③ COORDINATE GEOMETRY IN TWO DIMENSION: (2D)

MATH-I/1

(A) INTRODUCTION:

(a) The Geometry that we have studied is known as Euclidean Geometry. as it is the great work of Greek Mathematician Euclid.

(b) But in 1637 the French mathematician Rene Descartes published his work on co-ordinate geometry applying algebra to geometry. He is known as the father of Analytical Geometry.

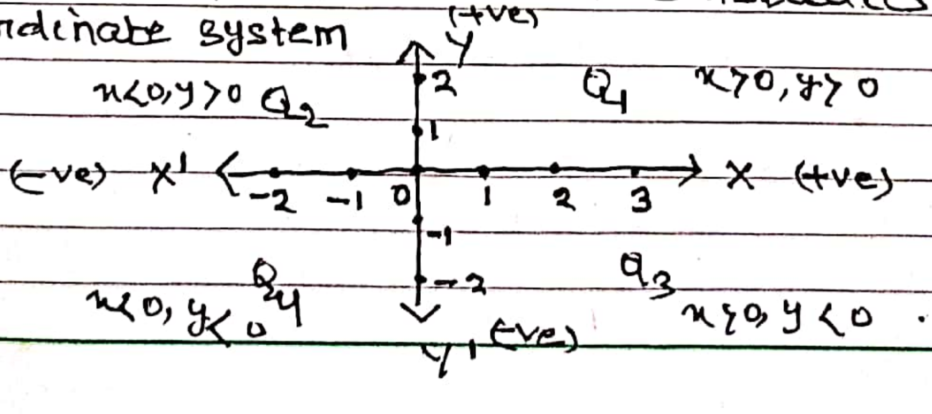
(c) He introduced the point representation in the plane by ordered pairs of real numbers.

(d) There are three co-ordinate system i.e (i) Cartesian co-ordinate system (ii) Polar co-ordinate system (iii) oblique cartesian co-ordinate system.

(B) Rectangular co-ordinate system:

(i) Let's consider two mutually perpendicular lines X'OX and Y'OY intersecting each other at the point 'O'. The point 'O' is called origin from which all measurements are done.

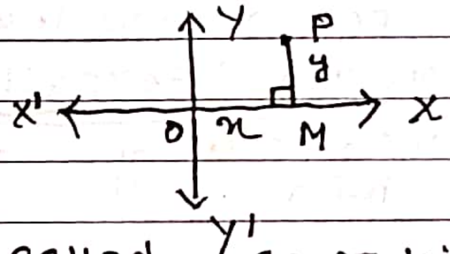
The line X'OX and Y'OY are called X-axis and Y-axis respectively. $\vec{OX}$ and $\vec{OY}$ represent the +ve direction of X axis and Y-axis respectively. $\vec{OX'}$ and $\vec{OY'}$ represents (-ve) direction of X-axis and Y-axis. This constitutes cartesian co-ordinate system



The branch of mathematics in which geometrical problems are solved through algebra using co-ordinate system is called co-ordinate geometry or analytical geometry. co-ordinate geometry relating to plane is called two dimensional geometry (2D).

(ii) Co-ordinate of a point:

(a) Let P be a point in the plane. draw perpendicular $\overline{PM}$ on X-axis. Let $OM = x$, $PM = y$.



Then order pair (x, y) is called co-ordinate of point P. x is called x-co-ordinate or abscissa and y is called y co-ordinate or ordinate.

Also Note that x = perpendicular distance of P from y-axis.

y = perpendicular distance of P from x-axis.

(b) co-ordinate of origin is $(0,0)$.

Any point on x axis is of the form $(a,0)$, $a \in R$

Any point on y-axis is of the form $(0,a)$, $a \in R$.

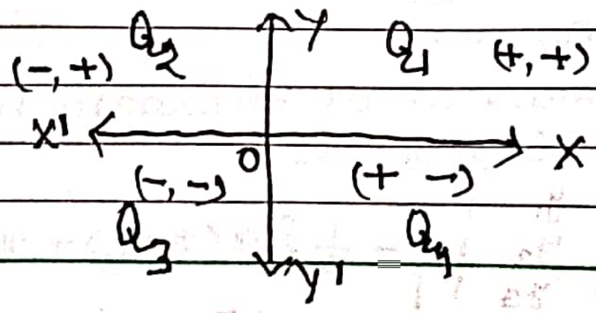
(iii) Quadrant: The co-ordinate axes XOX', $Y'OY$ divides the plane into four regions called quadrant.

The region XOY = first quadrant (Q_1) where $x>0, y>0$

The region $X'OY$ = 2nd quadrant (Q_2) where $x<0, y>0$

The region $X'OY'$ = 3rd quadrant (Q_3) where $x<0, y<0$

The region XOY' = 4th quadrant (Q_4) where $x>0, y<0$



MATH I / 3:

(C) DISTANCE FORMULA:

(i) Distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

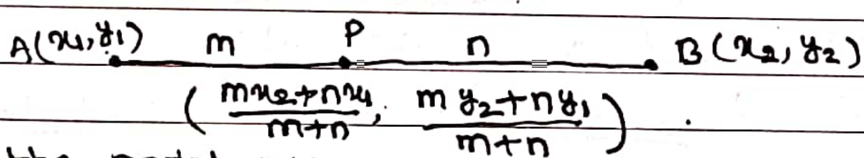
$$\boxed{AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}}$$

(ii) Distance of $P(x,y)$ from origin $(0,0)$ is $\boxed{\sqrt{x^2+y^2}}$

(D) DIVISION FORMULA:

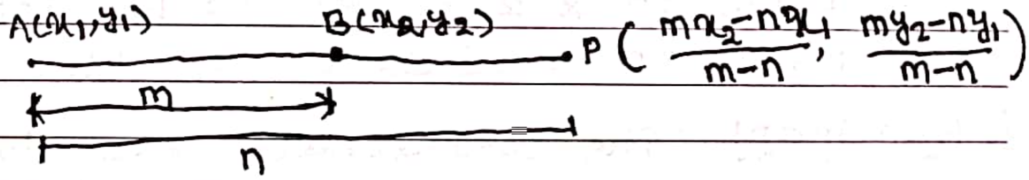
(i) If point $P(x,y)$ divides the line segment joining $A(x_1,y_1)$ and $B(x_2,y_2)$ internally in the ratio $m:n$ then

$$\boxed{x = \frac{mx_2+nx_1}{m+n},\ y = \frac{my_2+ny_1}{m+n}}$$



(ii) If the point $P(x,y)$ divides the line segment joining $A(x_1,y_1)$ and $B(x_2,y_2)$ externally in the ratio $m:n$ then

$$\boxed{x = \frac{mx_2-nx_1}{m-n},\ y = \frac{my_2-ny_1}{m-n}}$$



(iii) The mid point of line segment joining $A(x_1,y_1)$ and $B(x_2,y_2)$ is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

(iv) If P divides the line segment joining $A(x_1,y_1)$ and $B(x_2,y_2)$ in the ratio $k:1$ then co-ordinates of P is $\left(\frac{kx_2+x_1}{k+1}, \frac{ky_2+y_1}{k+1}\right)$. If $k < 0$ then P divides externally.

