} = \frac{\pi D^2}{2} \times \tau_c \times t_f$$

The thickness of flange is usually taken as half the diameter of shaft. Therefore from the above relation, the induced shearing stress in the flange may be checked.

4. *Design for bolts*

The bolts are subjected to shear stress due to the torque transmitted. The number of bolts (n) depends upon the diameter of shaft and the pitch circle diameter of bolts (D_1) is taken as 3 d. We know that

$$\text{Load on each bolt} = \frac{\pi}{4} (d_1)^2 \; \tau_b$$

$\therefore$ Total load on all the bolts

$$= \frac{\pi}{4} (d_1)^2 \; \tau_b \times n$$

and torque transmitted,

$$T = \frac{\pi}{4} (d_1)^2 \; \tau_b \times n \times \frac{D_1}{2}$$

From this equation, the diameter of bolt (d_1) may be obtained. Now the diameter of bolt may be checked in crushing.

We know that area resisting crushing of all the bolts

$$= n \times d_1 \times t_f$$

and crushing strength of all the bolts

$$= (n \times d_1 \times t_f) \, \sigma_{cb}$$

$$\therefore \text{Torque,} \quad T = (n \times d_1 \times t_f \times \sigma_{cb}) \frac{D_1}{2}$$

From this equation, the induced crushing stress in the bolts may be checked.

Example 13.6. *Design a cast iron protective type flange coupling to transmit 15 kW at 900 r.p.m. from an electric motor to a compressor. The service factor may be assumed as 1.35. The following permissible stresses may be used :*

Shear stress for shaft, bolt and key material	=	*40 MPa*
Crushing stress for bolt and key	=	*80 MPa*
Shear stress for cast iron	=	*8 MPa*

Draw a neat sketch of the coupling.

Solution. Given : P = 15 kW = 15 × 10^3 W ; N = 900 r.p.m. ; Service factor = 1.35 ; $\tau_s = \tau_b = \tau_k$ = 40 MPa = 40 N/mm^2 ; $\sigma_{cb} = \sigma_{ck}$ = 80 MPa = 80 N/mm^2 ; τ_c = 8 MPa = 8 N/mm^2

The protective type flange coupling is designed as discussed below :

1. *Design for hub*

First of all, let us find the diameter of the shaft (d). We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\,\pi N} = \frac{15 \times 10^3 \times 60}{2\,\pi \times 900} = 159.13 \text{ N-m}$$

Since the service factor is 1.35, therefore the maximum torque transmitted by the shaft,

$$T_{max} = 1.35 \times 159.13 = 215 \text{ N-m} = 215 \times 10^3 \text{ N-mm}$$

We know that the torque transmitted by the shaft (T),

$$215 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$\therefore$ $d^3 = 215 \times 10^3 / 7.86 = 27.4 \times 10^3$ or d = 30.1 say 35 mm **Ans.**

We know that outer diameter of the hub,

$$D = 2d = 2 \times 35 = 70 \text{ mm } \textbf{Ans.}$$

and length of hub, $L = 1.5\,d = 1.5 \times 35 = 52.5$ mm **Ans.**

Let us now check the induced shear stress for the hub material which is cast iron. Considering the hub as a hollow shaft. We know that the maximum torque transmitted (T_{max}).

$$215 \times 10^3 = \frac{\pi}{16} \times \tau_c \left[\frac{D^4 - d^4}{D}\right] = \frac{\pi}{16} \times \tau_c \left[\frac{(70)^4 - (35)^4}{70}\right] = 63\,147\, \tau_c$$

$\therefore$ $\tau_c = 215 \times 10^3 / 63\,147 = 3.4$ N/mm^2 = 3.4 MPa

Since the induced shear stress for the hub material (*i.e.* cast iron) is less than the permissible value of 8 MPa, therefore the design of hub is safe.

2. *Design for key*

Since the crushing stress for the key material is twice its shear stress (*i.e.* $\sigma_{ck} = 2\tau_k$), therefore a square key may be used. From Table 13.1, we find that for a shaft of 35 mm diameter,

Width of key, w = 12 mm **Ans.**

and thickness of key, $t = w$ = 12 mm **Ans.**

The length of key (l) is taken equal to the length of hub.

$\therefore$ $l = L$ = 52.5 mm **Ans.**

Let us now check the induced stresses in the key by considering it in shearing and crushing.

Considering the key in shearing. We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = l \times w \times \tau_k \times \frac{d}{2} = 52.5 \times 12 \times \tau_k \times \frac{35}{2} = 11\,025\, \tau_k$$

$\therefore$ $\tau_k = 215 \times 10^3 / 11\,025 = 19.5$ N/mm^2 = 19.5 MPa

Considering the key in crushing. We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = 52.5 \times \frac{12}{2} \times \sigma_{ck} \times \frac{35}{2} = 5512.5\, \sigma_{ck}$$

$\therefore$ $\sigma_{ck} = 215 \times 10^3 / 5512.5 = 39$ N/mm^2 = 39 MPa

Since the induced shear and crushing stresses in the key are less than the permissible stresses, therefore the design for key is safe.

3. *Design for flange*

The thickness of flange (t_f) is taken as 0.5 *d*.

∴ $t_f = 0.5\, d = 0.5 \times 35 = 17.5$ mm **Ans.**

Let us now check the induced shearing stress in the flange by considering the flange at the junction of the hub in shear.

We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = \frac{\pi D^2}{2} \times \tau_c \times t_f = \frac{\pi (70)^2}{2} \times \tau_c \times 17.5 = 134\ 713\ \tau_c$$

∴ $\tau_c = 215 \times 10^3 / 134\ 713 = 1.6\ \text{N/mm}^2 = 1.6$ MPa

Since the induced shear stress in the flange is less than 8 MPa, therefore the design of flange is safe.

4. *Design for bolts*

Let d_1 = Nominal diameter of bolts.

Since the diameter of the shaft is 35 mm, therefore let us take the number of bolts,

$n = 3$

and pitch circle diameter of bolts,

$D_1 = 3d = 3 \times 35 = 105$ mm

The bolts are subjected to shear stress due to the torque transmitted. We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = \frac{\pi}{4} (d_1)^2\ \tau_b \times n \times \frac{D_1}{2} = \frac{\pi}{4} (d_1)^2\ 40 \times 3 \times \frac{105}{2} = 4950\, (d_1)^2$$

∴ $(d_1)^2 = 215 \times 10^3 / 4950 = 43.43$ or $d_1 = 6.6$ mm

Assuming coarse threads, the nearest standard size of bolt is M 8. **Ans.**

Other proportions of the flange are taken as follows :

Outer diameter of the flange,

$D_2 = 4\, d = 4 \times 35 = 140$ mm **Ans.**

Thickness of the protective circumferential flange,

$t_p = 0.25\, d = 0.25 \times 35 = 8.75$ say 10 mm **Ans.**

Example 13.7. *Design and draw a protective type of cast iron flange coupling for a steel shaft transmitting 15 kW at 200 r.p.m. and having an allowable shear stress of 40 MPa. The working stress in the bolts should not exceed 30 MPa. Assume that the same material is used for shaft and key and that the crushing stress is twice the value of its shear stress. The maximum torque is 25% greater than the full load torque. The shear stress for cast iron is 14 MPa.*

Solution. Given : $P = 15$ kW $= 15 \times 10^3$ W ; $N = 200$ r.p.m. ; $\tau_s = 40$ MPa $= 40\ \text{N/mm}^2$; $\tau_b = 30$ MPa $= 30\ \text{N/mm}^2$; $\sigma_{ck} = 2\tau_k$; $T_{max} = 1.25\ T_{mean}$; $\tau_c = 14$ MPa $= 14\ \text{N/mm}^2$

