

**PNS SCHOOL OF ENGG & TECHNOLOGY MARSAGHAI KENDRAPARA**



**LECTURES NOTE ON**

**FLUID MECHANICS**

**4<sup>TH</sup> SEM MECHANICAL**

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**HOD MECHANICAL**

# Chapter-1

## Properties of Fluid

### Fluid

#### Definition:

A fluid is a substance which is capable of flowing or a substance which deforms continuously when subjected to external shearing force.

#### Characteristics:

- It has no definite shape of its own but will take the shape of the container in which it is stored.
- A small amount of shear force will cause a deformation.

### **Classification:**

A fluid can be classified as follows:

- Liquid
- Gas

#### **Liquid:**

It is a fluid which possesses a definite volume and assumed as incompressible

#### **GAS:**

It possesses no definite volume and is compressible.

Fluids are broadly classified into two types.

- Ideal fluids
- Real fluids

#### **Ideal fluid:**

An ideal fluid is one which has no viscosity and surface tension and is incompressible actually no ideal fluid exists.

#### **Real fluids:**

A real fluid is one which has viscosity, surface tension and compressibility in addition to the density.

### **PROPERTIES OF FLUIDS:**

#### **1. density or mass density : (S)**

Density of a fluid is defined as the ratio of the mass of a fluid to its volume. It is denoted by  $\rho$  The density of liquids are considered as constant while that of gases changes with pressure & temperature variations.

Mathematically

$$\rho = \frac{\text{mass}}{\text{volume}}$$

$$\text{Unit} = \frac{\text{kg}}{\text{m}^3}$$

$$\rho_{\text{water}} = 1000 \frac{\text{kg}}{\text{m}^3}$$

$$\text{or } \frac{\text{gm}}{\text{cm}^3}$$

2. **Specific weight or weight density((W):**

Specific weight of a fluid is defined as the ratio between the weights of a fluid to its volume. It is denoted by W.

$$\text{Mathematically } W = \frac{\text{weight of fluid}}{\text{volume of fluid}}$$

$$= \text{mg/v}$$

$$W = \rho g$$

$$\text{Unit} - \frac{N}{\text{m}^3}$$

3. **Specific volume:**

Specific volume of a fluid is defined as the volume of a fluid occupied by a unit mass or volume per unit mass of a fluid is called specific volume.

Mathematically

$$\text{Specific volume} = \frac{\text{Volume of fluid}}{\text{Mass of fluid}} = \frac{1}{\frac{\text{Mass of fluid}}{\text{Volume}}} = \frac{1}{\rho}$$

$$\text{Unit: } \frac{\text{m}^3}{\text{kg}}$$

4. **Specific gravity:**

Specific gravity is defined as the ratio of the weight density of a fluid to the density or when density standard fluid.

For liquids the standard fluid is water.

For gases the standard fluid is air.

It is denoted by the symbol  $S$

$$\text{Mathematically, } S(\text{for liquids}) = \frac{\text{Weight density (density) of liquid}}{\text{Weight density (density) of water}}$$

$$S(\text{for gases}) = \frac{\text{Weight density (density) of gas}}{\text{Weight density (density) of air}}$$

$$\begin{aligned} \text{Thus weight density of a liquid} &= S \times \text{Weight density of water} \\ &= S \times 1000 \times 9.81 \text{ N/m}^3 \end{aligned}$$

$$\begin{aligned} \text{The density of a liquid} &= S \times \text{Density of water} \\ &= S \times 1000 \text{ kg/m}^3. \end{aligned}$$

## Simple Problems:

### **Problem: - 1**

Calculate the specific weight, density and specific gravity of one litre of a liquid which weighs 7N.

**Solution.** Given :

$$\text{Volume} = 1 \text{ litre} = \frac{1}{1000} \text{ m}^3 \quad \left( \because 1 \text{ litre} = \frac{1}{1000} \text{ m}^3 \text{ or } 1 \text{ litre} = 1000 \text{ cm}^3 \right)$$

$$\text{Weight} = 7 \text{ N}$$

$$(i) \text{ Specific weight } (w) = \frac{\text{Weight}}{\text{Volume}} = \frac{7 \text{ N}}{\left(\frac{1}{1000}\right) \text{ m}^3} = 7000 \text{ N/m}^3. \text{ Ans.}$$

$$(ii) \text{ Density } (\rho) = \frac{w}{g} = \frac{7000}{9.81} \text{ kg/m}^3 = 713.5 \text{ kg/m}^3. \text{ Ans.}$$

$$(iii) \text{ Specific gravity} = \frac{\text{Density of liquid}}{\text{Density of water}} = \frac{713.5}{1000} \quad \{ \because \text{Density of water} = 1000 \text{ kg/m}^3 \}$$
$$= 0.7135. \text{ Ans.}$$

### **Problem: - 2**

Calculate the density, specific weight and specific gravity of one litre of petrol of specific gravity = 0.7

**Solution.** Given :  $\text{Volume} = 1 \text{ litre} = 1 \times 1000 \text{ cm}^3 = \frac{1000}{10^6} \text{ m}^3 = 0.001 \text{ m}^3$

$$\text{Sp. gravity} \quad S = 0.7$$

(i) Density ( $\rho$ )

Using equation (1.1.A),

$$\text{Density } (\rho) = S \times 1000 \text{ kg/m}^3 = 0.7 \times 1000 = 700 \text{ kg/m}^3. \text{ Ans.}$$

(ii) Specific weight ( $w$ )

$$\text{Using equation (1.1),} \quad w = \rho \times g = 700 \times 9.81 \text{ N/m}^3 = 6867 \text{ N/m}^3. \text{ Ans.}$$

(iii) Weight ( $W$ )

$$\text{We know that specific weight} = \frac{\text{Weight}}{\text{Volume}}$$

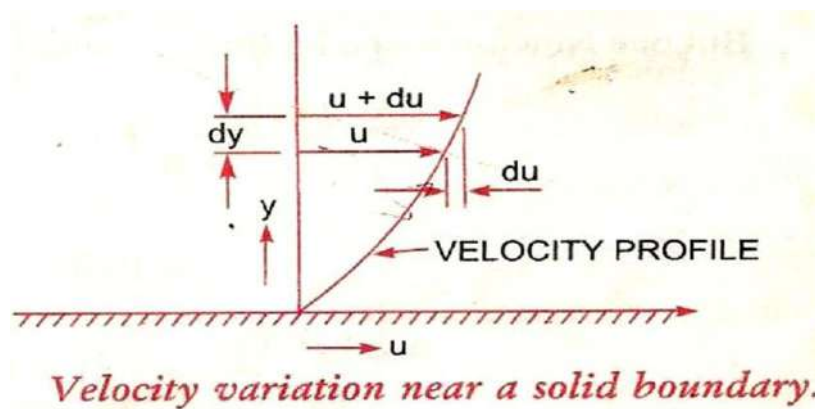
$$w = \frac{W}{0.001} \text{ or } 6867 = \frac{W}{0.001}$$

$$\therefore W = 6867 \times 0.001 = 6.867 \text{ N. Ans.}$$

**Viscosity:**

Viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid.

Let two layers of a fluid at a distance  $dy$  apart, move one over the other at different velocities  $u$  and  $u + du$ .



The viscosity together with the with the relative velocity between the two layers while causes a shear stress acting between the fluid layers, the top layer causes a shear stress on the adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer.

This shear stress is proportional to the rate of change of velocity with respect to  $y$ . It is denoted by  $r$ .

Mathematically

$$r \propto \frac{du}{dy}$$
$$r = \mu \frac{du}{dy}$$

Where  $\mu$  = co-efficient of dynamic viscosity or constant of proportionality or viscosity

$\frac{du}{dy}$  = rate of shear strain or velocity gradient

$$\mu = \frac{c}{\frac{du}{dy}}$$

If  $\frac{du}{dy} = 1$ ,

then  $\mu = r$

Viscosity is defined as the shear stress required to produce unit rate of shear strain.

Unit of viscosity in S.I system -  $\frac{Ns}{m^2}$

in C.G.S -  $\frac{Dyne\ s}{cm^2}$

in M.K.S. -  $\frac{kgfs}{m^2}$

$$\frac{Dyne\ s}{cm^2} = 1\ \text{Poise}$$

$$1\ \frac{Ns}{m^2} = 10\ \text{poise}$$

$$1\ \text{Centipoise} = \frac{1}{100}\ \text{poise}$$



### **Kinematic Viscosity:**

It is defined as the ratio between the dynamic viscosity and density of fluid.

It is denoted by  $\nu$ .

Mathematically

$$\nu = \frac{\text{Viscosity}}{\text{Density}} = \frac{\mu}{\rho} \quad \dots(1.4)$$

The units of kinematic viscosity is obtained as

$$\begin{aligned} \nu &= \frac{\text{Units of } \mu}{\text{Units of } \rho} = \frac{\text{Force} \times \text{Time}}{(\text{Length})^2 \times \frac{\text{Mass}}{(\text{Length})^3}} = \frac{\text{Force} \times \text{Time}}{\frac{\text{Mass}}{\text{Length}}} \\ &= \frac{\text{Mass} \times \frac{\text{Length}}{(\text{Time})^2} \times \text{Time}}{\left(\frac{\text{Mass}}{\text{Length}}\right)} \quad \left\{ \begin{array}{l} \because \text{Force} = \text{Mass} \times \text{Acc.} \\ = \text{Mass} \times \frac{\text{Length}}{\text{Time}^2} \end{array} \right\} \\ &= \frac{(\text{Length})^2}{\text{Time}} \end{aligned}$$

In MKS and SI, the unit of kinematic viscosity is metre<sup>2</sup>/sec or m<sup>2</sup>/sec while in CGS units it is written as cm<sup>2</sup>/s. In CGS units, kinematic viscosity is also known stoke.

$$\text{Thus, one stoke} = \text{cm}^2/\text{s} = \left(\frac{1}{100}\right)^2 \text{ m}^2/\text{s} = 10^{-4} \text{ m}^2/\text{s}$$

$$\text{Centistoke means} = \frac{1}{100} \text{ stoke.}$$

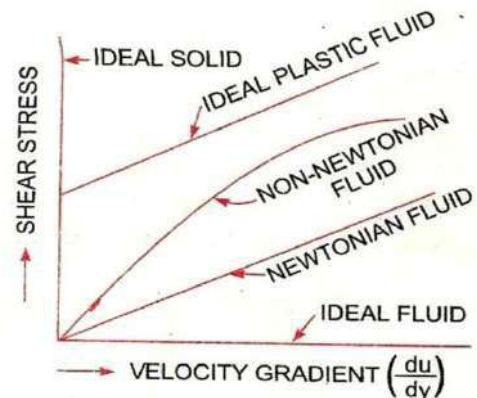
### **Newton's law of viscosity:**

It states that the shear stress on a fluid element layer is directly proportional to the rate of shear strain. The constant of proportionality is called the co-efficient of viscosity.

Mathematically

$$\tau = \mu \frac{du}{dy}$$

Fluids which obey the above equation or law are known as Newtonian fluids & the fluids which do not obey the law are called Non-Newtonian fluids.



### **Surface tension:**

Surface tension is defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquids such that the contact surface behaves like a stretched membrane under tension. The magnitude of this force per unit length of the free will has the same value as the surface energy per unit area.