(E) Area of a Triangle:

(i) Area of of triangle having vertices $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ is

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\}$$

(Take +ve sign)

(ii) Condition of Co-linearity: Three points $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ are colinear if and only if

$$\boxed{\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0} \text{ or } \boxed{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)=0}$$

NOTE

(i) Centroid of a Triangle having vertices $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ is $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

Centroid divides median in $2:1$

(ii) Incentre of a Triangle ABC having vertices $A(x_1,y_1)$, $B(x_2,y_2)$, $C(x_3,y_3)$ and sides a, b, c, is

$$\left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)$$

The point at which the bisectors of angles of a Triangle intersects is called incentre.

Problems:

① If the distance between points $(3,a)$ and $(6,1)$ is 5 then find a.

Soln Given points are $A(3,a)$ & $B(6,1)$

$A(3,a)$ —— 5 —— $B(6,1)$

Then $AB = 5 \Rightarrow \sqrt{(6-3)^2+(1-a)^2} = 5$

$\Rightarrow \sqrt{9+(1-a)^2} = 5 \Rightarrow 9+(1-a)^2 = 25 \Rightarrow (1-a)^2 = 16 \Rightarrow 1-a = \sqrt{16}$

$\Rightarrow 1-a = \pm 4 \Rightarrow 1-a = 4$ or $1-a = -4 \Rightarrow a = 1-4$ or $a = 1+4$

$\Rightarrow a = -3$ or $a = 5$ (Ans)

② In what ratio does the point $(3,-2)$ divide the line segment joining the points $(1,4)$ and $(-3,16)$.

Soln Given points are $A(3,-2)$, $B(1,4)$ and $C(-3,16)$.

$B(1,4)$ —k— $A(3,-2)$ —1— $C(-3,16)$

Let A Divides $\overline{BC}$ in the ratio $k:1$. Then

Co-ordinate of $A = \left(\frac{-3k+1}{k+1}, \frac{16k+4}{k+1}\right)$

$\because A = (3,-2)$

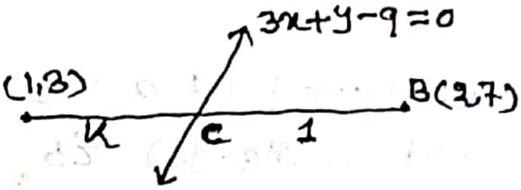
$\therefore \frac{-3k+1}{k+1} = 3$ ~~$\Rightarrow -3k+1 = 3k+3$~~

$\Rightarrow -3k+1 = 3k+3 \Rightarrow -6k = 2 \Rightarrow k = \frac{2}{-6} = -\frac{1}{3}$

Hence A divides BC in the ratio 1:3 externally.

③ Find the ratio in which the line $3x+y-9=0$ divides the line segment of the points $(1,3),(2,7)$.

Soln Given line is $3x+y-9=0$ —— ① and points are $A(1,3)$, $B(2,7)$



Let line ① divides $\overline{AB}$ in the ratio $k:1$ at point C.

Then co-ordinate of C are $\left(\frac{2k+1}{k+1}, \frac{7k+3}{k+1}\right)$

But C being a point on line ①,

$$3 \times \frac{2k+1}{k+1} + \frac{7k+3}{k+1} - 9 = 0 \Rightarrow \frac{6k+3}{k+1} + \frac{7k+3}{k+1} - 9 = 0$$

$$\Rightarrow \frac{6k+3+7k+3-9k-9}{k+1} = 0 \quad 4k-3=0 \Rightarrow k = \frac{3}{4}$$

$\therefore$ ~~The given line divides~~

The required ratio is 3:4.

④ For what values of x will the points $(x,-1),(2,1)$ and $(4,5)$ lie on a line?

Soln Given points are $A(x,-1)$, $B(2,1)$, $C(4,5)$.

If A, B, C lie on a line $\Rightarrow$ A, B, C are co-linear.

$$\Rightarrow \begin{vmatrix} x & -1 & 1 \\ 2 & 1 & 1 \\ 4 & 5 & 1 \end{vmatrix} = 0 \Rightarrow x(1-5) + 1(2-4) + 1(10-4) = 0$$

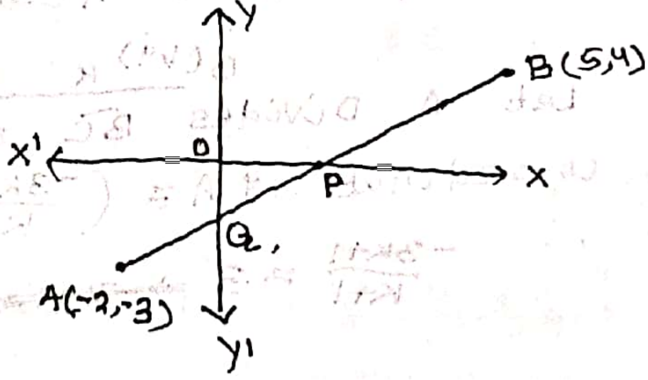
$$\Rightarrow -4x-2+6=0 \Rightarrow -4x+4=0 \Rightarrow 4x=4 \Rightarrow x = \frac{4}{4} = 1 \text{ (Ans)}$$

⑤ Find the ratio in which the line segment joining $(-2,-3)$ and $(5,4)$ is divided by co-ordinate axes and hence find the co-ordinates of the points.

Soln Given points are

$A(-2,-3)$ and $B(5,4)$

Let X-axis and Y-axis divides $\overline{AB}$ at P and Q respectively.