The protective type of cast iron flange coupling is designed as discussed below :

1. *Design for hub*

First of all, let us find the diameter of shaft (*d*). We know that the full load or mean torque transmitted by the shaft,

$$T_{mean} = \frac{P \times 60}{2\,\pi N} = \frac{15 \times 10^3 \times 60}{2\,\pi \times 200} = 716\ \text{N-m} = 716 \times 10^3\ \text{N-mm}$$

and maximum torque transmitted,

$$T_{max} = 1.25\ T_{mean} = 1.25 \times 716 \times 10^3 = 895 \times 10^3\ \text{N-mm}$$

We also know that maximum torque transmitted (T_{max}),

$$895 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$\therefore$ $d^3 = 895 \times 10^3 / 7.86 = 113\,868$ or $d = 48.4$ say 50 mm **Ans.**

We know that the outer diameter of the hub,

$D = 2\,d = 2 \times 50 = 100$ mm **Ans.**

and length of the hub, $L = 1.5\,d = 1.5 \times 50 = 75$ mm **Ans.**

Let us now check the induced shear stress for the hub material which is cast iron, by considering it as a hollow shaft. We know that the maximum torque transmitted (T_{max}),

$$895 \times 10^3 = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D}\right) = \frac{\pi}{16} \times \tau_c \left(\frac{(100)^4 - (50)^4}{100}\right) = 184\,100\, \tau_c$$

$\therefore$ $\tau_c = 895 \times 10^3 / 184\,100 = 4.86$ N/mm^2 = 4.86 MPa

Since the induced shear stress in the hub is less than the permissible value of 14 MPa, therefore the design for hub is safe.

2. *Design for key*

Since the crushing stress for the key material is twice its shear stress, therefore a square key may be used.

From Table 13.1, we find that for a 50 mm diameter shaft,

Width of key, $w = 16$ mm **Ans.**

and thickness of key, $t = w = 16$ mm **Ans.**

The length of key (l) is taken equal to the length of hub.

$\therefore$ $l = L = 75$ mm **Ans.**

Let us now check the induced stresses in the key by considering it in shearing and crushing. Considering the key in shearing. We know that the maximum torque transmitted (T_{max}),

$$895 \times 10^3 = l \times w \times \tau_k \times \frac{d}{2} = 75 \times 16 \times \tau_k \times \frac{50}{2} = 30 \times 10^3\, \tau_k$$

$\therefore$ $\tau_k = 895 \times 10^3 / 30 \times 10^3 = 29.8$ N/mm^2 = 29.8 MPa

Considering the key in crushing. We know that the maximum torque transmitted (T_{max}),

$$895 \times 10^3 = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = 75 \times \frac{16}{2} \times \sigma_{ck} \times \frac{50}{2} = 15 \times 10^3\, \sigma_{ck}$$

$\therefore$ $\sigma_{ck} = 895 \times 10^3 / 15 \times 10^3 = 59.6$ N/mm^2 = 59.6 MPa

Since the induced shear and crushing stresses in key are less than the permissible stresses, therefore the design for key is safe.

3. *Design for flange*

The thickness of the flange (t_f) is taken as $0.5\,d$.

$\therefore$ $t_f = 0.5 \times 50 = 25$ mm **Ans.**

Let us now check the induced shear stress in the flange, by considering the flange at the junction of the hub in shear. We know that the maximum torque transmitted (T_{max}),

$$895 \times 10^3 = \frac{\pi D^2}{2} \times \tau_c \times t_f = \frac{\pi\,(100)^2}{2} \times \tau_c \times 25 = 392\,750\, \tau_c$$

$\therefore$ $\tau_c = 895 \times 10^3 / 392\,750 = 2.5$ N/mm^2 = 2.5 MPa

Since the induced shear stress in the flange is less than the permissible value of 14 MPa, therefore the design for flange is safe.

4. Design for bolts

Let d_1 = Nominal diameter of bolts.

Since the diameter of shaft is 50 mm, therefore let us take the number of bolts,

$$n = 4$$

and pitch circle diameter of bolts,

$$D_1 = 3\,d = 3 \times 50 = 150 \text{ mm}$$

The bolts are subjected to shear stress due to the torque transmitted. We know that the maximum torque transmitted (T_{max}),

$$895 \times 10^3 = \frac{\pi}{4}(d_1)^2 \tau_b \times n \times \frac{D_1}{2} = \frac{\pi}{4}(d_1)^2\, 30 \times 4 \times \frac{150}{4} = 7070\,(d_1)^2$$

$\therefore$ $(d_1)^2 = 895 \times 10^3 / 7070 = 126.6$ or $d_1 = 11.25$ mm

Assuming coarse threads, the nearest standard diameter of the bolt is 12 mm (M 12). **Ans.**

Other proportions of the flange are taken as follows :

Outer diameter of the flange,

$$D_2 = 4\,d = 4 \times 50 = 200 \text{ mm } \textbf{Ans.}$$

Thickness of the protective circumferential flange,

$$t_p = 0.25\,d = 0.25 \times 50 = 12.5 \text{ mm } \textbf{Ans.}$$

Example 13.8. *Design and draw a cast iron flange coupling for a mild steel shaft transmitting 90 kW at 250 r.p.m. The allowable shear stress in the shaft is 40 MPa and the angle of twist is not to exceed 1° in a length of 20 diameters. The allowable shear stress in the coupling bolts is 30 MPa.*

Solution. Given : $P = 90$ kW $= 90 \times 10^3$ W ; $N = 250$ r.p.m. ; $\tau_s = 40$ MPa $= 40$ N/mm² ; $\theta = 1° = \pi / 180 = 0.0175$ rad ; $\tau_b = 30$ MPa $= 30$ N/mm²

First of all, let us find the diameter of the shaft (d). We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\,\pi\,N} = \frac{90 \times 10^3 \times 60}{2\,\pi \times 250} = 3440 \text{ N-m} = 3440 \times 10^3 \text{ N-mm}$$

Considering strength of the shaft, we know that

$$\frac{T}{J} = \frac{\tau_s}{d/2}$$

$$\frac{3440 \times 10^3}{\frac{\pi}{32} \times d^4} = \frac{40}{d/2} \quad \text{or} \quad \frac{35 \times 10^6}{d^4} = \frac{80}{d} \qquad \ldots \left(\because J = \frac{\pi}{32} \times d^4\right)$$

$\therefore$ $d^3 = 35 \times 10^6 / 80 = 0.438 \times 10^6$ or $d = 76$ mm

Considering rigidity of the shaft, we know that

$$\frac{T}{J} = \frac{C \times \theta}{l}$$

$$\frac{3440 \times 10^3}{\frac{\pi}{32} \times d^4} = \frac{84 \times 10^3 \times 0.0175}{20\,d} \quad \text{or} \quad \frac{35 \times 10^6}{d^4} = \frac{73.5}{d} \qquad \ldots (\text{Taking } C = 84 \text{ kN/mm}^2)$$

$\therefore$ $d^3 = 35 \times 10^6 / 73.5 = 0.476 \times 10^6$ or $d = 78$ mm

Taking the larger of the two values, we have

$$d = 78 \text{ say } 80 \text{ mm } \textbf{Ans.}$$

Let us now design the cast iron flange coupling of the protective type as discussed below :

1. *Design for hub*

We know that the outer diameter of hub,

$$D = 2d = 2 \times 80 = 160 \text{ mm } \textbf{Ans.}$$

and length of hub, $L = 1.5\, d = 1.5 \times 80 = 120$ mm **Ans.**

Let us now check the induced shear stress in the hub by considering it as a hollow shaft. The shear stress for the hub material (which is cast iron) is usually 14 MPa. We know that the torque transmitted (T),

$$3440 \times 10^3 = \frac{\pi}{16} \times \tau_c \left[\frac{D^4 - d^4}{D}\right] = \frac{\pi}{16} \times \tau_c \left[\frac{(160)^4 - (80)^4}{160}\right] = 754 \times 10^3\, \tau_c$$

$$\therefore \quad \tau_c = 3440 \times 10^3 / 754 \times 10^3 = 4.56 \text{ N/mm}^2 = 4.56 \text{ MPa}$$

Since the induced shear stress for the hub material is less than 14 MPa, therefore the design for hub is safe.