It is denoted by  $\sigma$

Mathematically 
$$\sigma = \frac{F}{L}$$

Unit in si system is N/m

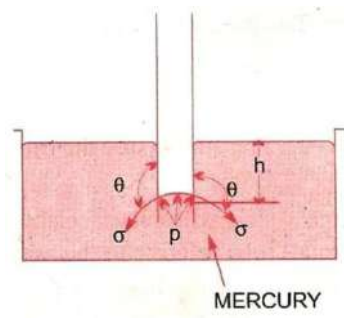
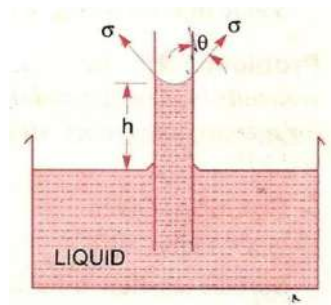
CGS system is Dyne/cm

MKS system is kgf/m

### **Capillarity:**

Capillarity is defined as a phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid. The rise of liquid surface is known as capillary rise while the fall of the liquid surface is known as capillary depression.

It is expressed in terms of cm or mm of liquid



Its value depends upon the specific weight of the liquid, diameter of the tube and surface tension of the liquid.



# Chapter-2

## Fluid Pressure And It's Measurements

### Syllabus:

- 2.1 Definitions and units of fluid pressure, pressure intensity and pressure head**
- 2.2 Concept of atmospheric pressure, gauge pressure, vacuum pressure and absolute pressure**
- 2.3 Pressure measuring instruments Manometers: Simple and differential Bourdon tube pressure gauge (Simple Numerical)**

### **Pressure of a Fluid:**

When a fluid is contained in a vessel, it exerts force at all points on the sides & bottoms of the container. The force exerted per unit area is called pressure.

If  $P$  = Pressure at any point

$F$  = Total force uniformly distributed over an area

$A$  = unit area

$P = F/A$

Unit of pressure -  $\frac{kgf}{m^2}$  in M.K.S.

-  $\frac{N}{m^2}$  in S.I.

-  $\frac{Dyne}{cm^2}$

1 pascal =  $1N/m^2$

1 kpa =  $1000 N/m^2$

### **Pressure head of a liquid:**

A liquid is subjected to pressure due to its own weight, this pressure increases as the depth of the liquid increases.

Let a bottomless cylinder stand in the liquid

Let  $w$  = specific weight of the liquid.

$H$  = height of the liquid in the cylinder.

$A$  = Area of the cylinder.

$$P = \frac{F}{A} = \frac{\text{weight of the liquid in the cylinder}}{\text{Area of the cylinder}}$$

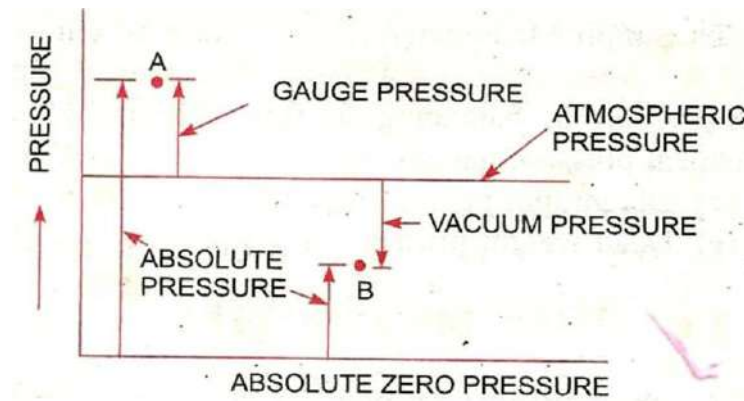
$$= \frac{W \times A h}{A}$$

$$= Wh$$

$$= \rho gh$$

So intensity of pressure at any point in a liquid is proportional to its depth.

## **ABSOLUTE, GAGUE, ATOMOSPHERIC, AND VACCUME PRESSURES:**



### **Atmospheric Pressure:**

The atmospheric air exerts a normal pressure upon all surfaces with which it is in contact & known as atmospheric pressure.

### **Absolute pressure:**

It is defined as the pressure which is measured with reference to absolute vacuum pressure or absolute zero pressure.

### **Gauge pressure:**

It is defined as the pressure which is measured with the help of a pressure measuring instrument in which the atmospheric pressure is taken as datum. The atmospheric pressure on the scale is marked as zero.

### **Vacuum pressure:**

It is defined as the pressure below the atmospheric pressure.

Mathematically:

$$\text{Absolute pressure} = \text{Atmospheric pressure} + \text{gauge pressure}$$

$$\text{Or } P_{\text{abs}} = P_{\text{atm}} + P_{\text{gauge}}$$

$$\text{Vacuum pressure} = \text{Atmospheric pressure} - \text{Absolute pressure}$$

$$P_{\text{vacuum}} = P_{\text{atm}} - P_{\text{abs}}$$

## **Pressure Measuring Instruments:**

The pressure of a fluid is measured by the following devices :

1. **Manometers**
2. **Mechanical Gauges.**

### **Manometers:**

Manometers are defined as the device used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same another column of the fluid. They are classified as:

- (a) **Simple manometers.**
- (b) **Differential Manometers.**

### **Mechanical Gauges:**

Mechanical gauges are defined as the device used for measuring the pressure by balancing the fluid column by the spring or dead weight. Commonly used mechanical pressure gauges are :

- **Diaphragm pressure gauge**
- **Bourdon tube pressure gauge**
- **Dead –weight pressure gauge**
- **Bellow pressure gauge**

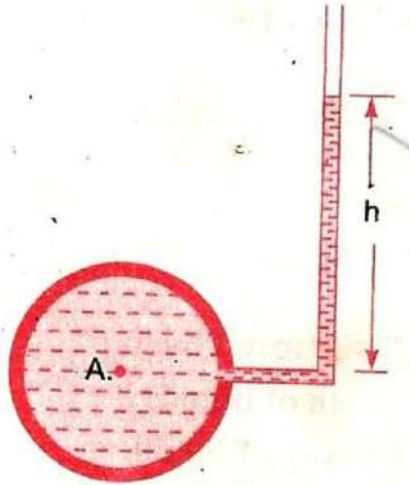
### **Simple Manometres:**

A simple manometer of a glass tube having one of its ends connected to a point where pressure is to be measured and other end remains open to atmosphere. Common types of simple manometers are :

- **Piezometer**
- **U- tube Manometer**
- **Single Column Manometer**



### Piezometer:

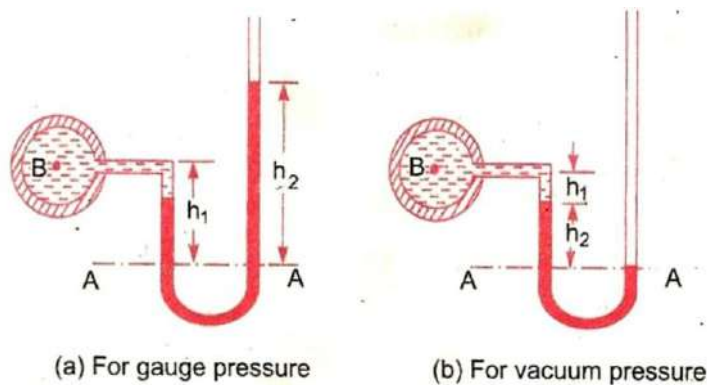


It is the simple form of manometer used for measuring gauge pressures. One end of this manometer is connected to the point where pressure is to be measured and other end is open to the atmosphere as shown in Figure. The rise of liquid gives the pressure head at that point A. Then pressure at A

$$P_A = pgh$$

### U – tube Manometer:

It consist of glass tube bent in U- shape , one end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere as shown in figure. The tube generally contains mercury.



**(a) For Gauge Pressure:**

Let  $p$  be the point which is to be measured, whose value is  $p$ . The datum line is A-A.

Let  $h_1$  = Height of light liquid above the datum line

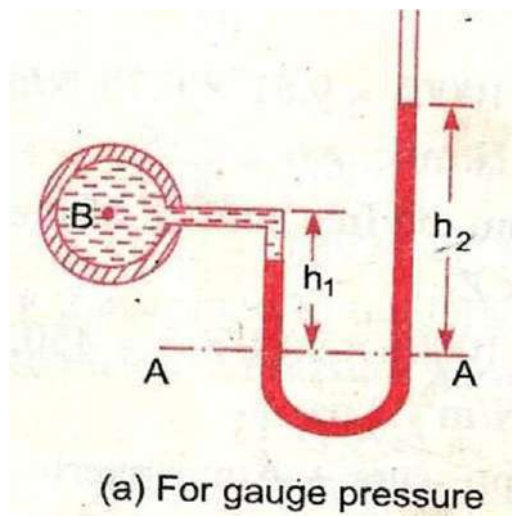
$h_2$  = Height of heavy liquid above the datum line

$S_1$  = Sp. gr. of light liquid

$\rho_1$  = Density of light liquid =  $1000 \times S_1$

$S_2$  = Sp. Gr. Of heavy weight

$\rho_2$  = density of heavy weight =  $1000 \times S_2$



Pressure is same in a horizontal surface. Hence pressure above the horizontal datum surface line A-A in the left column and in the right column of U-tube manometer should be same pressure above A-A in the left column

$$= p_A + \rho_1 \times g \times h_1$$

Pressure above A-A in the right column

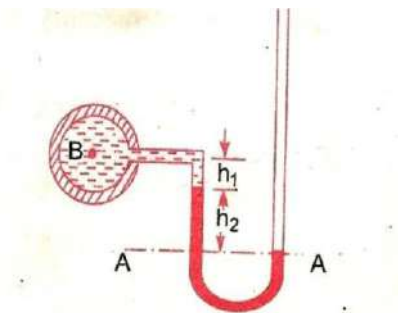
$$= \rho_2 \times g \times h_2$$

Hence equating the two pressures

$$p_A + \rho_1 g h_1 = \rho_2 g h_2$$

$$p_A = (\rho_2 g h_2 - \rho_1 g h_1).$$

**(b) For Vacuum Pressure:**



For measuring vacuum pressure, the level of the heavy liquid in the manometer will be as shown in figure. Then Pressure above A-A in the left column

$$= \rho_2 g h_2 + \rho_1 g h_1 + p_A$$

Pressure head in the right column above A - A = 0

$$\rho_2 g h_2 + \rho_1 g h_1 + p_A = 0$$

$$p_A = - (\rho_2 g h_2 + \rho_1 g h_1)$$

### **Single Column Manometer:**

Single column Manometer is modified form of a U- tube manometer in which a reservoir, having a large cross- sectional area (about 100 times as compared to the area of the tube) is connected to one of the limbs (say left limb) of the manometer as shown in figure. Due to large cross- sectional area of the reservoir, for any variation in pressure, the change in the liquid level in the reservoir will be very small which may be neglected and hence the pressure is given by the height of liquid in the other limb. The other limb may be vertical or inclined. Thus there are two types of single column manometer as:

- **Vertical Single Column Manometer**
- **Inclined Single Column Manometer**

**1. Vertical Single Column Manometer:**

Let X-X be the datum line in the reservoir and in the right limb of the manometer, when it is not connected to the pipe. When the manometer is connected to the pipe, due to high pressure at A, the heavy liquid in the reservoir will be pushed downward and will rise in the right limb.