Let P divides $\overline{AB}$ in K:1 ratio. Then co-ordinates of

$P = \left(\frac{5K-2}{K+1}, \frac{4K-3}{K+1}\right)$

P being a point on x-axis $\Rightarrow \frac{4K-3}{K+1} = 0 \Rightarrow 4K-3=0$

$\Rightarrow 4K=3 \Rightarrow K=\frac{3}{4}$.

Hence x-axis divides $\overline{AB}$ in 3:4 ratio.

Now $P = \left(\frac{5K-2}{K+1}, \frac{4K-3}{K+1}\right) = \left(\frac{5\times\frac{3}{4}-2}{\frac{3}{4}+1}, 0\right)$

$= \left(\frac{7/4}{7/4}, 0\right) = (1,0)$

Let Q divides $\overline{AB}$ in the ratio K:1

The $Q = \left(\frac{5K-2}{K+1}, \frac{4K-3}{K+1}\right)$

Q being a point on y-axis $\Rightarrow \frac{5K-2}{K+1} = 0 \Rightarrow 5K-2=0$

$\Rightarrow 5K=2 \Rightarrow K=\frac{2}{5}$

$\because$ Q divides $\overline{AB}$ in the ratio 2:5

Now $Q = \left(\frac{5K-2}{K+1}, \frac{4K-3}{K+1}\right) = \left(0, \frac{4\times\frac{2}{5}-3}{\frac{2}{5}+1}\right)$

$= \left(0, \frac{-7/5}{7/5}\right) = (0,-1)$

$\therefore$ x-axis divides the line segment in the ratio 3:4 and co-ordinates of the point on x-axis is (1,0)

y-axis divides the line segment in the ratio 2:5 and co-ordinates of the point on y-axis is (0,−1)

HOME TASK

(6) Prove that P(3,3), Q(9,0) and R(12,21) are the vertices of a right angled Triangle.

(7) Find the co-ordinates of the points which divides $\overline{PQ}$ in the ratio 2:3 internally and externally. Where P and Q are (2,−1), (−3,4)

(8) Find the area of the Triangle having vertices A(4,4), B(3,−2), C(−3,16).

(9) If the area of the triangle with vertices (0,0), (1,0), (0,a) is 10 units then find a.

(10) Find the value of a so that the points (1,4), (2,7), (3,a) are colinear.

(11) Two vertices of a triangle are (0,−4) and (6,0). If the median meets at the point (2,0) then find co-ordinate of third vertex.

(12) Show that the points (0,−1), (−2,3), (6,7), (8,3) are vertices of a rectangle.

(13) In a triangle one of the vertices is at (2,5). and centroid of the triangle is at (−1,1). Find the co-ordinate of mid point.

(14) Show that the points (1,1), (4,4), (4,8), (1,5) are vertices of a parallelogram.

(15) Three vertices of a parallelogram are (3,4), (3,8), (9,8). Find the fourth vertex.

(16) Show that the points (1,1), (1,−1), ($-\sqrt{3}$, $\sqrt{3}$) are the vertices of an equilateral triangle.

(17) Find the ratio in which the line segment joining (−2,−3) and (5,4) is divided by the co-ordinate axes.

(18) Find the ratio in which the line segment joining (2,−3) and (5,6) is divided by x-axis.

(19) Find the co-ordinates of the points of trisection of the segment joining the points (3,−8) and (9,4)

(F): INCLINATION & SLOPE OF A LINE:

(i) Angle of inclination of a line is the angle made by the line with positive direction of x-axis measured in anticlockwise direction. It is denoted by θ.

Note that $\boxed{0 \leq \theta \leq 180^\circ}$

(ii) (a) ~~Slope of a line is θ~~
If θ is the angle of inclination of a line then slope of the line is denoted by "m" and defined by $\boxed{m = \tan\theta}$

(b) Slope of a line joining two points (x_1, y_1) and (x_2, y_2) is $\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$

(iii) Let m_1 and m_2 are slopes of two lines L_1 and L_2, then

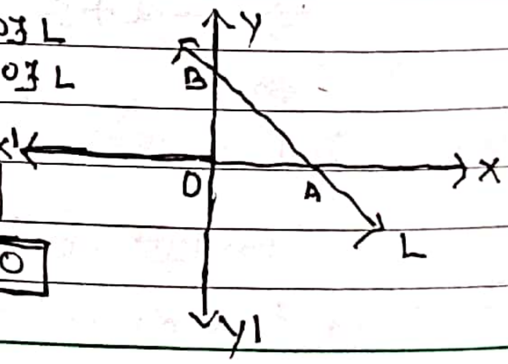
(a) $L_1 \parallel L_2 \Leftrightarrow m_1 = m_2$
(b) $L_1 \perp L_2 \Leftrightarrow m_1 \times m_2 = -1$

(iv)

Lines	Inclination	Slope
x-axis	0°	0
$\parallel$ to x-axis	0°	0
y-axis	90°	∞
$\parallel$ to y-axis	90°	∞

(F) INTERCEPTS:

(a) OA = x-intercepts of L
OB = y-intercepts of L



(b) To find x-intercepts $\boxed{\text{put } y = 0}$
To " y-intercepts, $\boxed{\text{put } x = 0}$

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PROBLEMS

(20) Find the slope of the line whose inclination is 60°

Soln Inclination $\theta = 60^\circ$

$\therefore$ slope $(m) = \tan\theta = \tan 60^\circ = \sqrt{3}$ (Ans)

(21) Find the inclination of the line whose slope is -1

Soln Here slope $m = -1$

$\Rightarrow \tan\theta = -1$ (θ = Inclination)

$\Rightarrow \tan\theta = \tan 135^\circ \Rightarrow \theta = 135^\circ$

$\therefore$ Inclination of the line is 135° (Ans)

(22) Find the inclination of the line joining the points $(1, 0)$, $(2, \sqrt{3})$.

Soln Given points are $A(1,0)$ & $B(2,\sqrt{3})$.

Slope of AB $(m) = \frac{\sqrt{3}-0}{2-1} = \frac{\sqrt{3}}{1} = \sqrt{3}$

$\Rightarrow \tan\theta = \sqrt{3}$ (θ = Inclination)

$\Rightarrow \tan\theta = \tan 60^\circ \Rightarrow \theta = 60^\circ$

$\therefore$ Inclination of the line is 60° (Ans)

(23) If slope of the line joining the points $(3,-4)$ and $(a, 2)$ is -1 then find a.