2. *Design for key*

From Table 13.1, we find that the proportions of key for a 80 mm diameter shaft are :

Width of key, $w = 25$ mm **Ans.**

and thickness of key, $t = 14$ mm **Ans.**

The length of key (l) is taken equal to the length of hub (L).

$$\therefore \quad l = L = 120 \text{ mm } \textbf{Ans.}$$

Assuming that the shaft and key are of the same material. Let us now check the induced shear stress in key. We know that the torque transmitted (T),

$$3440 \times 10^3 = l \times w \times \tau_k \times \frac{d}{2} = 120 \times 25 \times \tau_k \times \frac{80}{2} = 120 \times 10^3\, \tau_k$$

$$\tau_k = 3440 \times 10^3 / 120 \times 10^3 = 28.7 \text{ N/mm}^2 = 28.7 \text{ MPa}$$

Since the induced shear stress in the key is less than 40 MPa, therefore the design for key is safe.

3. *Design for flange*

The thickness of the flange (t_f) is taken as 0.5 d.

$$\therefore \quad t_f = 0.5\, d = 0.5 \times 80 = 40 \text{ mm } \textbf{Ans.}$$

Let us now check the induced shear stress in the cast iron flange by considering the flange at the junction of the hub under shear. We know that the torque transmitted (T),

$$3440 \times 10^3 = \frac{\pi D^2}{2} \times t_f \times \tau_c = \frac{\pi (160)^2}{2} \times 40 \times \tau_c = 1608 \times 10^3\, \tau_c$$

$$\therefore \quad \tau_c = 3440 \times 10^3 / 1608 \times 10^3 = 2.14 \text{ N/mm}^2 = 2.14 \text{ MPa}$$

Since the induced shear stress in the flange is less than 14 MPa, therefore the design for flange is safe.

4. *Design for bolts*

Let d_1 = Nominal diameter of bolts.

Since the diameter of the shaft is 80 mm, therefore let us take number of bolts,

$$n = 4$$

and pitch circle diameter of bolts,

$$D_1 = 3\, d = 3 \times 80 = 240 \text{ mm}$$

The bolts are subjected to shear stress due to the torque transmitted. We know that torque transmitted (T),

$$3440 \times 10^3 = \frac{\pi}{4} (d_1)^2\, n \times \tau_b \times \frac{D_1}{2} = \frac{\pi}{4} (d_1)^2 \times 4 \times 30 \times \frac{240}{2} = 11\,311\, (d_1)^2$$

$$\therefore \quad (d_1)^2 = 3440 \times 10^3 / 11\,311 = 304 \quad \text{or } d_1 = 17.4 \text{ mm}$$

Assuming coarse threads, the standard nominal diameter of bolt is 18 mm. **Ans.**

The other proportions are taken as follows :

Outer diameter of the flange,

$$D_2 = 4\,d = 4 \times 80 = 320 \text{ mm } \textbf{Ans.}$$

Thickness of protective circumferential flange,

$$t_p = 0.25\,d = 0.25 \times 80 = 20 \text{ mm } \textbf{Ans.}$$

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CH-4

Design of Coupling

Short Questions & Answers

Q.1. What do you mean by Coupling ?

Ans. A coupling is termed as a device used to make permanent or semi-permanent connection, where as a clutch permit rapid connection or disconnection at will of the operator.

Q.2. What is Shaft Coupling ?

Ans.
- ✷ To provide for the connection of shafts that are manufactured separately such as motor and generator.
- ✷ To provide for misalignment of the shaft or to introduce mechanical flexibility.
- ✷ To reduce the transmission of shock loads from one shaft to another.
- ✷ To introduce the protection against overloads.
- ✷ It should have no projecting parts.

Q.3. What are the requirements of a Good Shaft Coupling ?

Ans. The following requirements are :
- ✷ It should be easy to connect or disconnect.
- ✷ It should transmit the full power from one shaft to the other shaft without losses.
- ✷ It whould hold the shaft in perfect alignment.
- ✷ It should reduce the transmission of shock load from one shaft to another.
- ✷ It should have no projecting parts.

Q.4. Define Rigid Coupling ?

Ans. It is used to connect two shafts which are perfectly aligned.

The following types of Rigid Coupling are :
- ✷ Sleeve or Mulf Coupling.
- ✷ Clamp or Split-Muff or Compression Coupling.
- ✷ Flange Coupling.

Q.5. Define Flexible Coupling ?

Ans. It is used to connect two shafts having both lateral and angular misalignment.

The following types of Flexible Coupling are :
- ✷ Bushed Pin Type Coupling
- ✷ Universal Coupling
- ✷ Oldham Coupling

Long Questions :

Q.1. Write the design procedure of Sleeve or Muff Coupling with neat sketch.

Q.2. With neat sketch, write the design procedure of Clamp or Compression Coupling.

Q.3. Design a Muff Coupling to connect two shafts transmitting 40KW at 120 r.p.m. The permissible shear stress and crushing stress for the shaft and Key Material((M.S) are 30MPa and 80MPa respectively. The matrial of Muff is cast iron with permissible shear stress of 15MPa. Assume that the maximum torque transmitted is 25% greater than mean torque.

Q.4. Design a compression coupling for a shaft to transmit 1300N-M. The allowable shear stress for the shaft and key is 40MPa and the number of botts connecting the two halves are 4. The permissible tensile stress for the bott material is 70MPa. The co-efficient of froction between the Muff and the Shaft surface may be taken as 0.3.

DESIGN OF MACHINE ELEMENTS

CHAPTER-5

Design a closed coil helical spring

Prepared by :

Er. Ramesh Chandra Pradhan

Lecturer in

MECHANICAL DEPARTMENT

PNS School of Engineering & Technology

Nishamani Vihar, Marshaghai, Kendrapara

SPRING

Spring is defined as an elastic body, whose function is to distort when loaded and to recover its original shape when the load is removed. The various important applications of springs are as follows:

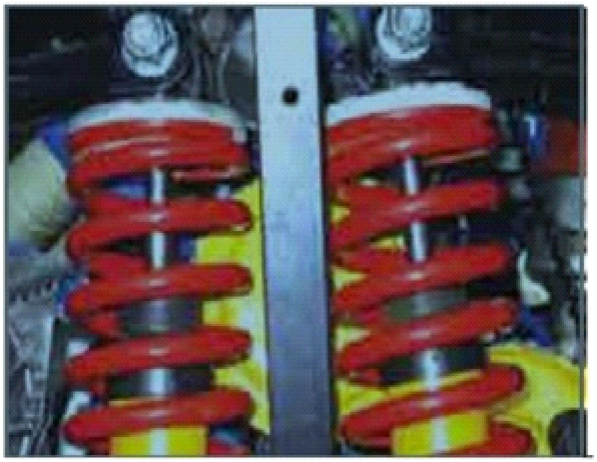
1. To cushion, absorb or control energy due to either shock or vibration as in car springs, railway buffers, air-craft landing gears, shock absorbers and vibration dampers.