Let  $\Delta h$  = Fall of heavy liquid in reservoir

$H_2$  = rise of heavy liquid in right limb

$H_1$  = height of center of pipe above X-X

$P_A$  = Pressure at A, which is to be measured

$A$  = Cross – sectional area of the reservoir

$a$  = Cross sectional area of the right limb

$S_1$  = Sp.gr.of liquid in pipe

$S_2$  = Sp.gr. of heavy weight liquid in reservoir and right limb

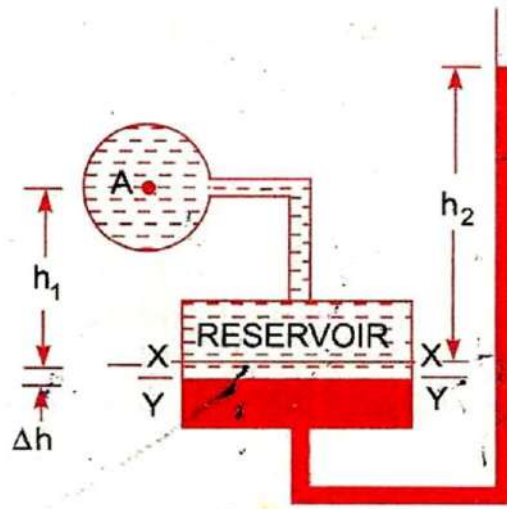
$P_1$  = Density in liquid in pipe

$P_2$  = Density of liquid in the reservoir

Fall of heavy liquid in the reservoir will cause a rise of heavy liquid level in the right limb.

$$\therefore A \times \Delta h = a \times h_2$$

$$\therefore \Delta h = \frac{a \times h}{A} \dots\dots\dots (i)$$



Now consider the datum line Y-Y as shown in Fig 2.15. Then pressure in the right limb above Y-Y.

$$= \rho_2 \times g \times (\Delta h + h_2)$$

Pressure in left limb above Y-Y  $= \rho_1 \times g \times (\Delta h + h_1) + p_A$

Equating the pressure, we have

$$\rho_2 \times g \times (\Delta h + h_2) = \rho_1 \times g \times (\Delta h + h_1) + P_A$$

$$P_A = \rho_2 g (\Delta h + h_2) - \rho_1 g (\Delta h + h_1)$$

$$= \Delta h [\rho_2 g - \rho_1 g] + h_2 \rho_2 g - h_1 \rho_1 g$$

But from equation (i),  $\Delta h = \frac{a \times h}{A}$

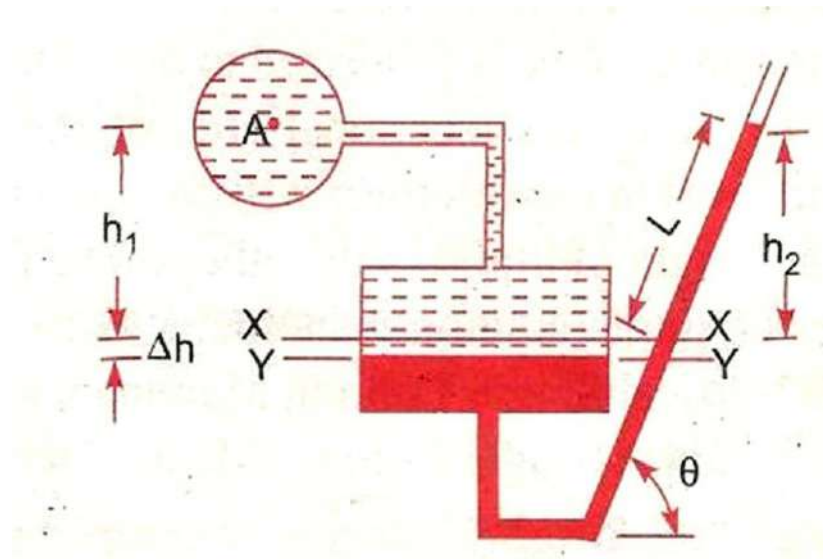
$$\text{So, } P_A = \frac{a \times h}{A} [\rho_2 g - \rho_1 g] + h_2 \rho_2 g - h_1 \rho_1 g$$

As the area A is very large as compared to a, hence ratio  $\frac{a}{A}$  becomes very small and can be neglected.

$$\text{Then } P_A = h_2 \rho_2 g - h_1 \rho_1 g$$

## 2. Inclined Single Column Manometer:

The given figure shows the inclined single column manometer which is more sensitive. Due to inclination the distance moved by the heavy liquid in the right limb will be more.



Let  $L$  = length of heavy liquid moved in right limb from X-X

$\theta$  = Inclination of right limb with horizontal

$h_2$  = Vertical rise of heavy liquid in right limb from X-X

$$= L \times \sin\theta$$

From the above equation for the pressure in the single column manometer the pressure at A is

$$P_A = h_2 \rho_2 g - h_1 \rho_1 g .$$

Substituting the value of  $h_2$ , we get

$$P_A = \sin\theta \rho_2 g L - h_1 \rho_1 g .$$

## **DIFFERENTIAL MANOMETERS:**

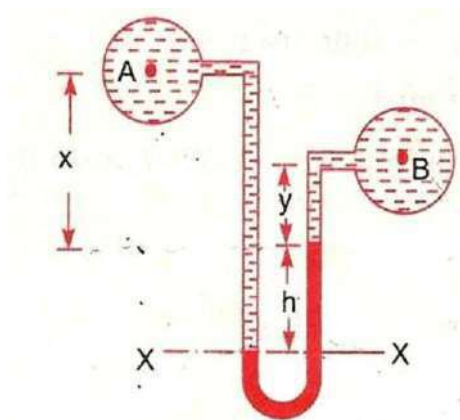
Differential manometers are the device use for measuring the difference of pressures between two points in a pipe or in two different pipes. A differential manometer consists of a U- tube, containing a heavy liquid, whose two ends are connected to the points, whose difference of pressure is to be measured. Most commonly used differential manometers are :

1. **U-tube differential manometer**
2. **Inverted U-tube differential manometer**

### **U-tube differential manometer:**

#### **Two points A and B are at different level**

The given figure shows the differential manometers of U-tube type.



Let the two points A and B are at different level also contains liquids of different sp.gr. These points are connected to the U-tube differential manometer. Let the pressure at A and B are  $P_A$  and  $P_B$ .

Let  $h$  = Difference of mercury level in the U- tube.

$y$  = Distance of the center of B, from the mercury level in the right limb.

$\rho_1$  = Density of liquid at A.

$\rho_2$  = Density of liquid at B.

$\rho_g$  = Density of heavy liquid or mercury.

Taking datum line at X-X .

Pressure above X-X in the limb

$$= \rho_1 g(h + x) + P_A$$

Where pressure  $P_A$  = Pressure at A.

Pressure above X-X in the right limb

$$= \rho_g \times g \times h + \rho_2 \times g \times y + p_B$$

Where pressure  $p_B$  = pressure at B.

Equating the two pressure, we have

$$P_1 g(h + x) + P_A = \rho_g \times g \times h + \rho_2 g y + p_B$$

$$\begin{aligned} \therefore P_A - p_B &= \rho_g \times g \times h + \rho_2 g y - \rho_1 g(h + x) \\ &= h \times g(\rho_g - \rho_1) + \rho_2 g y - \rho_1 g x \end{aligned}$$

$\therefore$  Different of pressure at A and B

$$= h \times g(\rho_g - \rho_1) + \rho_2 g y - \rho_1 g x$$

### **Two points A and B are at same level**

In the given figure A and B are the same level and contains the same liquid of density  $\rho_1$ , then

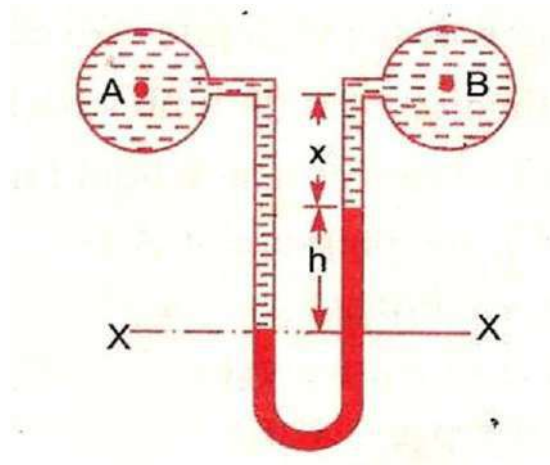
Pressure above X-X in right limb

$$= \rho_g \times g \times h + \rho_1 \times g \times X + p_B$$

Pressure above X-X in left limb

$$= P_1 \times g \times (h + x) + P_A$$





Equating the two pressure

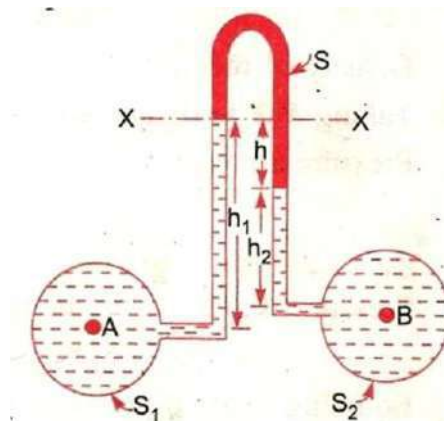
$$p_g \times g \times h + P_1 \times g \times X + p_B = P_1 \times g \times (h + x) + P_A$$

$$\therefore P_A - p_B = P_g \times g \times h + P_1 g x - P_1 g \times (h + x)$$

$$= g \times h (P_g - P_1)$$

**Inverted U-tube Differential Manometer:**

It consists of an inverted U-tube, containing a light liquid. The two ends of the U-tube are connected to the points whose difference of pressure is to be measured. It is used for measuring difference of low pressures. Fig 2.21 shows an inverted U-tube differential manometer connected to the points A and B. Let the pressure at A is more than the pressure at B.



Let  $h_1$ =Height of liquid in the left limb below the datum line X-X

$h_2$ = Height of liquid in the right limb

$h$ = Difference of light liquid

$\rho_1$ =Density of liquid at A

$\rho_2$ =Density of liquid at B

$\rho_s$ = Density of light liquid

$p_A$ =Pressure at A

$p_B$ = Pressure at B.

Taking X-X datum line.

Then pressure in the left limb below X-X

$$= P_A - \rho_1 \times g \times h_1.$$

Pressures in the right limb below X-X

$$= P_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

Equating the two pressure

$$P_A - \rho_1 \times g \times h_1 = P_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

$$P_A - P_B = \rho_1 \times g \times h_1 - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

### **Bourdon's Tube Pressure Gauge:**

- The pressure above or below the atmospheric pressure may be easily measured with the help of Bourdon tube pressure gauge.
- It consists of an elliptical tube ABC bent into an arc of a circle. This bent up tube is called Bourdon tube.
- When the gauge tube is connected to the C, the fluid under pressure flows into the tube the Bourdon tube as a result of the increased pressure tends to straighten itself.
- Since the tube is encased in a circular cover therefore it tends to become circular instead of straight.
- The elastic deformation of the Bourdon rotates the pointer.
- The pointer moves over a calibrated scale which directly gives the pressure.

**Numerical problems:**

**Q.1** The right limb of a simple U-tube manometer containing mercury is open to the atmosphere while the left limb is connected to a pipe in which a fluid of sp. gravity 0.9 is flowing. The centre of the pipe is 12 cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe if the difference of mercury level in the two limbs is 20 cm.

**Q.2** A single column manometer is connected to a pipe containing a liquid of sp. Gravity 0.9. Find the pressure in the pipe if the area of the reservoir is 100 times the area of the tube for manometer reading. The sp. Gravity of mercury is 13.6.

**Q.3** A differential manometer is connected at the two points A and B of two pipes. The pipe A contains a liquid of sp. Gravity = 1.5 while pipe B contains a liquid of sp. Gravity 0.9. The pressure at A and B are  $1 \text{ kg/cm}^2$  and  $1.80 \text{ kg/cm}^2$  respectively. Find the difference in mercury level in the differential manometer.

**Q.4** Water is flowing through two difference pipes to which an inverted differential manometer having an oil of sp. Gravity 0.8 is connected. The pressure head in the pipe A is 2 m of water, find the pressure in the pipe B for the manometer readings.