Soln Given points are $A(3,-4)$ & $B(a,2)$.

slope of AB $(m) = -1$

$\Rightarrow \frac{2-(-4)}{a-3} = -1 \Rightarrow \frac{6}{a-3} = -1$

$\Rightarrow -a+3 = 6 \Rightarrow -a = 6-3 = 3 \Rightarrow a = -3$ (Ans)

HOME TASK

(24) Find slopes of the lines whose inclinations are

(a) 30°, (b) 135° (c) 120° (d) 150° (e) 45°

(25) Find inclination of lines having slopes

(a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{3}$ (c) $-\sqrt{3}$ (d) 1 (e) $-\frac{1}{\sqrt{3}}$

(26) Find angle of inclination of the line joining the points (2, 3) and (3, 4)

(27) If slope of the line joining the points (a, 2) and (3, 4) is $\frac{1}{\sqrt{3}}$ then find a. Also find its inclination.

(G) LOCUS: (i) A path described by a moving point satisfying some geometrical conditions is called a Locus.

(ii) For example if a point in the plane moves in such a way that its distance from a fixed point always remains constant then the locus is a circle.

(H) EQUATION OF LINE:

- Equation of line is a relation in x and y to which all points on the line satisfies.

(H1) GENERAL FORM OF LINE Every first degree equation in x ~~and~~ y will represent a straight line.

Thus $\boxed{ax+by+c=0}$ where a and b are not simultaneously zero is the general form.

(H2) Slope-intercept form: $\boxed{y = mx+c}$ where m = slope, c = y-intercept

(H3) Slope-point form Equation of line having slope m and passing through the point (x_1, y_1) is

$$\boxed{y-y_1 = m(x-x_1)}$$

(H4) Two point form Equation of line passing through the points (x_1, y_1) and (x_2, y_2) is

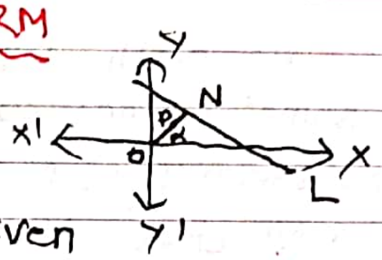
$$\boxed{y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)}$$

(H5) INTERCEPT FORM: Equation of line having x- and y-intercepts a and b respectively is

$$\boxed{\frac{x}{a}+\frac{y}{b}=1}$$

(H6) NORMAL (PERPENDICULAR) FORM

$$\boxed{x\cos\alpha + y\sin\alpha = p}$$



where p = length of the $\perp^{ar}$ drawn from origin to the given line.

α = angle made by that $\perp^{ar}$ with +ve direction of x-axis.

(H7)

LINES	**EQUATION**
x-axis	$y=0$
a line ∥ x-axis / $\perp^{ar}$ to y-axis	$y=k$ (constant)
y-axis	$x=0$
a line ∥ y-axis / $\perp^{ar}$ to x-axis.	$x=k$.

(I) CONVERSION OF GENERAL FORM INTO

(I1) SLOPE-INTERCEPT FORM

$ax+by+c=0 \Rightarrow by = -ax-c \Rightarrow y = \frac{-ax-c}{b}$

$\Rightarrow y = \left(-\frac{a}{b}\right)x + \left(-\frac{c}{b}\right)$ which is in the form of

$y = mx+c$ where $m = -\frac{a}{b}$, $c = -\frac{c}{b}$

Note that $\boxed{\text{slope }(m) = -\frac{a}{b},\ y\text{-intercept} = -\frac{c}{b}}$

(I2) INTERCEPTS FORM: $ax+by+c=0$

$\Rightarrow ax+by=-c$

$\Rightarrow \frac{ax+by}{-c} = 1$

$\Rightarrow \frac{ax}{-c} + \frac{by}{-c} = 1 \Rightarrow \frac{x}{(-c/a)} + \frac{y}{(-c/b)}$ which is in the form of $\frac{x}{A} + \frac{y}{B} = 1$, $A = -\frac{c}{a}$, $B = -\frac{c}{b}$

Note that $\boxed{\begin{array}{l} x\text{- Intercept} = -\frac{c}{a} \\ y\text{- intercept} = -\frac{c}{b} \end{array}}$

I3) NORMAL FORM

(i) Take the constant on RHS and make it (+ve)

(ii) Divide the equation by $\sqrt{a^2+b^2}$

ie $ax+by+c=0$

$\Rightarrow ax+by = -c$

$\Rightarrow -ax-by = c$ (If $c>0$)

$\Rightarrow \frac{-ax}{\sqrt{a^2+b^2}} - \frac{by}{\sqrt{a^2+b^2}} = \frac{c}{\sqrt{a^2+b^2}}$

which is in the form of $x\cos\alpha + y\sin\alpha = p$

where $\cos\alpha = \frac{-a}{\sqrt{a^2+b^2}}$, $\sin\alpha = \frac{-b}{\sqrt{a^2+b^2}}$, $p = \frac{c}{\sqrt{a^2+b^2}}$

J) ANGLE BETWEEN TWO LINES:

(i) If m_1 and m_2 are the slopes of two lines and θ be the acute angle between them then

$$\boxed{\theta = \tan^{-1}\left|\frac{m_1 - m_2}{1+m_1 m_2}\right|}$$

(ii) The angle θ between the lines $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ is

$$\boxed{\theta = \tan^{-1}\left|\frac{a_2b_1 - a_1b_2}{a_1a_2+b_1b_2}\right|}$$

(K) CONDITION OF PARALLELISM and PERPENDICULARITY

Let $L_1: a_1x+b_1y+c_1=0$

$L_2: a_2x+b_2y+c_2=0$ having slopes m_1 & m_2

then

(i) $L_1 \parallel L_2 \Leftrightarrow m_1 = m_2$

$L_1 \parallel L_2 \Leftrightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2}$

(ii) $L_1 \perp L_2 \Leftrightarrow m_1 \times m_2 = -1$

$L_1 \perp L_2 \Leftrightarrow a_1a_2 + b_1b_2 = 0$

(iii) L_1 & L_2 are identical or concident $\Leftrightarrow$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

(L) POINT OF INTERSECTION OF TWO LINES.

Let $L_1: a_1x+b_1y+c_1=0$

$L_2: a_2x+b_2y+c_2=0$

Then co-ordinate of point of intersection of L_1 & L_2 is

$$\left(\frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1}, \frac{c_1a_2-c_2a_1}{a_1b_2-a_2b_1}\right)$$

NOTE (i) Line parallel to the line $ax+by+c=0$ is $\boxed{ax+by+K=0}$.