2. To apply forces, as in brakes, clutches and spring loaded valves.

3. To control motion by maintaining contact between two elements as in cams and followers.

4. To measure forces, as in spring balances and engine indicators.

5. To store energy, as in watches, toys, etc.



TYPES OF SPRINGS

Though there are many types of the springs, yet the following, according to their shape, are important from the subject point of view

1. ***Helical springs.*** The helical springs are made up of a wire coiled in the form of a helix and is primarily intended for compressive or tensile loads. The cross-section of the wire from which the spring is made may be circular, square or rectangular. The two forms of helical springs are ***compression helical spring*** as shown in Fig. 23.1 (*a*) and ***tension helical spring*** as shown in Fig. 23.1 (*b*).

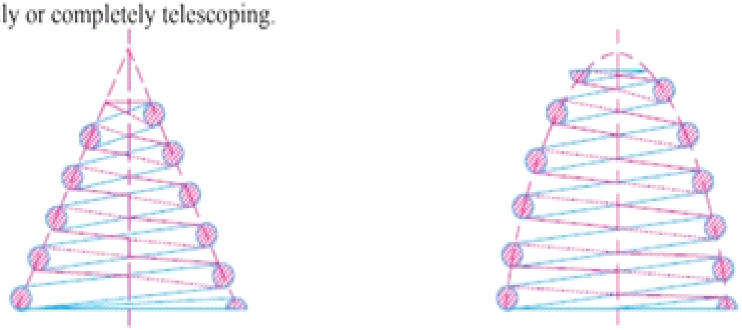
The helical springs are said to be ***closely coiled*** when the spring wire is coiled so close that the plane containing each turn is nearly at right angles to the axis of the helix and the wire is subjected to torsion. In other words, in a closely coiled helical spring, the helix angle is very small, it is usually less than 10°. The major stresses produced in helical springs are shear stresses due to twisting. The load applied is parallel to or along the axis of the spring.

In ***open coiled helical springs,*** the spring wire is coiled in such a way that there is a gap between the two consecutive turns, as a result of which the helix angle is large. Since the application of open coiled helical springs are limited, therefore our discussion shall confine to closely coiled helical springs only.

The helical springs have the following advantages:

(*a*) These are easy to manufacture.
(*b*) These are available in wide range.
(*c*) These are reliable.
(*d*) These have constant spring rate.
(*e*) Their performance can be predicted more accurately.
(*f*) Their characteristics can be varied by changing dimensions.

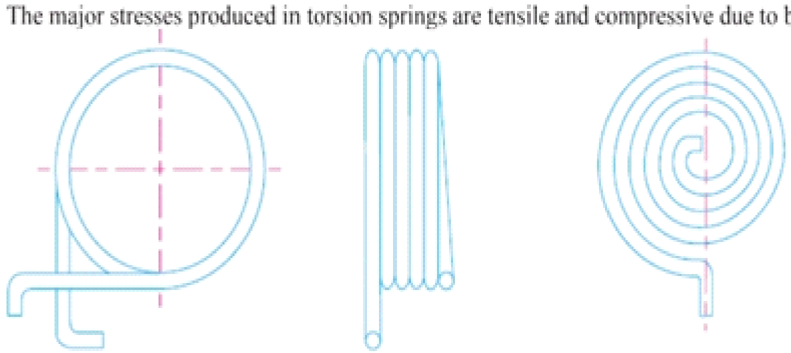
2. *Conical and volute springs.* The conical and volute springs, as shown in Fig. 23.2, are used in special applications where a telescoping spring or a spring with a spring rate that increases with the load is desired. The conical spring, as shown in Fig. 23.2 (*a*), is wound with a uniform pitch whereas the volute springs, as shown in Fig. 23.2 (*b*), are wound in the form of paraboloid with constant pitch and lead angles. The springs may be made either partially or completely telescoping.



(*a*) Conical spring.

(*b*) Volute spring.

3. *Torsion springs.* These springs may be of ***helical*** or ***spiral*** type as shown in Fig. 23.3. The **helical type** may be used only in applications where the load tends to wind up the spring and are used in various electrical mechanisms. The **spiral type** is also used where the load tends to increase the number of coils and when made of flat strip are used in watches and clocks. The major stresses produced in torsion springs are tensile and compressive due to bending.

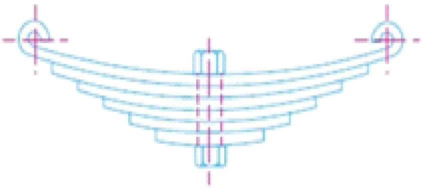


(*a*) Helical torsion spring.

(*b*) Spiral torsion spring.

4. *Laminated or leaf springs.* The laminated or leaf spring (also known as ***flat spring*** or ***carriage spring***) consists of a number of flat plates (known as leaves) of varying lengths held

together by means of clamps and bolts, as shown in Fig. 23.4. These are mostly used in automobiles.
The major stresses produced in leaf springs are tensile and compressive stresses.



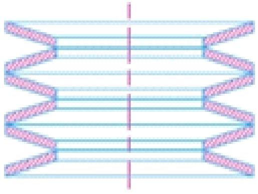


Fig. 23.4. Laminated or leaf springs. **Fig. 23.5.** Disc or bellevile springs.

5. *Disc or bellevile springs*. These springs consist of a number of conical discs held together against slipping by a central bolt or tube as shown in Fig. 23.5. These springs are used in applications where high spring rates and compact spring units are required.

The major stresses produced in disc or Belleville springs are tensile and compressive stresses.

6. Special purpose springs. These springs are air or liquid springs, rubber springs, ring springs etc. The fluids (air or liquid) can behave as a compression spring. These springs are used for special types of application only.

MATERIAL USED FOR HELICAL SPRING

The material of the spring should have high fatigue strength, high ductility, high resilience and it should be creep resistant. It largely depends upon the service for which they are used *i.e.* severe service, average service or light service.

Severe service means rapid continuous loading where the ratio of minimum to maximum load (or stress) is one-half or less, as in automotive valve springs.

Average service includes the same stress range as in severe service but with only intermittent operation, as in engine governor springs and automobile suspension springs.

Light service includes springs subjected to loads that are static or very infrequently varied, as in safety valve springs.

The springs are mostly made from oil-tempered carbon steel wires containing 0.60 to 0.70 per cent carbon and 0.60 to 1.0 per cent manganese. Music wire is used for small springs. Non-ferrous materials like phosphor bronze, beryllium copper, monel metal, brass etc., may be used in special cases to increase fatigue resistance, temperature resistance and corrosion resistance.

STANDARD SIZE OF SPRING WIRE

The standard size of spring wire may be selected from the following table:

Standard wire gauge (SWG) number and corresponding diameter of spring wire.

