

**PNS SCHOOL OF ENGINEERING & TECHNOLOGY**  
NISHMANI VIHAR , MARSHAGHAI, KENDRAPARA-754213



**SUBJECT-HYDRAULICS AND IRRIGATION  
ENGINEERING (4THthSemester )**

**Prepared by Er Gayatree sahuo  
DEPARTMENT OF CIVIL  
ENGINEERING**

# Hydraulics I

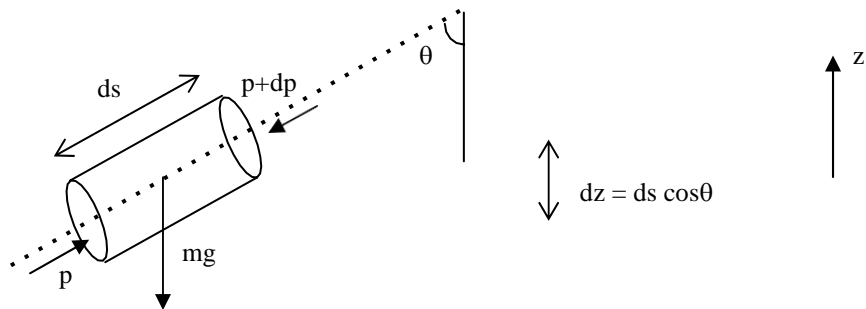
## A Hydrostatics

### B1 Hydrostatic pressure and the hydrostatic equation

#### Hydrostatics:

Hydrostatics deals with the study of fluids at rest and in equilibrium (like statics in mechanics). Net forces are zero, and there is no flow.

Consider a small static cylinder of fluid, with axis of the cylinder (s-axis) tilted at an angle  $\theta$  to the vertical, z-axis.



Volume of fluid element =  $dA \, ds$   
 Mass of fluid element  $m = \rho \, dA \, ds$

#### Resolving forces in the s-direction

Gravitational force =  $-mg = -\rho g \, dA \, ds$  (downwards).

Pressure force (s-direction) =  $p \, dA - (p+dp) \, dA = -dp \, dA$

Gravitational force (s-direction) =  $-mg \cos\theta = -\rho g \, dA \, ds \cos\theta = -\rho g \, dA \, dz$

So total force in s-direction

$$-dp \, dA - \rho g \, dA \, dz = 0.$$

Rearranging this gives us:

Hydrostatic equation  $\frac{dp}{dz} = -\rho g$
---

Note the pressure increases as  $z$  decreases (or depth increases). Integrating downwards (in the negative  $z$ -direction) from a point  $z_0$  of known pressure  $p_0$  to a point  $z < z_0$

$$p(z) = p_0 - \int_{z_0}^z \rho g \, dz = p_0 + \int_z^{z_0} \rho g \, dz$$

Thus the pressure (relative to the reference pressure) is given by the weight of fluid per unit area above that point.

**Pressure underwater**

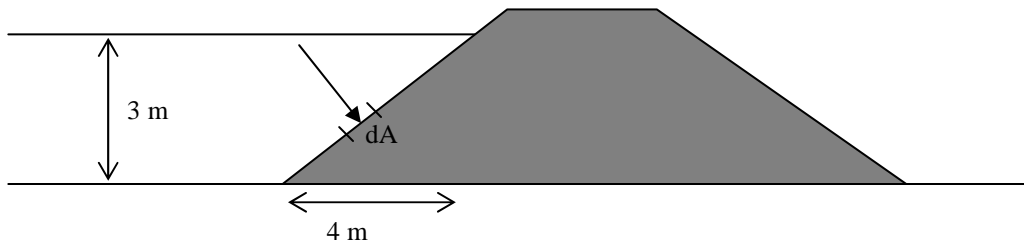
We will usually deal with simple uniform constant pressure. E.g. the pressure under water at a depth  $h$  below the free surface (where the pressure is  $p_{atm}$ ) is given by

$$p = p_{atm} + \rho gh$$

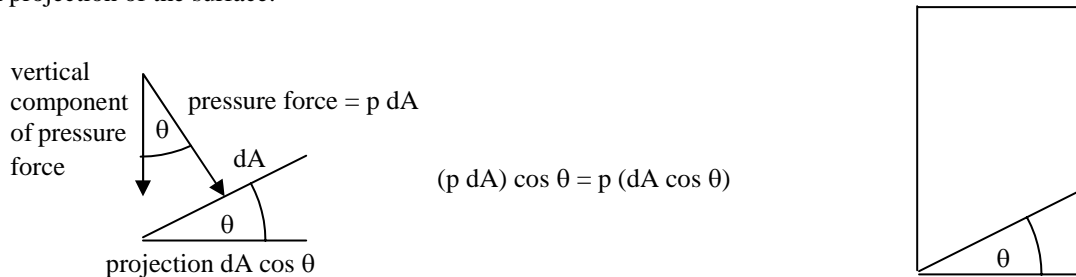
Example: What is the pressure (relative to atmospheric pressure) at a depth of 10 m and 1000 m underwater? [98.1 kPa, 9.81 MPa]

**Example: vertical force on a dam.**

Dam length 40 m.



The pressure force will act on the surface of the dam in a direction normal to the face of the dam. Thus to find the total force we would need to integrate the pressure force (which varies with depth) over the sloping dam face. However, the vertical component of this force is the local pressure multiplied by the plan projection of the surface:



Since the pressure is equal to the total weight per unit area of the water above that point, the downward component of the force on the element  $dA$  is equal to the total weight of water above  $dA$ . Thus the total vertical force on the dam is simply the weight of water above the dam [= 2.354 MN].

**Absolute, gauge and vacuum pressure**

Absolute pressure is the pressure measured relative to a total vacuum, and thus is always positive.

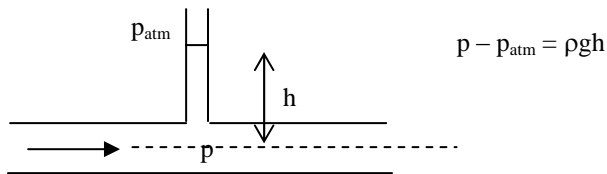
We often measure pressures relative to the local atmospheric pressure (e.g. the pressure underwater above) and this is known as the gauge pressure. Since pressure differences are what gives net forces, subtracting a constant reference pressure is not generally important in fluid dynamics (but we need the absolute pressure for the gas equation, for example).

If the pressure is less than atmospheric pressure but given as relative to atmospheric pressure this is sometimes referred to as a vacuum pressure (e.g. a vacuum pressure of  $10 \text{ kN/m}^2$  is a pressure of  $10 \text{ kN/m}^2$  below atmospheric pressure, or a gauge pressure of  $-10 \text{ kN/m}^2$ ).

## B2 Pressure measurement

### Piezometer

The pressure in a liquid can be measured (relative to atmospheric pressure) using a piezometer tube, e.g. by tapping a tube into a pipe:



### Manometer

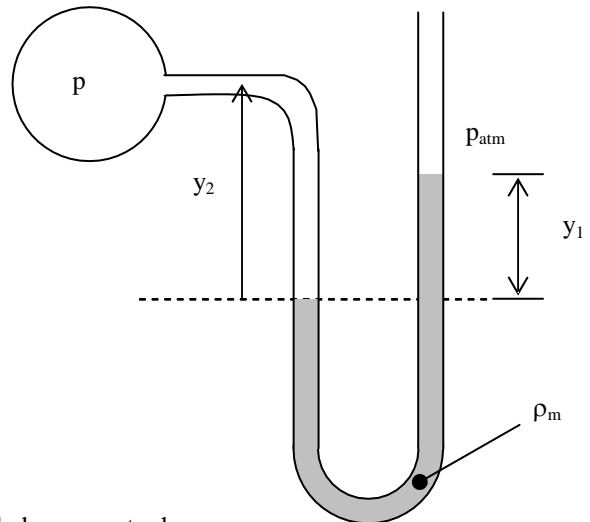
A U-tube manometer can measure the pressure difference between two points or relative to atmospheric pressure (as shown).

The pressure on the horizontal line must be the same in both arms of the tube:

$$p + \rho g y_2 = p_{atm} + \rho_m g y_1$$

thus

$$p - p_{atm} = \rho_m g y_1 - \rho g y_2$$

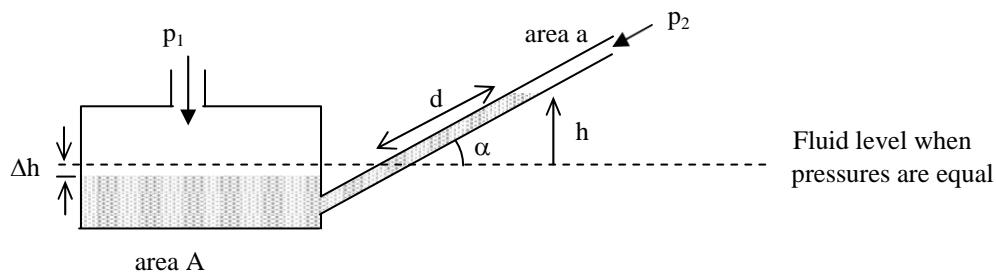


E.g. what is  $p$  (relative to atmospheric) if a water filled manometer has  $y_1 = 4\text{cm}$  and  $y_2 = 10\text{cm}$ ? (assume  $\rho = 1.2\text{ kg m}^{-3}$ .)

[answer 391 Pa]

### Inclined manometer

When measuring small pressures using a manometer, the height moved by the fluid can be small. To make measurement easier an inclined manometer can be used:



$$p_1 = p_2 + \rho g(h + \Delta h)$$

$$h = d \sin \alpha$$

$$A \Delta h = a d$$

$$p_1 - p_2 = \rho g d (\sin \alpha + a/A)$$

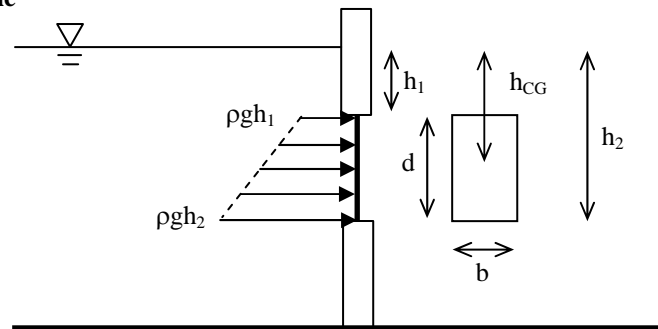
If  $a \ll A$ , then

$$p_1 - p_2 = \rho g d \sin \alpha$$

### B3 Hydrostatic force on a plane surface

The force on a submerged surface is always normal to surface, whatever its orientation or shape.

#### Vertical rectangle



The magnitude of the total force is given by the area under the pressure diagram times the width (b).

$$F = \frac{1}{2} (\rho g h_1 + \rho g h_2) d b = \rho g h_{CG} d b$$

The force is the average pressure multiplied by the area (A), and the average pressure is the pressure at the centroid of the submerged shape (here just the centre of the rectangle).

$$F = p_{CG} A = \rho g h_{CG} A$$

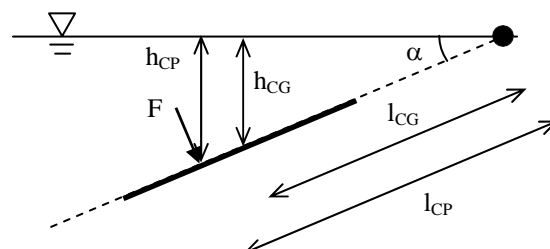
Although the force can be found from the pressure at the centroid of the rectangle, the force does not act through this point. Instead it acts through a point known as the centre of pressure, which is below the centroid as the pressures are higher with depth. We can find the centre of pressure by considering moments of the forces about an axis at the surface.



$$\int h p \, dA = \int h \rho g h \, dA \quad = \quad h_{CP} F = h_{CP} \rho g h_{CG} A$$

$$h_{CP} = \frac{\int h^2 dA}{A h_{CG}}$$

#### Inclined rectangle



$$F = \rho g h_{CG} A = \rho g l_{CG} \sin \alpha A$$

$$l_{CP} = \frac{\int l^2 dA}{A l_{CG}}$$

$$h_{CP} = l_{CP} \sin \alpha$$

**Examples: rectangles**

Vertical rectangle, height 2m, width 1.5 m, with the centre of the rectangle 3m below the surface.

[Total force = 88.3 kN (in horizontal direction), acting at a point 3.11 m below the surface.]

Inclined rectangle: same as before but this time inclined with  $\alpha = 60^\circ$ .

[Total force is still 88.3 kN, but this time acting  $30^\circ$  below the horizontal, at a point 3.08 m below the surface (a distance 9.6 cm along the rectangle from its centre).]

**General plane shape**

For any submerged inclined plane shape, we find the same relationships as for the inclined rectangle.

$$F = \rho g h_{CG} A = \rho g l_{CG} \sin \alpha A \qquad I_{CP} = \frac{\int l^2 dA}{A l_{CG}} \qquad h_{CP} = l_{CP} \sin \alpha$$

$\int l^2 dA$  is the second moment of area for the shape about the axis on the water surface as shown above.