# Chapter-3

## Hydrostatics

### Syllabus:

- 3.1 Definition of hydrostatic pressure**
- 3.2 Total pressure and centre of pressure on immersed bodies (Simple Numericals)**
- 3.3 Archimedis' principle, concept of buoyancy, metacentre and metacentric height**
- 3.4 Concept of floatation**

### Hydrostatics:

Hydrostatics means the study of pressure exerted by the liquid at rest & the direction of such a pressure is always right angle to the surface on which it acts.

### Total pressure and center of pressure:

#### Total pressure

Total pressure is defined as the force exerted by a static fluid on a surface either plane or curved when the fluid comes in contact with surfaces. This force always acts normal to the surface.

### **Center of pressure:**

Center of pressure is defined as the point of application of the total pressure on the surface.

There are four cases of submerged surfaces on which the total pressure force and center of pressure is to be determined. The submerged surfaces may be:

1. **Vertical plane surface**
2. **Horizontal plane surface**
3. **Inclined plane surface**
4. **Curved surface.**

### **Vertical plane surface submerged in liquid**

Consider a plane vertical surface of arbitrary shape immersed in a liquid as shown in figure

Let  $A$  = total area of the surface

$H$  = distanced of C.G. of the area from free surface of liquid

$G$  = center of gravity of plane surface

$P$  = center of pressure

$h^*$  = distance of center of pressure from free surface of liquid.

### **Total pressure(F):**

The total pressure on the surface may be determined by dividing the entire surface into a number of small parallel strips. The force on surface is then calculated by integrating the force on small strip.

Consider a strip of thickness  $dh$  & width  $b$  at a depth of  $h$  from free surface of liquid.

Pressure intensity on the strip

$$p = \rho gh$$

Area of the strip,  $dA = b \times dh$

Total pressure force on strip,  $dF = qdA$

$$= qgh \times b \times dh$$

Total pressure force on the whole surface

$$F = \int dF = \int qgh \times b \times dh$$

$$= qg \int h \times b \times dh$$

$\int h \times dA =$  moment of surface area about the free surface of liquid

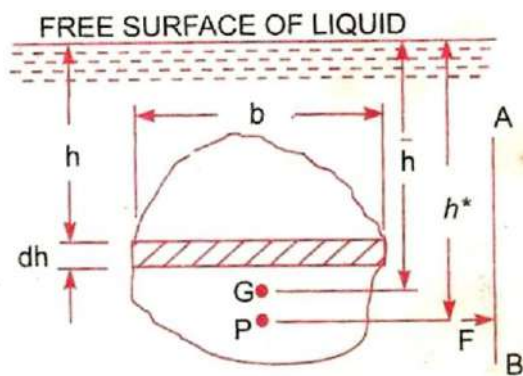
$=$  Area of surface  $\times$  distance of C.G. from the free surface

$$= A \times \bar{h}$$

So,  $F = qgA\bar{h}$

**Centre of the pressure: (  $h^*$  )**

Centre of pressure is calculated by using the principle of moments which states that the moment of resultant force about an axis is equal to the sum of moments of the components about the same axis.



The resultant force  $F$  is acting at  $P$ , at a distance  $h^*$  from the free surface of liquid.

Hence moment of force  $F$  about free surface of liquid  $= F \times h^*$

But moment force  $dF$  acting on a strip about the free surface of liquid =  $dF \times h$

Sum of moments of all such forces about free surface of liquid

$$= \int qgh \times b \times dh \times h$$

$$= qg \int h \times b \times dh \times h$$

$$= qg \int bh^2 dh$$

$$= qg \int h^2 dA$$

$\int h^2 dA$  = moment of inertia of the surface area about the free surface of liquid =  $I_0$

Sum of the moments about free surface

$$= qg I_0$$

$$F \times h^* = qg I_0$$

$$qg A \bar{h} \times h^* = qg I_0$$

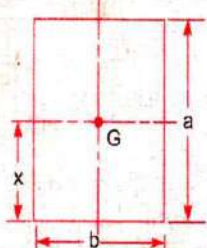
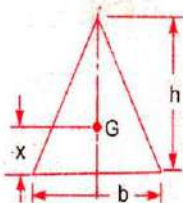
$$h^* = \frac{qg I_0}{qg A \bar{h}}$$

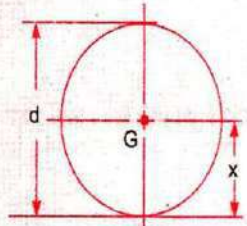
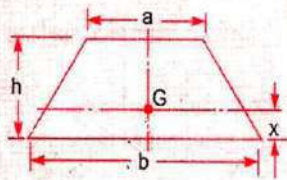
$$= \frac{I_0}{A \bar{h}}$$

By the parallel axis theorem, we have

$$I_0 = I_G + A \times (\bar{h})^2$$

$$h^* = \frac{I_G + A \bar{h}^2}{A \bar{h}} = \frac{I_G}{A \bar{h}} + \bar{h}$$

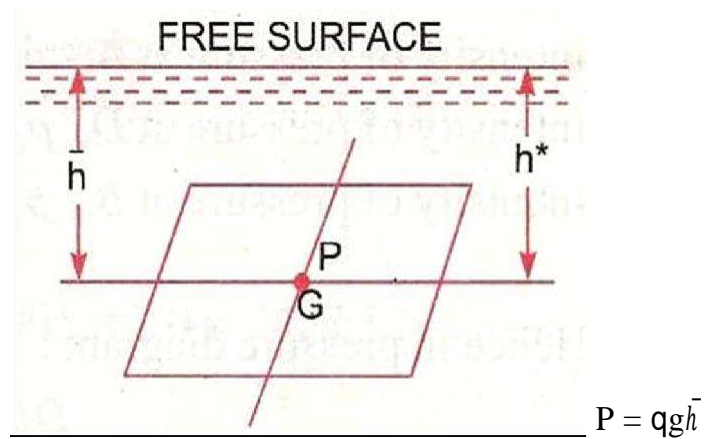
Plane surface	C.G. from the base	Area	Moment of inertia about an axis passing through C.G. and parallel to base ( $I_G$ )	Moment of inertia about base ( $I_0$ )
1. Rectangle 	$x = \frac{d}{2}$	$bd$	$\frac{bd^3}{12}$	$\frac{bd^3}{3}$
2. Triangle 	$x = \frac{h}{3}$	$\frac{bh}{2}$	$\frac{bh^3}{36}$	$\frac{bh^3}{12}$

Plane surface	C.G. from the base	Area	Moment of inertia about an axis passing through C.G. and parallel to base ( $I_G$ )	Moment of inertia about base ( $I_0$ )
3. Circle 	$x = \frac{d}{2}$	$\frac{\pi d^2}{4}$	$\frac{\pi d^4}{64}$	—
4. Trapezium 	$x = \left(\frac{2a+b}{a+b}\right) \frac{h}{3}$	$\frac{(a+b)}{2} \times h$	$\left(\frac{a^2 + 4ab + b^2}{36(a+b)}\right) \times h^3$	—



### Horizontal plane surface submerged in liquid:

Consider a plane horizontal surface immersed in a static fluid as every point of the surface is at the same depth from the free surface of the liquid, the pressure intensity will be equal on the entire surface.

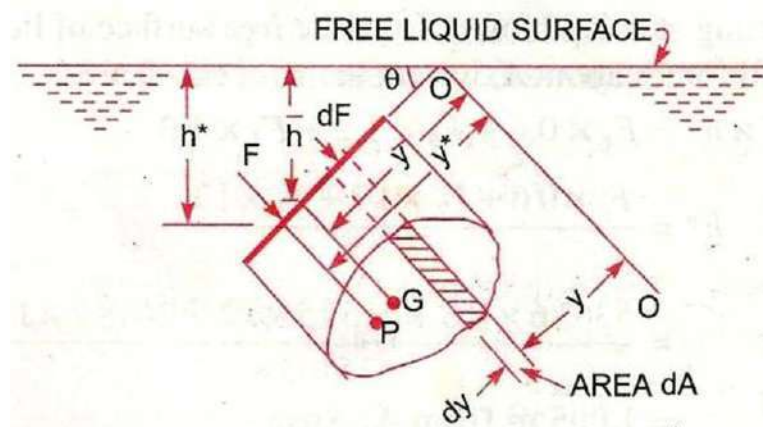


$A$  = total area

$F = P \times A$

$= qgA\bar{h}$

### Inclined plane surface submerged in liquid:



Let  $A$  = total area of the inclined surface

$H$  = depth of C.G. of inclined area from free surface.

$h^*$  = distance of center of pressure from free surface of liquid.

$\theta$  = angle made by the plane of surface with free liquid surface.

Let the plane of the surface if produced meet the free liquid surface at  $O$ . Then  $O-O$  is the axis parallel to the plane of the surface

$\bar{y}$  = distance of C.G of the inclined surface from  $O-O$ .

$y^*$  = distance of the centre of pressure from  $O-O$ .

Consider a small strip of area  $dA$  at a depth 'h' from free surface & at a distance  $y$  from axis  $O-O$ .

$$P = qgh$$

$$dF = pdA$$

$$= qgh dA$$

Total pressure force

$$F = \int dF = \int qgh dA$$

$$h = y \sin \theta$$

$$F = \int qgy \sin \theta dA$$

$$= qg \sin \theta \int y dA$$

$$= qg \sin \theta I_o$$

$$= qg \sin \theta A \bar{y}$$

$$= qgA \bar{y} \sin \theta$$

$$= qgA \bar{h}$$

**Centre of pressure:**

Pressure force on the strip  $dF = qgh \, dA$

$$= qg \sin \theta \, dA$$

Moment of the force  $dF$  about  $O - O$

$$= dF \times y = qgy^2 \sin \theta \, dA$$

Sum of moments of all such forces about  $O - O$

$$= qg \sin \theta \int y^2 dA$$

$\int y^2 dA =$  moment of inertia of the surface about  $O - O = I_o$

$$= qg \sin \theta I_o$$

Moment of total force about  $O - O$

$$= F y^*$$

$$F y^* = qg \sin \theta I_o$$

$$qg A \bar{h} \times \frac{h^*}{\sin \theta} = qg \sin \theta I_o$$

$$h^* = \frac{\sin^2 \theta}{A \bar{h}} I_o$$

$$= \frac{\sin^2 \theta}{A \bar{h}} [I_G + A \times (\bar{y})^2]$$

Here  $\frac{\bar{y}}{\bar{h}} = \sin \theta$

$$\bar{y} = \frac{\bar{h}}{\sin \theta}$$

$$h^* = \frac{\sin^2 \theta}{A \bar{h}} \left[ I_G + A \times \left( \frac{\bar{h}}{\sin \theta} \right)^2 \right]$$

$$h^* = \frac{I_G \sin^2 \theta}{A \bar{h}} + \bar{h}$$

### **Archimedes principle:**

When a body is immersed in a fluid either wholly or partially, it is buoyed or lifted up by a force, which is equal to the weight of fluid displaced by the body.

### **Buoyancy:**

Whenever a body is immersed wholly or partially in a fluid it is subjected to an upward force which tends to lift it up. This tendency for an immersed body to be lifted up in the fluid due to an upward force opposite to action of gravity is known as buoyancy this upward force is known as force of buoyancy.

### **Centre of Buoyancy:**

It is defined as the point through which the force of buoyancy is supposed to act. The force of buoyancy is a vertical force and is equal to the weight of the fluid displaced by the body.

Center of buoyancy will be the centre of gravity of the fluid displaced.

### **Problem-1:**

Find the volume of the water displaced & position of centre of buoyancy for a wooden block of width 2.5m & of depth 1.5m when it floats horizontally in water. The density of wooden block is 650 kg/m<sup>3</sup> & its length 6.0m.