SWG	*Diameter (mm)*	*SWG*	*Diameter (mm)*	*SWG*	*Diameter (mm)*	*SWG*	*Diameter (mm)*
7/0	12.70	7	4.470	20	0.914	33	0.2540
6/0	11.785	8	4.064	21	0.813	34	0.2337
5/0	10.973	9	3.658	22	0.711	35	0.2134
4/0	10.160	10	3.251	23	0.610	36	0.1930
3/0	9.490	11	2.946	24	0.559	37	0.1727
2/0	8.839	12	2.642	25	0.508	38	0.1524
0	8.229	13	2.337	26	0.457	39	0.1321
1	7.620	14	2.032	27	0.4166	40	0.1219
2	7.010	15	1.829	28	0.3759	41	0.1118
3	6.401	16	1.626	29	0.3454	42	0.1016
4	5.893	17	1.422	30	0.3150	43	0.0914
5	5.385	18	1.219	31	0.2946	44	0.0813
6	4.877	19	1.016	32	0.2743	45	0.0711

TERMS USED IN COMPRESSION SPRING

The following terms used in connection with compression springs are important from the subject point of view.

1. ***Solid length.*** When the compression spring is compressed until the coils come in contact with each other, then the spring is said to be ***solid***. The solid length of a spring is the product of total number of coils and the diameter of the wire. Mathematically,

Solid length of the spring,

$$L_S = n'.d$$

where n' = Total number of coils, and

d = Diameter of the wire.

2. ***Free length.*** The free length of a compression spring, as shown in Fig. 23.6, is the length of the spring in the free or unloaded condition. It is equal to the solid length plus the maximum deflection or compression of the spring and the clearance between the adjacent coils (when fully compressed). Mathematically,

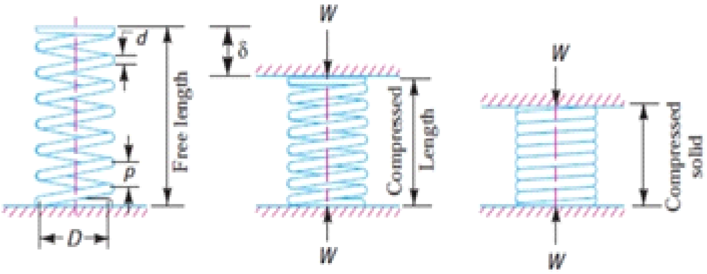


Fig. 23.6. Compression spring nomenclature.

Free length of the spring,

L_F = Solid length + Maximum compression + *Clearance between adjacent coils (or clash allowance)

$= n'.d + \delta_{max} + 0.15\,\delta_{max}$

The following relation may also be used to find the free length of the spring, *i.e.*

$$L_F = n'.d + \delta_{max} + (n' - 1) \times 1 \text{ mm}$$

In this expression, the clearance between the two adjacent coils is taken as 1 mm.

3. *Spring index*. The spring index is defined as the ratio of the mean diameter of the coil to the diameter of the wire. Mathematically,

Spring index, $C = D / d$

where D = Mean diameter of the coil, and

d = Diameter of the wire.

4. *Spring rate*. The spring rate (or stiffness or spring constant) is defined as the load required per unit deflection of the spring. Mathematically,

Spring rate, $k = W / \delta$

where W = Load, and

δ = Deflection of the spring.

5. *Pitch*. The pitch of the coil is defined as the axial distance between adjacent coils in uncompressed state. Mathematically,

Pitch of the coil, $p = \dfrac{\text{Free length}}{n' - 1}$

The pitch of the coil may also be obtained by using the following relation, *i.e.*

Pitch of the coil, $p = \dfrac{L_F - L_S}{n'} + d$

where L_F = Free length of the spring,

L_S = Solid length of the spring,

n' = Total number of coils, and

d = Diameter of the wire.

In choosing the pitch of the coils, the following points should be noted :

(*a*) The pitch of the coils should be such that if the spring is accidently or carelessly compressed, the stress does not increase the yield point stress in torsion.

(*b*) The spring should not close up before the maximum service load is reached.

Note : In designing a tension spring (See Example 23.8), the minimum gap between two coils when the spring is in the free state is taken as 1 mm. Thus the free length of the spring,

$L_F = n.d + (n - 1)$

and pitch of the coil, $p = \dfrac{L_F}{n - 1}$

Total number of turns, solid length and free length fordifferent types of end connections.

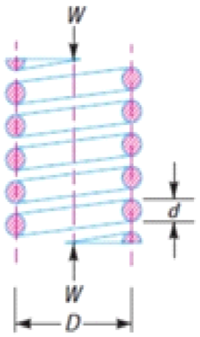
Type of end	Total number of turns (n')	Solid length	Free length
1. Plain ends	n	$(n+1)\,d$	$p \times n + d$
2. Ground ends	n	$n \times d$	$p \times n$
3. Squared ends	$n + 2$	$(n+3)\,d$	$p \times n + 3d$
4. Squared and ground ends	$n + 2$	$(n+2)\,d$	$p \times n + 2d$

where n = Number of active turns,
p = Pitch of the coils, and
d = Diameter of the spring wire.

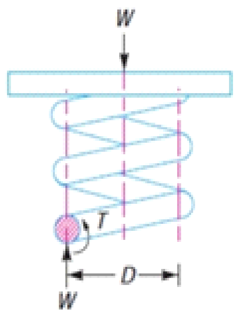
STRESS IN HELICAL SPRING OF CIRCULAR WIRE

Consider a helical compression spring made of circular wire and subjected to an axial load W, as shown in Fig. 23.10 (*a*).

Let D = Mean diameter of the spring coil,
d = Diameter of the spring wire,
n = Number of active coils,
G = Modulus of rigidity for the spring material,
W = Axial load on the spring,
τ = Maximum shear stress induced in the wire,
C = Spring index = D/d,
p = Pitch of the coils, and
δ = Deflection of the spring, as a result of an axial load W.



(*a*) Axially loaded helical spring.



(*b*) Free body diagram showing that wire is subjected to torsional shear and a direct shear.

Fig. 23.10

Now consider a part of the compression spring as shown in Fig. 23.10 (*b*). The load W tends to rotate the wire due to the twisting moment (T) set up in the wire. Thus torsional shear stress is induced in the wire.

A little consideration will show that part of the spring, as shown in Fig. 23.10 (*b*), is in equilibrium under the action of two forces W and the twisting moment T. We know that the twisting moment,

$$T = W \times \frac{D}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$\therefore \quad \tau_1 = \frac{8 W.D}{\pi d^3} \quad \text{...}(i)$$

The torsional shear stress diagram is shown in Fig. 23.11 (*a*).

In addition to the torsional shear stress (τ_1) induced in the wire, the following stresses also act on the wire :

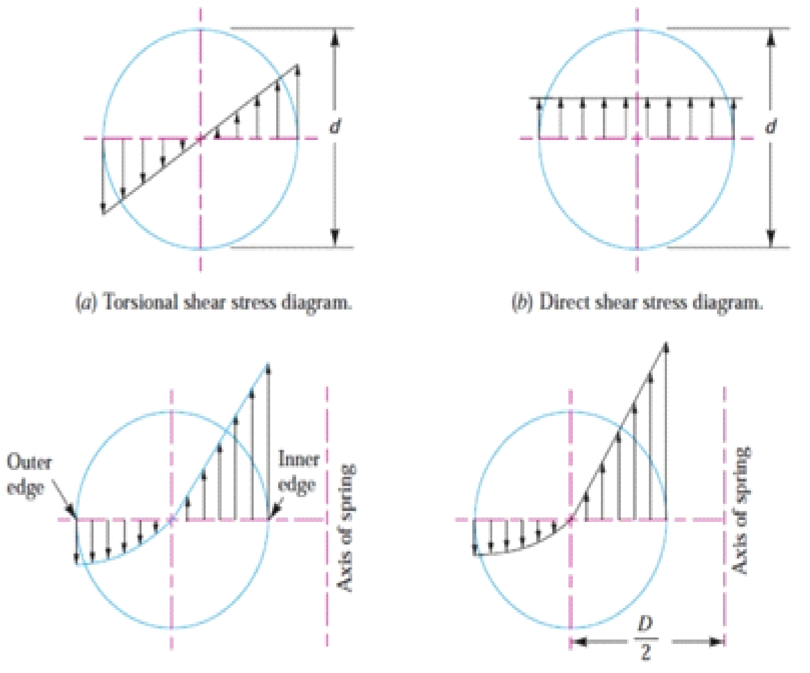
1. Direct shear stress due to the load W, and
2. Stress due to curvature of wire.

