LECTURER NOTES

ON

ENGINEERING MECHANICS
(Theory-4)

FOR

1ST & 2nd SEMESTER CIVIL, COMP.SC, ETC, ELECTRICAL & MECHANICAL ENGINEERING
(As per SCTE&VT, Odisha Syllabus)

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1 Basics of Mechanics and Force System

UNIT SPECIFICS

Following topics are discusses in this unit:

- Significance and relevance of mechanics
- Fundamental concepts related to engineering mechanics
- Types of units for measurement and fundamental SI units
- Force: Unit, Characteristics, Effect, System & Classification, Moments & its type
- Principles of static for force: Equilibrium law, Superposition law & Transmissibility law
- Resolution of force and Composition of force
- Analytical and Graphical methods to find resultant force

This unit gives details for different types of quantities and their units related to the whole course. Again, some fundamental concepts were also explain here for better clarity of the course. Some basic practice (other method than discussed) related to examples were also encouraged to calculate.

Basic activity for moment with help of some day-to-day examples, taken as an activity to the students. Based on the restriction of pages, importance has given to the quality of content.

The practical applications of the unit topics are discuss for generating further curiosity and creativity as well as improving capacity for problem solving of the student. After the related practical, based on the unit content, there is a "KNOW MORE" section, which has been design for supplementary information for benefit of the users of this book. MCQ and other subjective questions were also included for further interest to the unit, for the betterment of the students.

RATIONALE

This unit covers significance and relevance of mechanics as well as type of quantity. For any technician or engineer the knowledge of different system of units is essential. These basic information on System of units has been provided here along with information on scalar and vector quantities. Various principle of static are explained in this unit, which are used to solve the problems. Again, we are concentrating only on coplanar forces and study of non-coplanar forces is beyond the scope of this course.

PRE-REQUISITES

Basic knowledge of Physics and Math's from Secondary Education [Standard 8 to standard 10]

UNIT OUTCOMES

After completing this unit, you will be able to-

1. Explain the significance and relevance of engineering mechanics.
2. Restate scalar and vector quantities.
3. Illustrate S.I. system of unit.
4. Summarize the concept of the force, force system and its classification.
5. Apply principle of transmissibility of force.
6. Analyze the resolution and composition of force.
7. Apply law of triangle, parallelogram and polygon of force.
8. Evaluate resultant force of coplanar concurrent force system, parallel force system and non-concurrent force system.

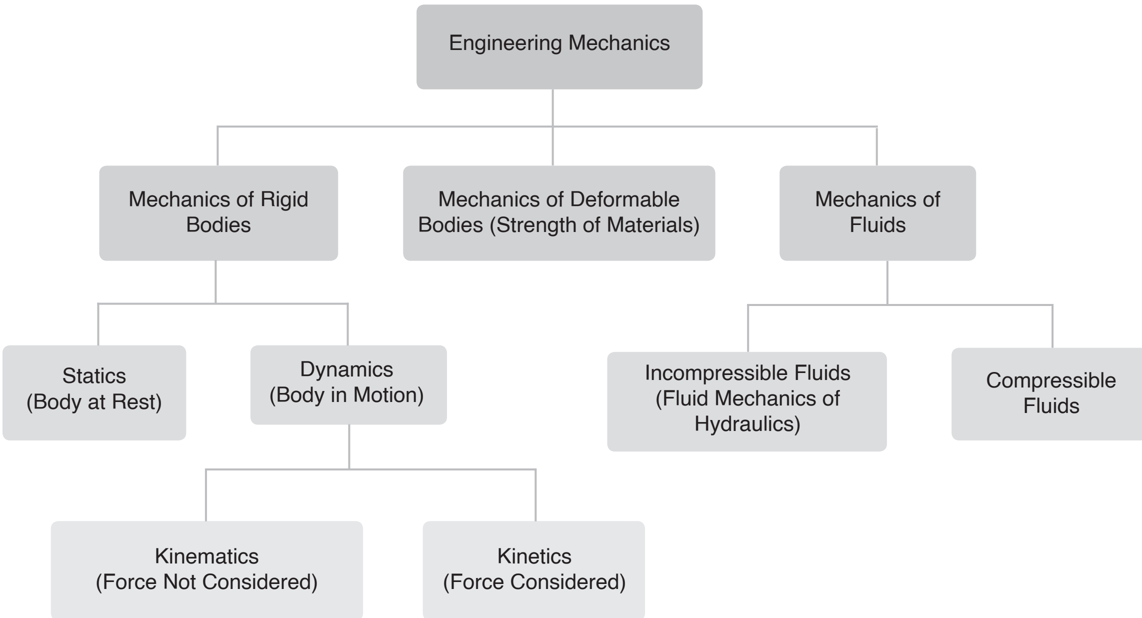
MAPPING UNIT OUTCOMES WITH COURSE OUTCOMES

