(4) CIRCLE

MATH-I/1

(A) CIRCLE: (i) The Locus of a point which moves in a plane such that its distance from a fixed point is always constant is called a circle.

(ii) The fixed point is called centre and the fixed distance is called radius.

C = centre, CP = r = radius.

Any chord passing through centre is called diameter. PQ = Diameter.

(B) EQUATION OF CIRCLE

(B1) centre-radius form (standard form)

(i) Equation of circle having centre at (h, k) and radius r is $\boxed{(x-h)^2+(y-k)^2=r^2}$

(ii) Equation of circle with centre at origin and radius r is $\boxed{x^2+y^2=r^2}$

(B2) EQUATION OF CIRCLE with ENDS OF A DIAMETER

Equation of circle having End points (x_1, y_1) and (x_2, y_2) of a diameter is

$$\boxed{(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0}$$

(B3): General form:

(i) $\boxed{x^2+y^2+2gx+2fy+c=0}$ is the General form

where $g^2+f^2>c$

(ii) centre $=(-g,-f)$, radius $=\sqrt{g^2+f^2-c}$

(iii) Equation of circle is a

(a) 2nd degree equation in x and y.

(b) It contains no terms of xy

(c) co-efficient of x^2 = coefficient of y^2.

Hence $ax^2+2hxy+by^2+2gx+2fy+c=0$ will represent a circle if $\boxed{a=b,\ h=0}$

problems:

① Find centre and radius of the circle $2x^2+2y^2-5x+3y-11=0$ $\left(\frac{5}{4}, -\frac{3}{4}\right)$, $\frac{\sqrt{122}}{4}$

② Find equation of circle whose centre is at (1,4) and passing through (-2,1)
$(x-1)^2+(y-4)^2=18$.

③ Find equation of circle concentric with the circle $x^2+y^2-8x+6y-5=0$ and passing through the point (-2,-7). $x^2+y^2-8x+6y-27=0$

④ Find equation of circle whose ends of a diameter are the points of intersections of the lines $x+y-1=0$, $4x+3y+1=0$ and $4x+y+3=0$, $x-2y+3=0$
$(x^2+y^2+5x-6y+9=0)$

⑤ Find equation of circle whose centre is on the line $8x+5y=0$ and passing through the points (2,1), (3,5). $(27x^2+27y^2+145x-232y-193=0)$

⑥ Find equation of circle which passes through origin and cut off intercepts a and b from the axes.
$x^2+y^2-ax-by=0$

⑦ For what value of K the equation $2x^2+(K-1)y^2-6x+4y-1=0$ will represent a circle.

SOLUTION

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① Given circle is

$$2x^2+2y^2-5x+3y-11=0$$

$$\Rightarrow x^2+y^2-\frac{5}{2}x+\frac{3}{2}y-\frac{11}{2}=0 \text{ —— ①}$$

Comparing with general form,

$$2g=-\frac{5}{2} \Rightarrow g=-\frac{5}{4}$$

$$2f=\frac{3}{2} \Rightarrow f=\frac{3}{4}$$

and $c=-\frac{11}{2}$

$\therefore$ centre $=(-g,-f)=\left(\frac{5}{4},-\frac{3}{4}\right)$ (Ans)

radius $r=\sqrt{g^2+f^2-c}=\sqrt{\left(-\frac{5}{4}\right)^2+\left(\frac{3}{4}\right)^2-\left(-\frac{11}{2}\right)}$

$=\sqrt{\frac{25}{16}+\frac{9}{16}+\frac{11}{2}}=\sqrt{\frac{25+9+88}{16}}=\frac{\sqrt{122}}{4}$ (Ans)

② Centre of circle is at $C(1,4)=(h,k)$ (say)

Circle passes through $P(-2,1)$

$\therefore$ radius of circle $r=CP=\sqrt{(-2-1)^2+(1-4)^2}$

$=\sqrt{9+9}=\sqrt{18}$

Equation of circle is

$$(x-h)^2+(y-k)^2=r^2$$

$$\Rightarrow (x-1)^2+(y-4)^2=(\sqrt{18})^2=18 \text{ (Ans)}$$

③ Given circle is $x^2+y^2-8x+6y-5=0$ —— ①

Here $2g=-8 \Rightarrow g=-\frac{8}{2}=-4$

$2f=6 \Rightarrow f=6/2=3$

$\therefore$ centre of circle ① $=(-g,-f)=(4,-3)$

Since required circle is concentric with circle ①, centre of required circle

$=C(4,-3)=C(h,k)$ (say)