Solution:

Width = 2.5 m

Density of wooden block = 650kg/m<sup>3</sup>

Depth = 1.5m

Length = 6m

Volume of the block

$$= 2.5 \times 1.5 \times 6$$

$$= 22.50 \text{m}^3$$

Volume of the block = Wt of water displaced

$$= W \times V$$

$$= \rho g \times V$$

$$= 650 \times 9.81 \times 6$$

$$= 143471 \text{ N}$$

Volume of water displaced

$$= \frac{\text{weight}}{\rho \times g}$$

$$= \frac{143471}{1000 \times 9.81}$$

$$= 14.625 \text{ m}^3$$

Position of centre of buoyancy

Volume of wooden block in water = volume of water displaced

$$2.5 \times 6 \times h = 14.625$$

$$\Rightarrow h = \frac{14.625}{2.5 \times 6}$$

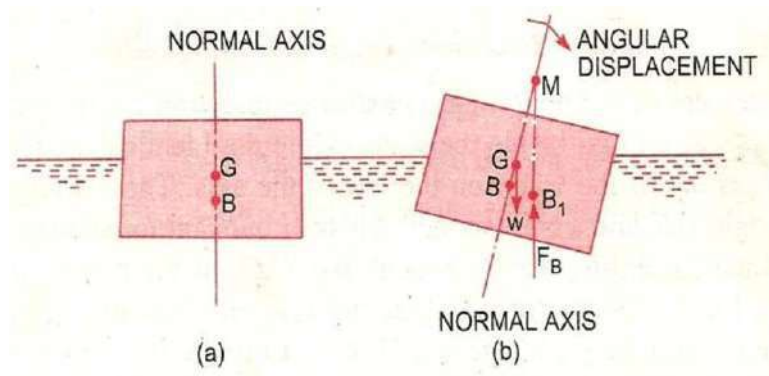
$$= 0.975 \text{m}$$

$$\text{Centre of buoyancy} = \frac{0.975}{2}$$

$$= 0.4875 \text{ m from base.}$$

### **Meta-centre:**

It is defined as the point about which a body starts oscillating when the body is tilted by a small angle. The meta centre may also be defined as the point at which the line of action of the force of buoyancy will meet the normal axis. Of the body when the body is given a small angular displacement.



### **Meta centre height:**

The distance between the meta centre of a floating body and the centre of gravity of the body is called meta-centric height i.e the distance  $MG$ .

### **Concept of flotation:**

#### **Flotation:**

When a body is immersed in any fluid, it experiences two forces. First one is the weight of body  $W$  acting vertically downwards, second is the buoyancy force  $F_b$  acting vertically upwards in case  $W$  is greater than  $F_b$ , the weight will cause the body to sink in the fluid. In case  $W = F_b$  the body will remain in equilibrium at any level. In case  $W$  is small than  $F_b$  the body will move upwards in fluid. The body moving up will come to rest or top moving up in fluid when the fluid displaced by it's submerged part is equal to its weight  $W$ , the body in this situation is said to be floating and this phenomenon is known as flotation.

### **Principle of flotation:**

The principle of flotation states that the weight of the floating body is equal to the weight of the fluid displaced by the body.

Consider a body floating at the free surface of the liquid. The shaded part of the body is inside the fluid and it has volume  $V_1$ . The other part of the body is in air and it has volume  $V_2$ . Now the body can be considered to be in two fluids viz. air and liquid. Hence buoyant force

$$F_p = \rho_{\text{liquid}} V_1 g_1 + \rho_{\text{air}} V_2 g_2 = W$$

Since  $\rho_{\text{air}} \ll \rho_{\text{liquid}}$

$$F_p = \rho_{\text{liquid}} V_1 g = W$$

Buoyancy force is equal to weight of the liquid displaced

### **The ways to make the body float:**

The body can be made to float:

1. Decreasing the weight of the body while keeping the volume same. For example, making body hollow.
2. Increasing the volume of the body while keeping the body same. For example, attaching life jacket to a person fixed the person floating.





# Chapter-4

## KINEMATICS

### Syllabus:

**Types of fluid flow****Continuity equation (Statement and proof for one dimensional flow)****Bernoulli's theorem (Statement and proof)****Applications and limitations of Bernoulli's theorem (Venturimeter, pitot tube) (Simple Numerical)**

### **Introduction:-**

This chapter includes the study of forces causing fluid flow. The dynamics of fluid flow is the study of fluid motion with the forces causing flow. The dynamic behaviour of the fluid flow is analysed by the Newton second law of motion, which relates the acceleration with the forces. The fluid is assumed to be incompressible and non-viscous.

## **TYPES OF FLOW:-**

The fluid flow is classified as follows:

- **STEADY AND UNSTEADY FLOW**
- **UNIFORM AND NON- UNIFORM FLOWS**
- **LAMINAR AND TURBULANT FLOWS**
- **COMPRESSIBLE AND INCOMPRESSIBLE FLOWS**
- **ROTATIONAL AND IRROTATIONAL FLOWS**
- **ONE, TWO, THREE DIMENSIONAL FLOW**

### ➤ **STEADY AND UNSTEADY FLOW:-**

#### 1. **Steady flow:-**

Steady flow is defined as that type of flow in which the fluid characteristics like velocity, pressure, density at a point do not change with time.

Thus, mathematically

$$\left(\frac{\partial v}{\partial t}\right)_{0,y_0,z_0} = 0$$

$$\left(\frac{\partial p}{\partial t}\right)_{0,y_0,z_0} = 0$$

$$\left(\frac{\partial \rho}{\partial t}\right)_{0,y_0,z_0} = 0$$

Where  $x_0, y_0, z_0$  is a point in fluid flow .

#### 2. **Unsteady flow:-**

Unsteady flow is defined as that type of flow in which the velocity, pressure, and density at a point changes w.r.t time.

Thus, mathematically

$$\left(\frac{\partial v}{\partial t}\right)_{0,y_0,z_0} \neq 0,$$

$$\left(\frac{\partial p}{\partial t}\right)_{0,y_0,z_0} \neq 0,$$

$$\left(\frac{\partial \rho}{\partial t}\right)_{0,y_0,z_0} \neq 0$$

➤ **UNIFORM AND NON- UNIFORM FLOWS:-**

1. **Uniform flow:-**

It is defined as the flow in which velocity of flow at any given time does not change w.r.t length of flow or space.

Mathematically,

$$\left(\frac{dv}{ds}\right)_{=constant} = 0$$

where  $\partial v$  = velocity of flow ,

$\partial s$  = length of flow ,

T = time

2. **Non- uniform flows:-**

It is defined as the flow in which velocity of flow at any given time changes w.r.t length of flow.

Mathematically,

$$\left(\frac{dv}{ds}\right)_{=constant} \neq 0$$

➤ **LAMINAR AND TURBULANT FLOWS:-**

1. **Laminar flow:-**

Laminar flow is that type of flow in which the fluid particles are moved in a well defined path called streamlines. The paths are parallel and straight to each other.

2. **Turbulent flow:-**

Turbulent flow is that type of flow in which the fluid particles are moved in a zig-zag manner.

For a pipe flow the type of flow is determined by Reynolds number ( $R_e$ )

Mathematically

$$R_e = \frac{VD}{\nu}$$

Where  $V$  = mean velocity of flow

$D$  = diameter of pipe

$\nu$  = kinematic viscosity

If  $R_e < 2000$ , then flow is laminar flow.

If  $R_e > 4000$ , then flow is turbulent flow.

If  $R_e$  lies in between 2000 and 4000, the flow may be laminar or turbulent.

➤ **COMPRESSIBLE AND INCOMPRESSIBLE FLOWS :-**

1. **Compressible flow:-**

Compressible flow is that type of flow in which the density of fluid changes from point to point.

So,  $\rho \neq \text{constant}$ .

2. **Incompressible flow:-**

Incompressible flow is that type of flow in which the density is constant for the fluid flow.

So,  $\rho = \text{constant}$

➤ **ROTATIONAL AND IRROTATIONAL FLOWS:-**

1. **Rotational flow:-**

Rotational flow is that of flow in which the fluid particles while flowing along stream lines also rotate about their own axis.

2. **Ir-rotational flow:-**

Irrotational flow is that type of flow in which the fluid particles while flowing along streamlines do not rotate about their own axis.

➤ **ONE, TWO, THREE DIMENSIONAL FLOW:-**

1. **One dimensional flow:-**

One dimension flow is defined as that type of flow in which velocity is a function of time and one space co-ordinate only.

For a steady one dimensional flow, the velocity is a function of one space co-ordinate only.

$$\text{So, } U = f(x),$$

$$V = 0,$$

$$W = 0$$

U, V, W are velocity components in x, y, z direction respectively.

2. **Two-dimensional flow:-**

Two-dimensional flow is the flow in which velocity is a function of time and 2- space co- ordinates only. For a steady 2- dimensional flow the velocity is a function of two – space co-ordinate only.

$$\text{So, } U = f_1(x,y) ,$$

$$V = f_2(x,y) ,$$

$$W = 0$$

3. **Three-dimensional flow:-**

Three – dimensional flow is the flow in which velocity is a function of time and 3- space co-ordinates only. For steady three- dimensional flow, the velocity is a function of three space co-ordinates only.

So  $U = f_1(x, y, z)$   
 $V = f_2(x, y, z)$   
 $W = f_3(x, y, z)$

### **RATE OF FLOW OR DISCHARGE**

It is defined as the quantity of a fluid flowing per second through a section of pipe.

For an incompressible fluid the rate of flow or discharge is expressed as the volume of fluid flowing across the section per second.

For compressible fluids, the rate of flow is usually expressed as the weight of fluid flowing across the section.

$$Q = A \cdot V$$

Where A = cross sectional area of the pipe

V = velocity of fluid across the section

Unit:-

1. For incompressible fluid

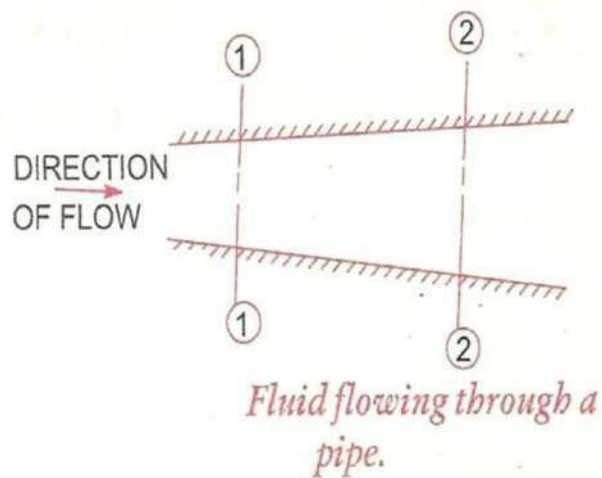
$$\frac{m^3}{sec} \text{ or } \frac{litre}{sec}$$

2. For compressible fluid:

$$\frac{newton}{sec} \text{ (S.I units), } \frac{kgf}{sec} \text{ (M.K.S units)}$$

**EQUATION OF CONTINUITY:-**

It is based on the principle of conservation of mass. For a fluid flowing through the pipe at all the cross-section, the quantity of fluid per second is constant.



Let  $V_1$  = average velocity at cross-section 1-1.

$\rho_1$  = density at cross-section 1-1

$A_1$  = area of pipe at section 1-1

$V_2$  = average velocity at cross-section 2-2

$\rho_2$  = density at cross-section 2-2

$A_2$  = area of pipe at section 2-2

The rate of flow at section 1-1 =  $\rho_1 A_1 V_1$

The rate of flow at section 2-2 =  $\rho_2 A_2 V_2$

According to laws of conservation of mass rate of flow at section 1-1 is equal to the rate of flow at section 2-2 ,

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

This is called continuity equation.