We know that direct shear stress due to the load W,

$$\tau_2 = \frac{\text{Load}}{\text{Cross-sectional area of the wire}}$$

$$= \frac{W}{\frac{\pi}{4} \times d^2} = \frac{4W}{\pi d^2} \quad \text{...}(ii)$$

The direct shear stress diagram is shown in Fig. 23.11 (*b*) and the resultant diagram of torsional shear stress and direct shear stress is shown in Fig. 23.11 (*c*).



(*a*) Torsional shear stress diagram.

(*b*) Direct shear stress diagram.

(*c*) Resultant torsional shear and direct shear stress diagram.

(*d*) Resultant torsional shear, direct shear and curvature shear stress diagram.

Fig. 23.11. Superposition of stresses in a helical spring.

We know that the resultant shear stress induced in the wire,

$$\tau = \tau_1 \pm \tau_2 = \frac{8W.D}{\pi d^3} \pm \frac{4W}{\pi d^2}$$

The ***positive*** sign is used for the inner edge of the wire and ***negative*** sign is used for the outer lge of the wire. Since the stress is maximum at the inner edge of the wire, therefore

Maximum shear stress induced in the wire,

$$= \text{Torsional shear stress} + \text{Direct shear stress}$$

$$= \frac{8W.D}{\pi d^3} + \frac{4W}{\pi d^2} = \frac{8W.D}{\pi d^3}\left(1 + \frac{d}{2D}\right)$$

$$= \frac{8\,W.D}{\pi d^3}\left(1 + \frac{1}{2C}\right) = K_S \times \frac{8\,W.D}{\pi d^3} \quad ...(iii)$$

... (Substituting $D/d = C$)

where K_S = Shear stress factor $= 1 + \frac{1}{2C}$

From the above equation, it can be observed that the effect of direct shear $\left(\frac{8\,WD}{\pi d^3} \times \frac{1}{2C}\right)$ is appreciable for springs of small spring index C. Also we have neglected the effect of wire curvature in equation *(iii)*. It may be noted that when the springs are subjected to static loads, the effect of wire curvature may be neglected, because yielding of the material will relieve the stresses.

In order to consider the effects of both direct shear as well as curvature of the wire, a Wahl's stress factor (K) introduced by A.M. Wahl may be used. The resultant diagram of torsional shear, direct shear and curvature shear stress is shown in Fig. 23.11 (*d*).

∴ Maximum shear stress induced in the wire,

$$\tau = K \times \frac{8\,W.D}{\pi d^3} = K \times \frac{8\,W.C}{\pi d^2} \quad ...(iv)$$

where $K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}$

The values of K for a given spring index (C) may be obtained from the graph as shown in Fig. 23.12.

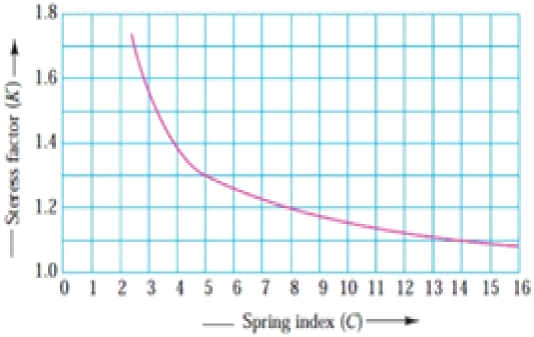


Fig. 23.12. Wahl's stress factor for helical springs.

We see from Fig. 23.12 that Wahl's stress factor increases very rapidly as the spring index decreases. The spring mostly used in machinery have spring index above 3.

Note: The Wahl's stress factor (K) may be considered as composed of two sub-factors, K_S and K_C, such that

$$K = K_S \times K_C$$

where K_S = Stress factor due to shear, and

K_C = Stress concentration factor due to curvature.

DEFLECTION OF HELICAL SPRING OF CIRCULAR WIRE

In the previous article, we have discussed the maximum shear stress developed in the wire. We know that

Total active length of the wire,

$$l = \text{Length of one coil} \times \text{No. of active coils} = \pi D \times n$$

Let θ = Angular deflection of the wire when acted upon by the torque T.

$\therefore$ Axial deflection of the spring,

$$\delta = \theta \times D/2 \quad \text{...(i)}$$

We also know that

$$\frac{T}{J} = \frac{\tau}{D/2} = \frac{G.\theta}{l}$$

$$\therefore \quad \theta = \frac{T\,l}{J.G} \quad \text{...}\left(\text{considering } \frac{T}{J} = \frac{G.\theta}{l}\right)$$

where J = Polar moment of inertia of the spring wire

$= \frac{\pi}{32} \times d^4$, d being the diameter of spring wire.

and G = Modulus of rigidity for the material of the spring wire.

Now substituting the values of l and J in the above equation, we have

$$\theta = \frac{T\,l}{J.G} = \frac{\left(W \times \frac{D}{2}\right)\pi\,D.n}{\frac{\pi}{32} \times d^4\,G} = \frac{16\,W.D^2.n}{G.d^4} \quad \text{...(ii)}$$

Substituting this value of θ in equation (i), we have

$$\delta = \frac{16\,W.D^2.n}{G.d^4} \times \frac{D}{2} = \frac{8\,W.D^3.n}{G.d^4} = \frac{8\,W.C^3.n}{G.d} \quad \text{...}(\because C = D/d)$$

and the stiffness of the spring or spring rate,

$$\frac{W}{\delta} = \frac{G.d^4}{8\,D^3.n} = \frac{G.d}{8\,C^3.n} = \text{constant}$$

23.10 Eccentric Loading of Springs

Sometimes, the load on the springs does not coincide with the axis of the spring, *i.e.* the spring is subjected to an eccentric load. In such cases, not only the safe load for the spring reduces, the stiffness of the spring is also affected. The eccentric load on the spring increases the stress on one side of the spring and decreases on the other side. When the load is offset by a distance e from the spring axis, then the safe load on the spring may be obtained by multiplying the axial load by the factor $\frac{D}{2\,e + D}$, where D is the mean diameter of the spring.

23.11 Buckling of Compression Springs

It has been found experimentally that when the free length of the spring (L_F) is more than four times the mean or pitch diameter (D), then the spring behaves like a column and may fail by buckling at a comparatively low load as shown in Fig. 23.13. The critical axial load (W_{cr}) that causes buckling may be calculated by using the following relation, *i.e.*

$$W_{cr} = k \times K_B \times L_F$$

where k = Spring rate or stiffness of the spring = W/δ,

L_F = Free length of the spring, and

K_B = Buckling factor depending upon the ratio L_F / D.

The buckling factor (K_B) for the hinged end and built-in end springs may be taken from the following table.

