(5) CO-ORDINATE GEOMETRY IN THREE DIMENSION (3D)

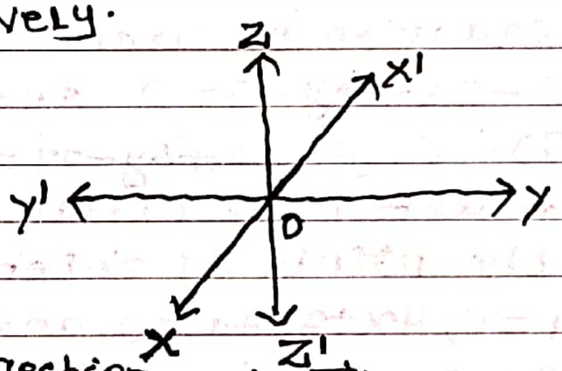
MATH-I/1:

A: Rectangular Co-ordinate axes:

→ To locate the position of a point in space two numbers x, y are not sufficient. It requires three numbers x, y, z.

→ Let's consider three mutually perpendicular lines $X'OX$, $Y'OY$ and $Z'OZ$ intersecting with eachother at O. point 'O' is called origin

→ The lines $X'OX, Y'OY, Z'OZ$ are called x-axis, y-axis and z axis respectively.



→ $\overrightarrow{OX}$ represents +ve direction and $\overrightarrow{OX'}$ represents −ve direction of x-axis. similarly for y axis and z-axis.

→ Two axes in pair gives a plane. Thus we have three planes i.e XY, YZ, ZX. These planes are called co-ordinate planes

→ position of any point 'p' in space is determined by an order triple of real numbers (x, y, z) called Co-ordinate of P. where

x = length of $\perp^{ar}$ from P on YZ plane.

y = " " " " P on ZX plane

z = " " " " P on XY plane.

→ Three planes divides the space into 8 parts. Each parts is known as octant.

MATH-I/2

These octants and signs of x, y, z of any point are given below.

$OXYZ\,(+\,+\,+)$, $OX'YZ\,(-\,+\,+)$, $OXY'Z\,(+\,-\,+)$, $OXYZ'\,(+\,+\,-)$,
$OX'Y'Z\,(-\,-\,+)$, $OXY'Z'\,(+\,-\,-)$, $OX'YZ'\,(-\,+\,-)$, $OX'Y'Z'\,(-\,-\,-)$

NOTE:

(i) Axis

Axis	co-ordinate of any point.	Equation
X	$(a, 0, 0)$	$y=0, z=0$
Y	$(0, b, 0)$	$x=0, z=0$
Z	$(0, 0, c)$	$x=0, y=0$
origin	$(0, 0, 0)$	—

(ii) Plane

Plane	co-ordinates	Equations
xy / ∥ to xy	$(a, b, 0)$	$z=0$ / $z=k$
yz / ∥ to yz	$(0, b, c)$	$x=0$ / $x=k$
zx / ∥ to zx	$(a, 0, c)$	$y=0$ / $y=k$

B: DISTANCE FORMULA:

(i) Distance between two points $A(x_1, y_1, z_1)$ & $B(x_2, y_2, z_2)$ is

$$\boxed{AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}}$$

(ii) Distance of $P(x, y, z)$ from origin is $\boxed{\sqrt{x^2+y^2+z^2}}$

(C) DIVISION FORMULA

(i) Co-ordinates of a point which divides the line segment joining two points (x_1, y_1, z_1) and (x_2, y_2, z_2) internally in the ratio $m:n$ is

$$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right)$$

(ii) If it divides externally then its co-ordinates is

$$\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}, \frac{mz_2-nz_1}{m-n}\right)$$

(iii) Mid point of the line joining (x_1, y_1, z_1) and (x_2, y_2, z_2) is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$

(iv) co-ordinates of the point which divides the join of (x_1, y_1, z_1) and (x_2, y_2, z_2) is $\left(\frac{kx_2+x_1}{k+1}, \frac{ky_2+y_1}{k+1}, \frac{kz_2+z_1}{k+1}\right)$.

If $k < 0$ then it divides externally and if $k > 0$ then it divides internally.

(D) CENTROID OF A TRIANGLE:

(i) The point of intersection of medians of a triangle is called centroid.