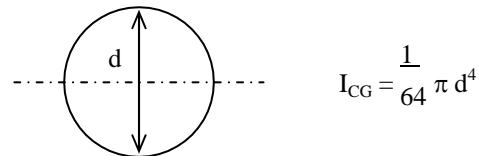
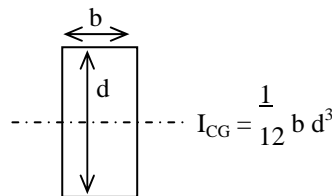
Parallel axis theorem

There are standard results for the second moments of area for various shapes but generally about an axis through their centre,  $I_{CG}$ . We can use the parallel axis theorem to find second moments about other axes, such as at the surface:

$$\int l^2 dA = I_{CG} + A l_{CG}^2$$

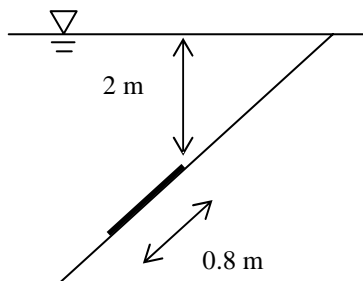
Note we are measuring the lengths  $l$  on the inclined surface (the pressure force acts normal to this surface). We can use this result to find the **centre of pressure**:

$$l_{CP} = \frac{I_{CG}}{A l_{CG}} + l_{CG}$$



**Example: Circular gate**

What is the force on a circular gate of diameter 0.8m mounted on a sloping dam face of angle  $45^\circ$ , and where does it act?



Distance along the slope to the top of the gate is 2.828 m, so  $l_{CG} = 2.828 + 0.4 = 3.228$  m. Thus  $h_{CG} = 2.283$  m.

$$A = 0.503 \text{ m}^2, \qquad F = 11.27 \text{ kN}$$

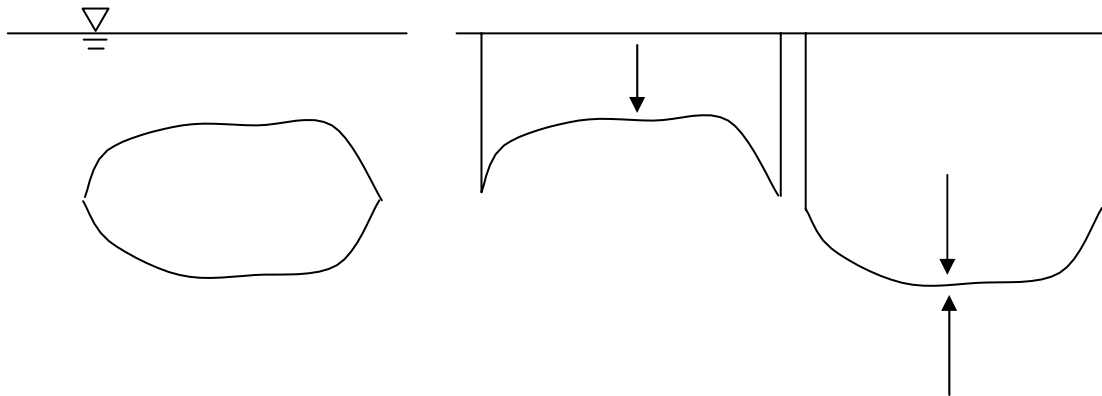
$$I_{CG} = 0.02011 \text{ m}^4$$

$$l_{CP} = 0.0124 \text{ m} + l_{CG} = 3.240 \text{ m}$$

This is a distance of 0.412 m along the slope from the top of the gate, so, for example, the moment of the force about the top of the gate is 4.64 kN m.

## B4 Buoyancy and Archimedes Principle

### Forces on a submerged object

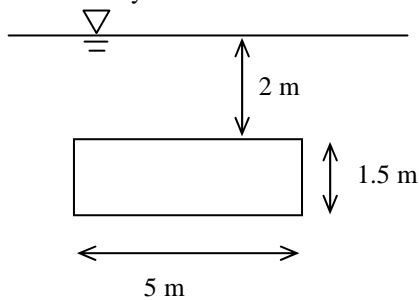


The vertical force on a submerged object can be thought of as a downward force on the upper surface of the object together with an upward force on the lower surface. The force on the upper surface is the weight of the water above this surface. The force on the lower surface is of the same magnitude (but opposite direction) as the weight of water that would be above this surface if the object were not present. The difference is an upward force equal to the weight of water displaced by the object (Archimedes Principle).

Alternatively consider the forces acting on a submerged object of volume  $V$  and density  $\rho_0$ . The gravitational force on the object is  $mg = \rho_0 Vg$  downwards. If the object is denser than water we expect the object to sink (net force downwards), while if it is less dense we expect it to float (net force upwards). If the object has the same density as water ( $\rho_0 = \rho$ ) we expect the net force to be zero, so the buoyancy force must balance the gravitational force. Thus  $B = mg = \rho Vg$ , or again the buoyancy force is equal to the weight of water of the same volume as the object.

### Example: Forces on a cuboid

What are the hydrostatic forces on the cuboid shown, if the other dimension is 3 m?



Pressure on top face = 19.62 kPa

Force on top face = 294.3 kN (downwards)

Pressure on bottom face = 34.34 kPa

Force on bottom face = 515.0 kN (upwards)

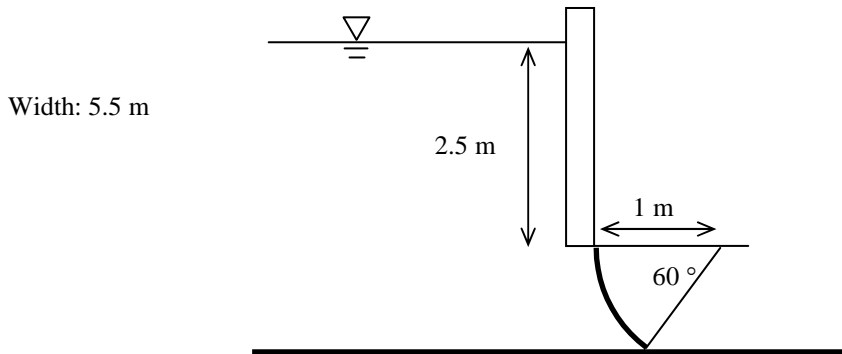
Thus total force = 220.7 kN (upwards)

Volume of cuboid =  $22.5 \text{ m}^3$ , weight of water displaced = 220.7 kN.

## B5 Hydrostatic force on a curved surface

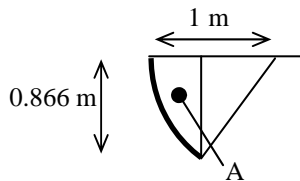
The hydrostatic pressure force is always normal to the surface, so for a simple curved surface of uniform curvature, the total force must pass through centre of curvature.

It is generally easiest to calculate the vertical and horizontal components of the total force separately, and then the point of application of the force can be found by ensuring the total force passes through the centre of curvature.



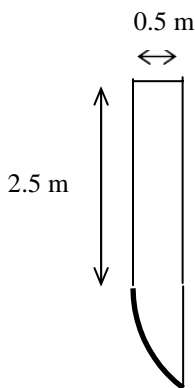
### Example: forces on a curved gate

First, some geometry.



Area of sector =  $0.524 \text{ m}^2$ .  
 Area of triangle =  $0.217 \text{ m}^2$ .  
 Area A =  $0.307 \text{ m}^2$ .

### Vertical force

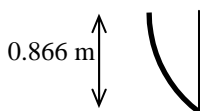


Vertical force equal to weight of fluid displaced by following shape:

Area of shape =  $0.5 \times 2.5 + A = 1.557 \text{ m}^2$

Weight =  $84.01 \text{ kN}$

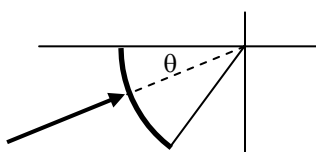
### Horizontal force



Horizontal force is the same as that on a vertical surface of the same height. Average pressure =  $(2.5 + \frac{1}{2} \times 0.866) \rho g = 28.77 \text{ kPa}$

Force =  $28.77 \times 0.866 \times w = 137.0 \text{ kN}$

### Total force



magnitude =  $160.7 \text{ kN}$ ,  $\theta = 31.5^\circ$



## B6 Tutorial: Hydrostatics

Rectangular gate

Curved gate

## B Kinematics and continuity

### C1 Kinematics

**Velocity** of a fluid is a vector function of position and time.

$$\mathbf{u}(\mathbf{x}, t) = (u, v, w) \text{ or } (u_x, u_y, u_z) \text{ where } \mathbf{x} = (x, y, z).$$

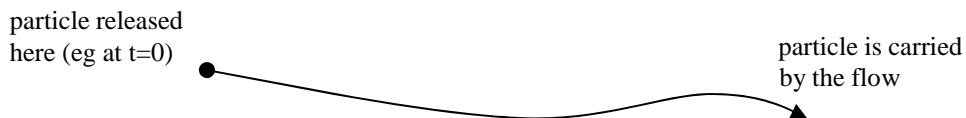
Typically, velocity is measured by an instrument placed at a particular position  $\mathbf{x}$  in the flow. For example: hot wire probes, pitot tubes, anemometers, current meters, LDA, acoustic Doppler. This way of thinking of the velocity as a function of position is called **Eulerian** representation. The disadvantage of the Eulerian representation is that you're not measuring the same piece of fluid.

Alternatively, flow can be measured by following the motion of particular fluid elements (e.g. using tracers, dyes, floats, bubbles, or small particles). Thinking of the flow in this way is called the **Lagrangian** representation.

**Pressure** is a scalar function of position and time.  $p(\mathbf{x}, t)$

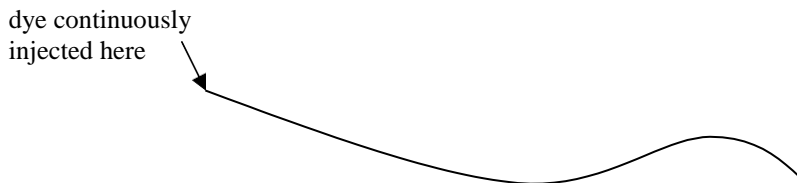
#### Path line or particle path

The line followed by a particle in the fluid released at some point in the flow (what you would see with a long exposure photograph of the flow).



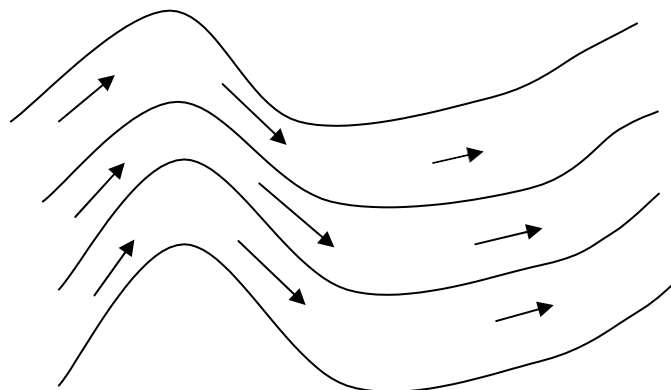
#### Streakline or dyeline

The line of dye resulting by continuously injecting dye at a particular point in the flow.



#### Streamlines

Lines everywhere tangent to the flow direction. This gives an overall picture of the flow field (at a particular instant in time).



**Steady flow**

Flow that doesn't change in time (though the velocity may be different in different parts of the fluid).

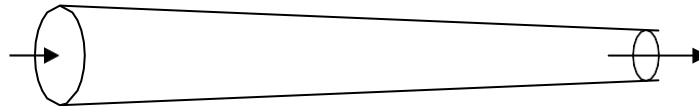
$$\frac{\partial \mathbf{u}}{\partial t} = 0 \text{ (at constant } \mathbf{x} \text{)}$$

For steady flow, streamlines, streaklines and particle-paths are all the same.

**Acceleration**

Just because the velocity at a point isn't changing, this doesn't mean that fluid passing that point isn't accelerating.

Example: steady flow along a contracting pipe:



Flow in = flow out, so the velocity increases.

Following a fluid element (Lagrangian) in steady flow,

$$a = \frac{du}{dt} = \frac{\partial u}{\partial x} \frac{dx}{dt} = \frac{\partial u}{\partial x} u$$

acceleration = velocity × (velocity gradient)

In general,

$$a = u \frac{\partial u}{\partial x} + \frac{\partial u}{\partial t}$$

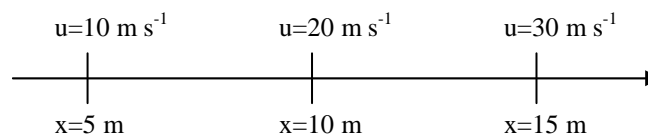
( In 3D, we write  $\frac{D\mathbf{u}}{Dt} = \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}$  )

$$\frac{\partial u}{\partial x} = \frac{1}{2} \frac{\partial(u^2)}{\partial x}$$

Note,  $a = u \frac{\partial u}{\partial x} = \frac{1}{2} \frac{\partial(u^2)}{\partial x}$ , which (for constant acceleration) leads to  $v^2 - u^2 = 2as$ .

Example

$u = kx$  (with  $k=2 \text{ s}^{-1}$ )



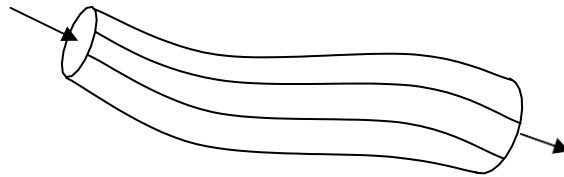
Velocity gradient =  $\frac{\partial u}{\partial x} = 2 \text{ s}^{-1}$  (units are [m/s] / [m]).