If the fluid is compressible, then  $\rho_1 = \rho_2$ ,

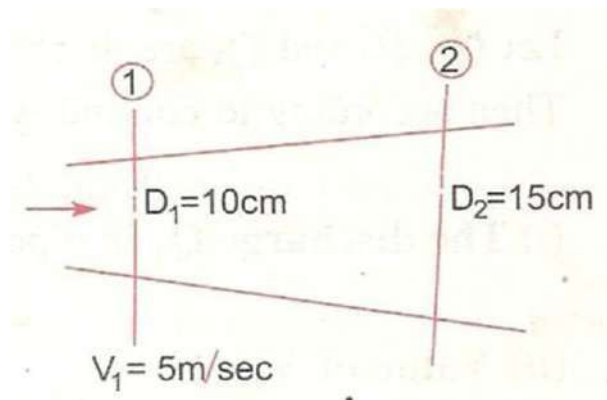
$$\text{so } A_1 V_1 = A_2 V_2$$

“If no fluid is added removed from the pipe in any length then the mass passing across different sections shall be same”

## Simple Problems

### Problem:-1

The diameters of a pipe at the sections 1 and 2 are 10cm and 15cm respectively. Find the discharge through the pipe if the velocity of the water flowing through the pipe at section 1 is 5m/s. Determine also the velocity at section 2.



**Solution.** Given :

At section 1,

$$D_1 = 10\text{ cm} = 0.1\text{ m}$$

$$A_1 = \frac{\pi}{4} (D_1)^2 = \frac{\pi}{4} (.1)^2 = .007854\text{ m}^2$$

$$V_1 = 5\text{ m/s.}$$

At section 2,

$$D_2 = 15\text{ cm} = 0.15\text{ m}$$

$$A_2 = \frac{\pi}{4} (.15)^2 = 0.01767\text{ m}^2$$

(i) Discharge through pipe is given by equation (5.1)

or 
$$Q = A_1 \times V_1$$
$$= .007854 \times 5 = \mathbf{0.03927\text{ m}^3/\text{s. Ans.}}$$

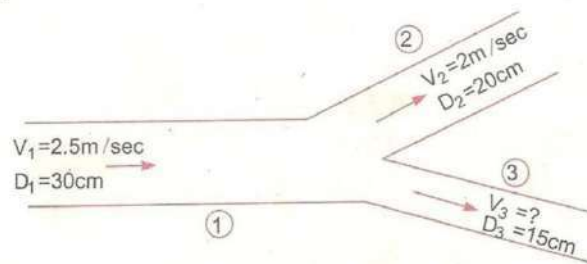
Using equation (5.3), we have  $A_1 V_1 = A_2 V_2$

(ii) 
$$\therefore V_2 = \frac{A_1 V_1}{A_2} = \frac{.007854}{.01767} \times 5.0 = \mathbf{2.22\text{ m/s.}}$$



**Problem:-2**

A 30cm diameter pipe conveying water branches into two pipes of diameter 20cm and 15cm respectively. If the average velocity in the 30cm diameter pipe is 2.5 m/s, find the discharge in this pipe. Also determine the velocity in 15cm pipe if the average velocity in 20cm diameter pipe is 2m/s

**Solution:****Given Data:**

$$D_1 = 30\text{cm} = 0.30\text{m}$$

$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (0.3)^2 = 0.07068 \text{ m}^2$$

$$V_1 = 2.5 \text{ m/s}$$

$$D_2 = 20\text{cm} = 0.2\text{m}$$

$$A_2 = \frac{\pi}{4} 0.2^2 = 0.0314 \text{ m}^2$$

$$V_2 = 2\text{m/s}$$

$$D_3 = 15\text{cm} = 0.15\text{m}$$

$$A_3 = \frac{\pi}{4} 0.15^2 = 0.01767 \text{ m}^2$$

Let  $Q_1$ ,  $Q_2$ ,  $Q_3$  are discharges in pipe 1, 2, 3 respectively

$$Q_1 = Q_2 + Q_3$$

The discharge  $Q_1$  in pipe 1 is given as

$$Q_1 = A_1 V_1$$

$$= 0.07068 \times 2.5 \text{ m}^3/\text{s}$$

$$Q_2 = A_2 V_2$$

$$= 0.0314 \times 2.0 \text{ m}^3/\text{s}$$

Substituting the values of  $Q_1$  and  $Q_2$  on the above equation we get

$$0.1767 = 0.0628 + Q_3$$

$$Q_3 = 0.1767 - 0.0628$$
$$= 0.1139 \text{ m}^3/\text{s}$$

Again  $Q_3 = A_3 V_3$

$$= 0.01767 \times V_3$$

Or  $0.1139 = 0.01767 \times V_3$

$$V_3 = \frac{0.1139}{0.01767}$$

$$= 6.44 \text{ m/s}$$

**Problem:-3**

A 25 cm diameter pipe carries oil of sp. Gr. 0.9 at a velocity of 3m/s. At another section the diameter is 20cm. Find the velocity at this section and also mass rater of flow of oil.

**Solution.** Given :

at section 1,

$$D_1 = 25 \text{ cm} = 0.25 \text{ m}$$

$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times .25^2 = 0.049 \text{ m}^2$$

$$V_1 = 3 \text{ m/s}$$

at section 2,

$$D_2 = 20 \text{ cm} = 0.2 \text{ m}$$

$$A_2 = \frac{\pi}{4} (.2)^2 = 0.0314 \text{ m}^2$$

$$V_2 = ?$$

Mass rate of flow of oil = ?

Applying continuity equation at sections 1 and 2,

$$A_1 V_1 = A_2 V_2$$

or  $0.049 \times 3.0 = 0.0314 \times V_2$

$$\therefore V_2 = \frac{0.049 \times 3.0}{.0314} = \mathbf{4.68 \text{ m/s. Ans.}}$$

Mass rate of flow of oil = Mass density  $\times Q = \rho \times A_1 \times V_1$

Sp. gr. of oil =  $\frac{\text{Densit of oil}}{\text{Densit of water}}$

$\therefore$  Density of oil = Sp. gr. of oil  $\times$  Density of water

$$= 0.9 \times 1000 \text{ kg/m}^3 = \frac{900 \text{ kg}}{\text{m}^3}$$

$\therefore$  Mass rate of flow =  $900 \times 0.049 \times 3.0 \text{ kg/s} = \mathbf{132.23 \text{ kg/s. Ans.}}$

**Bernoulli's equation:**

**Statement:** It states that in a steady ideal flow of an incompressible fluid, the total energy at any point of flow is constant.

The total energy consists of pressure energy, kinetic energy & potential energy or datum energy. These energies per unit weight are

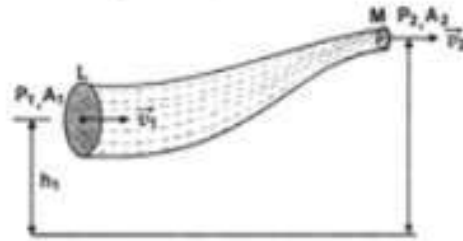
$$\text{Pressure energy} = \frac{P}{\rho g}$$

$$\text{Kinetic energy} = \frac{v^2}{2g}$$

$$\text{Datum energy} = z$$

Mathematically

$$\frac{P}{\rho} + gh + \frac{1}{2}v^2 = \text{Constant}$$



Proof: Let us consider the ideal liquid of density  $\rho$  flowing through the pipe LM of varying cross-section. Let  $P_1$  and  $P_2$  be the pressures at ends L and M and  $A_1$  and  $A_2$  be the areas of cross-sections at ends L and M respectively. Let the liquid enter with velocity  $v_1$  and leave with velocity  $v_2$ . Let  $A_1 > A_2$ . By equation of continuity,

$$A_1 v_1 = A_2 v_2$$

Since  $A_1 > A_2$ ,

$$\therefore v_2 > v_1 \quad \text{and} \quad P_1 > P_2$$

Let  $m$  be mass of liquid entering at end L in time  $t$ . In time  $t$ , the liquid will cover a distance of  $v_1 t$ .

Therefore the work done by pressure on the liquid at end L in time  $t$  is

$$\begin{aligned} W_1 &= \text{force} \times \text{displacement} \\ &= P_1 A_1 v_1 t \end{aligned} \quad \dots(1)$$

Since same mass  $m$  leaves the pipe at end M in same time  $t$ , in which liquid will cover the distance  $v_2 t$ , therefore work done by liquid against the force due to pressure  $P_2$  is

$$W_2 = P_2 A_2 v_2 t \quad \dots(2)$$

Net work done by pressure on the liquid in time  $t$  is,

$$W = W_1 - W_2 = P_1 A_1 v_1 t - P_2 A_2 v_2 t \quad \dots(3)$$

This work done on liquid by pressure increases its kinetic and potential energy.

Increase in kinetic energy of liquid is,

$$\Delta K = \frac{1}{2} m (v_2^2 - v_1^2) \quad \dots(4)$$

According to work-energy relation,

$$P_1 A_1 v_1 t - P_2 A_2 v_2 t = \frac{1}{2} m (v_2^2 - v_1^2) + mg(h_2 - h_1) \quad \dots(6)$$

If there is no source and sink of liquid, then mass of liquid entering at end L is equal to the mass of liquid leaving the pipe at end M and is given by

$$A_1 v_1 \rho t = A_2 v_2 \rho t = m$$

$$\text{or} \quad A_1 v_1 t = A_2 v_2 t = \frac{m}{\rho} \quad \dots(7)$$

From (6) and (7)

$$P_1 \frac{m}{\rho} - P_2 \frac{m}{\rho} = \frac{1}{2} m (v_2^2 - v_1^2) + mg(h_2 - h_1)$$

$$\text{or} \quad P_1 \frac{m}{\rho} + \frac{1}{2} m v_1^2 + mg h_1 = P_2 \frac{m}{\rho} + \frac{1}{2} m v_2^2 + mg h_2$$

$$\text{or} \quad \frac{P}{\rho} + gh + \frac{1}{2} v^2 = \text{Constant}$$

**Problem:- 5**

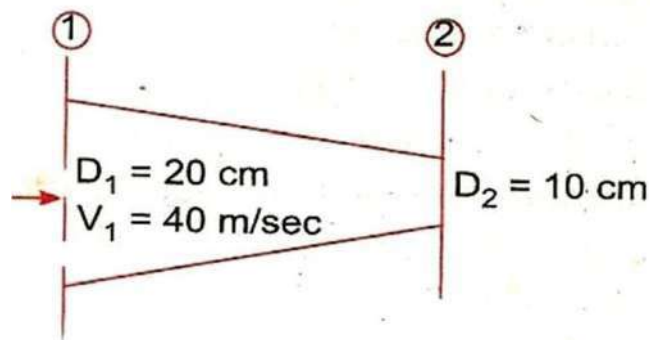
Water is flowing through a pipe of 5cm diameter under a pressure of 29.43 N/cm<sup>2</sup> (gauge) and with mean velocity of 2.0 m/s. Find the total head or total energy per unit weight of the water at a cross-section, which is 5m above the datum line.

**Solution.** Given :

Diameter of pipe	= 5 cm = 0.5 m	
Pressure,	$p = 29.43 \text{ N/cm}^2 = 29.43 \times 10^4 \text{ N/m}^2$	
Velocity,	$v = 2.0 \text{ m/s}$	
Datum head,	$z = 5 \text{ m}$	
Total head	= pressure head + kinetic head + datum head	
Pressure head	$= \frac{p}{\rho g} = \frac{29.43 \times 10^4}{1000 \times 9.81} = 30 \text{ m}$	$\left\{ \rho \text{ for water} = 1000 \frac{\text{kg}}{\text{m}^3} \right\}$
Kinetic head	$= \frac{v^2}{2g} = \frac{2 \times 2}{2 \times 9.81} = 0.204 \text{ m}$	
$\therefore$ Total head	$= \frac{p}{\rho g} + \frac{v^2}{2g} + z = 30 + 0.204 + 5 = 35.204 \text{ m. Ans.}$	

**Problem:- 6**

A pipe, through which water is flowing, is having diameters, 20cm and 10cm at the cross sections 1 and 2 respectively. The velocity of water at section 1 is given 4.0 m/s. Find the velocity head at sections 1 and 2 and also rate of discharge.