Unit-1 Outcome	Expected Mapping with Programme Outcomes 1- Weak Correlation; 2- Medium correlation; 3- Strong Correlation				
	CO-1	CO-2	CO-3	CO-4	CO-5
U1-O1	2	-	-	-	-
U1-O2	2	-	-	-	-
U1-O3	2	-	-	-	-
U1-O4	3	1	-	-	-
U1-O5	2	2	-	-	-
U1-O6	-	3	-	-	-
U1-O7	-	2	-	-	-
U1-O8	-	3	-	-	-

1.1 SIGNIFICANCE AND RELEVANCE OF MECHANICS

Mechanics can be defined as branch of science, which deals with behavior of a body under the action of forces. Engineering mechanics refer to practical applications of principles of mechanics to engineering problem. Engineering mechanics is also called as applied mechanics.

In practice we encounter three types of bodies namely (a) Rigid body (b) Deformable body (c) Fluid.



1.2 FUNDAMENTAL CONCEPTS

Before we study the mechanic, certain Basic concept should be clearly understood.

Space : It is a region, which extends in all direction and contains everything in it. Examples : Sun, moon, star etc. In space position of body is located with respect to a reference system. The position of an aircraft in space found with respect to earth.

Time : It is measure of succession of events. The time is measured in second(s) and other related units. An event can be describe by position of point.

Mass : It is an indication of the quantity of matter present in a system. The more mass means more matter.

Flexible body : A body, which deform, under the action of applied force, is call flexible body.

Rigid body : A body, which does not deform, under the action of applied forces, is call rigid body.

1.3 SCALAR AND VECTOR

The physical quantities in mechanics can be Express mathematically as follows :

Scalar Quantity : Quantities, which described by their magnitude known as scalar quantity. Examples are mass, length, time, volume, temperature etc.

Vector Quantity : Quantities, which described by their magnitude and direction (both) known as vector quantity. Examples are velocity, force, acceleration, momentum etc.

A vector quantity can be represented by straight line with an arrow head. The length of straight line represents the magnitude while direction of line represents the direction of vector and arrow head indicate the sense of direction.

1.4 UNITS OF MEASUREMENT [SI UNITS]

Fundamental units : Length, Mass and Time are the basic fundamental quantities and unit of these quantities are known as fundamental units.

Derived units : Units of other than fundamental quantities may be derive from the basic units referred as derived units. Examples: (1) Area is result of multiplication of two lengths quantity as L^2. (2) Velocity is length divided by time as $\frac{L}{T}$. (3) Force is product of mass & acceleration as kg · m/sec^2 [N].

SI units : By international agreement in in 1960, the international system of units known as S.I. Unit is accepted and used all over the worldwide. The symbols and notation of SI units and their derivatives are standardize to avoid any possibility of confusion.

Table 1.1: Fundamental SI units

Sr. No.	Fundamental Quantity	Name of SI unit	Symbol
1	Length	Meter	m
2	Mass	Kilogram	kg
3	Time	Second	s
4	Electrical current	Ampere	A
5	Temperature	Kelvin	K
6	Luminous intensity	Candela	cd

1.5 FORCE

1.5.1 Introduction

You have studied about force at your high school level and also in Science - Applied Physics course. Let us recall, what is force ? Suppose you are driving the nail in a wall. Naturally you are required to push the nail in the wall. Then what is this "push" ? It is the force. Now consider another situation in which drum is rolling down and you want to stop it. Then naturally you will apply some resistance to its motion. This resistance is nothing but the force. Hence force is an external agent which tends to changes the state of body at rest or in motion.