Acceleration =  $u \frac{\partial u}{\partial x}$ . At  $x = 5 \text{ m}$ , the acceleration is  $10 \times 2 = 20 \text{ m s}^{-2}$ , while at  $x = 10 \text{ m}$ ,  $a = 40 \text{ m s}^{-2}$ .

## C2 Conservation of mass: continuity

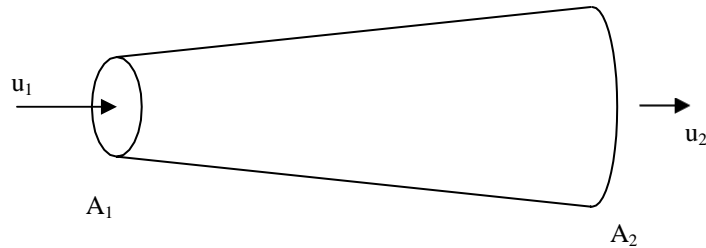
### Streamtube

Bounded by a set of streamlines, so fluid remains within the tube (a "virtual pipe").



### Pipes

A pipe with changing cross-section:



If  $u_1$  is the average velocity where the cross-sectional area is  $A_1$ , and similarly for  $u_2$  and  $A_2$ , then (for incompressible flow),

Mass flow in = mass flow out

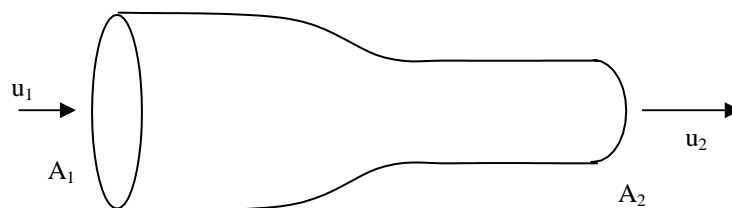
$$\rho u_1 A_1 = \rho u_2 A_2 \text{ (kg s}^{-1}\text{)}$$

The volume flow rate or discharge  $Q = u_1 A_1 = u_2 A_2 \text{ (m}^3 \text{s}^{-1}\text{)}$ .

Thus  $u_2 = u_1(A_1/A_2)$ .

### Example

A pipe of internal diameter 10mm is connected to a pipe of internal diameter 5mm. If the fluid speed entering the larger diameter pipe is  $1 \text{ m s}^{-1}$ , what is the speed of the fluid as it flows through the smaller pipe?

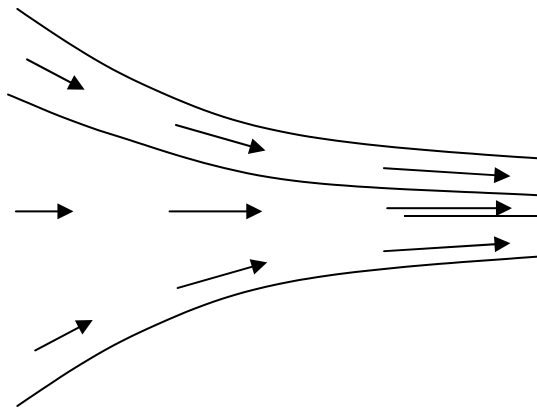


$$u_1 = 1 \text{ m s}^{-1}, \quad A_1 = \pi \times 0.01^2 / 4 \text{ m}^2, \quad A_2 = \pi \times 0.005^2 / 4 \text{ m}^2.$$

$$u_2 = u_1(A_1/A_2) = 1 (0.01^2/0.005^2) = 4 \text{ m s}^{-1}.$$

### Going back to streamlines

Flow speed increases where streamlines converge.



[ $u_x$  increases in x direction,  
 $u_y$  decreases in y direction.  
In general,

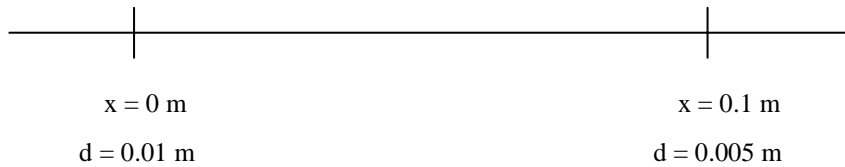
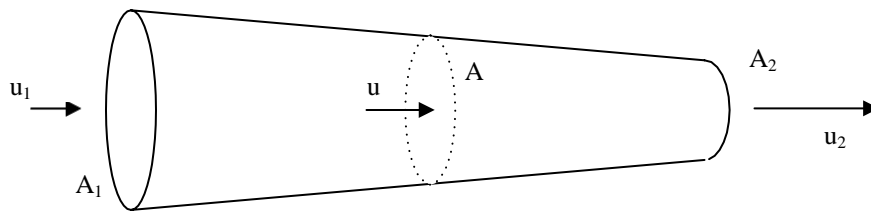
$$\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} = 0$$

or  $\nabla \cdot \mathbf{u} = 0$

for incompressible flows.]

### Acceleration example

In the example of the contracting flow ( $u_1 = 1 \text{ m s}^{-1}$ ), if the diameter changes linearly from 10 mm to 5 mm over a distance of 100 mm, what is the velocity and acceleration as a function of distance along the contraction?



$$d = 0.01 - 0.05x, \quad \text{area} = (\pi/4)d^2, \quad \Rightarrow \quad \text{area} = (\pi/4)(0.01 - 0.05x)^2.$$

$$Q = u A = u_1 A_1 = 1 \times (\pi/4) \times 0.01^2, \text{ constant.}$$

$$\text{So } u = u_1 (A_1/A) = 1 \times 0.01^2 / (0.01 - 0.05x)^2 = 1 / (1 - 5x)^2 = (1 - 5x)^{-2}$$

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x} (1 - 5x)^{-2} = 10(1 - 5x)^{-3}$$

$$a = u \frac{\partial u}{\partial x} = 10(1 - 5x)^{-5}$$

At  $x = 0, 0.05, 0.1 \text{ m}$ ,  $u = 1, 1.78, 4 \text{ m s}^{-1}$ ,  $a = 10, 42.1, 320 \text{ m s}^{-2}$ .

## C3 Tutorial: Kinematics and continuity

### Streamlines, particles paths and streaklines in unsteady flow.

#### Example

$$u_x = V_0 \cos(2\pi t/T)$$

$$u_y = V_0 \sin(2\pi t/T)$$

What do streamlines look like (at a particular value of  $t$ )?

Release particle at  $(x,y) = (0,0)$  at  $t = 0$ , what is the path?

Release dye at  $(0,0)$  from  $t = 0$  to  $t = T$ , what is the resulting dyeline?

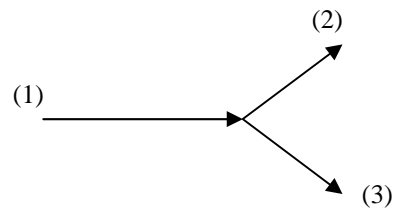
#### **Continuity in pipe networks**

Flow in = flow out.

Flow in (1) 10mm ID,  $0.1 \text{ m s}^{-1}$ .

Flow out (2) 5mm ID,  $0.05 \text{ m s}^{-1}$ .

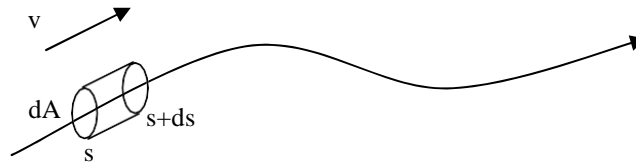
Flow out (3) 7mm ID, what is  $u_3$ ?



## C Energy and momentum: Principles

### D1 Conservation of energy: Bernoulli's Equation

Steady flow, no friction

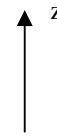


Fluid element moves along a fixed streamline (like a bead on a wire). Distance measured along streamline denoted by  $s$ , velocity along the streamline by  $v (=ds/dt)$ .

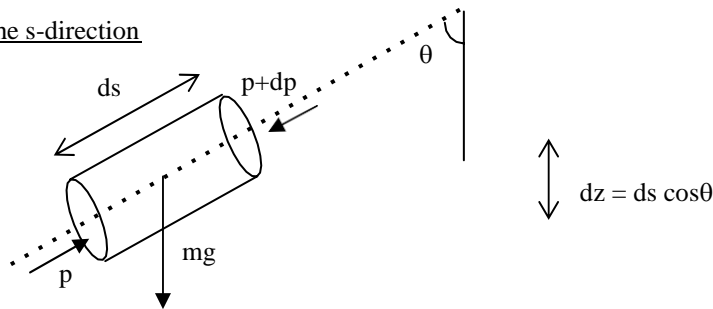
Volume of fluid element =  $dA ds$

Mass of fluid element  $m = \rho dA ds$

Gravitational force =  $-mg = -\rho g dA ds$  (downwards).



#### Resolving forces in the s-direction



Pressure force (s-direction) =  $p dA - (p+dp) dA = - dp dA$

Gravitational force (s-direction) =  $-mg \cos\theta = -\rho g dA ds \cos\theta = -\rho g dA dz$

$F = ma$ , and  $a = \frac{dv}{dt}$  so,

$$\rho dA ds \frac{dv}{dt} = - dp dA - \rho g dA dz.$$

$$\frac{dv}{dt} = -\frac{1}{\rho} \frac{dp}{ds} - g \frac{dz}{ds}.$$

Now,  $\frac{dv}{dt} = \frac{ds}{dt} \frac{dv}{ds} = v \frac{dv}{ds} = \frac{1}{2} \frac{d(v^2)}{ds}$ , so we have,

$$\frac{1}{2} \frac{d(v^2)}{ds} + \frac{1}{\rho} \frac{dp}{ds} + g \frac{dz}{ds} = 0.$$

Integrate  $\int ds$  (along streamline)

$$\frac{1}{2} v^2 + \frac{p}{\rho} + gz = \text{constant}$$

$$\frac{1}{2} v^2 + \frac{p}{\rho} + gz = gH$$

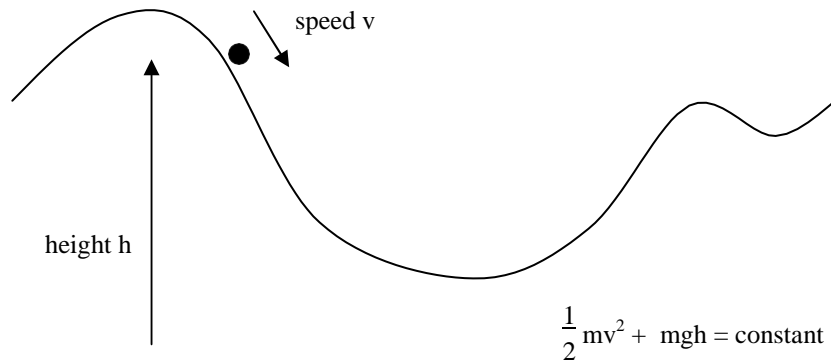
Bernoulli's Equation  
(along streamline)

Kinetic energy + potential energy = constant (no friction)

(This form is energy per unit mass.)

### Analogy

Particle on a smooth surface



### Head

The constant  $H$  in Bernoulli's Equation is known as the **total head** (relative to some datum  $z=0$ ).

Dividing the equation above by  $g$  gives Bernoulli's Equation in the form of energy per unit weight or "head" (the form most commonly used by engineers):

$$\frac{v^2}{2g} + \frac{p}{\rho g} + z = H$$

The kinetic energy term  $\frac{v^2}{2g}$  is known as the **dynamic head**.

The sum of the potential energy terms  $\left(\frac{p}{\rho g} + z\right)$  is known as the **piezometric head**, while the pressure

term on its own  $\frac{p}{\rho g}$  is known as the **pressure energy, pressure head or static head**.

At constant  $z$ , high pressure corresponds to low speeds and low pressure to high speeds.

### Power

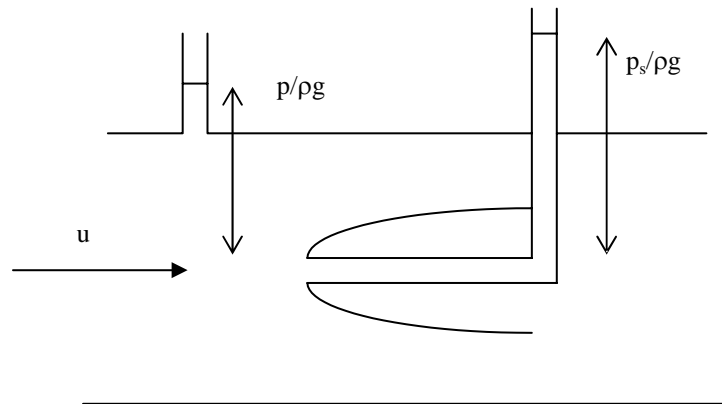
The maximum available power = energy per unit time =  $\rho gHQ$ .

E.g. a static reservoir 10 m above a hydroelectric power station with a flow rate of  $20 \text{ m}^3 \text{ s}^{-1}$  has a maximum available power of  $1000 \times 9.81 \times 10 \times 20 = 1.96 \text{ MW}$  (mega watts).