(ii) centroid divides a median in the ratio $2:1$

(iii) Co-ordinate of centroid of a triangle having vertices (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3) is

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$$

(E) Direction cosine of a line: (dcs) & Direction ratios (drs)

(i) Direction cosines of a line are the cosines of angles that the line makes with positive direction of x- y- and z- axis respectively.

(ii) If α, β, γ are the angles made by the line with +ve direction of x-, y-, & z-axis respectively then $\cos\alpha$, $\cos\beta$, $\cos\gamma$ are called direction cosines which are denoted by l, m, n.

$\therefore$ $\boxed{l = \cos\alpha,\ m = \cos\beta,\ n = \cos\gamma}$

α, β, γ are called direction angle.

(iii) If l, m, n are dcs of a line then a, b, c are called drs if $\boxed{\frac{a}{l} = \frac{b}{m} = \frac{c}{n}}$ i.e. drs are proportional to dcs.

(iv) dcs of a line are unique but a line may have infinite number of sets of drs.

(v) If $\langle a, b, c\rangle$ are drs and $\langle l, m, n\rangle$ are dcs of a line then

$$\boxed{l = \frac{a}{\sqrt{a^2+b^2+c^2}},\ m = \frac{b}{\sqrt{a^2+b^2+c^2}},\ n = \frac{c}{\sqrt{a^2+b^2+c^2}}}$$

(vi) If l, m, n are dcs of a line then $\boxed{l^2+m^2+n^2=1}$

(vii) Drs of a line joining two points (x_1, y_1, z_1) and (x_2, y_2, z_2) are $\boxed{\langle x_2-x_1,\ y_2-y_1,\ z_2-z_1\rangle}$

(viii) If O be the origin and $P(x, y, z)$ be any point such that $OP = r$. Then co-ordinates of P are $\boxed{(lr, mr, nr)}$

(ix) dcs of x, y, z axes are $\langle 1,0,0\rangle, \langle 0,1,0\rangle, \langle 0,0,1\rangle$

(F) Angle between two lines

(i) If $\langle l_1, m_1, n_1\rangle$ and $\langle l_2, m_2, n_2\rangle$ are dcs of two lines and θ be the angle between them then

$$\boxed{\theta = \cos^{-1}(l_1l_2 + m_1m_2 + n_1n_2)}$$

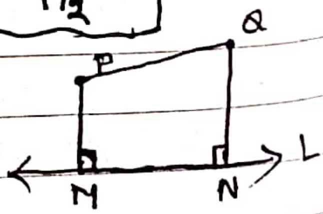
(ii) If $\langle a_1, b_1, c_1\rangle$ and $\langle a_2, b_2, c_2\rangle$ are drs of two lines and θ is the angle between them then

$$\boxed{\theta = \cos^{-1}\left(\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right)}$$

(iii) $L_1 \perp L_2 \Leftrightarrow \boxed{a_1a_2+b_1b_2+c_1c_2=0}$ or $\boxed{l_1l_2+m_1m_2+n_1n_2}=0$

$L_1 \parallel L_2 \Leftrightarrow \boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}}$ or $\boxed{\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}}$

G: projection of a line segment on another line:



(i) MN = projection of $\overline{PQ}$ on line L

(ii) If $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are two points, $\langle l, m, n\rangle$ are dcs of line L then

$$\boxed{MN = (x_2-x_1)l + (y_2-y_1)m + (z_2-z_1)n}$$

problems:

(1) Find dcs of the line joining the points (2,1,2), (4,2,0) A- $\langle \frac{2}{3}, \frac{1}{3}, -\frac{2}{3} \rangle$

(2) Find the angle between two lines having direction ratios 2,3,6 and 1,2,2. $\theta = \cos^{-1}(\frac{20}{21})$

(3) Find the angle between the lines having dcs $\langle \frac{\sqrt{3}}{4}, \frac{1}{4}, \frac{\sqrt{3}}{2} \rangle$ and $\langle \frac{\sqrt{3}}{4}, \frac{1}{4}, -\frac{\sqrt{3}}{2} \rangle$ $(\theta = 120°)$

(4) Find the ratio in which the line joining the points (2,4,5) and (3,5,-4) is divided by xy plane. (5:4)

(5) Find the foot of the perpendicular from A(1,1,1) on the line joining B(1,4,6) and C(5,4,4) (3,4,5)

(6) Show that the points (3,2,4), (4,5,2), (5,8,0) are co-linear.