1.5.2 Unit of force in SI system

The force is measure in Newton (N). A Newton is defined as a force, which can produce an acceleration of 1 meter per $second^2$ in a body of 1 kg mass. The larger units of force are –

1 Kilo Newton (kN) = 1000 Newton = 10^3 N

1 Mega Newton (MN) = 1000 × 1000 Newton = 10^6 N

1 Giga Newton (GN) = 1000 × 1000 × 1000 Newton = 10^9 N

1.5.3 Characteristics of force

As you know, force is a vector quantity, it means, it can identify by magnitude as well as direction. To represent force completely it is required following four elements, which are known as characteristics of force.

(A) Magnitude (B) Direction (C) Sense - Type of force - and (D) Point of application.

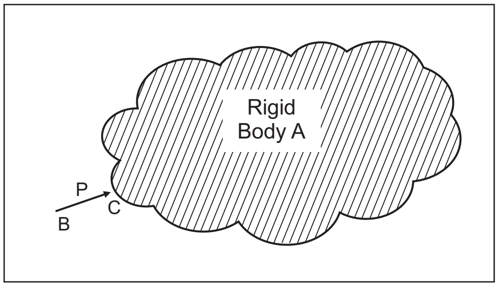


Fig. 1.1: Characteristics of force

Figure 1.1 show a rigid body A on which force P act at point C, which is point of application,while line BC show direction of force P with magnitude shown as P above the line and arrow head at point C show sense (type of force).

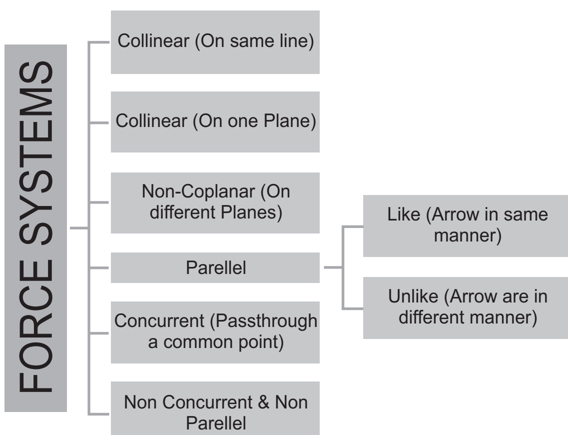
1.5.4 Effect of Force

The force when applied, following effects may happens on rigid body.

i. Change its state of rest OR motion.
ii. Accelerate OR retard its motion.
iii. Change its shape and size.
iv. Turn OR rotate it.
v. Keep it in equilibrium.

1.5.5 Force system and Classification

When several forces or group of force act simultaneously on a body, the body is said to be under the action of force system. These force systems are further classify according to the line of action and arrangements of the forces as shown below.



Collinear force system

The line of action of all forces lies along the same straight line as shown in fig. 1.2, then that force system is known as collinear force system.

Fig. 1.2: Collinear force system

Coplanar force system

All the forces in this system are lie in one plane, system is known as coplanar force system. Forces F_1, F_2 and F_3 only of force system acting on plan XY i.e., on one (same) plane shown in fig. 1.3, is example of coplanar force system.

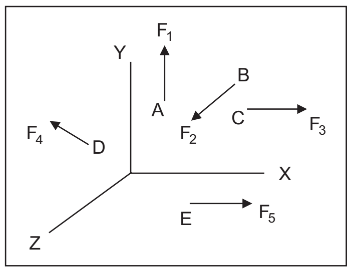


Fig. 1.3: Coplanar force system and non-coplanar force system

Basic of force and force system

Non-coplanar force system

All the force in the system are not lie in the same plane but act on different planes as shown in fig. 1.3 acting on planes XY, YZ and ZX. (Forces F_1, F_2, F_3, F_4 and F_5)

Parallel force system

The line of action means direction of all the forces are parallel to each other and do not intersect. This system is further sub classified as, Like parallel forces and Unlike parallel forces. If forces acting in the same direction, and are parallel to each other, are known as like parallel forces, where as if they are acting in opposite direction, and are parallel to each other, are known as unlike parallel forces as shown in fig. 1.4.