## D2 Bernoulli's Equation: Applications to flow measurement

### Pitot tube

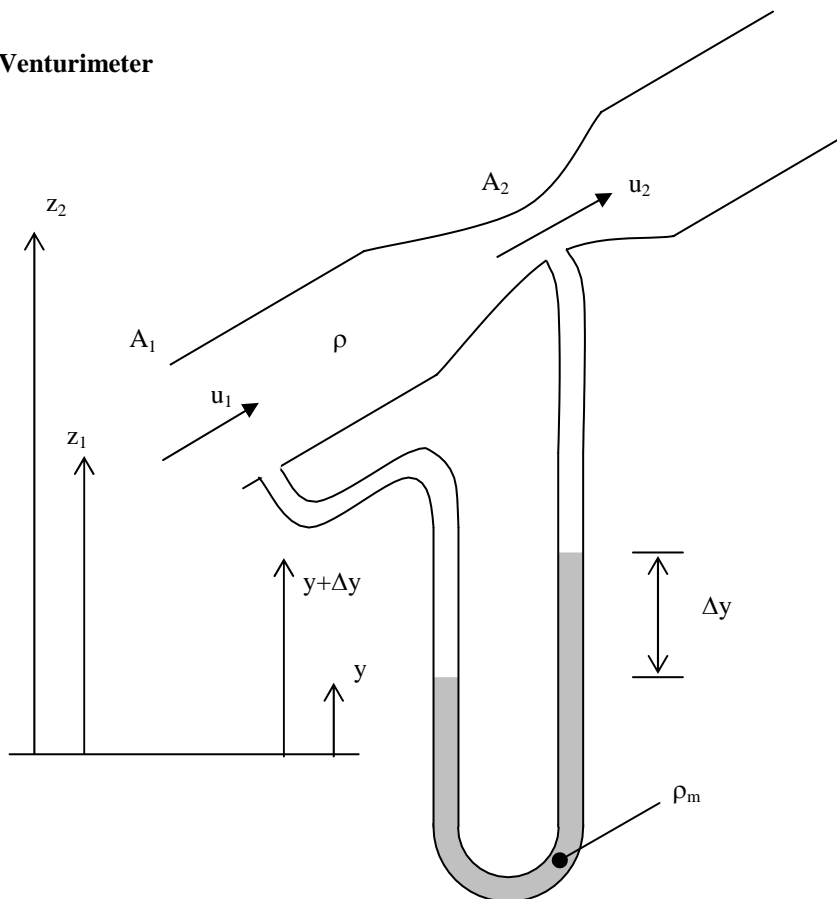


At the tip of the probe there is a stagnation point and the flow speed is zero. The height  $z$  is the same for both measurements, so we have,

$$\frac{u^2}{2g} + \frac{p}{\rho g} = H = \frac{0^2}{2g} + \frac{p_s}{\rho g}$$

$$u = \sqrt{\frac{2(p_s - p)}{\rho}}$$

### Venturimeter



Continuity (conservation of mass)

$$Q = u_1 A_1 = u_2 A_2, \quad \Rightarrow \quad u_2 = u_1 (A_1/A_2).$$

Bernoulli's Equation (conservation of energy)

$$\frac{u_1^2}{2g} + \frac{p_1}{\rho g} + z_1 = H = \frac{u_2^2}{2g} + \frac{p_2}{\rho g} + z_2$$

or,

$$\frac{u_2^2}{2g} - \frac{u_1^2}{2g} = \left( \frac{p_1}{\rho g} + z_1 \right) - \left( \frac{p_2}{\rho g} + z_2 \right) \equiv \Delta h$$

Pressures in the manometer

Write  $p_0$  for the pressure in the manometer at  $z=0$ .

$$p_0 = p_1 + \rho g(z_1 - y) + \rho_m g y \quad \text{and} \quad p_0 = p_2 + \rho g(z_2 - (y + \Delta y)) + \rho_m g(y + \Delta y)$$

$$(p_1 + \rho g z_1) - (p_2 + \rho g z_2) = + \rho g y - \rho_m g y - \rho g(y + \Delta y) + \rho_m g(y + \Delta y) = \Delta y(\rho_m - \rho)g$$

$$\Delta h = \Delta y(\rho_m - \rho)/\rho = \Delta y(\rho_m/\rho - 1).$$

Now,

$$u_2^2 - u_1^2 = 2g\Delta h,$$

using continuity gives,

$$u_1^2 \left( \frac{A_1^2}{A_2^2} - 1 \right) = 2g\Delta h,$$

and so,

$$u_1 = \sqrt{\frac{2g\Delta h}{\left( \frac{A_1^2}{A_2^2} - 1 \right)}}$$

and  $Q = u_1 A_1$ . However, this assumes no energy loss. In practice we find,

$$Q = A_1 C_d \sqrt{\frac{2g\Delta h}{\left( \frac{A_1^2}{A_2^2} - 1 \right)}},$$

where  $C_d \approx 0.95$  to  $0.99$  for a smooth contraction.

$C_d$  is known as the **Discharge Coefficient**.

### D3 Momentum principle: control volumes

#### Control volume

Identify a region of the flow.

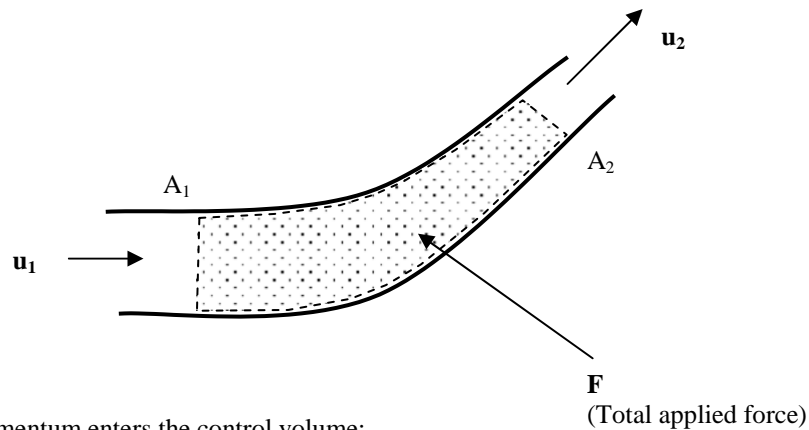
Identify all the forces acting on the fluid in this region.

Calculate the rate of change of momentum of the fluid passing through this region (out - in).

**Important:** velocity, momentum and force are all **vector** quantities. On the other hand, energy equations (like the Bernoulli Equation) and conservation of volume (in pipe flows) can only give speeds and not directions.

#### Example

Flow in a pipe



Rate at which momentum enters the control volume:

$$\rho_1 \mathbf{u}_1 Q_1$$

Rate at which momentum leaves the control volume

$$\rho_2 \mathbf{u}_2 Q_2$$

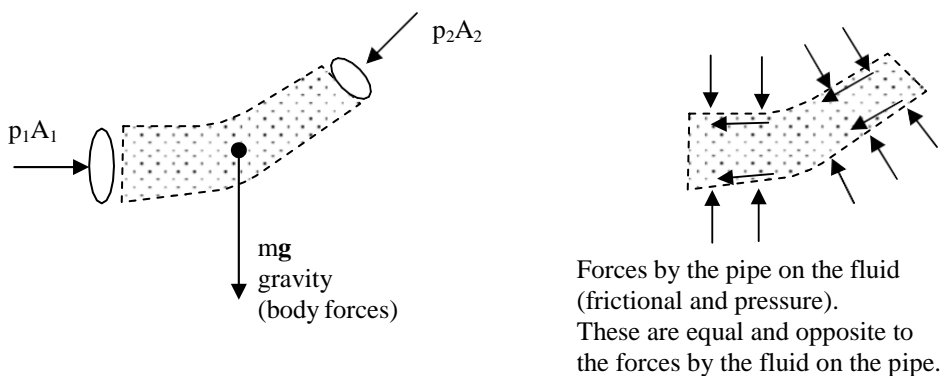
For incompressible flows  $\rho_1 = \rho_2$  and  $Q_1 = Q_2 = Q$ , so

$$(\mathbf{u}_2 - \mathbf{u}_1) \rho Q = \mathbf{F}$$

(rate of change of momentum = total force)

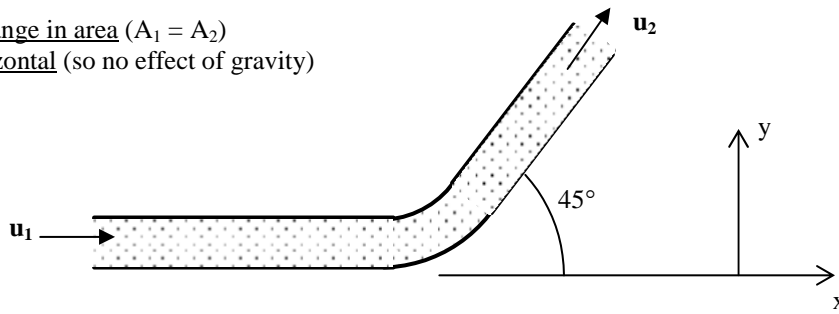
The total force includes:

Fluid pressure acting at each end of the control volume



## Flow around a pipe bend

a) no change in area ( $A_1 = A_2$ )  
and horizontal (so no effect of gravity)



Assume no change in pressure ( $p_1 = p_2 = p$ ).

Rate of change of momentum

$$(\mathbf{u}_2 - \mathbf{u}_1) \rho Q = (\mathbf{u}_2 - \mathbf{u}_1) \rho A u$$

Where  $u$  is the speed ( $u = |\mathbf{u}_2| = |\mathbf{u}_1|$ ), and so  $\mathbf{u}_1 = (u, 0)$  and  $\mathbf{u}_2 = (u \cos 45^\circ, u \sin 45^\circ)$

So the x-component of the total force on the fluid is:

$$F_x = (u \cos 45^\circ - u) \rho A u = -(1 - \cos 45^\circ) \rho A u^2,$$

and the y-component is

$$F_y = u \sin 45^\circ \rho A u = \sin 45^\circ \rho A u^2.$$

The sum of the pressure forces at the two ends is:

$$(pA, 0) + (-pA \cos 45^\circ, -pA \sin 45^\circ) = pA(1 - \cos 45^\circ, -\sin 45^\circ)$$

total force on fluid = pressure force at ends + force by pipe on fluid

force by fluid on pipe = -force by pipe on fluid = pressure force - total force

$$= pA(1 - \cos 45^\circ, -\sin 45^\circ) - (F_x, F_y)$$

$$= (pA + \rho A u^2)(1 - \cos 45^\circ, -\sin 45^\circ)$$

### Example

Pipe carrying water, diameter 10 mm, pressure  $10^4$  Pa, flow rate  $Q = 1.20 \times 10^{-4} \text{ m}^3 \text{ s}^{-1}$ .

$$A = (\pi/4)(0.01)^2 = 7.85 \times 10^{-5} \text{ m}^2. \quad u = Q/A = 1.53 \text{ m s}^{-1}.$$

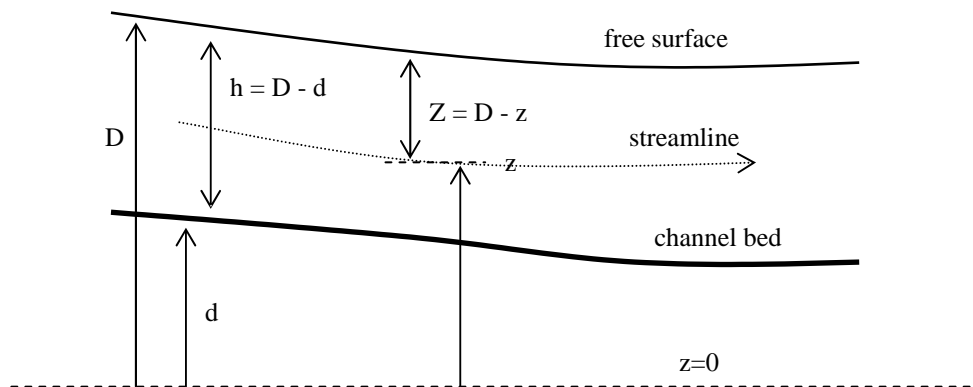
$$(pA + \rho A u^2) = 7.85 \times 10^{-5} \times (10^4 + 1000 \times 1.53^2) = 0.968 \text{ N}.$$

Force on pipe:

$$(0.28, -0.68) \text{ N}$$

If there is a change in area and if we can ignore energy losses, then we can use Bernoulli's Equation to find the relation between changes in pressure and changes in speed.

**D4 Momentum principle: open channel flow**  
**Conservation of energy (Bernoulli's Equation) - steady flow**



If the channel bed and the flow is slowly-varying, so that there are no strong vertical velocities or accelerations, then the pressure is approximately hydrostatic:

$$p = \rho g Z = \rho g(D - z)$$

Thus the total head (for steady flow) on a streamline through  $z$  is given by,

$$H = \frac{u^2}{2g} + \frac{p}{\rho g} + z = \frac{u^2}{2g} + \frac{\rho g(D - z)}{\rho g} + z = \frac{u^2}{2g} + D.$$

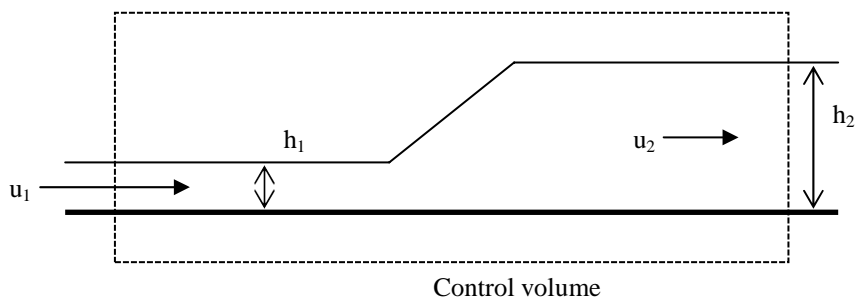
Note this depends on the height of the free-surface, but not the actual height of the streamline.