Also required circle passes through $P(-2,-7)$

Hence its radius

$$r = CP = \sqrt{(-2-4)^2+(-7+3)^2}$$

$$= \sqrt{36+16} = \sqrt{52}$$

$\therefore$ Equation of required circle is

$$(x-h)^2+(y-k)^2 = r^2$$

$$\Rightarrow (x-4)^2+(y+3)^2 = (\sqrt{52})^2 = 52$$

$$\Rightarrow x^2-8x+16+y^2+6y+9 = 52$$

$$\Rightarrow x^2+y^2-8x+6y+25-52 = 0$$

$$\Rightarrow x^2+y^2-8x+6y-27 = 0 \text{ (Ans)}$$

(4) Given lines are $x+y-1=0$ —— (1)

$4x+3y+1=0$ —— (2)

$4x+y+3=0$ —— (3)

$x-2y+3=0$ —— (4)

point of intersection of line (1) and (2)

$$= A\left(\frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1}, \frac{c_1a_2-c_2a_1}{a_1b_2-a_2b_1}\right)$$

$$= A\left(\frac{(1)(1)-(3)(-1)}{(1)(3)-(4)(1)}, \frac{(-1)(4)-(1)(1)}{(1)(3)-(4)(1)}\right)$$

$$= A\left(\frac{4}{-1}, \frac{-5}{-1}\right) = A(-4,5)$$

point of intersection of (3) and (4) =

$$B\left(\frac{(1)(3)-(-2)(3)}{(4)(-2)-(1)(1)}, \frac{(3)(1)-(3)(4)}{(4)(-2)-(1)(1)}\right)$$

$$= B\left(\frac{9}{-9}, \frac{-9}{-9}\right) = B(-1,1)$$

$\therefore$ Ends of a diameter of the required circle are $A(-4,5)$ and $B(-1,1)$

Here $x_1=-4,\ y_1=5$

$x_2=-1,\ y_2=1$

Hence equation of required circle is

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

$$\Rightarrow (x+4)(x+1)+(y-5)(y-1)=0$$

$$\Rightarrow x^2+5x+4+y^2-6y+5=0$$

$$\Rightarrow x^2+y^2+5x-6y+9=0 \quad \text{(Ans)}$$

(5) Given line is $8x+5y=0$ —— (1)

and points are $A(2,1), B(3,5)$.

Let equation of required circle be

$$x^2+y^2+2gx+2fy+c=0 \text{ —— (2)}$$

centre of circle (2) is $C(-g,-f)$

point C lies on line (1)

$$\Rightarrow -8g-5f=0 \Rightarrow 8g+5f=0 \text{ —— (3)}$$

circle (2) passes through $A(2,1)$ and $B(3,5)$

Hence

$$2^2+1^2+2g\times2+2f\times1+c=0$$

$$\Rightarrow 4g+2f+c+5=0 \text{ —— (4)}$$

and $3^2+5^2+2g\times3+2f\times5+c=0$

$$\Rightarrow 6g+10f+c+34=0 \text{ —— (5)}$$

$Eq^n(5)-Eq^n(4) \Rightarrow 2g+8f+29=0$ —— (6)

Eqⁿ (3)×1 ⇒ $8g+5f = 0$

Eqⁿ (6)×4 ⇒ $8g+32f+116 = 0$

Substracting (−) (−) (−) (−)

$\Rightarrow -27f-116 = 0$

$\Rightarrow 27f = -116 \Rightarrow f = -\frac{116}{27}$

From (3) $8g = -5f = (-5)\times\left(-\frac{116}{27}\right) = \frac{580}{27}$

$\Rightarrow g = \frac{580}{27}\times\frac{1}{8} = \frac{145}{54}$

From (4) $c = -4g-2f-5$

$= -4\times\frac{145}{54} - 2\left(-\frac{116}{27}\right) - 5$

$= -\frac{290}{27} + \frac{232}{27} - 5 = \frac{-290+232-135}{27}$

$= -\frac{193}{27}$

Equation of required circle is

$x^2+y^2+2gx+2fy+c=0$

$\Rightarrow x^2+y^2+2\left(\frac{145}{54}\right)x+2\left(-\frac{116}{27}\right)y+\left(-\frac{193}{27}\right)=0$