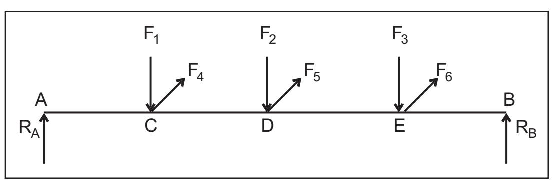


Fig.1.4: Parallel force system

Forces F_1, F_2 and F_3 is like parallel forces for this force system, while all forces F_1, F_2, F_3 and R_A & R_B of force system is unlike parallel forces. Here all the forces lie in one plane, so form Co-planar parallel force system. But if we add F_4, F_5 & F_6 forces which lies on another plane, then all the forces F_1, F_2, F_3, F_4, F_5, F_6 and R_A & R_B form Non- coplanar parallel force system.

Concurrent force system

All the forces have different direction but their line of action (Direction) passes through a single common point. Such force system known as concurrent force system. The point, where the line of action of all the forces meet is known as point of concurrency of the force system.

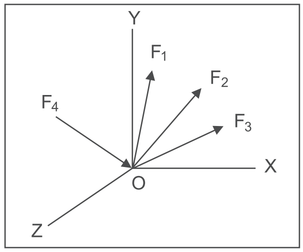


Fig.1.5: Concurrent force system

In Fig. 1.5, Force system of forces F_1, F_2 & F_3 pass through common point O and all lies on same plan XY. Such force system known as coplanar concurrent force system. If now, we add F_4 in same force system, which lies on another plane YZ, then force system of forces F_1, F_2, F_3 and F_4 is known as non-coplanar concurrent force system.

Non-concurrent and Non-parallel force system

If the forces of force systems are not lie in same line and not pass-through common point as well as line of action are not parallel to each other, it means, force system which not satisfy the condition of parallel, concurrent and linear force system, then such system is known as Non-concurrent & Non-parallel force system. If all the forces lie on same plane, it's known as coplanar non-concurrent non-parallel force system and if all the force lies on different planes, then is non coplanar non-concurrent non-parallel force system.

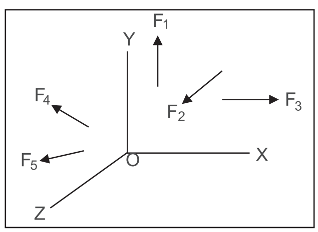


Fig.1.6: Non-concurrent non-parallel force system

In fig. 1.6, force system of forces F_1, F_2 and F_3 not lies in the same line and not parallel to each other as well as not pass-through common point but lies on same plane XY is the example of coplanar non-concurrent non parallel force system. Now if we add two forces F_4 and F_5, which lies on another plane YZ, then forces F_1, F_2, F_3, F_4 and F_5 of force system is example of non-coplanar non concurrent non parallel force system.

1.5.6 Principles of static for force

Following law or principles for force are required to study the coplanar concurrent force system.

(A) Equilibrium law (B) Principle of superposition (C) Principle of transmissibility.

(A) Equilibrium law of force

Two forces can be in equilibrium only, if they are equal in magnitude, opposite in direction and collinear in action.

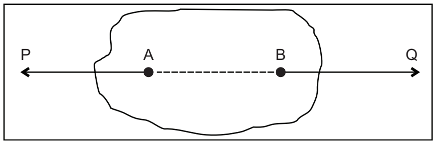
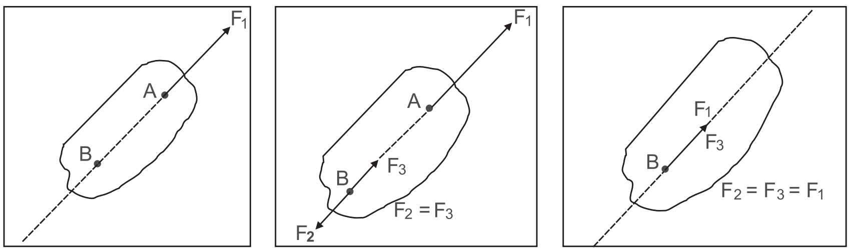


Fig. 1.7: Equilibrium law of force

A body shown in fig. 1.7 acted upon by two forces P and Q with line of action is same AB at point A and B respectively. Now what will happen if; (i) The magnitude of P is greater than Q (ii) The magnitude of P is smaller than Q & (iii) P and Q are equal. You can say that in case (i) body move in the direction of force P and in case (ii) body will move in the direction force Q, but in case (iii) body will not move or we can also say that body is at rest, it means, body is in equilibrium.