If the flow is uniform (the same at all depths in the fluid), we simply have

$$H = \frac{u^2}{2g} + D,$$

for all positions along the channel. (In practice there are frictional losses along the channel.)

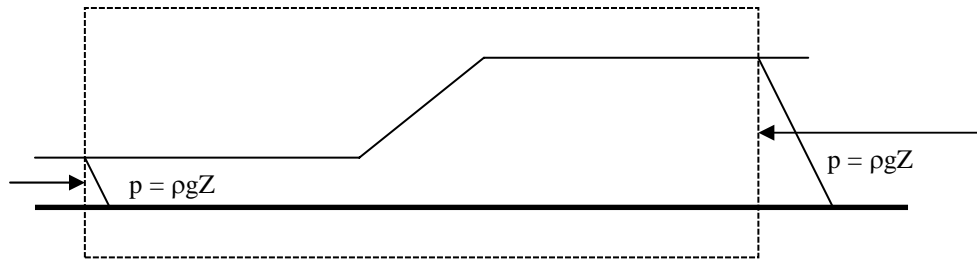
**Momentum: Hydraulic jumps**



Assume the width,  $w$ , is constant.

$$Q = w u_1 h_1 = w u_2 h_2, \quad \text{and so,} \quad u_2 = u_1 h_1 / h_2.$$

Pressure forces: hydrostatic pressure giving a horizontal force at each end.



Total force (per unit width),

$$F = \int_0^{h_1} \rho g Z dZ - \int_0^{h_2} \rho g Z dZ = \frac{1}{2} \rho g (h_1^2 - h_2^2) = u_1 h_1 \rho (u_2 - u_1)$$

$$\frac{1}{2} g (h_1^2 - h_2^2) = u_1^2 h_1 \left( \frac{h_2}{h_1} - 1 \right)$$

$$u_1 = \sqrt{gh_1} \sqrt{\frac{1}{2r(1+r)}},$$

where  $r = h_2/h_1$  is the size of the jump.

Note that for a jump ( $r > 1$ ), we need  $u_1 > \sqrt{gh_1}$  ("supercritical flow").

Energy loss

$$H_1 = u_1^2/(2g) + h_1, \quad H_2 = u_2^2/(2g) + h_2$$

This gives the head loss:

$$H_1 - H_2 = (h_1/4r) (r - 1)^3.$$

Power loss is given by,

$$\rho g (H_1 - H_2) Q.$$

Froude number

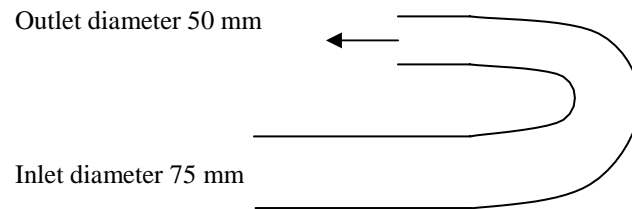
The ratio  $Fr = u/\sqrt{gh}$  is known as the Froude number. Since  $\sqrt{gh}$  is the speed of surface waves, the Froude number can be thought of as analogous to the Mach number, in that it is a ratio of a flow speed to a wave speed.

- Fr > 1    Supercritical flow
- Fr < 1    Subcritical flow
- Fr = 1    Critical flow

## D5 Tutorial: forces and hydraulic jumps

(a) Flow in a bend (no energy losses)

Water flows at a rate of  $0.1 \text{ m}^3 \text{ s}^{-1}$  through a contracting U-tube (in the horizontal plane) into the atmosphere as shown in the figure. What is the force on the tube? (You may ignore energy losses.)



(b) Hydraulic jump

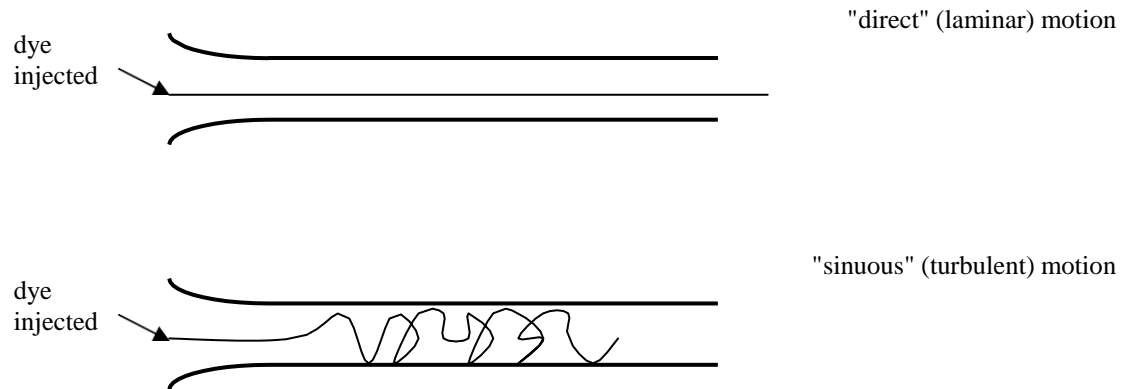
Flow in a 150 mm wide channel jumps from a depth of 50 mm to 150 mm. What is the flow rate and what is the rate of energy loss at the jump?

## D Pipeflow

### E1 Reynolds Experiment: Laminar and turbulent flow

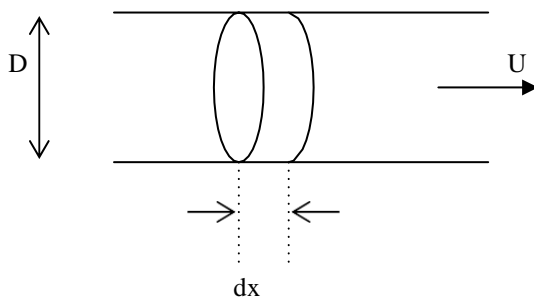
**Osborne Reynolds** (1842-1912), Professor at Manchester University

Derived an expression for the relative size of viscous and other accelerations (Reynolds number: see below) and demonstrated the importance of this relation by examining the nature of flow in a pipe.



### Reynolds Number (Re)

Consider flow in a pipe of diameter  $D$  with typical speed  $U$ .



Volume of fluid element

$$V = (\pi/4)D^2 dx.$$

So mass of fluid element

$$m = \rho(\pi/4)D^2 dx.$$

Area of fluid element in contact with pipe wall

$$\pi D dx.$$

Typical size of velocity gradient

$$\frac{\partial u}{\partial r} \sim \frac{U}{D}$$

So typical size of viscous force on fluid element (ignore  $\pi$ )

$$E \quad v \sim \mu \frac{U}{D} D dx \sim \mu U dx$$

Thus accelerations due to viscous forces (again ignoring constants)

$$a = \frac{F}{m} \sim \frac{\mu U dx}{\rho D^2 dx} = \frac{\nu U}{D^2},$$

where  $\nu = \frac{\mu}{\rho}$  is the kinematic viscosity.



But we can find another scale for accelerations

$$a \sim u \frac{\partial u}{\partial x} \sim U \frac{U}{D} = \frac{U^2}{D}$$

The ratio of this to the viscous acceleration is given by

$$\frac{U^2 D^2}{D \nu U} = \frac{UD}{\nu} \equiv Re$$

The Reynolds number (usually denoted by Re) is the ratio of inertial to viscous accelerations and is equal to a velocity scale  $\times$  length scale / kinematic viscosity. Note that the dimensions cancel out ( $\nu$  has units  $m^2 s^{-1}$ ), so that the Reynolds number is a dimensionless number and will be the same whatever units are used to measure lengths and times.

High Re - Turbulent flow (Re > 4000 for typical pipe flows). Inertial forces dominate.

Low Re - Laminar flow (Re < 2000 for typical pipe flows). Viscous forces dominate.

At intermediate Re the flow is "transitional" and may have intermittent "bursts" of turbulence in an otherwise smooth flow.

For flow in a pipe, the nature of the flow doesn't depend on the individual parameters (U, D,  $\nu$ ) but on the combination Re (the velocity scale used is usually the average velocity  $U=Q/A$ ).

#### Examples

Flow rate  $Q = 0.01 \text{ m}^3 \text{ s}^{-1}$  of water ( $\nu = 10^{-6} \text{ m}^2 \text{ s}^{-1}$ ), in a pipe of diameter  $D = 100 \text{ mm}$ . What is the Reynolds number and the expected type of flow?

$$A = (\pi/4) 0.1^2 = 7.85 \times 10^{-3} \text{ m}^2. \quad U = Q/A = 1.27 \text{ m s}^{-1}.$$

$Re = UD/\nu = 1.27 \times 0.1 / 10^{-6} = 127\,000$ , so the flow will be turbulent.

Flow rate  $Q = 10^{-6} \text{ m}^3 \text{ s}^{-1}$  ( $= 1 \text{ cm}^3 \text{ s}^{-1}$ ) of olive oil ( $\nu = 10^{-4} \text{ m}^2 \text{ s}^{-1}$ ), in a pipe of diameter  $D = 5 \text{ mm}$ . What is the Reynolds number and the expected type of flow?

$$A = (\pi/4) 0.005^2 = 1.96 \times 10^{-5} \text{ m}^2. \quad U = Q/A = 0.051 \text{ m s}^{-1}.$$

$Re = UD/\nu = 0.051 \times 0.005 / 10^{-4} = 2.6$ , so the flow will be laminar.

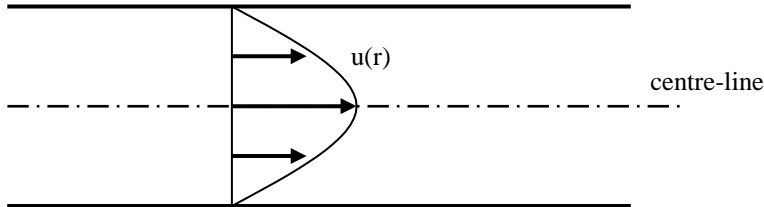
Most fluid flows in civil engineering are at high Reynolds number (an exception is flows through porous media, e.g. groundwater, water filtration).

Flow	U (m s <sup>-1</sup> )	L (m)	$\nu$ (m <sup>2</sup> s <sup>-1</sup> )	Re
Water in a domestic pipe	2	0.01	10 <sup>-6</sup>	2 × 10 <sup>4</sup>
Air flow past a building	5	10	1.5 × 10 <sup>-5</sup>	3.3 × 10 <sup>6</sup>
River flow	2	5	10 <sup>-6</sup>	10 <sup>7</sup>
Air flow through a doorway	0.5	2	1.5 × 10 <sup>-5</sup>	6.7 × 10 <sup>4</sup>
Water percolating through sand	6 × 10 <sup>-5</sup>	10 <sup>-4</sup>	10 <sup>-6</sup>	6 × 10 <sup>-3</sup>

## E2 Pipeflow: laminar flow

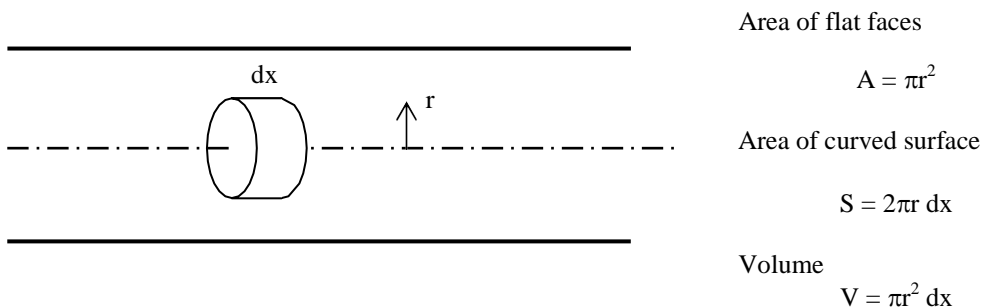
Low Re, viscous forces are important.

### Laminar flow in a pipe (Poiseuille Flow)

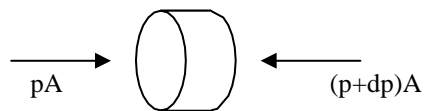


Smooth, steady flow. Expect velocity  $u$  to be a function of radius  $r$ , with  $u = 0$  at  $r = R$  (no flow at the pipe wall) but do not expect the speed to vary along the pipe (at a given radius).

Consider the forces on a cylindrical fluid element within the pipe:

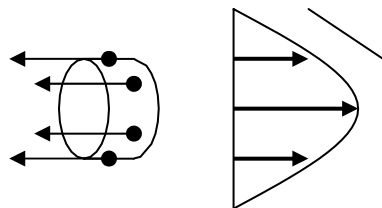


#### Pressure forces



Total pressure force =  $-dp A = -dp \pi r^2$ .

#### Viscous forces



The viscous force depends on the shear at that radius.

Force per unit area (viscous stress)

$$= \mu \frac{\partial u}{\partial r}$$

So total force =  $\mu \frac{\partial u}{\partial r} S = \mu \frac{\partial u}{\partial r} \times 2\pi r dx$

Here velocity decreases as  $r$  increases, so this is negative - the viscous forces tend to slow the fluid down.