(7) The projection of a line on the axes are 6, 2, 3. Find the length of the line and its direction cosines.

(8) Find image of the point (6,3,-4) w.r.t yz-plane. (-6,3,-4)

(9) Find projection of line segment joining (1,3,-1) and (3,2,4) on z-axis.

HOME TASK:

(10) If A, B, C are the points (1,4,2), (2,1,2), (2,-3,4). Find angles of the triangle ABC.

(11) Find co-ordinates of the point which divides the join (1,2,3) and (3,-5,6) in the ratio 3:-5

(12) Find the ratio in which the line through (2,4,5) and (3,5,-4) is divided by xy plane.

(13) Find the ratio in which the line joining (2,-3,1) and (3,-4,-5) is divided by the line $2x+y+z=7$.

(14) Show that A(0,0,0), B(3,4,5), C(-3,-4,-5) are co-linear.

(15) Find the ratio in which the join of the points (-2,4,7), (3,-5,8) is divided by co-ordinate planes

MATH-2/6

① Given points are $A(2,1,2)$ and $B(4,2,0)$

drs of $\overleftrightarrow{AB} = \langle 4-2, 2-1, 0-2 \rangle = \langle 2,1,-2 \rangle$

dcs of $\overleftrightarrow{AB} = \left\langle \frac{2}{\sqrt{2^2+1^2+(-2)^2}}, \frac{1}{\sqrt{2^2+1^2+(-2)^2}}, \frac{-2}{\sqrt{2^2+1^2+(-2)^2}} \right\rangle$

$= \left\langle \frac{2}{3}, \frac{1}{3}, -\frac{2}{3} \right\rangle$ (Ans)

② drs of lines are $\langle 2,3,6 \rangle$ and $\langle 1,2,2 \rangle$

Here $a_1 = 2$, $b_1 = 3$, $c_1 = 6$

$a_2 = 1$, $b_2 = 2$, $c_2 = 2$

Let θ be the angle between the lines.

Then
$$\theta = \cos^{-1}\left[\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right]$$

$$= \cos^{-1}\left[\frac{2\times1+3\times2+6\times2}{\sqrt{2^2+3^2+6^2}\sqrt{1^2+2^2+2^2}}\right]$$

$$= \cos^{-1}\left[\frac{20}{\sqrt{49}\sqrt{9}}\right] = \cos^{-1}\left[\frac{20}{7\times3}\right]$$

$$= \cos^{-1}\left(\frac{20}{21}\right)$$

③ dcs of lines are $\left\langle \frac{\sqrt{3}}{4}, \frac{1}{4}, \frac{\sqrt{3}}{2} \right\rangle$ and $\left\langle \frac{\sqrt{3}}{4}, \frac{1}{4}, -\frac{\sqrt{3}}{2} \right\rangle$

Here $l_1 = \frac{\sqrt{3}}{4}, m_1 = \frac{1}{4}, n_1 = \frac{\sqrt{3}}{2}$

$l_2 = \frac{\sqrt{3}}{4}, m_1 = \frac{1}{4}, n_1 = -\frac{\sqrt{3}}{2}$

Let θ be the angle between the lines. Then

$$\theta = \cos^{-1}[l_1l_2+m_1m_2+n_1n_2]$$

$$= \cos^{-1}\left[\frac{\sqrt{3}}{4}\times\frac{\sqrt{3}}{4}+\frac{1}{4}\times\frac{1}{4}+\frac{\sqrt{3}}{2}\left(-\frac{\sqrt{3}}{2}\right)\right]$$

$= \cos^{-1}\left[\frac{3}{16}+\frac{1}{16}-\frac{3}{4}\right] = \cos^{-1}\left[\frac{3+1-12}{16}\right] = \cos^{-1}\left[\frac{-8}{16}\right]$

$= \cos^{-1}\left[-\frac{1}{2}\right] = 120^\circ$ (Ans)

(4) Given points are $A(2,4,5)$ and $B(3,5,-4)$

Let xy-plane divides $\overline{AB}$ at P in the ratio $K:1$.