(B) Principle of superposition of force

The action of a given system of forces on a body will not change, if we add or subtract from it another system of the forces in equilibrium.



(a) (b) (c)

Fig. 1.8: Principle of superposition of force

Consider the body shown in fig. 1.8(a), under the action of force F_1. Force F_1 is applied at point A acting alone along line AB. Now let us add a system of forces in equilibrium at point B as force $F_2 = F_3$ as shown in fig. 1.8(b), then effect of force F_1 on body will not change as force F_2 and F_3 are nullify each other due to equilibrium.

Now see what is the effect, if $F_2 = F_3 = F_1$ as shown in fig. 1.8(c), then F_1 and F_2 are equal and opposite in nature nullify each other and only F_3 acting point B, but $F_3 = F_1$; it means, at point B force F_1 is acting. Thus, effect of force F_1 acting at point A is transferred to point B acting in the same form.

(C) Principle of transmissibility of force

We can now state principle of transmissibility of Forces from above phenomena shown in fig. 1.8(c) as follow : *The point of application of force may be transmitted along its line of action without changing* *the effect of forces on the body.* Thus, the principle of transmissibility is understood by principle of superposition force.

While applying this principle in practice, you should remember that external effect of force remains the same, when point of application is changed on the body, but change of position affects the internal affects; it means, stresses induced in the body, which is out of scope of this course. You will study it, in another course of 2^{nd} year.

1.6 COPLANNER CONCURRENT FORCES

1.6.1 Resolution of force

A force can be split up into two given direction such that the resultant of these forces is a given force. These components forces will give the same effect on the body as given by a single force. The procedure is known as resolution of force and resolved forces are known as components of forces. The resolution of the force into two components can be made as follow.

(A) Orthogonal components and (B) Non-orthogonal components.

(A) Orthogonal components

Generally, force is split up into two mutually perpendicular co-ordinate axes X and Y known as horizontal and vertical components respectively as shown in in fig. 1.9.

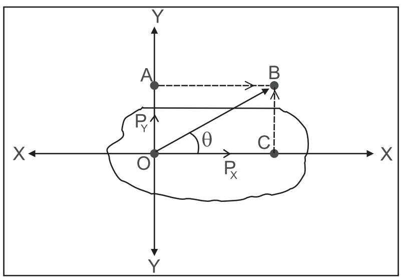


Fig. 1.9: Orthogonal components

In fig. 1.9, you can see that a force P is acting at a point O at angle θ (theta) with the X axis on a body. Now let us say that P is represent in magnitude and direction by vector OB. Two perpendicular axis XX and YY are also drawn through point O. Drop normal means perpendicular BC from point B on XX axis and BA on YY axis. Now consider Triangle OBC in which side OB represent the force P acting at angle θ with X direction and sides OC and BC represents components along X and Y axis as P_x and P_y respectively.

Here, $\cos\theta = \dfrac{OC}{OB}$ therefore $OC = P_x = OB \cdot \cos\theta = P \cdot \cos\theta$

$$\therefore\ P_x = P \cdot \cos\theta \qquad \ldots \text{(i)}$$

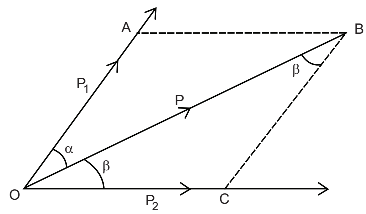
And $\sin\theta = \dfrac{BC}{OB}$ therefore $BC = P_y = OB \cdot \sin\theta = P \cdot \sin\theta$

$$\therefore\ P_y = P \cdot \sin\theta \qquad \ldots \text{(ii)}$$

Equation (i) & (ii) gives components of force P along X & Y direction respectively.

(B) Non-orthogonal components

Here force is resolved at any two given directions, which are not mutually perpendicular to each other. A given force P, which represent by line OB can be resolved into two components P_1 & P_2 along direction at angle α (Alpha) and β (Beta) with force P as shown in fig. 1.10.



Resolution of force

Fig. 1.10: Non-orthogonal components

1.7 COMPOSITION OF FORCE (RESULTANT FORCE)

If number of forces in force system are applied on a body, then we can replace it in a single force, which produce the same effect as force system, then this replaced single force is known as resultant force and the process by which the resultant force is found out is known as composition of forces. Its reverse process then the resolution as we have already studied in Topic 1.6.1. There are two methods for finding out resultant force. (I) Analytical method and (II) Graphical method. We have study the graphical methods in practical.