**Solution.** Given :

$$D_1 = 20 \text{ cm} = 0.2 \text{ m}$$

∴ Area,

$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (.2)^2 = 0.0314 \text{ m}^2$$

$$V_1 = 4.0 \text{ m/s}$$

$$D_2 = 0.1 \text{ m}$$

∴

$$A_2 = \frac{\pi}{4} (.1)^2 = .00785 \text{ m}^2$$

(i) Velocity head at section 1

$$= \frac{V_1^2}{2g} = \frac{4.0 \times 4.0}{2 \times 9.81} = \mathbf{0.815 \text{ m. Ans.}}$$

(ii) Velocity head at section 2 =  $V_2^2/2g$

To find  $V_2$ , apply continuity equation at 1 and 2

$$\therefore A_1 V_1 = A_2 V_2 \quad \text{or} \quad V_2 = \frac{A_1 V_1}{A_2} = \frac{.0314}{.00785} \times 4.0 = 16.0 \text{ m/s}$$

$$\therefore \text{Velocity head at section 2} = \frac{V_2^2}{2g} = \frac{16.0 \times 16.0}{2 \times 9.81} = \mathbf{83.047 \text{ m. Ans.}}$$

(iii) Rate of discharge

$$\begin{aligned} &= A_1 V_1 \quad \text{or} \quad A_2 V_2 \\ &= 0.0314 \times 4.0 = 0.1256 \text{ m}^3/\text{s} \end{aligned}$$



### **Application of Bernoulli's equation:**

Bernoulli's equation is applied in all problems of incompressible fluid flow where energy consideration are involved. It is also applied to following measuring devices

#### **1. Venturimeter**

#### **2. Pitot tube**

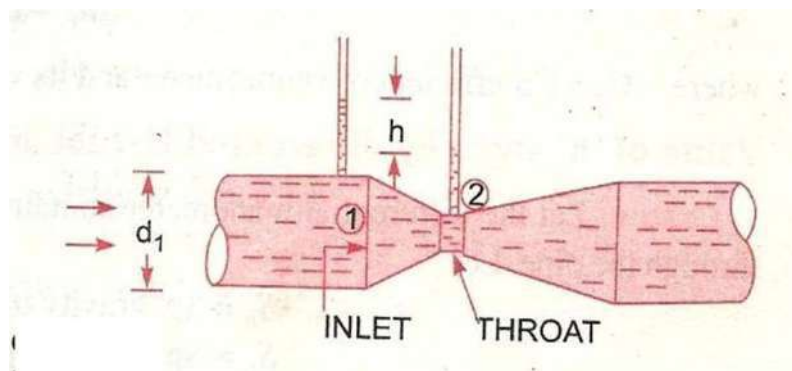
### **Venturimeter:**

A venturimeter is a device used for measuring the rate of a flow of a fluid flowing through a pipe it consists of three parts.

- I. Short converging part**
- II. Throat**
- III. Diverging part**

### **Expression for rate of flow through venturimeter:**

Consider a venturimeter is fitted in a horizontal pipe through which a fluid flowing



Let  $d_1$  = diameter at inlet or at section (i)-(ii)

$P_1$  = pressure at section (1)-(1)

$V_1$  = velocity of fluid at section (1) – (1)

$A_1$  = area at section (1) – (1) =  $\frac{\pi d_1^2}{4}$

$D_2, p_2, v_2, a_2$  are corresponding values at section 2 applying Bernouli's equation at sections 1 and 2 we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

As pipe is horizontal, hence  $z_1 = z_2$

$$\therefore \frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} \quad \text{or} \quad \frac{p_1 - p_2}{\rho g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

But  $\frac{p_1 - p_2}{\rho g}$  is the difference of pressure heads at sections 1 and 2

and it is equal to h

$$\text{So, } h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

Now applying continuity equation at sections 1 & 2  $a_1 v_1 = a_2 v_2$

$$\text{Or } v_1 = \frac{a_2 v_2}{a_1}$$

Substituting this value

$$h = \frac{v_2^2}{2g} - \frac{\left(\frac{a_2 v_2}{a_1}\right)^2}{2g} = \frac{v_2^2}{2g} \left[1 - \frac{a_2^2}{a_1^2}\right] = \frac{v_2^2}{2g} \left[\frac{a_1^2 - a_2^2}{a_1^2}\right]$$

$$v_2^2 = 2gh \frac{a_1^2}{a_1^2 - a_2^2}$$

$$v_2 = \sqrt{2gh \frac{a_1^2}{a_1^2 - a_2^2}} = \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

$$Q = a_2 v_2$$

$$= a_2 \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

Where Q = Theoretical discharge

Actual discharge will be less than theoretical discharge

$$Q_{act} = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

Where  $C_d$  = co-efficient of venturimeter and value is less than 1

### Value of 'h' given by differential U-tube manometer:

#### Case-i:

Let the differential manometer contains a liquid which is heavier than the liquid flowing through the pipe

Let  $S_h$  = Sp. Gravity of the heavier liquid

$S_0$  = Sp. Gravity of the liquid flowing through pipe

x = difference of the heavier liquid column in U-tube

$$P_A - P_B = gx(\rho_g - \rho_0)$$

$$\frac{P_A - P_B}{\rho_0 g} = x \left(\frac{\rho_g}{\rho_0} - 1\right)$$

$$h = x \left[\frac{S_h}{S_0} - 1\right]$$

**Case-ii**

If the differential manometer contains a liquid lighter than the liquid flowing through the pipe

Where  $S_1$  = Specific gravity of lighter liquid in U-tube manometre

$S_0$  = Specific gravity of fluid flowing through in U-tube manometre

$x$  = Difference of lighter liquid columns in U- tube

The value of  $h$  is given by

$$h = x \left[ 1 - \frac{S_1}{S_0} \right]$$

**Case-iii:**

Inclined venturimeter with differential U-tube manometre

Let the differential manometer contains heavier liquid

Then  $h$  is given as

$$\begin{aligned} h &= \left[ \frac{P_1}{\rho g} + z_1 \right] - \left[ \frac{P_2}{\rho g} + z_2 \right] \\ &= x \left[ \frac{S_0 h}{S_0} - 1 \right] \end{aligned}$$

**Case-iv:**

Similarly for inclined venturimeter in which differential manometer contains a liquid which is heavier than the liquid flowing through the pipe.

Then

$$h = \left[ \frac{P_1}{\rho g} + z_1 \right] - \left[ \frac{P_2}{\rho g} + z_2 \right]$$

$$h = x \left[ 1 - \frac{S_1}{S_0} \right]$$

### Limitations:

- Bernoulli's equation has been derived under the assumption that no external force except the gravity force is acting on the liquid. But in actual practice some external forces always act on the liquid when effect the flow of liquid
- If the liquid is flowing in a curved path the energy due to centrifugal force should also be taken into account.

### Pitot-tube:

It is a device used for measuring the velocity of flow at any point in a pipe or a channel.

It is based on the principle that if the velocity flow at a point becomes zero, the pressure there is increased due to conversion of the kinetic energy into pressure energy.

The pitot-tube consists of a glass tube, bent at right angles. Consider two points 1 and 2 at the same level. Such a way that 2 is at the inlet of pitot tube and one is far away from the tube

Let  $P_1$  = pressure at point 1

$V_1$  = velocity of fluid at point 1

$P_2$  = pressure at 2

$V_2$  = velocity of fluid at point 2

H = Depth of tube in the liquid

h = Rise of the liquid in the tube above the free surface

Applying Bernoulli's theorem

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{P_1}{\rho g} = H \quad \frac{P_2}{\rho g} = (h + H)$$

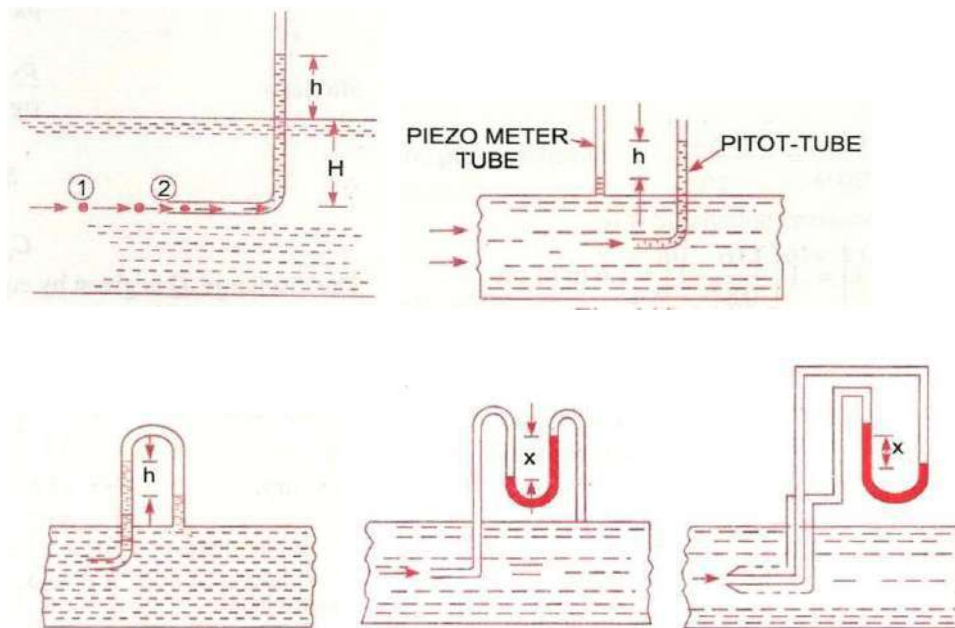
$$H + \frac{V_1^2}{2g} = h + H$$

$$V_1 = \sqrt{2gh}$$

Actual velocity,  $V_{act} = C_v \sqrt{2gh}$

$C_v$  = co-efficient of Pitot-tube

### Different Arrangement of Pitot tubes



### Numerical Problems:

#### **Problem:- 7**

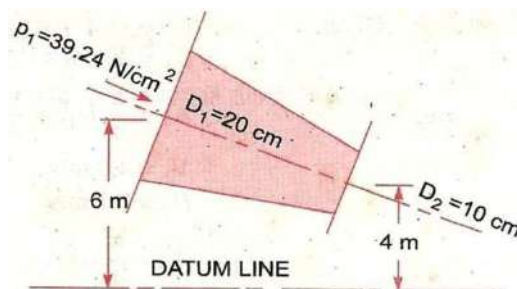
Water is flowing through a pipe of 5cm diameter under a pressure of 29.43 N/cm<sup>2</sup> (gauge) and with mean velocity of 2.0 m/s. Find the total head or total energy per unit weight of the water at a cross-section, which is 5m above the datum line.