$\Rightarrow x^2+y^2+\frac{145x}{27}-\frac{232y}{27}-\frac{193}{27}=0$

$\Rightarrow 27x^2+27y^2+145x-232y-193=0$ (Ans)

(6) Let equation of required circle be

$x^2+y^2+2gx+2fy+c=0$ —— (1)

It passes through origin $\Rightarrow 0^2+0^2+2g\times0+2f\times0+c=0 \Rightarrow c=0$

It cuts off intercepts a and b from

Co-ordinate axes

MATH-i/7

Hence circle ① passes through the points $(a,0)$ and $(0,b)$

$\therefore \quad a^2+0^2+2g\times a+2f\times 0+c=0$

$\Rightarrow a^2+2ga=0 \Rightarrow a(a+2g)=0$

$\Rightarrow a=0$ or $a+2g=0 \Rightarrow a+2g=0 \quad (\because a\neq 0)$

$\Rightarrow 2g=-a \Rightarrow \boxed{g=-\frac{a}{2}}$

and $0^2+b^2+2g\times 0+2f\times b+c=0$

$\Rightarrow b^2+2fb=0 \Rightarrow b(b+2f)=0$

$\Rightarrow b+2f=0 \quad (\because b\neq 0)$

$\Rightarrow \boxed{f=-\frac{b}{2}}$

Eqn of required circle is

$x^2+y^2+2gx+2fy+c=0$

$\Rightarrow x^2+y^2+2\left(-\frac{a}{2}\right)x+2\left(-\frac{b}{2}\right)y+0=0$

$\Rightarrow x^2+y^2-ax-by=0$ (Ans)

(7) Given equation is $2x^2-(k-1)y^2-6x+4y=0$ —— ①

① will represent a circle if

$-(k-1)=2 \Rightarrow -k+1=2$

$\Rightarrow -k=2-1=1$

$\Rightarrow k=-1$ (Ans)

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HOME TASK

⑧ Find equation of circle passing through $(0,1),(1,0)$ and $(0,0)$

(9) Find equation of circle passing through $(0,2),(3,0)$, and $(3,2)$

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(10) If equation of two diameter of a circle are $x-y=5$ and $2x+y=4$ and radius of the circle is 5 then find equation of circle.

(11) Find equation of circle whose centre lies on +ve direction of y-axis at a distance 5 from origin and whose radius is 3.

(12) Find centre and radius of the circle $25x^2+25y^2-30x-10y-6=0$

(13) Prove that the points $(2,-4)$, $(3,-1)$, $(3,-3)$, $(0,0)$ are concyclic (i.e lie on a circle)

(14) Find equation of circle whose centre is at $(2,3)$ and radius 4.

(15) Find equation of circle with centre at $(1,4)$ and passing through a point $(2,6)$

(16) Find equation of circle whose centre is at $(5,5)$ and touches both axes.

(17) Determine which circle is greater?
$x^2+y^2-3x+4y=0$ and $x^2+y^2-6x+8y=0$

(18) Find equation of circle concentric with the circle $2x^2+2y^2+8x+12y-25=0$ and having its circumference 6π unit.

(19) Find equation of circle concentric with the circle $4x^2+4y^2-24x+16y-9=0$ and whose area is 9π sq. units.

MATH-2/9

(20) Find equation of circle which passes through $(3,-2)$, $(-2,0)$ and whose centre lies on the line $2x-y=3$

(21) Find equation of circle circumscribing the the triangle having vertices $(1,-5)$, $(5,7)$, $(-5,1)$

(22) Find equation of circle whose end points of a diameter are $(1,2)$ and $(-3,4)$

~~(23) Find equation of circle passing through origin and~~ making intercepts 4 and 5 on co-ordinate axes.