1.7.1 Analytical methods for concurrent force system

The resultant force of a given force system can be find out by following three methods:

(A) Law of parallelogram of force (B) Law of triangle of force (C) Method of resolution of forces.

1.7.1.1 Law of parallelogram of forces (Resultant force of two coplanar concurrent forces)

This method is use to find resultant of two coplanar concurrent forces acts on a body. Law of parallelogram of force state as below.

Two forces acting simultaneously on a body, if represent in magnitude and direction by two adjacent sides of a parallelogram, then diagonal of parallelogram from the point of intersection of two forces represent the resultant force in magnitude as well as in direction.

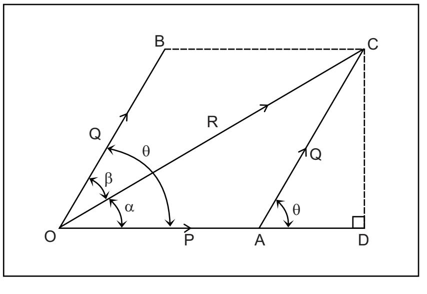


Fig. 1.11: Law of parallelogram of force

Let consider fig. 1.11, force P and Q acting on a body at a point O, with angle θ between two forces P and Q. The resultant force R of two forces P & Q can be mathematically represent by:

Magnitude of resultant is obtained by $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$ ...(iii)

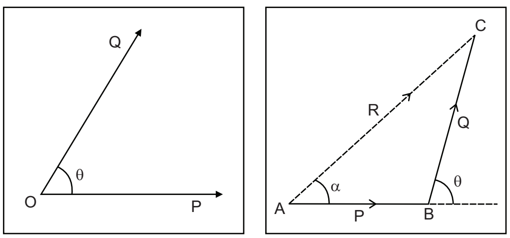
Direction of resultant (α) is obtained by $\tan\alpha = \dfrac{Q\sin\theta}{P + Q\cos\theta}$ from force P direction ...(iv)

Try these : (1) Above equation (iv) can calculate angle of resultant α, between R and P. Can you calculate angle (β) between resultant R and another force Q?
(2) Above equations (iii) and (iv) are applicable for pull type forces. Can you imagine what should be done if any or both forces are push type forces?

1.7.1.2 Law of triangle of force

When only two and two forces are acting on common point, we can apply this law to find out resultant force of force system. It States "*If two forces acting at a point are represented in magnitude and direction by two sides of a triangle taken in order; the third side of the triangle taken in opposite order represent the resultant force of two forces in magnitude and direction.*"

Let consider fig. 1.12, force P and Q acting on a body at a point O, with angle θ between two forces P and Q. This method is generally use as graphical method, which we have to study in practical.



Force System-II

Fig. 1.12: Law of triangle of force

SOME SOLVED EXAMPLES

Example 1. *Find the resultant force of two forces 30 N and 40 N acting at a point with an angle of 60° with one another.*

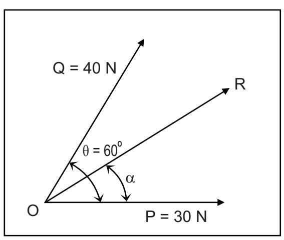


Fig.1.13

Solution:

P = 30 N, Q = 40 N & Angle (between P and Q) θ = 60°.

Put this value in equations, we get magnitude as well as direction of resultant force.

(i) Magnitude $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$$R = \sqrt{(30)^2 + (40)^2 + 2 \times 30 \times 40 \times \cos 60} = \sqrt{3700}$$

Hence, magnitude of resultant R = 60.83 N **(Answer)**

(ii) Direction $\tan\alpha = \dfrac{Q\sin\theta}{P + Q\cos\theta}$

$$= \frac{40 \times \sin 60}{30 + (40 \times \cos 60)} = \frac{34.64}{50} = 0.6928$$

Hence, α = 34.71° from direction force P. **(Answer)**

Example 2. *A push of 180 N and pull of 350 N acting at angle of 135° at one point. Find resultant force.*