Then co-ordinate of $P = \left(\frac{3K+2}{K+1}, \frac{5K+4}{K+1}, \frac{-4K+5}{K+1}\right)$

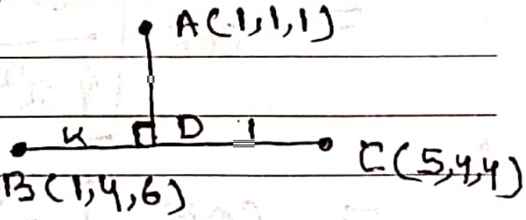
But P being a point on xy-plane,

$$\frac{-4K+5}{K+1} = 0 \Rightarrow -4K+5=0 \Rightarrow K=\frac{5}{4}$$

Required ratio $= K:1 = \frac{5}{4}:1 = 5:4$ (Ans)

(5) Given points are $A(1,1,1)$, $B(1,4,6)$, $C(5,4,4)$

$\overline{AD} \perp \overline{BC}$ and 'D' is the foot of the perpendicular.



Let 'D' divides $\overline{BC}$ in the $K:1$ ratio. Then

Co-ordinate of $D = \left(\frac{5K+1}{K+1}, \frac{4K+4}{K+1}, \frac{4K+6}{K+1}\right)$

Dr's of $\overleftrightarrow{BC} = \langle 5-1, 4-4, 4-6\rangle = \langle 4, 0, -2\rangle$

Dr's of $\overline{AD} = \left\langle \frac{5K+1}{K+1}-1, \frac{4K+4}{K+1}-1, \frac{4K+6}{K+1}-1\right\rangle$

$= \left\langle \frac{4K}{K+1}, \frac{3K+3}{K+1}, \frac{3K+5}{K+1}\right\rangle$

But $\overleftrightarrow{AD} \perp \overleftrightarrow{BC}$

$$\Rightarrow 4 \times \frac{4K}{K+1} + 0 \times \frac{3K+3}{K+1} + (-2) \times \frac{3K+5}{K+1} = 0$$

$$\Rightarrow \frac{16K - 6K - 10}{K+1} = 0 \Rightarrow 10K - 10 = 0 \Rightarrow K = 1$$

$$\therefore D = \left(\frac{5K+1}{K+1}, \frac{4K+4}{K+1}, \frac{4K+6}{K+1}\right)$$

$$= \left(\frac{5\times1+1}{1+1}, \frac{4\times1+4}{1+1}, \frac{4\times1+6}{1+1}\right) = (3, 4, 5)$$

$\therefore$ Foot of the perpendicular is $(3,4,5)$ (Ans)

(6) Given points are $A(3,2,4)$, $B(4,5,2)$, $C(5,8,0)$

dr's of $\overleftrightarrow{AB} = \langle 4-3, 5-2, 2-4 \rangle = \langle 1, 3, -2 \rangle = \langle a_1, b_1, c_1 \rangle$

dr's of $\overleftrightarrow{AC} = \langle 5-3, 8-2, 0-4 \rangle = \langle 2, 6, -4 \rangle = \langle a_2, b_2, c_2 \rangle$

Here $\frac{a_1}{a_2} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$, $\frac{c_1}{c_2} = \frac{-2}{-4} = \frac{1}{2}$

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\Rightarrow \overleftrightarrow{AB} \parallel \overleftrightarrow{AC}$, But they have a point A in common. Hence A, B, C lie on same line i.e A, B, C are colinear.

(7) Let the line be $\overleftrightarrow{OP}$. projection of $\overleftrightarrow{OP}$ on axes are $\langle 6, 2, 3 \rangle$. Hence co-ordinate of P is $(6,2,3)$

If l, m, n are dc's of $\overleftrightarrow{OP}$ and $\overline{OP} = r$ then co-ordinate of P is (lr, mr, nr)

$\therefore lr = 6,\ mr = 2,\ nr = 3$ —(1) Now

$$l^2r^2 + m^2r^2 + n^2r^2 = 6^2 + 2^2 + 3^2 = 49$$

$$\Rightarrow r^2(l^2+m^2+n^2) = 49 \Rightarrow r = \sqrt{49} = 7 \quad (\because l^2+m^2+n^2=1)$$

From (1) $l = \frac{6}{r} = \frac{6}{7}$, $m = \frac{2}{r} = \frac{2}{7}$, $n = \frac{3}{r} = \frac{3}{7}$

$\therefore$ Length $= 7$ unit and dc's are $\left\langle \frac{6}{7}, \frac{2}{7}, \frac{3}{7} \right\rangle$ (Ans)

(8) We know that image of the point (x, y, z) w.r.t yz plane is $(-x, y, z)$.