(ii) Line perpendicular to the line $ax+by+c=0$ is $\boxed{bx-ay+K=0}$

(M) Equation of line passing through point of intersection of two lines:

Let $L_1: a_1x+b_1y+c_1=0$

$L_2: a_2x+b_2y+c_2=0$

Then equation of line passing through point of intersection of L_1 and L_2 is

$$\boxed{L_1 + \lambda L_2 = 0}$$

(N) (i) Equation of line passing through a point and parallel or perpendicular to a line:

step-i Find slope of given line.

step-ii Find slope of Required line using condition of parallelism or perpendicularity.

step-iii Apply slope-point formula to find required line.

(O) DISTANCE OF a POINT from a LINE.

(i) Distance of point (x_1, y_1) from the line $ax+by+c=0$ is

$$\boxed{d = \left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|}$$

(ii) Distance of line $ax+by+c=0$ from origin is

$$\boxed{d = \left|\frac{c}{\sqrt{a^2+b^2}}\right|}$$

(iii) Distance between two parallel lines $ax+by+c_1=0$ & $ax+by+c_2=0$ is

$$d = \left|\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right|$$

PROBLEMS

NO 28) Find the equation of line which passes through the point (3,4) and sum of its intercepts is 14 $(4x+3y=24)$ or $(x+y-7=0)$ (Ans)

(29) Find the equation of line Bisecting the line segment joining (3,−4) and (1,2) at right angles $(x-3y-5=0)$ (Ans)

(30) Find the equation of line passing through (1,−2) and making intercept 2 on y-axis.
Ans: $(4x+y-2=0)$ (Ans)

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(31) Find equation of line passing through the point (2,3) and parallel to the join of (4,−5) and (−7,3)
$(8x+11y-49=0)$

(32) Reduce $2x-3y+7=0$ to the slope form and find its intercepts on y-axis. $(c=7/3, m=\frac{2}{3})$

(33) Reduce $3x+5y+4=0$ to intercept form and hence find its intercepts $(a=-\frac{4}{3}, b=-4/5)$.

(34) Reduce $3x-y+2=0$ to normal form and find the length of the perpendicular segment from origin to the line. $(p=\frac{2}{\sqrt{10}})$

(35) Find the equation of line parallel to x-axis and passing through (1,3). $(y=3)$

(36) Find equation of line passing through (−4,2), and parallel to the line $4x-3y=0$. $(4x-3y+22=0)$

(37) Find equation of line passing through the point (−2,3) and perpendicular to the line $3x+4y-11=0$
$(4x-3y+17=0)$

(38) Find equation of line passing through the point of intersection of lines $x+3y+2=0$ and $x-2y-4=0$ and perpendicular to the line $2y+5x-9=0$.
$(10x-25y-46=0)$

(39) Find the length of the ⊥ar drawn from the point (−3,−4) to the line $12x-5y+65=0$ $(\frac{49}{13})$

(40) Find the distance of the point (3,2) from the line $x+3y-1=0$, measured parallel to the line $3x-4y+1=0$ $(A-\frac{40}{13})$

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(41) Find the equation of line which passes through the point (3, -4) and is such that its portion between the axes is divided at that point internally in the ratio 2:3 $(8x-9y-60=0)$

(42) Find k if $2x-3y+4=0$ and $4x+ky+1=0$ are parallel $(k=-6)$

(43) Find k if $2x+3y-1=0$ and $kx-4y+2=0$ are $\perp$ar. $(k=6)$

(44) Find k if $2x+3y+k=0$ and $kx-3y+2=0$ are identical. $(k=-2)$

(45) Find the distance between the lines $2y-3=0$ and $3y-2=0$ $(\frac{5}{6})$

(46) Find the distance between the lines $2x-3y+9=0$ and $4x-6y+1=0$ $(\frac{17}{2\sqrt{13}})$

(47) Find the angle between the lines $x+y+7=0$ and $x-y+1=0$ 90°

(48) Find the foot of the perpendicular from the point (2,3) on the line $3x-4y+7=0$ $(\frac{47}{25}, \frac{79}{25})$

SOLUTION:

NO28) Let equation of required line be

$$\frac{x}{a}+\frac{y}{b}=1 \quad \text{——} (1)$$

Line (1) passes through the point (3,4)

$$\therefore \quad \frac{3}{a}+\frac{4}{b}=1 \quad \text{——} (2)$$

Again sum of intercepts of line (1) is 14

$$\therefore \quad a+b=14 \Rightarrow b=14-a \quad \text{——} (3)$$

from eq^n(2), $\frac{3}{a}+\frac{4}{b}=1 \Rightarrow \frac{3}{a}+\frac{4}{14-a}=1$

$\Rightarrow \frac{42-3a+4a}{a(14-a)}=1 \Rightarrow \frac{42+a}{14a-a^2}=1 \Rightarrow 42+a=14a-a^2$

$\Rightarrow$ ~~$a^2 + 42 - 14 = 0$~~ $\Rightarrow a^2+a-14a+42=0$

$\Rightarrow a^2-13a+42=0 \Rightarrow a^2-6a-7a+42=0$

$\Rightarrow a(a-6)-7(a-6)=0 \Rightarrow (a-6)(a-7)=0$

$\Rightarrow (a-6)=0$ or $a-7=0 \Rightarrow a=6$ or $a=7$

If $a=6$ then $b=14-a=14-6=8$

$\therefore$ Equation of Required line is

$\frac{x}{a}+\frac{y}{b}=1 \Rightarrow \frac{x}{6}+\frac{y}{8}=1 \Rightarrow \frac{4x+3y}{24}=1$

$\Rightarrow 4x+3y-24=0$ ——— (4)

If $a=7$ then $b=14-a=14-7=7$

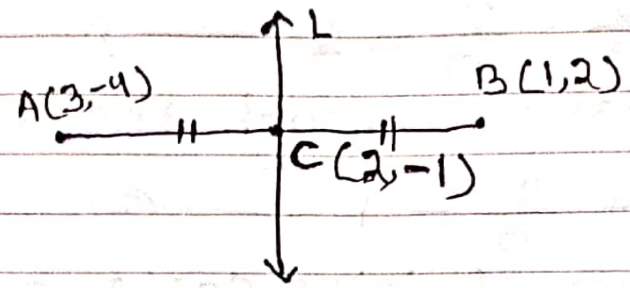
$\therefore$ Equation of Required line is $\frac{x}{a}+\frac{y}{b}=1$