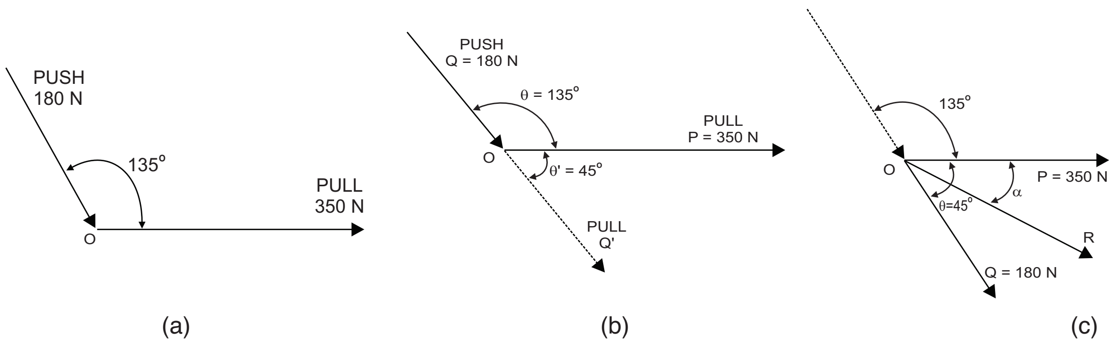


Fig. 1.14: (a) Data (b) All forces in Pull sense(c) Resultant force

Solution :

P = 350 N (Pull); Q =180 N (Push); Angle θ = 135°, Q′ = 180 N (Pull), θ′ = 45°

Here force Q need to be converted into pull type force by extending the line of action for push force, so that now the angle between two forces P and Q are 45° as shown in fig. (b).

(i) Magnitude $R = \sqrt{P^2 + Q'^2 + 2PQ\cos\theta'}$

$$R = \sqrt{(350)^2 + (180)^2 + (2 \times 350 \times 180 \times \cos 45^\circ)} = \sqrt{243982}$$

R = 493.96 N **(Answer)**

(ii) Direction $\tan\alpha = \dfrac{Q\sin\theta}{P + Q\cos\theta} = \dfrac{180\sin 45^\circ}{350 + 180\cos 45^\circ} = \dfrac{127.26}{477.26} = 0.26667$

Hence α = 14.93° with force P **(Answer)**

Try this : *Above Ex gives angle for resultant (α) with P. Can you calculate angle (β) between resultant R and another force Q?*

Example 3. *Two forces equal to P and 2P respectively act on a particle. When the first force is increased by 120 Newton and the second force is doubled, the direction of resultant force remains the same in both cases. Determine the value of force P.*

Solution :

Case (i) $P_1 = P; Q_1 = 2P; \theta_1 = \theta; \alpha_1 = \alpha$

Case (ii) $P_2 = P+120; Q_2 = 4P; \theta_2 = \theta; \alpha_2 = \alpha$

Apply Condition of direction of resultant force remains same in both case.

For Case (i) $$\tan\alpha_1 = \frac{Q_1 \sin\theta_1}{P_1 + (Q_1 \cos\theta_1)} = \frac{2P \sin\theta}{P + (2P\cos\theta)} = \tan\alpha$$

Case (ii) $$\tan\alpha_2 = \frac{Q_2 \sin\theta_2}{P_2 + (Q_2 \cos\theta_2)} = \frac{4P \sin\theta}{(P+120)(4P\cos\theta)} = \tan\alpha$$

Equating both cases; $$\frac{2P\sin\theta}{P + (2P\cos\theta)} = \frac{4P\sin\theta}{(P+120)(4P\cos\theta)}$$

$$\frac{2P\sin\theta}{4P\sin\theta} = \frac{P + (2P\cos\theta)}{(P+120)(4P\cos\theta)}$$

$$\frac{1}{2} = \frac{P + (2P\cos\theta)}{(P+120)(4P\cos\theta)}$$

$$(P+120) + (4P\cos\theta) = 2P + (4P\cos\theta)$$

$$P + 120 = 2P$$

$P = 120$ N **(Answer)**

Example 4. *Two equal forces of magnitude P are acting on a particle. (i) Find the angle between these two forces when their resultant is 1.5P. (ii) What can be maximum value of R and when it occurs?*

Solution:

Part (i) $P = P; Q = P; R = 1.5P; \theta = ?$

As we know law of parallelogram of forces

Magnitude $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$$(1.5P)^2 = P^2 + P^2 + (2PP\cos\theta)$$

$$2.25P^2 = 2P^2 + 2P^2\cos\theta$$

$$2.25P^2 = 2P^2(1+\cos\theta)$$

$$\frac{2.25}{2} = (1+\cos\theta)$$

$$\cos\theta = 0.125$$

$\theta = 82.81°$ **(Answer)**

Part (ii) Maximum R = ? ; θ_{max} = ?