For steady flow which isn't changing along the pipe ( $du/dx = 0$ ), the sum of the viscous and pressure forces must be zero.

$$-dp \pi r^2 + \mu \frac{\partial u}{\partial r} \times 2\pi r dx = 0$$

Rearranging gives,

$$2\mu \frac{\partial u}{\partial r} = r \frac{dp}{dx}$$

If the pressure gradient driving the flow along the pipe is constant  $G = \frac{dp}{dx}$ , then,

$$\frac{\partial u}{\partial r} = \frac{Gr}{2\mu}$$

If the flow is being driven in the x-direction we expect the pressure to decrease as x increases, so G would be negative.

Integrating this up we get  $u = \frac{Gr^2}{4\mu} + \text{constant}$ . We find the constant by using  $u(R) = 0$ :

$$u = -\frac{FR}{4\mu} (R^2 - r^2)$$

The velocity profile for laminar flow is parabolic. This flow is known as Poiseuille flow. The total flow rate is given by,

$$Q = \int_0^R u \cdot 2\pi r \, dr = -\frac{\pi GR^4}{8\mu}$$

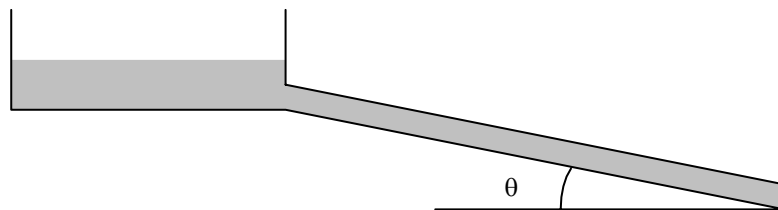
The average flow speed  $U = Q/A$  is

$$U = -\frac{GR^2}{8\mu} = \frac{1}{2} U_{\max}$$

For viscous flows, velocity and flow rates are proportional to the pressure gradient.

Note that energy is lost and Bernoulli's Equation no longer applies (or has to be adjusted to include energy losses).

Example: viscous flow down inclined tube



Here there is no significant pressure gradient driving the flow, but the component of gravity along the tube has the same effect:

$$\rho g \sin \theta = -G$$

For  $d = 3 \text{ mm}$ ,  $\theta = 10^\circ$ , with water we get,

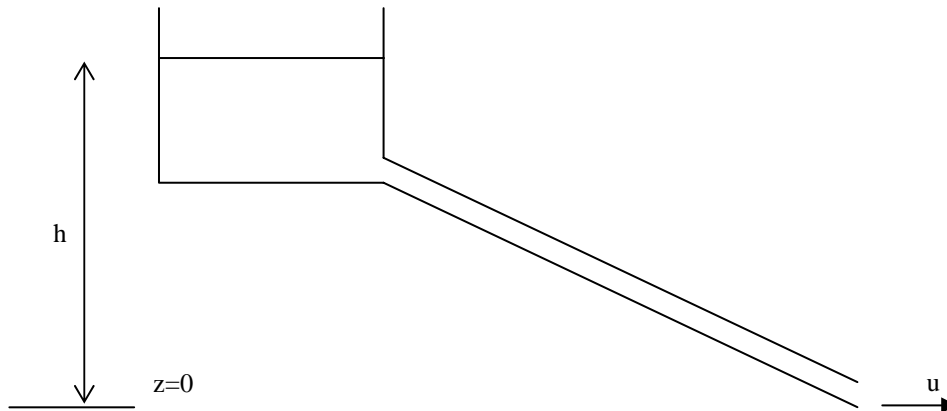
$$U = -\frac{GR^2}{8\mu} = \frac{\rho g \sin \theta d^2}{32\mu} = \frac{g \sin \theta d^2}{32\nu} = \frac{9.81 \times \sin 10^\circ \cdot 0.003^2}{32 \times 10^{-6}} = 0.47 \text{ m s}^{-1}$$

Check Reynolds number:

$$Re = UD/\nu = 1410 \text{ (just about laminar).}$$

### E3 Flow from static reservoir (no energy losses)

Now consider the case where we can ignore viscosity and energy losses.



If the water is discharged into the atmosphere, then the pressure there is atmospheric (approximately the same as at the surface of the reservoir). Bernoulli's Equation reduces to,

$$h = \frac{u^2}{2g}, \text{ so } u = \sqrt{2gh}. \text{ (This is the speed a body falling from height } h \text{ would have.)}$$

The total head is given by the height of the water surface in the static reservoir, i.e.  $H = h$ .

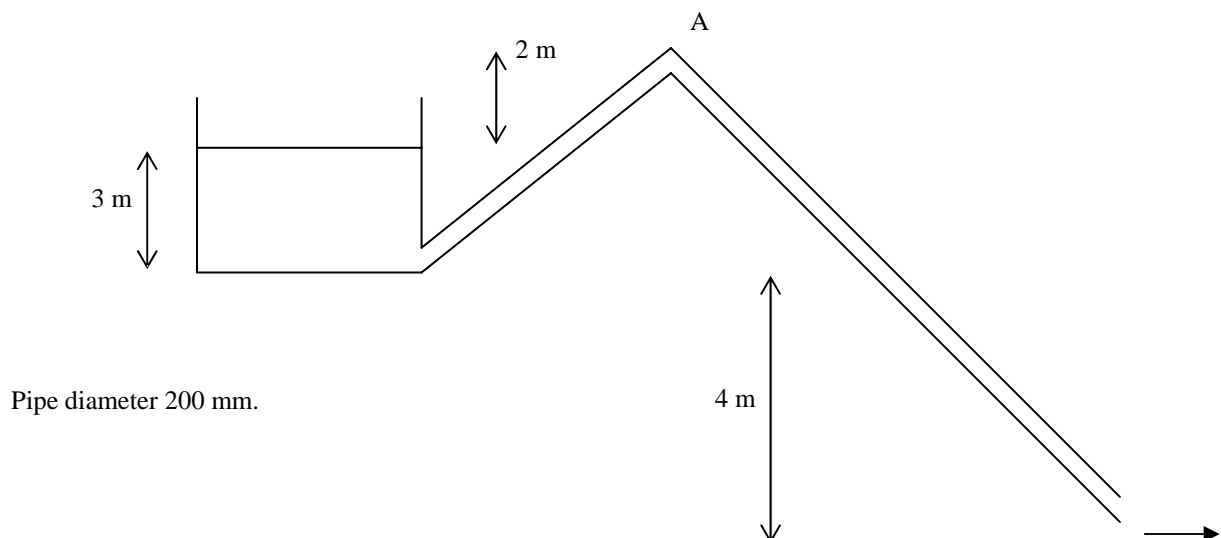
$$H = \frac{u^2}{2g} + \frac{p}{\rho g} + z,$$

where  $p$  is the gauge pressure (relative to atmospheric).

Thus in the pipe, the pressure is simply given by

$$p = -\rho g z.$$

#### Example



Height of surface above outlet  $H = 3 + 4 = 7$  m.

Speed of flow  $u = \sqrt{2 \times g \times 7} = 11.7 \text{ m s}^{-1}$ . Area  $A = (\pi/4)d^2 = 0.314 \text{ m}^2$ .

Thus flow rate  $Q = 11.7 \times 0.314 = 3.67 \text{ m}^3 \text{ s}^{-1}$  (= 3670 litres/s).

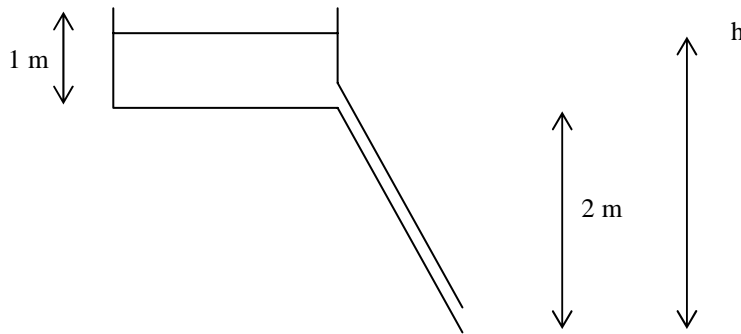
Pressure at A (relative to atmospheric)

$$p = -\rho g z = -1000 \times 9.81 \times (4 + 3 + 2) = -88.3 \text{ kN m}^{-2} = -88.3 \text{ kPa}.$$

(Note: for total heights over about 10 m, predicted pressures would be less than absolute zero, which is not possible.)

Example

An open rectangular tank  $2\text{m} \times 3\text{m} \times 1\text{m}$  high is full of water. It is emptied through a tube of diameter 15mm, discharging to the atmosphere 2m below the bottom of the tank. How long does it take to empty?



When the depth of water in the tank is  $h$ , the speed of the water coming out of the tube is given by,

$$u = \sqrt{2gh},$$

(assuming the vertical speed of the water surface is negligible and ignoring any energy losses).

The flow rate is  $Q = ua = \sqrt{2gh} \frac{\pi}{4} d^2$ , where  $d = 0.015$  m and  $a$  is the tube area.

The rate at which the water level descends is  $Q/A$  where  $A$  is the area of the tank ( $A = 6 \text{ m}^2$ ).

$$\frac{dh}{dt} = -\frac{Q}{A} = -\frac{\sqrt{2gh} \left( \frac{\pi}{4} \right) 0.015^2}{6} \text{ or } \frac{dh}{dt} = -1.305 \times 10^{-4} \sqrt{h}.$$

So

$$\int_{h=3}^{h=2} h^{-1/2} dh = - \int_{t=0}^{t=t_e} 1.305 \times 10^{-4} dt.$$

Thus  $2 \times (3^{1/2} - 2^{1/2}) = 1.305 \times 10^{-4} t_e$  or  $t_e = 4870$  s (approximately one hour twenty minutes).

## E4 Turbulent flow and head loss

### Head loss in pipes: Darcy's formula

In Civil Engineering applications, flows are usually at high Reynolds numbers. Conservation of energy (Bernoulli's Equation) gave

$$H = \frac{u^2}{2g} + \frac{p}{\rho g} + z,$$

with H a constant.

For flow in a pipe of constant diameter u is constant and so the dynamic head is constant. If Bernoulli's Equation applied we would expect the piezometric head to also remain constant. However, experiments show that the piezometric head falls, with the fall proportional to the length of the tube and the square of the speed (at high Reynolds number).

This can be thought of as a drop in the total head. For a pipe of length l and diameter d, the drop is found to be,

$$h_f = \left(\frac{\lambda l}{d}\right) \left(\frac{u^2}{2g}\right),$$

where  $\lambda$  is a constant, known as the **friction factor**. This expression is known as Darcy's formula. (It is convenient to keep it as a multiple of the dynamic head.) The constant  $\lambda$  depends on the relative roughness of the pipe ( $k_s/d$ , where  $k_s$ , or just k, is the size of the roughness - "bumps") and on the Reynolds number of the flow. For very large values of Re,  $\lambda$  becomes independent of Re and is approximately given by,

$$\lambda = (2 \log_{10}(3.7 d/k_s))^{-2}$$

Darcy's formula is sometimes expressed as

$$h_f = \left(\frac{4fl}{d}\right) \left(\frac{u^2}{2g}\right),$$

where  $f = \lambda/4$ . **WARNING:** Sometimes (especially in the USA) the symbol f is used for  $\lambda$ , giving a different f from that we've defined (by a factor of 4). When using published graphs and tables it is important to check which version of f or  $\lambda$  is used.

### Example

For galvanized steel, the roughness scale  $k_s = 0.15$  mm. For a pipe of diameter 100 mm,  $d/k_s = 667$ .

Thus the friction factor (for high Re) is  $\lambda = 0.0217$ .

Typical roughness scales  $k_s$  (mm)

Riveted steel	1 - 10
Concrete	0.3 - 3
Wood stave	0.2 - 1
Cast iron	0.25
Galvanized steel	0.15
Steel or wrought iron	0.045
Drawn tubing	0.0015

Roughness tends to increase with age because of deposits and corrosion.

### Friction factor for laminar flow

If there is a head loss of  $h_f$ , then the corresponding pressure change is simply

$$\Delta p = h_f \rho g.$$

This gives a pressure gradient (along a pipe of length l) of

$$G = - \frac{\Delta p}{l} = - \frac{h_f \rho g}{l}.$$

For low Re, laminar flow, we found the average velocity was given by

$$U = \frac{\rho g d^2}{32\mu} = \frac{h_f \rho g d^2}{32 \mu l}$$

Rearranging gives

$$h_f = \frac{32\mu U}{\rho g d^2} = \left(\frac{l}{d}\right) \left(\frac{64\nu}{Ud}\right) \left(\frac{U^2}{2g}\right) = \left(\frac{l}{d}\right) \left(\frac{64}{Re}\right) \left(\frac{U^2}{2g}\right)$$

Which is equivalent to Darcy's formula with  $\lambda = 64/Re$ .

### Moody's Diagram

At low Re (laminar flow) the pressure drop (and head loss) along a pipe is proportional to the velocity, while at very large Re the pressure drop is proportional to the square of the velocity. For a given relative roughness ( $k_s/d$ ), a plot of friction factor  $\lambda$  as a function of Reynolds number will have  $\lambda = 64/Re$  at low Re, tending towards a constant  $\lambda = (2 \log_{10}(3.7 d/k_s))^{-2}$  at high Re. At intermediate values of Re there is a transition between the laminar and very turbulent flows. For very smooth pipes, where the boundary layer is larger than the roughness elements,  $\lambda = 0.316 Re^{-1/4}$  gives good agreement with experiment up to  $Re = 10^5$ . A Moody Diagram shows the relationship between  $\lambda$  and Re for various relative roughnesses ( $k_s/d$ ). The plot is usually on logarithmic axes, so that the laminar formula  $\lambda = 64/Re$  is a straight line.