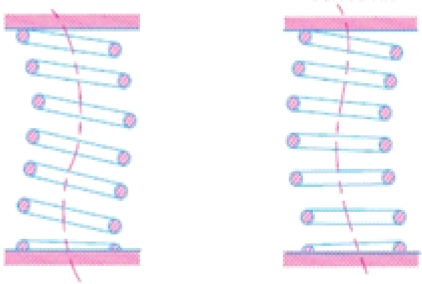


Fig. 23.13. Buckling of compression springs.

Table 23.4. Values of buckling factor (K_B).

L_F/D	*Hinged end spring*	*Built-in end spring*	L_F/D	*Hinged end spring*	*Built-in end spring*
1	0.72	0.72	5	0.11	0.53
2	0.63	0.71	6	0.07	0.38
3	0.38	0.68	7	0.05	0.26
4	0.20	0.63	8	0.04	0.19

It may be noted that a ***hinged end spring*** is one which is supported on pivots at both ends as in case of springs having plain ends where as a ***built-in end spring*** is one in which a squared and ground end spring is compressed between two rigid and parallel flat plates.

It order to avoid the buckling of spring, it is either mounted on a central rod or located on a tube. When the spring is located on a tube, the clearance between the tube walls and the spring should be kept as small as possible, but it must be sufficient to allow for increase in spring diameter during compression.

SURGE IN SPRINGS

When one end of a helical spring is resting on a rigid support and the other end is loaded suddenly, then all the coils of the spring will not suddenly deflect equally, because some time is required for the propagation of stress along the spring wire. A little consideration will show that in the beginning, the end coils of the spring in contact with the applied load takes up whole of the deflection and then it transmits a large part of its deflection to the adjacent coils. In this way, a wave of compression propagates through the coils to the supported end from where it is reflected back to the deflected end. This wave of compression travels along the spring indefinitely. If the applied load is of fluctuating type as in the case of valve spring in internal combustion engines and if the time interval between the load applications is equal to the time required for the wave to travel from one end to the other end, then resonance will occur. This results in very large deflections of the coils and correspondingly very high stresses. Under these conditions, it is just possible that the spring may fail. This phenomenon is called ***surge.***

It has been found that the natural frequency of spring should be atleast twenty times the frequency of application of a periodic load in order to avoid resonance with all harmonic frequencies upto twentieth order. The natural frequency for springs clamped between two plates is given by

$$f_n = \frac{d}{2\pi D^2 n}\sqrt{\frac{6\,G.g}{\rho}} \text{ cycles/s}$$

where d = Diameter of the wire,

D = Mean diameter of the spring,

n = Number of active turns,

G = Modulus of rigidity,

g = Acceleration due to gravity, and

ρ = Density of the material of the spring.

The surge in springs may be eliminated by using the following methods :

1. By using friction dampers on the centre coils so that the wave propagation dies out.
2. By using springs of high natural frequency.
3. By using springs having pitch of the coils near the ends different than at the centre to have different natural frequencies.

Example 23.1. *A compression coil spring made of an alloy steel is having the following specifications :*

Mean diameter of coil = 50 mm ; Wire diameter = 5 mm ; Number of active coils = 20.

If this spring is subjected to an axial load of 500 N ; calculate the maximum shear stress (neglect the curvature effect) to which the spring material is subjected.

Solution. Given : D = 50 mm ; d = 5 mm ; *n = 20 ; W = 500 N

We know that the spring index,

$$C = \frac{D}{d} = \frac{50}{5} = 10$$

∴ Shear stress factor,

$$K_S = 1 + \frac{1}{2C} = 1 + \frac{1}{2\times 10} = 1.05$$

and maximum shear stress (neglecting the effect of wire curvature),

$$\tau = K_S \times \frac{8W.D}{\pi d^3} = 1.05 \times \frac{8\times 500\times 50}{\pi \times 5^3} = 534.7 \text{ N/mm}^2$$

$= 534.7$ MPa **Ans.**

Example 23.2. *A helical spring is made from a wire of 6 mm diameter and has outside diameter of 75 mm. If the permissible shear stress is 350 MPa and modulus of rigidity 84 kN/mm², find the axial load which the spring can carry and the deflection per active turn.*

Solution. Given : d = 6 mm ; D_o = 75 mm ; τ = 350 MPa = 350 N/mm² ; G = 84 kN/mm² = 84×10^3 N/mm²

We know that mean diameter of the spring,

$$D = D_o - d = 75 - 6 = 69 \text{ mm}$$

∴ Spring index,
$$C = \frac{D}{d} = \frac{69}{6} = 11.5$$

Let W = Axial load, and

δ / n = Deflection per active turn.

1. Neglecting the effect of curvature

We know that the shear stress factor,

$$K_S = 1 + \frac{1}{2C} = 1 + \frac{1}{2\times 11.5} = 1.043$$

and maximum shear stress induced in the wire (τ),

$$350 = K_S \times \frac{8\,W.D}{\pi d^3} = 1.043 \times \frac{8\,W \times 69}{\pi \times 6^3} = 0.848\,W$$

∴ $W = 350 / 0.848 = 412.7$ N **Ans.**

We know that deflection of the spring,

$$\delta = \frac{8\,W.D^3.n}{G.d^4}$$

∴ Deflection per active turn,

$$\frac{\delta}{n} = \frac{8\,W.D^3}{G.d^4} = \frac{8 \times 412.7\,(69)^3}{84 \times 10^3 \times 6^4} = 9.96 \text{ mm } \textbf{Ans.}$$

2. Considering the effect of curvature

We know that Wahl's stress factor,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4 \times 11.5 - 1}{4 \times 11.5 - 4} + \frac{0.615}{11.5} = 1.123$$

We also know that the maximum shear stress induced in the wire (τ),

$$350 = K \times \frac{8W.C}{\pi\,d^2} = 1.123 \times \frac{8 \times W \times 11.5}{\pi \times 6^2} = 0.913\,W$$

∴ $W = 350 / 0.913 = 383.4$ N **Ans.**

and deflection of the spring,

$$\delta = \frac{8\,W.D^3.n}{G.d^4}$$

∴ Deflection per active turn,

$$\frac{\delta}{n} = \frac{8\,W.D^3}{G.d^4} = \frac{8 \times 383.4\,(69)^3}{84 \times 10^3 \times 6^4} = 9.26 \text{ mm } \textbf{Ans.}$$

Example 23.5. *Design a helical compression spring for a maximum load of 1000 N for a deflection of 25 mm using the value of spring index as 5.*

The maximum permissible shear stress for spring wire is 420 MPa and modulus of rigidity is 84 kN/mm².

Take Wahl's factor, $K = \frac{4C-1}{4C-4} + \frac{0.615}{C}$, *where C = Spring index.*

Solution. Given : $W = 1000$ N ; $\delta = 25$ mm ; $C = D/d = 5$; $\tau = 420$ MPa $= 420$ N/mm² ; $G = 84$ kN/mm² $= 84 \times 10^3$ N/mm²

1. Mean diameter of the spring coil

Let D = Mean diameter of the spring coil, and

d = Diameter of the spring wire.

We know that Wahl's stress factor,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4 \times 5 - 1}{4 \times 5 - 4} + \frac{0.615}{5} = 1.31$$

and maximum shear stress (τ),

$$420 = K \times \frac{8\,W.C}{\pi\,d^2} = 1.31 \times \frac{8 \times 1000 \times 5}{\pi\,d^2} = \frac{16\,677}{d^2}$$

∴ $d^2 = 16\,677 / 420 = 39.7$ or $d = 6.3$ mm

From Table 23.2, we shall take a standard wire of size *SWG* 3 having diameter (d) = 6.401 mm.