Hence image of the point $(6, 3, -4)$ w.r.t yz plane is $(-6, 3, -4)$ (Ans)

(9) Given points are $P(1, 3, -1) = P(x_1, y_1, z_1)$ (say)

and $Q(3, 2, 4) = Q(x_2, y_2, z_2)$ (say)

DCs of z-axis $\langle l, m, n \rangle = \langle 0, 0, 1 \rangle$

Hence projection of ~~z-axis~~ $\overline{PQ}$ on z-axis

$= l(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1)$

$= 0 \cdot (3-1) + 0(2-3) + 1(4+1) = 5$ (Ans)

PLANE

G: A plane is defined as a surface such that the line joining any two points on the surface lies wholly on it

H: EQUATION OF PLANE

H₁: GENERAL FORM: $\boxed{ax+by+cz+d=0}$ where a,b,c are not all zero. Note that equation of plane contains three independent constant.

H₂: PLANE PASSING THROUGH A POINT & HAVING dcs/drs OF ITS NORMALS

(i) Equation of plane passing throug a point (x_1, y_1, z_1) and having dcs of its Normal $\langle l, m, n\rangle$ is

$$\boxed{l(x-x_1)+m(y-y_1)+n(z-z_1)=0}$$

(ii) If $\langle a, b, c\rangle$ are drs of Normal to the plane then equation of plane is

$$\boxed{a(x-x_1)+b(y-y_1)+c(z-z_1)=0}$$

H₃: INTERCEPT FORM:

Equation of plane making off intercepts a, b, c on co-ordinate axes is

$$\boxed{\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1}$$

H₄: THREE POINTS FORM Equation of plane passing through three non-colinear points (x_1,y_1,z_1), (x_2,y_2,z_2) and (x_3,y_3,z_3) is

$$\boxed{\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0}$$

Note that four points (x_1,y_1,z_1), (x_2,y_2,z_2), (x_3,y_3,z_3) and (x_4,y_4,z_4) will be co planar if

$$\begin{vmatrix} x_4-x_1 & y_4-y_1 & z_4-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0$$

H5: NORMAL FORM If p = Length of the perpendicular drawn from origin to the plane and $\langle l,m,n\rangle$ are dcs of the perpendicular then its equation is

$$\boxed{lx+my+nz=p}$$

H6: EQUATION OF PLANE PARALLEL TO A PLANE:

Equation of plane parallel to a plane $ax+by+cz+d=0$ is $\boxed{ax+by+cz+k=0}$

H7: EQUATION OF PLANE PASSING THROUGH LINE OF INTERSECTION OF TWO PLANES:

Let $P_1: a_1x+b_1y+c_1z+d_1=0$

$P_2: a_2x+b_2y+c_2z+d_2=0$ are two planes. Then Eq^n of plane passing through Line of intersection of $P_1=0$ & $P_2=0$ is $\boxed{P_1+KP_2=0}$, K = constant.

I: ANGLE BETWEEN TWO PLANES:

If $a_1x+b_1y+c_1z+d_1=0$

$a_2x+b_2y+c_2z+d_2=0$ are two planes and θ be the angle between these two planes then

$$\boxed{\theta = \cos^{-1}\left[\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right]}$$

J: CONDITION OF PARALLELITY & PERPENDICULARITY OF TWO PLANES:

Let $P_1: a_1x+b_1y+c_1z+d_1=0$

$P_2: a_2x+b_2y+c_2z+d_2=0$ Then

(i) $\boxed{P_1 \parallel P_2 \Leftrightarrow \frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}}$

(ii) P_1 and P_2 identical $\Leftrightarrow$ $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}=\frac{d_1}{d_2}$

(iii) $\boxed{P_1 \perp P_2 \Leftrightarrow a_1a_2+b_1b_2+c_1c_2=0}$

K: DISTANCE OF A POINT FROM A PLANE:

(i) Distance of a point $P(x_1, y_1)$ from a plane $ax+by+cz+d=0$ is

$$\boxed{d = \left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|}$$

(ii) Distance between two parallel planes $ax+by+cz+d_1=0$ and $ax+by+cz+d_2=0$ is

$$\boxed{d = \left|\frac{d_1-d_2}{\sqrt{a^2+b^2+c^2}}\right|}$$

PROBLEMS

1. Find the perpendicular distance of the point $(1,-1,-1)$ from the plane $2x+y+2z+4=0$ (A-1)
2. Find the distance between the planes $2x-3y+6z+1=0$ and $4x-6y+12z+5=0$ $(A \to \frac{3}{14})$
3. Find equation of plane parallel to zx plane such that the y-intercept is equal to 6. $(A \to y=6)$
4. Find equation of plane which passes through the point $(1,-1,4)$ and parallel to the plane $2x-3y+7z=11$ $(A \to 2x-3y+7z-33=0)$
5. Reduce the plane $3x-4y+z+5=0$ into normal form.
6. Find equation of plane which is perpendicular to the plane $5x+3y+6z+8=0$ and contains the line of intersection of the planes $x+2y+3z-4=0$ & $2x+y-z+5=0$ $(A \to 51x+15y-50z+173=0)$

⑦ Find equation of plane passing through the points $(-1,1,1)$ and $(1,-1,1)$ and perpendicular to the plane $x+2y+2z=5$. (A→ $2x+2y-3z+3=0$)

⑧ Find equation of ~~points~~ plane passing through the points $(2,1,0)$, $(3,-2,-2)$, $(3,1,7)$
(A→ $7x+3y-z-17=0$)

⑨ Find the angle between the planes $x+2y+2z-7=0$ and $2x-y+z=0$ (A→ $\cos^{-1}\left(\frac{2}{3\sqrt{6}}\right)$)

⑩ Show that four points $(-6,3,2)$, $(3,-2,4)$, $(5,7,3)$ and $(-13,17,-1)$ are co-planar.

HOME TASK

⑪ Find equation of plane which passes through $(4,-2,1)$ and is perpendicular to the line whose direction ratios are $7, 2, -3$.

(12) Find equation of plane passing through the points $(2,-3,2)$ and is normal to the line through the points $(3,4,-1)$ and $(2,-1,5)$

(13) Find equation of plane parallel to y-axis such that x-intercept is -3 and z intercept is 4

(14) Find intercept made on the co-ordinate axes by the plane $x+2y-2z=9$. Find d.c.s des of the normal to the plane.

(15) The foot of the perpendicular drawn from origin to a plane is $(12,-4,-3)$. Find equation of plane.

(16) The line drawn from $(4,-1,2)$ to the point $(-3,2,3)$ meets a plane at right angle at the point $(-10,5,4)$. Find the equation of plane.

(17) Find the equation of the plane containing line of intersection of the planes $x+y+z+1=0$ and $2x-3y+5z-2=0$ and passing through the point $(-1,2,1)$.

(18) Find equation of plane passing through the points $(-1,3,2)$ and perpendicular to the planes $x+2y+2z=5$ and $3x+3y+2z=8$

(19) Find equation of plane perpendicular to yz-plane and passing through the points $(1,-2,4)$ and $(3,-4,5)$

(20) Find distance between to planes $x-y+2z-4=0$ and $2x-2y+4z+5=0$

(21) Find equation of plane passing through the point $(3,4,-1)$ and is parallel to the plane $2x-3y+5z+7=0$. Also calculate the distance between the two planes.

(22) Find the value of K such that the planes $x+3y+Kz=5$ and $Kx+y+2z=0$ are perpendicular to each other.

MATH-i/15

(1) Given plane is $2x+y+2z+4=0$ —— ①

Given point is $P(1,-1,-1)$.

perpendicular distance of P from ① is

$$d = \left|\frac{2(1)+(-1)+2(-1)+4}{\sqrt{(2)^2+(-1)^2+(2)^2}}\right| = \left|\frac{3}{3}\right| = 1 \text{ (Ans)}$$

(2) Given planes are $2x-3y+6z+1=0$ —— ①

and $4x-6y+12z+5=0$

$\Rightarrow 2(2x-3y+6z+\frac{5}{2})=0 \Rightarrow 2x-3y+6z+\frac{5}{2}=0$ —— ②

Plane ① and ② are parallel.