(WARNING: If you see a plot where the laminar formula is friction factor  $= 16/Re$ , then this is a plot of  $f = \lambda/4$ , as described earlier. Older exam papers also give  $f$  instead of  $\lambda$ , and you will need to use  $\lambda = 4f$  to get the correct results.)

For the present course, we will assume that the flow can be described by a constant  $\lambda$  (rough pipe flow), or by laminar flow. At intermediate values calculations become more complicated because the flow depends on  $\lambda$ , but  $\lambda$  depends on Re which in turn depends on the flow. Thus an iterative procedure is usually needed to calculate the flow. Formulae that match the smooth pipe flow equation to the high Re (constant  $\lambda$ ) values and the methods of using them will be dealt with in later courses.

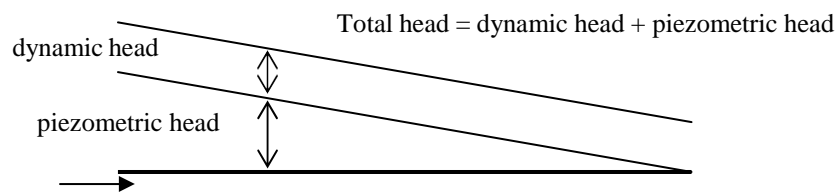
### Example

A flow  $Q = 0.1 \text{ m}^3 \text{ s}^{-1}$  is flowing through a galvanized steel pipe of diameter 100 mm. What is the head loss over a distance of 15 m?

$$A = (\pi/4) 0.1^2 = 7.85 \times 10^{-3} \text{ m}^2, \quad k_s = 0.15 \text{ mm}, \quad \lambda = 0.0217, \quad u = Q/A = 12.7 \text{ m s}^{-1}$$

$$h_f = \left(\frac{\lambda l}{d}\right) \left(\frac{u^2}{2g}\right) = \left(\frac{0.0217 \times 15}{0.1}\right) \left(\frac{12.7^2}{2 \times 9.81}\right) = 26.8 \text{ m}$$

Power loss =  $Q \rho g h_f = 26.3 \text{ kW}$ .



## E5 Pipeflow: other head losses

### Head loss coefficient

We've already seen that the head loss along a pipe of length  $l$  is given by

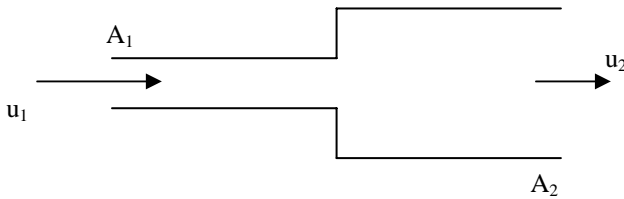
$$h_f = \left(\frac{\lambda l}{d}\right) \left(\frac{u^2}{2g}\right).$$

There are other head losses (energy losses) caused by enlargements, contractions, bends, valves and other fittings. These are generally expressed in the form

$$h_l = k \left(\frac{u^2}{2g}\right).$$

In principle, the head loss coefficient,  $k$ , varies with  $Re$  but at large  $Re$  the head loss coefficient is effectively constant. (**Warning:** don't confuse the non-dimensional head loss coefficient  $k$  with the roughness length also unfortunately often denoted by  $k$  or  $k_s$ .)

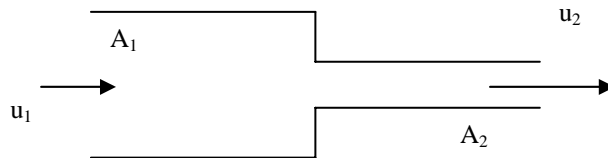
### Abrupt enlargement



$$h_l = \left(1 - \frac{A_1}{A_2}\right)^2 \frac{u_1^2}{2g} = \left(\frac{A_2}{A_1} - 1\right)^2 \frac{u_2^2}{2g}$$

(Proof later in F1.) As  $A_2 \rightarrow \infty$ , the "exit loss" (e.g. from a pipe into a reservoir)  $h_l \rightarrow \frac{u^2}{2g}$ .

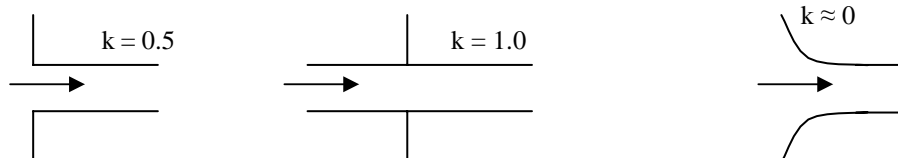
### Abrupt contraction



$h_l = k \frac{u_2^2}{2g}$ . As  $A_1 \rightarrow \infty$ , the "entry loss" (e.g. from a reservoir into a pipe)  $h_l \rightarrow 0.5 \frac{u^2}{2g}$

$d_2/d_1$ (diameter ratio)	0	0.2	0.4	0.6	0.8
$k$	0.5	0.45	0.38	0.28	0.14

### Other entry losses

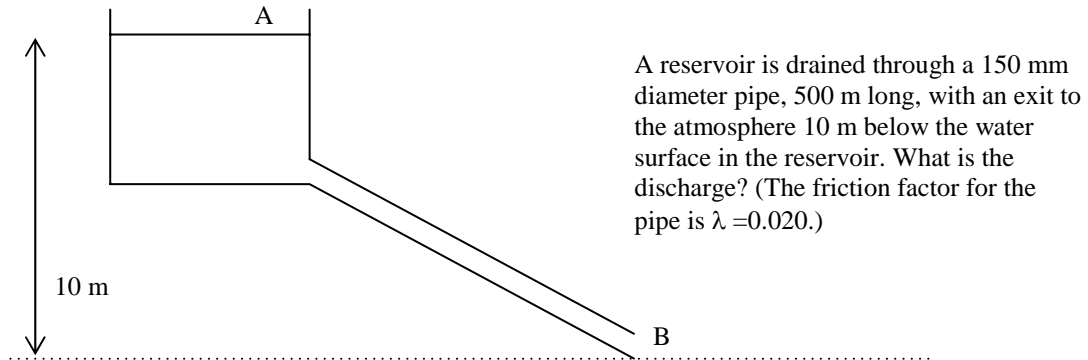




### Pipe fittings, typical losses

Fitting	90° bend	90° corner	45° bend	T (in-line)	T (side)
k	0.9	1.1	0.4	0.4	1.2

### Example



Energy at A = Energy at B + Energy losses from A to B

$$H_A = H_B + H_{\text{losses}}$$

$H_{\text{losses}}$  = entry loss + pipe loss (assume no significant loss on exit to atmosphere)

$$10 = \frac{u^2}{2g} + k_{\text{entry}} \frac{u^2}{2g} + \left( \frac{\lambda l}{d} \right) \frac{u^2}{2g}$$

$$10 = \frac{u^2}{2g} + 0.5 \frac{u^2}{2g} + \left( \frac{0.020 \times 500}{0.15} \right) \frac{u^2}{2g}$$

$$10 = (1 + 0.5 + 66.7) \frac{u^2}{2g}$$

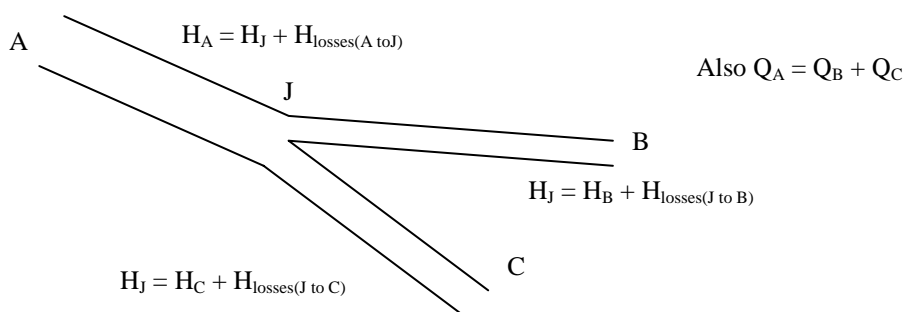
$$\frac{u^2}{2g} = 10/68.2 = 0.147, \quad \text{so } u = 1.70 \text{ m s}^{-1}$$

This gives  $Q = 0.030 \text{ m}^3 \text{ s}^{-1}$ .

(Check  $Re = ud/v = 2.6 \times 10^5$ .)

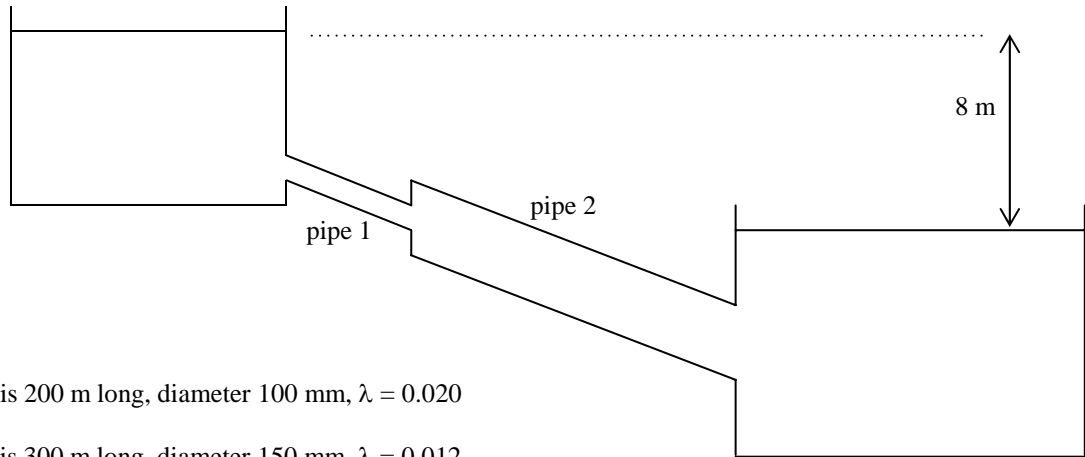
Note that most of the losses come from flow along the pipe. In practice ignoring entry and exit losses, and losses at bends and fittings does not usually lead to significant errors in most civil engineering applications.

### Junctions



## E6 Tutorial: Pipeflow

Calculate the flow between the two reservoirs connected as shown.



Pipe 1 is 200 m long, diameter 100 mm,  $\lambda = 0.020$

Pipe 2 is 300 m long, diameter 150 mm,  $\lambda = 0.012$

## *Hydraulic pump*

There are typically three types of hydraulic pump constructions found in mobile hydraulic applications. These include gear, piston, and vane; however, there are also clutch pumps, dump pumps, and pumps for refuse vehicles such as dry valve pumps and Muncie Power Products' Live Pak™.

The hydraulic pump is the component of the hydraulic system that takes mechanical energy and converts it into fluid energy in the form of oil flow. This mechanical energy is taken from what is called the prime mover (a turning force) such as the power take-off or directly from the truck engine.

With each hydraulic pump, the pump will be of either a uni-rotational or bi-rotational design. As its name implies, a uni-rotational pump is designed to operate in one direction of shaft rotation. On the other hand, a bi-rotational pump has the ability to operate in either direction.

### **GEAR PUMPS**

For truck-mounted hydraulic systems, the most common design in use is the [gear pump](#). This design is characterized as having fewer moving parts, being easy to service, more tolerant of contamination than other designs and relatively inexpensive. Gear pumps are fixed displacement, also called positive displacement, pumps. This means the same volume of flow is produced with each rotation of the pump's shaft. Gear pumps are rated in terms of the pump's maximum pressure rating, cubic inch displacement and maximum input speed limitation.

Generally, gear pumps are used in [open center hydraulic systems](#). Gear pumps trap oil in the areas between the teeth of the pump's two gears and the body of the pump, transport it around the circumference of the gear cavity and then force it through the outlet port as the gears mesh. Behind the brass alloy thrust plates, or wear plates, a small amount of pressurized oil pushes the plates tightly against the gear ends to improve pump efficiency.

### **PISTON PUMPS**

When high operating pressures are required, [piston pumps](#) are often used. Piston pumps will traditionally withstand higher pressures than gear pumps with comparable displacements; however, there is a higher initial cost associated with piston pumps as well as a lower resistance to contamination and increased complexity. This complexity falls to the equipment designer and service technician to understand in order to ensure the piston pump is working correctly with its additional moving parts, stricter filtration requirements and closer tolerances. Piston pumps are often used with truck-mounted cranes, but are also found within other applications such as snow and ice control where it may be desirable to vary system flow without varying engine speed.

A cylinder block containing pistons that move in and out is housed within a piston pump. It's the movement of these pistons that draw oil from the supply port and then force it through the outlet.

The angle of the swash plate, which the slipper end of the piston rides against, determines the length of the piston's stroke. While the swash plate remains stationary, the cylinder block, encompassing the pistons, rotates with the pump's input shaft. The pump displacement is then determined by the total volume of the pump's cylinders. [Fixed and variable displacement](#) designs are both available.

### **Main components of reciprocating pump**

---

Reciprocating pump has wide application and to clear the basic idea it is necessary to know the basic parts.