$\Rightarrow \frac{x}{7}+\frac{y}{7}=1 \Rightarrow x+y-7=0$ ——— (5)

Hence equation of required line is

$4x+3y-24=0$ or $x+y-7=0$ (Ans)

(29) Given points are $A(3,-4)$ & $B(1,2)$



Line 'L' Bisect $\overline{AB}$ at C at right angle.
Hence "C" is the mid point of $\overline{AB}$

$\therefore$ Coordinate of C $= \left(\frac{3+1}{2}, \frac{-4+2}{2}\right) = (2,-1)$

Slope of $\overline{AB}$ $m_1 = \frac{2-(-4)}{1-3} = \frac{6}{-2} = -3$

$\because$ $L \perp \overline{AB}$, slope of L, $m_2 = -\frac{1}{m_1} = -\frac{1}{-3} = \frac{1}{3}$

Now Line L passes through point $C(2,-1)$ having slope m_2.

Hence Equation of Line L is

$$y - y_1 = m_2(x - x_1)$$

$$\Rightarrow y-(-1) = \frac{1}{3}(x-2)$$

$$\Rightarrow 3y+3 = x-2 \Rightarrow x-3y-2-3=0$$

$$\Rightarrow x-3y-5=0$$

$\therefore$ Equation of required line is $x-3y-5=0$ (Ans)

(30) Let equation of required line be $y = mx + c$ —— ①
where m = slope, c = y-intercepts.
It is given that $c = 2$.
Also line① passes through the point $(1,-2)$
Hence from①, $-2 = m(1)+2 \Rightarrow m = -4$

Hence Equation of Required line is

$$y = mx + c \Rightarrow y = -4x + 2$$

$$\Rightarrow 4x + y - 2 = 0 \quad (\text{Ans})$$

(36) Given point is $P(-4, 2) = P(x_1, y_1)$ (say)

Given line is $4x - 3y = 0$ —— (1)

slope of line (1) $m_1 = -\frac{a}{b} = -\frac{4}{-3} = \frac{4}{3}$

$\because$ Required line is parallel to line (1),

slope of required line is

$$m_2 = \frac{4}{3}$$

$\therefore$ Equation of required line is

$$y - y_1 = m_2(x - x_1)$$

$$\Rightarrow y - 2 = \frac{4}{3}(x + 4) \Rightarrow 3y - 6 = 4x + 16$$

$$\Rightarrow 4x - 3y + 6 + 16 = 0 \Rightarrow 4x - 3y + 22 = 0 \quad (\text{Ans})$$

(31) Given points are $A(2, 3)$, $B(4, -5)$ & $C(-7, 3)$

Slope of line $\overleftrightarrow{BC}$ $m_1 = \frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{3 - (-5)}{-7 - 4} = \frac{-8}{11}$$

Since required line is parallel to $\overleftrightarrow{BC}$,

Slope of required line is

$$m = m_1 = -\frac{8}{11}$$

Required line passes through $A(2, 3)$

$\leq A(x_1, y_1)$

Equation of required line is

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow y - 3 = -\frac{8}{11}(x-2)$$

$$\Rightarrow 11y - 33 = -8x + 16$$

$$\Rightarrow 8x + 11y - 33 - 16 = 0$$

$$\Rightarrow 8x + 11y - 49 = 0 \quad \text{(Ans)}$$

(32) $2x - 3y + 7 = 0 \Rightarrow 3y = 2x + 7 \Rightarrow y = \frac{2x+7}{3}$

$$\Rightarrow y = \frac{2x}{3} + \frac{7}{3} \Rightarrow y = \left(\frac{2}{3}\right)x + \frac{7}{3} \quad \text{——(1)}$$

which is the required slope form $y = mx + c$ where slope $m = \frac{2}{3}$ and intercept on y-axis is $\frac{7}{3}$ (Ans)

(33) $3x + 5y + 4 = 0 \Rightarrow 3x + 5y = -4$

$$\Rightarrow \frac{3x + 5y}{-4} = \frac{-4}{-4}$$

$$\Rightarrow \frac{3x}{-4} + \frac{5y}{-4} = 1 \Rightarrow \frac{x}{(-4/3)} + \frac{y}{(-4/5)} = 1 \quad \text{——(1)}$$

which is the required intercept form

From (1) x-intercept $a = -4/3$

and y-intercept $b = -4/5$ (Ans)

(34) $3x - y + 2 = 0$

$$\Rightarrow 3x - y = -2$$

$$\Rightarrow -3x + y = 2 \Rightarrow \frac{-3}{\sqrt{(-3)^2+(1)^2}}x + \frac{1}{\sqrt{(-3)^2+(1)^2}}y = \frac{2}{\sqrt{(-3)^2+(1)^2}}$$

$\Rightarrow \left(-\frac{3}{\sqrt{10}}\right)x + \left(\frac{1}{\sqrt{10}}\right)y = \frac{2}{\sqrt{10}}$ which is ——(1)

the required Normal form.

From (1) Length of the perpendicular segment from origin is $\frac{2}{\sqrt{10}}$ (Ans)

35》 Equation of line parallel to x-axis is $y = K$ ——(1)

since line (1) passes through the point (1, 3), $3 = K$

$\Rightarrow K = 3$

$\therefore$ Equation of required line is

$y = K \Rightarrow y = 3$ (Ans)

37 Given line is $3x + 4y - 11 = 0$ ——(1)

and point is $P(-2, 3) = P(x_1, y_1)$ (say)

slope of line (1) is

$m_1 = -\frac{a}{b} = -\frac{3}{4}$

Required line is $\perp$ to line (1)

$\therefore$ slope of ~~line~~ Required line is

$m = -\frac{1}{m_1} = \frac{4}{3}$

Hence equation of required line is

$y - y_1 = m(x - x_1)$ ($\because$ Required line passes through P)

$\Rightarrow y - 3 = \frac{4}{3}(x + 2)$

$\Rightarrow 3y - 9 = 4x + 8$

$\Rightarrow 4x - 3y + 9 + 8 = 0 \Rightarrow 4x - 3y + 17 = 0$ (Ans)