Hence distance between them

$$d = \left|\frac{1-\frac{5}{2}}{\sqrt{(2)^2+(-3)^2+(6)^2}}\right| = \left|\frac{-3/2}{\sqrt{49}}\right| = \frac{3}{2}\times\frac{1}{7}$$

$= \frac{3}{14}$ unit (Ans)

(3) Since the plane is parallel to zx plane,

Let its equation be $y=k$ —— ①

Its y-intercept is 6

$\Rightarrow$ It passes through the point $(0,6,0)$ on y axis.

Hence $6=k \Rightarrow k=6$

$\therefore$ Eqn of plane is $y=k \Rightarrow y=6$ (Ans)

(4) Given plane is $2x-3y+7z=11$

$\Rightarrow 2x-3y+7z-11=0$ —— ①

Equation of plane parall to the plane ① is

$2x-3y+7z+k=0$ —— ②

Scanned with CamScanner

Plane ② passes through the point $(1,-1,4)$
Hence

$$2(1)-3(-1)+7(4)+K=0$$

$$\Rightarrow 33+K=0 \Rightarrow \boxed{K=-33}$$

$\therefore$ Equation of required plane is

$$2x-3y+7z+K=0$$

$$\Rightarrow 2x-3y+7z-33=0 \text{ (Ans)}$$

⑤ Given plane is $3x-4y+z+5=0$

$$\Rightarrow 3x-4y+z=-5$$

$$\Rightarrow -3x+4y-z=5$$

$$\Rightarrow \frac{-3x}{\sqrt{(3)^2+(-4)^2+(1)^2}}+\frac{4y}{\sqrt{(3)^2+(-4)^2+(1)^2}}-\frac{z}{\sqrt{(3)^2+(-4)^2+(1)^2}}$$

$$=\frac{5}{\sqrt{(3)^2+(-4)^2+(1)^2}}$$

$$\Rightarrow \left(\frac{-3}{\sqrt{26}}\right)x+\left(\frac{4}{\sqrt{26}}\right)y+\left(-\frac{1}{\sqrt{26}}\right)z=\frac{5}{\sqrt{26}}$$

which is in Normal form. Here

$$l=-\frac{3}{\sqrt{26}},\ m=\frac{4}{\sqrt{26}},\ n=-\frac{1}{\sqrt{26}},\ p=\frac{5}{\sqrt{26}} \text{ (Ans)}$$

(6) Given planes are $5x+3y+6z+8=0$ —①

$$x+2y+3z-4=0 \text{ —②}$$

$$2x+y-z+5=0 \text{ —③}$$

Equation of plane passing through line of intersection of plane ② and ③ is

$$(x+2y+3z-4)+k(2x+y-z+5)=0$$

$$\Rightarrow (1+2k)x+(2+k)y+(3-k)z-4+5k=0 \quad \text{——} ④$$

But plane ① ⊥ plane ④

$$\Rightarrow 5(1+2k)+3(2+k)+6(3-k)=0$$

$$\Rightarrow 7k+29=0 \Rightarrow k=-\frac{29}{7}$$

Eq^n of required plane is

$$(1+2k)x+(2+k)y+(3-k)z-4+5k=0$$

$$\Rightarrow \left(1-\frac{58}{7}\right)x+\left(2-\frac{29}{7}\right)y+\left(3+\frac{29}{7}\right)z-4-\frac{145}{7}=0$$

$$\Rightarrow \left(-\frac{51}{7}\right)x+\left(-\frac{15}{7}\right)y+\left(\frac{50}{7}\right)z-\frac{145}{7}-4=0$$

$$\Rightarrow -51x-15y+50z-145-28=0$$

$$\Rightarrow 51x+15y-50z+173=0 \text{ (Ans)}$$

(7) Given points are $P(-1,1,1)$ and $Q(1,-1,1)$.

Given plane is $x+2y+2z=5$ —— ①

Equation of plane passing through P is

$$a(x+1)+b(y-1)+c(z-1)=0 \quad \text{——} ②$$

plane ② passes through Q.