The basic parts along with its function;

- Water reservoir – it is not a part of reciprocating pump, however, it is the main source where from the reciprocating pump takes the water. It may be a source of other fluid as well.
- Strainer – It removes all impurities from the liquid to avert choking the pump.
- Suction Pipe – It is a pipe by which pump takes the water from the reservoir.
- Suction Valve – It is a non-return type valve installed on the suction pipe and helps to flow from reservoir to pump not the vice versa.
- Cylinder or liquid cylinder – The main component where pressure is increased. It is a hollow cylinder with coatings. It consists of a piston along with piston rings.
- Piston or plunger and Piston rod – Piston is directly connected to a rod that is the piston rod. This piston rod is again connected to the connecting rod. Piston makes the reciprocating motion in forward and backward motion and creates pressure inside the cylinder.
- Piston rings – Piston rings are small but one of the vital parts to protect the piston surface as well as cylinder inner surface from wear and tear. It helps to operate the pump smoothly.
- Packing – Packing is necessary for all pumps, to have a proper sealing between cylinder and piston. It helps to stop leakage.
- Crank and Connecting rod – Crank is connected to the power source and connecting rod makes connection between crank and piston rod. These component helps to change the circular motion into linear motion.
- Delivery valve (non-return valve) – Like suction valve delivery valve is also non return type and it helps to built up the pressure. It protect the pump from back flow.
- Delivery pipe – It helps to supply the fluid at destination.
- Air Vessel – Few reciprocating pumps may have an air vessel, it helps to reduce the frictional head or acceleration head.

### **Reciprocating Pump Application**[\[edit\]](#)

---

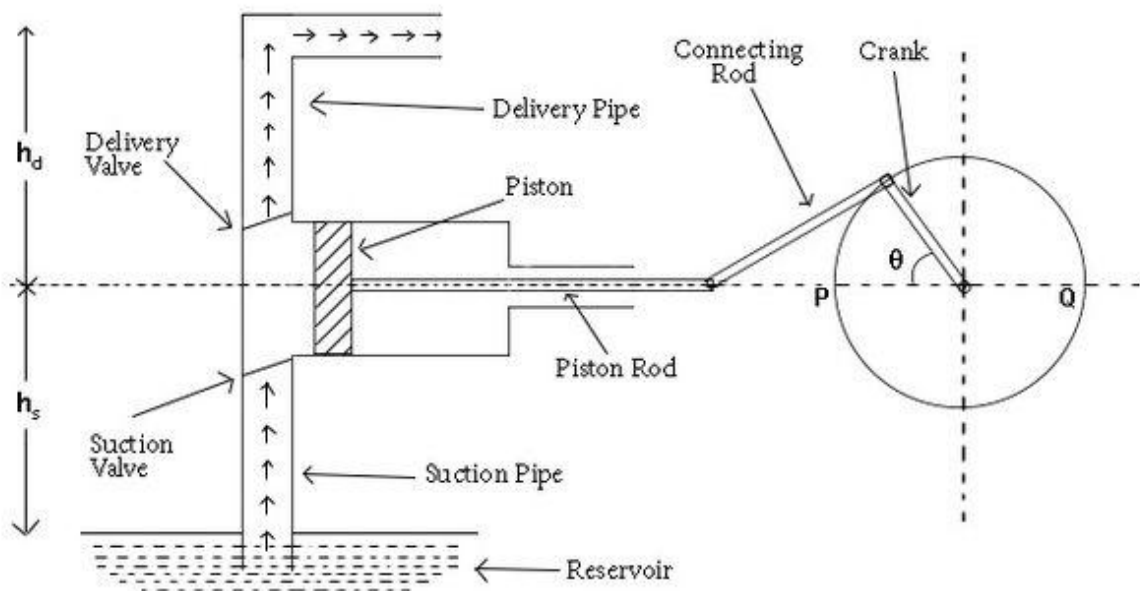
Application of Reciprocating pumps, are as follows:

- Vessel, pipe, tank, tube, condensate pipe, heat exchanger etc. cleaning,
- Oil drilling, refineries, production, disposal, injections.
- Pneumatic pressure applications.
- Vehicle cleaning.
- Sewer line cleaning.
- Wet sandblasting
- Boiler feeding
- High-pressure pumps for the RO system (Reverse osmosis)
- Hydro testing of tanks, vessels, etc.
- Firefighting system.
- Wastewater treatment system.

### Examples[\[edit\]](#)

Examples of reciprocating pumps include

- [Wind mill](#) water and oil pump
- [Hand pump](#)
- [Axial piston pump](#)



**Reciprocating Pump**

### What is a centrifugal pump?

A [centrifugal pump](#) is a mechanical device designed to move a fluid by means of the transfer of rotational energy from one or more driven rotors, called impellers. Fluid enters the rapidly rotating impeller along its axis and is cast out by centrifugal force along its circumference through the impeller's vane tips. The action of the impeller increases the fluid's velocity and pressure and also directs it towards the pump outlet. The pump casing is specially designed to constrict the fluid from the pump inlet, direct it into the impeller and then slow and control the fluid before discharge.

### How does a centrifugal pump work?

The impeller is the key component of a centrifugal pump. It consists of a series of curved vanes. These are normally sandwiched between two discs (an enclosed impeller). For fluids with entrained solids, an open or semi-open impeller (backed by a single disc) is preferred (Figure 1).



*Figure 1. Impeller Types (l to r): Open, Semi-Enclosed (or Semi-Open), Enclosed.*

Fluid enters the impeller at its axis (the 'eye') and exits along the circumference between the vanes. The impeller, on the opposite side to the eye, is connected through a drive shaft to a motor and rotated at high speed (typically 500–5000rpm). The rotational motion of the impeller accelerates the fluid out through the impeller vanes into the pump casing.

There are two basic designs of pump casing: volute and diffuser. The purpose in both designs is to translate the fluid flow into a controlled discharge at pressure.

In a volute casing, the impeller is offset, effectively creating a curved funnel with an increasing cross-sectional area towards the pump outlet. This design causes the fluid pressure to increase towards the outlet (Figure 2).

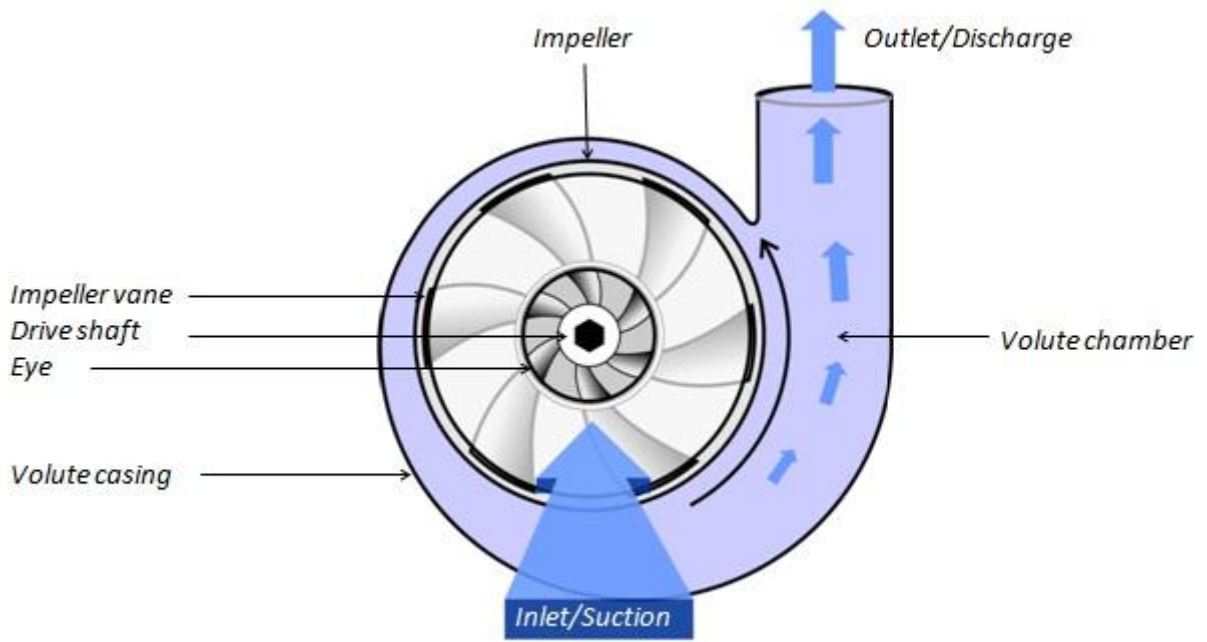


Figure 2. Volute case design

The same basic principle applies to diffuser designs. In this case, the fluid pressure increases as fluid is expelled between a set of stationary vanes surrounding the impeller (Figure 3). Diffuser designs can be tailored for specific applications and can therefore be more efficient. Volute cases are better suited to applications involving entrained solids or high viscosity fluids when it is advantageous to avoid the added constrictions of diffuser vanes. The asymmetry of the volute design can result in greater wear on the impeller and drive shaft.

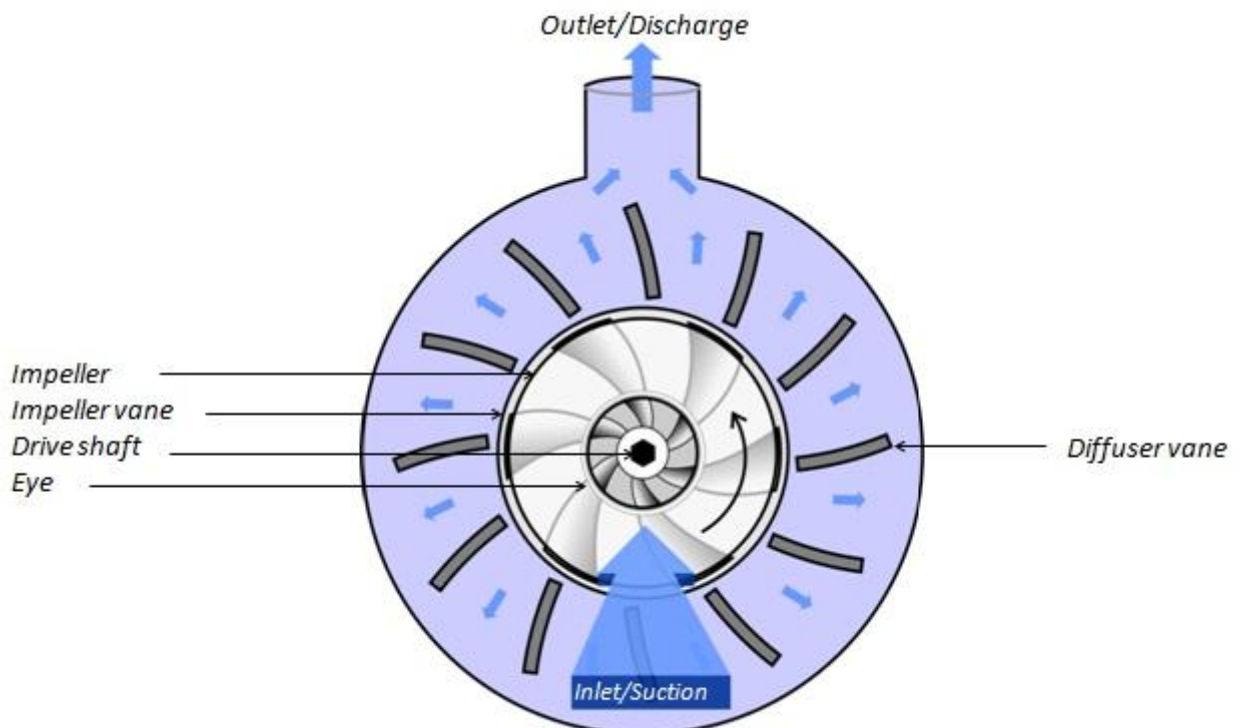


Figure 3. Diffuser case design

### **What are the main features of a centrifugal pump?**

There are two main families of pumps: centrifugal and [positive displacement](#) pumps. In comparison to the latter, centrifugal pumps are usually specified for higher flows and for pumping lower viscosity liquids, down to 0.1 cP. In some chemical plants, 90% of the pumps in use will be centrifugal pumps. However, there are a number of applications for which positive displacement pumps are preferred.

### **What are the limitations of a centrifugal pump?**

The efficient operation of a centrifugal pump relies on the constant, high speed rotation of its impeller. With high viscosity feeds, centrifugal pumps become increasingly inefficient: there is greater resistance and a higher pressure is needed to maintain a specific flow rate. In general, centrifugal pumps are therefore suited to low pressure, high capacity, pumping applications of liquids with viscosities between 0.1 and 200 cP.

Slurries such as mud, or high viscosity oils can cause excessive wear and overheating leading to damage and premature failures. Positive displacement pumps often operate at considerably lower speeds and are less prone to these problems.

Any pumped medium that is sensitive to shearing (the separation of emulsions, slurries or biological liquids) can also be damaged by the high speed of a centrifugal pump's impeller. In such cases, the lower speed of a positive displacement pump is preferred.

A further limitation is that, unlike a positive displacement pump, a centrifugal pump cannot provide suction when dry: it must initially be primed with the pumped fluid. Centrifugal pumps are therefore not suited to any application where the supply is intermittent. Additionally, if the feed pressure is variable, a centrifugal pump produces a variable flow; a positive displacement pump is insensitive to changing pressures and will provide a constant output. So, in applications where accurate dosing is required, a positive displacement pump is preferred.