∴ Mean diameter of the spring coil,

$D = C.d = 5\,d = 5 \times 6.401 = 32.005$ mm **Ans.** ... ($\because C = D/d = 5$)

and outer diameter of the spring coil,

$D_o = D + d = 32.005 + 6.401 = 38.406$ mm **Ans.**

2. Number of turns of the coils

Let n = Number of active turns of the coils.

We know that compression of the spring (δ),

$$25 = \frac{8W.C^3.n}{G.d} = \frac{8 \times 1000\,(5)^3 n}{84 \times 10^3 \times 6.401} = 1.86\,n$$

$\therefore$ $n = 25 / 1.86 = 13.44$ say 14 **Ans.**

For squared and ground ends, the total number of turns,

$$n' = n + 2 = 14 + 2 = 16 \text{ Ans.}$$

3. Free length of the spring

We know that free length of the spring

$$= n'.d + \delta + 0.15\,\delta = 16 \times 6.401 + 25 + 0.15 \times 25$$

$$= 131.2 \text{ mm Ans.}$$

4. Pitch of the coil

We know that pitch of the coil

$$= \frac{\text{Free length}}{n' - 1} = \frac{131.2}{16 - 1} = 8.75 \text{ mm Ans.}$$

Example 23.6. *Design a close coiled helical compression spring for a service load ranging from 2250 N to 2750 N. The axial deflection of the spring for the load range is 6 mm. Assume a spring index of 5. The permissible shear stress intensity is 420 MPa and modulus of rigidity, G = 84 kN/mm².*

Neglect the effect of stress concentration. Draw a fully dimensioned sketch of the spring, showing details of the finish of the end coils.

Solution. Given : $W_1 = 2250$ N ; $W_2 = 2750$ N ; $\delta = 6$ mm ; $C = D/d = 5$; $\tau = 420$ MPa $= 420$ N/mm² ; $G = 84$ kN/mm² $= 84 \times 10^3$ N/mm²

1. Mean diameter of the spring coil

Let D = Mean diameter of the spring coil for a maximum load of $W_2 = 2750$ N, and

d = Diameter of the spring wire.

We know that twisting moment on the spring,

$$T = W_2 \times \frac{D}{2} = 2750 \times \frac{5d}{2} = 6875\,d \qquad \ldots\left(\because C = \frac{D}{d} = 5\right)$$

We also know that twisting moment (T),

$$6875\,d = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 420 \times d^3 = 82.48\,d^3$$

$\therefore$ $d^2 = 6875 / 82.48 = 83.35$ or $d = 9.13$ mm

From Table 23.2, we shall take a standard wire of size *SWG* 3/0 having diameter (d) = 9.49 mm.

$\therefore$ Mean diameter of the spring coil,

$$D = 5d = 5 \times 9.49 = 47.45 \text{ mm Ans.}$$

We know that outer diameter of the spring coil,

$$D_o = D + d = 47.45 + 9.49 = 56.94 \text{ mm Ans.}$$

and inner diameter of the spring coil,

$$D_i = D - d = 47.45 - 9.49 = 37.96 \text{ mm Ans.}$$

2. Number of turns of the spring coil

Let n = Number of active turns.

It is given that the axial deflection (δ) for the load range from 2250 N to 2750 N (*i.e.* for $W = 500$ N) is 6 mm.

We know that the deflection of the spring (δ),

$$6 = \frac{8\,W.C^3.n}{G.d} = \frac{8 \times 500\,(5)^3\,n}{84 \times 10^3 \times 9.49} = 0.63\,n$$

$\therefore \quad n = 6 / 0.63 = 9.5$ say 10 **Ans.**

For squared and ground ends, the total number of turns,

$$n' = 10 + 2 = 12 \textbf{ Ans.}$$

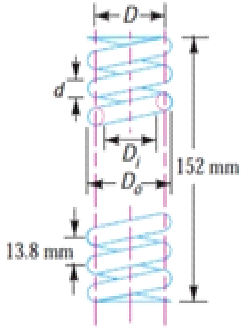


Fig. 23.14

3. Free length of the spring

Since the compression produced under 500 N is 6 mm, therefore maximum compression produced under the maximum load of 2750 N is

$$\delta_{max} = \frac{6}{500} \times 2750 = 33 \text{ mm}$$

We know that free length of the spring,

$$L_F = n'.d + \delta_{max} + 0.15\,\delta_{max}$$
$$= 12 \times 9.49 + 33 + 0.15 \times 33$$
$$= 151.83 \text{ say } 152 \text{ mm } \textbf{Ans.}$$

4. Pitch of the coil

We know that pitch of the coil

$$= \frac{\text{Free length}}{n' - 1} = \frac{152}{12 - 1} = 13.73 \text{ say } 13.8 \text{ mm } \textbf{Ans.}$$

The spring is shown in Fig. 23.14.

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CH-5

Design a Closed Coil Helical Spring

Short Questions & Answers

Q.1. Define Spring ?

Ans. A spring is defened as an elastic body, whose function is to distort when loaded and to recover its original shape, when the load is removed.

Q.2. Define Helical Spring ?

Ans. The helical spring are made up of a wire coiled in the form of a helix and primarily intended for compressive or tensile loads. The cross sectin of the wire for which the spring made may be circular, square or reatangular.

Q.3. Define Solid Length of Spring ?

Ans. When the compression spring is compressed. Until the coils come in contact with each other, then spring is called to be solid. The solid length of a spring is the product of total number of coils and the diameter of the wire.

$L_s = n^1 \times d$ where, n^1 = total number of coils
d = diameter of wire
Ls = Solid Length of Spring.

Q.4. Define Spring Index and Spring Rate ?

Ans. **Spring Index :** It is defined as the ratio of mean diameter of coil to the diameter of wire.

Mathematically, $C = \frac{D}{d}$ Where, C = Spring Index
D = Mean diameter of Coil
d = diameter of wire.

Spring Rate : The Spring Rate or Stiffiness or Spring Constant is defined as the load required per unit deflection of the spring.

Mathematically, $K = \frac{W}{\delta}$ Where, W = Load
δ = deflection of the spring

Q.5. What are the methods to eliminate Surge in Spring ?

Ans. The following methods are :

- By using friction dampers on the centre coil, so that wave propagation dies out.
- By using spring of high natural frequency.
- By using spring having pitch of the coils near the ends different than at the to have different natural frequencies.

Long Questions :

Q.1. Briefly explain Surge in Springs ?

Q.2 Derive Stresses in Helical Springs of Circular Wire ?

Q.3. A Helical spring is made from a wire of 6mm diameter and has outside diameter of 75mm. If the permissible shear stress is 350MPa and modulus of regidity is 84KN/mm^2. Find the axial oad which the spring can carry and the deflection per active turn ?

Q.4. Design a spring for a balance to measure 0 to 1000N over the length of 80mm. The spring is to be enclosed in a casing of 25mm diameter. The approximate number of turns is 30. The modules of regidity is 85KN/m^2. Also calculate the maximum shear stress induced ?

Q.5. Design a closed helical compression for a service load rasging from 2250N to 2750N. The axial deflection of the spring for the load range is 6mm. Assume a spring Index of 5. The permissible shear stress is 420MPa and modulus of rigidity is 84KN/mm^2. Neglect the effect of stress concentration ?