Hence,

$a(1+1)+b(-1-1)+c(1-1)=0$

$\Rightarrow 2a-2b+0.c=0$ ——— (3)

Also plane (2) $\perp$ plane (1)

$1\times a+2\times b+2\times c=0$

$\Rightarrow a+2b+2c=0$ ——— (4)

solving (3) and (4) by cross multiplication,

$$\frac{a}{(-2)(2)-(2)(0)}=\frac{b}{(0)(1)-(2)(2)}=\frac{c}{(2)(2)-(1)(-2)}$$

$$\Rightarrow \frac{a}{-4}=\frac{b}{-4}=\frac{c}{6}$$

$$\Rightarrow \frac{a}{-2}=\frac{b}{-2}=\frac{c}{3}=K \text{ (say)}$$

$\Rightarrow a=-2K,\ b=-2K,\ c=3K.$

Hence eqn of required plane is

$a(x+1)+b(y-1)+c(z-1)=0$

$\Rightarrow -2K(x+1)-2K(y-1)+3K(z-1)=0$

$\Rightarrow -K[2(x+1)+2(y-1)-3(z-1)]=0$

$\Rightarrow 2x+2+2y-2-3z+3=0$

$\Rightarrow 2x+2y-3z+3=0$ (Ans)

(8) Given points are $A(2,1,0)$, $B(3,-2,-2)$, $C(3,1,7)$

Here $x_1 = 2,\ y_1 = 1,\ z_1 = 0$

$x_2 = 3,\ y_2 = -2,\ z_2 = -2$

$x_3 = 3,\ y_3 = 1,\ z_3 = 7$

Equation of plane passing through A, B, C is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-2 & y-1 & z-0 \\ 3-2 & -2-1 & -2-0 \\ 3-2 & 1-1 & 7-0 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x-2 & y-1 & z \\ 1 & -3 & -2 \\ 1 & 0 & 7 \end{vmatrix} = 0.$$

$$\Rightarrow 1\begin{vmatrix} y-1 & z \\ -3 & -2 \end{vmatrix} + 7\begin{vmatrix} x-2 & y-1 \\ 1 & -3 \end{vmatrix} = 0$$ Expanding w.r.t R_3

$$\Rightarrow -2y+2+3z+7(-3x+6-y+1)=0$$

$$\Rightarrow -2y+2+3z-21x-7y+49=0$$

$$\Rightarrow -21x-9y+3z+51=0$$

$$\Rightarrow 21x+9y-3z-51=0$$

$$\Rightarrow 7x+3y-z-17=0 \text{ (Ans)}$$

(9) Given planes are

$$x+2y+2z-7=0 \quad \text{——} \ (1)$$
$$2x-y+z=0 \quad \text{——} \ (2)$$

Here $a_1=1, b_1=2, c_1=2$

$a_2=2, b_2=-1, c_2=1$

Let θ be the angle between plane (1) and (2)

$$\therefore \theta = \cos^{-1}\left[\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right]$$

$$= \cos^{-1}\left[\frac{(1)(2)+(2)(-1)+(2)(1)}{\sqrt{(1)^2+(2)^2+(2)^2}\sqrt{(2)^2+(-1)^2+(1)^2}}\right]$$

$$= \cos^{-1}\left[\frac{2}{\sqrt{9}\sqrt{6}}\right] = \cos^{-1}\left[\frac{2}{3\sqrt{6}}\right] \text{ (Ans)}$$

(10) Given points are $A(-6,3,2)$, $B(3,-2,4)$, $C(5,7,3)$ and $D(-13,17,-1)$

Here $x_1=-6, y_1=3, z_1=2$

$x_2=3, y_2=-2, z_2=4$

$x_3=5, y_3=7, z_3=3$

$x_4=-13, y_4=17, z_4=-1$

Now
$$\begin{vmatrix} x_4-x_1 & y_4-y_1 & z_4-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = \begin{vmatrix} -13+6 & 17-3 & -1-2 \\ 3+6 & -2-3 & 4-2 \\ 5+6 & 7-3 & 3-2 \end{vmatrix}$$

$$= \begin{vmatrix} -7 & 14 & -3 \\ 9 & -5 & 2 \\ 11 & 4 & 1 \end{vmatrix} = -7(-5-8)-14(9-22)-3(36+55)$$

$$= 91+182-273 = 273-273 = 0$$

Hence A, B, C, D are co-planar $\square$