**Solution.** Given :

Diameter of pipe	= 5 cm = 0.5 m	
Pressure,	$p = 29.43 \text{ N/cm}^2 = 29.43 \times 10^4 \text{ N/m}^2$	
Velocity,	$v = 2.0 \text{ m/s}$	
Datum head,	$z = 5 \text{ m}$	
Total head	= pressure head + kinetic head + datum head	
Pressure head	$= \frac{p}{\rho g} = \frac{29.43 \times 10^4}{1000 \times 9.81} = 30 \text{ m}$	$\left\{ \rho \text{ for water} = 1000 \frac{\text{kg}}{\text{m}^3} \right\}$
Kinetic head	$= \frac{v^2}{2g} = \frac{2 \times 2}{2 \times 9.81} = 0.204 \text{ m}$	
$\therefore$ Total head	$= \frac{p}{\rho g} + \frac{v^2}{2g} + z = 30 + 0.204 + 5 = 35.204 \text{ m. Ans.}$	

#### **Problem:- 8**

The water is flowing through a pipe having diameters 20 cm and 10 cm at sections 1 and 2 respectively. The rate of flow through pipe is 35lit/s. The section 1 is 6m above datum and section 2 is 4m above datum. If the pressure at section 1 is 39.24 N/cm<sup>2</sup>. Find the intensity of pressure at section 2



**Solution:**

Given

At section 1,

$$D_1 = 20 \text{ cm} = 0.2 \text{ m}$$

$$A_1 = \frac{\pi}{4} (.2)^2 = .0314 \text{ m}^2$$

$$p_1 = 39.24 \text{ N/cm}^2 \\ = 39.24 \times 10^4 \text{ N/m}^2$$

$$z_1 = 6.0 \text{ m}$$

At section 2,

$$D_2 = 0.10 \text{ m}$$

$$A_2 = \frac{\pi}{4} (0.1)^2 = .00785 \text{ m}^2$$

$$z_2 = 4 \text{ m}$$

$$p_2 = ?$$

Rate of flow,

$$Q = 35 \text{ lit/s} = \frac{35}{1000} = .035 \text{ m}^3/\text{s}$$

Now

$$Q = A_1 V_1 = A_2 V_2$$

 $\therefore$ 

$$V_1 = \frac{Q}{A_1} = \frac{.035}{.0314} = 1.114 \text{ m/s}$$

and

$$V_2 = \frac{Q}{A_2} = \frac{.035}{.00785} = 4.456 \text{ m/s}$$

Applying Bernoulli's equation at sections 1 and 2, we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\text{or } \frac{39.24 \times 10^4}{1000 \times 9.81} + \frac{(1.114)^2}{2 \times 9.81} + 6.0 = \frac{p_2}{1000 \times 9.81} + \frac{(4.456)^2}{2 \times 9.81} + 4.0$$

$$\text{or } 40 + 0.063 + 6.0 = \frac{p_2}{9810} + 1.012 + 4.0$$

$$\text{or } 46.063 = \frac{p_2}{9810} + 5.012$$

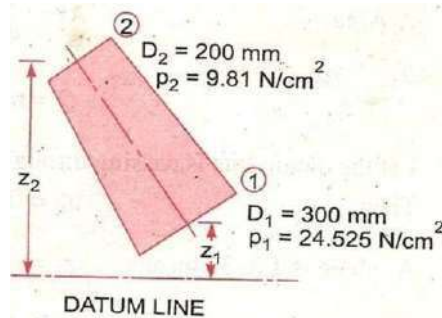
$$\therefore \frac{p_2}{9810} = 46.063 - 5.012 = 41.051$$

$$\therefore p_2 = 41.051 \times 9810 \text{ N/m}^2 \\ = \frac{41.051 \times 9810}{10^4} \text{ N/cm}^2 = \mathbf{40.27 \text{ N/cm}^2}$$



### Problem:- 9

Water is flowing through a pipe having diameter 300mm and 200 mm at the bottom and upper end respectively. The intensity of pressure at the bottom end is  $9.81\text{N/m}^2$ . Determine the difference in datum head if the rate of flow through pipe is 40 lit/s



**Solution.** Given :

**Section 1,**  $D_1 = 300 \text{ mm} = 0.3 \text{ m}$   
 $p_1 = 24.525 \text{ N/cm}^2 = 24.525 \times 10^4 \text{ N/m}^2$

**Section 2,**  $D_2 = 200 \text{ mm} = 0.2 \text{ m}$   
 $p_2 = 9.81 \text{ N/cm}^2 = 9.81 \times 10^4 \text{ N/m}^2$

Rate of flow = 40 lit/s

or  $Q = \frac{40}{1000} = 0.04 \text{ m}^3/\text{s}$

Now  $A_1 V_1 = A_2 V_2 = \text{rate of flow} = 0.04$

$$\therefore V_1 = \frac{.04}{A_1} = \frac{.04}{\frac{\pi}{4} D_1^2} = \frac{0.04}{\frac{\pi}{4} (0.3)^2} = 0.5658 \text{ m/s}$$

$$\approx 0.566 \text{ m/s}$$

$$V_2 = \frac{.04}{A_2} = \frac{.04}{\frac{\pi}{4} (D_2)^2} = \frac{0.04}{\frac{\pi}{4} (0.2)^2} = 1.274 \text{ m/s}$$

Applying Bernoulli's equation at (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

or  $\frac{24.525 \times 10^4}{1000 \times 9.81} + \frac{.566 \times .566}{2 \times 9.81} + z_1 = \frac{9.81 \times 10^4}{1000 \times 9.81} + \frac{(1.274)^2}{2 \times 9.81} + z_2$

or  $25 + .32 + z_1 = 10 + 1.623 + z_2$

or  $25.32 + z_1 = 11.623 + z_2$

$\therefore z_2 - z_1 = 25.32 - 11.623 = 13.697 = 13.70 \text{ m}$

$\therefore$  Difference in datum head =  $z_2 - z_1 = 13.70 \text{ m. Ans.}$

**Problem:- 10**

A horizontal venturimeter with inlet and throat diameters 10cm and 15 cm respectively is used to measure the flow of water. The reading of differential manometer connected to the inlet and throat is 20cm of mercury. Determine the rate of flow. Take  $C_d = 0.98$

**Solution.** Given :

Dia. at inlet,  $d_1 = 30 \text{ cm}$

$\therefore$  Area at inlet,  $a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} (30)^2 = 706.85 \text{ cm}^2$

Dia. at throat,  $d_2 = 15 \text{ cm}$

$\therefore$   $a_2 = \frac{\pi}{4} \times 15^2 = 176.7 \text{ cm}^2$

$C_d = 0.98$

Reading of differential manometer =  $x = 20 \text{ cm}$  of mercury.

$\therefore$  Difference of pressure head is given by (6.9)

or 
$$h = x \left[ \frac{S_h}{S_o} - 1 \right]$$

where  $S_h = \text{Sp. gravity of mercury} = 13.6$ ,  $S_o = \text{Sp. gravity of water} = 1$

$$= 20 \left[ \frac{13.6}{1} - 1 \right] = 20 \times 12.6 \text{ cm} = 252.0 \text{ cm of water.}$$

The discharge through venturimeter is given by eqn. (6.8)

$$\begin{aligned} Q &= C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} \\ &= 0.98 \times \frac{706.85 \times 176.7}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 9.81 \times 252} \\ &= \frac{86067593.36}{\sqrt{499636.9 - 31222.9}} = \frac{86067593.36}{684.4} \\ &= 125756 \text{ cm}^3/\text{s} = \frac{125756}{1000} \text{ lit/s} = \mathbf{125.756 \text{ lit/s.}} \end{aligned}$$

**Problem:- 11**

An oil of Sp.gr. 0.8 is flowing through a horizontal venturimeter having inlet diameter 20cm and throat diameter 10 cm. The oil mercury differential manometer shows a reading of 25cm. Calculate the discharge of oil through the horizontal venturimeter. Take  $C_d = 0.98$

**Solution.** Given :

$$\text{Sp. gr. of oil, } S_o = 0.8$$

$$\text{Sp. gr. of mercury, } S_h = 13.6$$

$$\text{Reading of differential manometer, } x = 25 \text{ cm}$$

$$\begin{aligned} \therefore \text{ Difference of pressure head, } h &= x \left[ \frac{S_h}{S_o} - 1 \right] \\ &= 25 \left[ \frac{13.6}{0.8} - 1 \right] \text{ cm of oil} = 25 [17 - 1] = 400 \text{ cm of oil.} \end{aligned}$$

$$\text{Dia. at inlet, } d_1 = 20 \text{ cm}$$

$$\therefore a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} \times 20^2 = 314.16 \text{ cm}^2$$

$$d_2 = 10 \text{ cm}$$

$$\therefore a_2 = \frac{\pi}{4} \times 10^2 = 78.54 \text{ cm}^2$$

$$C_d = 0.98$$

$\therefore$  The discharge  $Q$  is given by equation (6.8)

or

$$\begin{aligned} Q &= C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} \\ &= 0.98 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times 400} \\ &= \frac{21421375.68}{\sqrt{98696 - 6168}} = \frac{21421375.68}{304} \text{ cm}^3/\text{s} \\ &= 70465 \text{ cm}^3/\text{s} = \mathbf{70.465 \text{ litres/s. Ans.}} \end{aligned}$$

**Problem:- 12**

A horizontal venturimeter with inlet and throat diameters 20cm and 10 cm respectively is used to measure the flow of oil of Sp. gr. The discharge of oil through venturimeter is 60lit/s . Find the reading of oil-mercury differential manometer. Take  $C_d = 0.98$

**Solution.** Given :

$$d_1 = 20 \text{ cm}$$

$$\therefore a_1 = \frac{\pi}{4} 20^2 = 314.16 \text{ cm}^2$$

$$d_2 = 10 \text{ cm}$$

$$\therefore a_2 = \frac{\pi}{4} \times 10^2 = 78.54 \text{ cm}^2$$

$$C_d = 0.98$$

$$Q = 60 \text{ litres/s} = 60 \times 1000 \text{ cm}^3/\text{s}$$

Using the equation (6.8), 
$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

or 
$$60 \times 1000 = 0.98 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times h}$$

$$= \frac{1071068.78\sqrt{h}}{304}$$

or 
$$\sqrt{h} = \frac{304 \times 60000}{1071068.78} = 17.029$$

$\therefore h = (17.029)^2 = 289.98 \text{ cm of oil}$

But 
$$h = x \left[ \frac{S_h}{S_o} - 1 \right]$$

where  $S_h = \text{Sp. gr. of mercury} = 13.6$   
 $S_o = \text{Sp. gr. of oil} = 0.8$   
 $x = \text{Reading of manometer}$

$\therefore 289.98 = x \left[ \frac{13.6}{0.8} - 1 \right] = 16x$

$\therefore x = \frac{289.98}{16} = 18.12 \text{ cm.}$

$\therefore \text{Reading of oil-mercury differential manometer} = \mathbf{18.12 \text{ cm.}}$

### Problem:-13

A static pitot-tube placed in the centre of a 300 mm pipe line has one orifice pointing upstream and is perpendicular to it. The mean velocity in the pipe is 0.80 of the central velocity. Find the discharge through the pipe if the pressure difference between the two orifices is 60mm of water. Take  $C_v = 0.98$

**Solution.** Given :

Dia. of pipe,  $d = 300 \text{ mm} = 0.30 \text{ m}$   
 Diff. of pressure head,  $h = 60 \text{ mm of water} = .06 \text{ m of water}$   
 $C_v = 0.98$

Mean velocity,  $\bar{V} = 0.80 \times \text{Central velocity}$   
 Central velocity is given by equation (6.14)

$$= C_v \sqrt{2gh} = 0.98 \times \sqrt{2 \times 9.81 \times .06} = 1.063 \text{ m/s}$$

$\therefore \bar{V} = 0.80 \times 1.063 = 0.8504 \text{ m/s}$

Discharge,  $Q = \text{Area of pipe} \times \bar{V}$

$$= \frac{\pi}{4} d^2 \times \bar{V} = \frac{\pi}{4} (.30)^2 \times 0.8504 = \mathbf{0.06 \text{ m}^3/\text{s. Ans.}}$$