(38) Given lines are

$$x+3y+2=0 \quad \text{---} (1)$$

&

$$x-2y-4=0 \quad \text{---} (2)$$

$$2y+5x-9=0 \quad \text{---} (3)$$

Required line passes through point of intersection of line (1) & (2). Hence,

Let Equation of required line be

$$(x+3y+2)+K(x-2y-4)=0$$

$$\Rightarrow (K+1)x+(3-2K)y+2-4K=0 \quad \text{---} (4)$$

Slope of line (3) $m_1 = -\frac{a}{b} = -\frac{5}{2}$

Slope of line (4) $m_2 = -\frac{a}{b} = -\frac{(K+1)}{3-2K}$

But line (3) $\perp^{ar}$ line (4)

$$\Rightarrow m_1 \times m_2 = -1$$

$$\Rightarrow \left(-\frac{5}{2}\right) \times \left[-\frac{(K+1)}{3-2K}\right] = -1$$

$$\Rightarrow \frac{5K+5}{6-4K} = -1 \Rightarrow 5K+5 = 4K-6$$

$$\Rightarrow 5K-4K = -6-5 \Rightarrow K = -11$$

$\therefore$ Equation of required line is

$$(K+1)x+(3-2K)y+2-4K=0$$

$$\Rightarrow (-11+1)x+(3+22)y+2+44=0$$

$$\Rightarrow -10x+25y+46=0$$

$$\Rightarrow 10x-25y-46=0 \text{ (Ans)}$$

(39) Given line is

$$12x - 5y + 65 = 0 \quad \text{——} (1)$$

and point is $P(-3,-4)$

$$d = \left|\frac{12(-3) - 5(-4) + 65}{\sqrt{(12)^2 + (-5)^2}}\right| = \left|\frac{-36+20+65}{\sqrt{144+25}}\right|$$

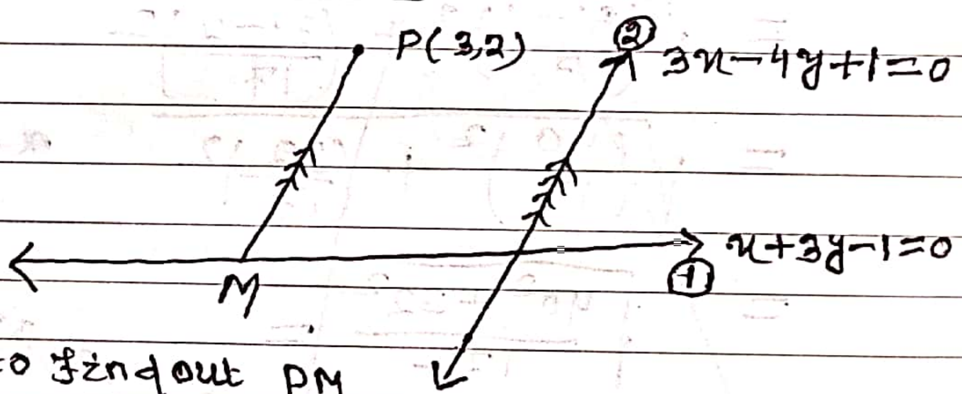
$$= \left|\frac{49}{\sqrt{169}}\right| = \left|\frac{49}{13}\right| = \frac{49}{13} \text{ unit}$$

$\therefore$ Length of the perpendicular drawn from the point $(-3,-4)$ to the line $12x - 5y + 65 = 0$ is $\frac{49}{13}$ unit

Ans.

(40) Given point is $P(3,2)$ and lines are $x + 3y - 1 = 0$ ——(1)

$$3x - 4y + 1 = 0 \quad \text{——} (2)$$



We have to find out PM

Slope of line (2) $m_1 = -\frac{a}{b} = \frac{-3}{-4} = \frac{3}{4}$

$\overline{PM} \parallel$ line(2)

$\Rightarrow$ slope of $\overline{PM}$ $m_2 = m_1 = \frac{3}{4}$

Also $\overline{PM}$ passes through $P(3,2) = (x_1, y_1)$ (say)

Hence equation of $\overleftrightarrow{PM}$ is

$$y - y_1 = m_2(x - x_1)$$

$$\Rightarrow y - 2 = \frac{3}{4}(x - 3)$$

$4y-8 = 3x-9 \Rightarrow 3x-4y-9+8=0$

$\Rightarrow 3x-4y-1=0$ ——— (3)

Solving Eqⁿ (1) & (3) coordinate of M

$$= \left(\frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1}, \frac{c_1a_2-c_2a_1}{a_1b_2-a_2b_1}\right)$$

$$= \left(\frac{(3)(-1)-(-4)(-1)}{(1)(-4)-(3)(3)}, \frac{(-1)(3)-(-1)(1)}{(1)(-4)-(3)(3)}\right)$$

$$= \left(\frac{-7}{-13}, \frac{-2}{-13}\right) = \left(\frac{7}{13}, \frac{2}{13}\right)$$

Now $PM = \sqrt{\left(3-\frac{7}{13}\right)^2 + \left(2-\frac{2}{13}\right)^2}$

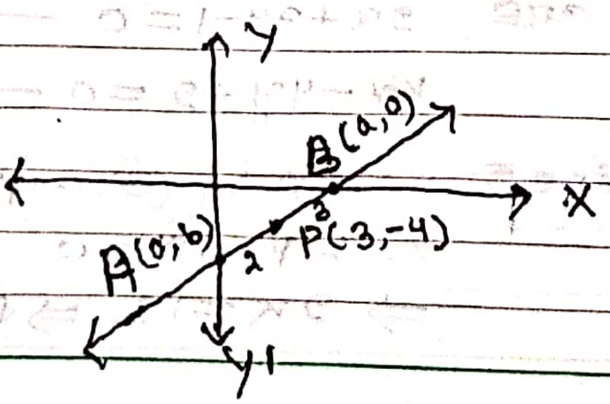
$$= \sqrt{\left(\frac{32}{13}\right)^2 + \left(\frac{24}{13}\right)^2}$$

$$= \sqrt{\frac{32^2+24^2}{13^2}} = \sqrt{\frac{1024+576}{13^2}}$$

$$= \sqrt{\frac{1600}{13^2}} = \frac{40}{13} \text{ unit.}$$

$\therefore$ The required distance is $\frac{40}{13}$ unit. (Ans)

(41) Given point is $P(3,-4)$. Given ratio is $2:3$



MATH-iii/25:

Let equation of required line be $\frac{x}{a}+\frac{y}{b}=1$ —— (1)

It cuts X-axis at A(a,0) and Y axis at B(0,b)

Line (1) passes through P(3,-4) s.t AP : BP = 2 : 3

$\therefore$ co-ordinate of $p = \left(\frac{2\times a+3\times 0}{2+3}, \frac{2\times 0+3\times b}{2+3}\right)=\left(\frac{2a}{5}, \frac{3b}{5}\right)$

$\therefore \left(\frac{2a}{5}, \frac{3b}{5}\right) = (3,-4)$

$\Rightarrow \frac{2a}{5} = +3$ and $\frac{3b}{5} = -4$

$\Rightarrow 2a = +15$ and $3b = -20$

$\Rightarrow a = \frac{15}{2}$ and $b = -\frac{20}{3}$

From (1) Equation of Required line is

$\frac{x}{a}+\frac{y}{b}=1 \Rightarrow \frac{x}{(15/2)}+\frac{y}{(-20/3)}=1$

$\Rightarrow \frac{2x}{15}-\frac{3y}{20}=1 \Rightarrow \frac{8x-9y}{60}=1$

$\Rightarrow 8x-9y=60 \Rightarrow 8x-9y-60=0$ (Ans)

42) Given lines are $2x-3y+4=0$ —— (1)

$4x+ky+1=0$ —— (2)

Line (1) $\parallel$ Line (2)

$\Rightarrow \frac{2}{4} = -\frac{3}{k} \Rightarrow \frac{1}{2} = -\frac{3}{k} \Rightarrow k=-6$ (Ans)

(43) Given lines are $2x+3y-1=0$ —— (1)

and $kx-4y+2=0$ —— (2)

Line (1) $\perp^{ar}$ Line (2) $\Rightarrow (2)(k)+(3)(-4)=0$

$\Rightarrow 2k-12=0$

$\Rightarrow 2k=12 \Rightarrow k=6$ (Ans)

44) Given lines are $2x+3y+K=0$ —— (1)
and $Kx-3y+2=0$ —— (2)

Line (1) & (2) are identical

$\Rightarrow \frac{2}{K} = \frac{3}{-3} = \frac{K}{2}$

$\Rightarrow \frac{2}{K} = -1 = \frac{K}{2}$ —— (3)

From (3) $\frac{2}{K} = \frac{K}{2} \Rightarrow K^2 = 4 \Rightarrow K = \sqrt{4} = \pm 2$

But $K = +2$ doesnot satisfy (3) Hence
$K = -2$ (Ans)

(45) Given lines are $2y-3=0$ —— (1)
and $3y-2=0$
$\Rightarrow 2y - \frac{4}{3} = 0$ —— (2) multiplying $\frac{2}{3}$

Line (1) ∥ (2) $\Rightarrow d = \left|\frac{-3-(-\frac{4}{3})}{\sqrt{0^2+2^2}}\right| = \left|\frac{-3+\frac{4}{3}}{2}\right|$

$= \left|\frac{-\frac{5}{3}}{2}\right| = \frac{5}{6}$ unit.

(46) Given lines are $2x-3y+9=0$ —— (1)
and $4x-6y+1=0$
$\Rightarrow 2x-3y+\frac{1}{2}=0$ —— (2)
(Dividing by 2)

Line (1) ∥ Line (2). Here $c_1 = 9,\ c_2 = \frac{1}{2},\ a=2, b=-3$

Distance $d = \left|\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right| = \left|\frac{9-\frac{1}{2}}{\sqrt{2^2+3^2}}\right| = \left|\frac{17/2}{\sqrt{13}}\right|$

$= \frac{17}{2\sqrt{13}}$ units (Ans)

MATH-I/27

(47) Given lines are

$$x+y+7=0 \text{ ——— } (1)$$

and $x-y+1=0$ ——— (2)

Here $a_1=1, b_1=1, c_1=7$

$a_2=1, b_2=-1, c_2=1$

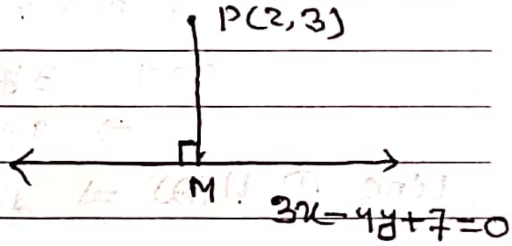
Let θ be the angle between (1) & (2)

$$\therefore \theta = \tan^{-1}\left|\frac{a_2b_1-a_1b_2}{a_1a_2+b_1b_2}\right| = \tan^{-1}\left|\frac{(1)(1)-(1)(-1)}{(1)(1)+(1)(-1)}\right|$$

$$= \tan^{-1}\left|\frac{2}{0}\right| = \tan^{-1}\infty = \frac{\pi}{2}$$

(48) Given point is $P(2,3)$ and the line is

$$3x-4y+7=0 \text{ ——— } (1)$$



$\overline{PM} \perp$ line (1) and M is the foot of the perpendicular

Slope of line (1) $m_1 = -\frac{a}{b}$

$$= \frac{-3}{-4} = \frac{3}{4}$$

Slope of $\overline{PM} = m_2 = -\frac{1}{m_1}$ ($\because \overline{PM} \perp$ line (1))

$$= -\frac{4}{3}$$

Equation of PM is

$$y-y_1 = m_2(x-x_1)$$

$\Rightarrow y-3 = -\frac{4}{3}(x-2)$ $\because \overline{PM}$ passes through $P(2,3) = P(x_1,y_1)$

$\Rightarrow 3y-9 = -4x+8$

$\Rightarrow 4x+3y-9-8=0$

$\Rightarrow 4x+3y-17=0$ ——— (2)

Now Eq^n(1)$\times 4 \Rightarrow 12x-16y+28=0$

Eq^n(2)$\times 3 \Rightarrow 12x+9y-51=0$

$\quad (-) \quad (-) \quad (+) \quad (-)$

$$-25y+79=0$$

$$\Rightarrow y = \frac{79}{25}$$

From ① $3x - 4y + 7 = 0$

$$\Rightarrow 3x = 4y - 7 = 4 \times \frac{79}{25} - 7 = \frac{316 - 175}{25}$$

$$= \frac{141}{25}$$

$$\Rightarrow x = \frac{141}{25} \times \frac{1}{3} = \frac{47}{25}$$

$\therefore$ co-ordinate of M $= (x, y) = \left(\frac{47}{25}, \frac{79}{25}\right)$

$\therefore$ Foot of the perpendicular is $\left(\frac{47}{25}, \frac{79}{25}\right)$ (Ans)

→END←