(a) For Resultant to be maximum cos θ should be maximum. Now maximum cos θ = 1.

$R^2 = P^2+Q^2 + (2PQ \cos \theta)$

$R^2 = P^2 + P^2 + (2 \cdot P \cdot P \cdot 1)$

$R^2= P^2 + P^2 + 2P^2 = 4P^2$

Hence, Max R = 2P **(Answer)**

(b) Now, here, maximum cos θ = 1 only when θ = 0° (Answer)

Hence, maximum magnitude of resultant of two equal forces is 2P and obtained when the angle between those two forces is 0°.

1.7.1.3 Method of Resolution

It becomes very lengthy and tedious process to find the resultant force by law of parallelogram of forces, when more than two concurrent forces acting at a point. Method of resolution is very helpful to determine resultant of such force system. Method of resultant is given in following steps.

Step-1: If necessary, rearrange all the forces in either pull or push form and gives notation F_1, F_2… & so on in anticlockwise manner from positive X axis. Also compute angle of all forces with the positive X axis in anticlockwise manner.

Step-2: Find algebraic sum of horizontal component of all the force with relevant sign and give notation as ΣH. [+ve as: □ and -ve as: □]

Step-3: Find algebraic sum of vertical component of all the force with relevant sign andgive notation as ΣV. [+ve as: □ and -ve as: □]

Step-4: Find the magnitude of resultant force R by equation. $R^2 = \Sigma H^2 + \Sigma V^2$

Step-5: Find angle (α) of resultant force with horizontal by equation. $\tan \alpha = \dfrac{\Sigma V}{\Sigma H}$

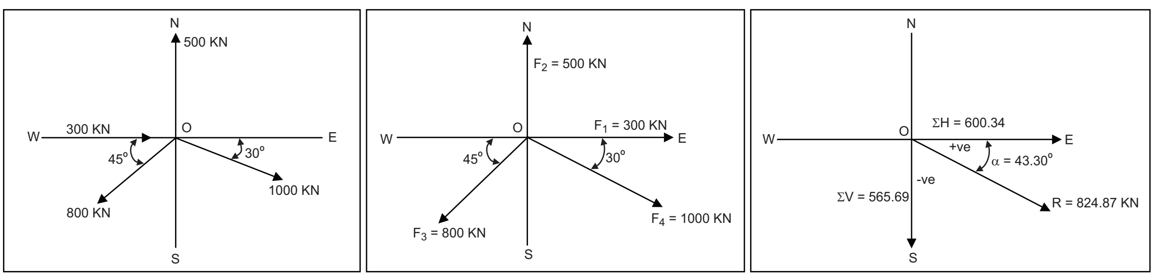
Example 5. *A system of four coplanar concurrent forces are acting at a point as given below. Find the magnitude and direction of resultant force.*

(i) 500 kN acting due North

(ii) 1000 kN acting 30° South of East

(iii) 800 kN acting South-West

(iv) 300 kN acting from West



(a) Data (b) All forces in same sense (c) Resultant force

Fig. 1.15

Solution:

From above data, first draw diagram shows all the forces as shown in fig. (a).

Step-1: In This case, 300 kN force acting as push force on the point & all others are pull, so this 300 kN force should re-arranged as pull force by extending line of action as shown fig (b).

Step-2 & 3: Now to Find out Horizontal and vertical components of all the forces it is easy way to workout in tabular form as shown below.

Sr. No.	Magnitude of Forces (kN)	Angle θ with respect to +X axis	Horizontal Component $F_x = F\cos\theta$ (kN)	Vertical Component $F_y = F\sin\theta$ (kN)
1	$F_1 = 300$	0°	300 cos 0° = 300.00	300 sin 0° = 0.00
2	$F_2 = 500$	90°	500 cos 90° = 0.00	500 sin 90° = 500.00
3	$F_3 = 800$	180+45 = 225°	800 cos 225° = –565.69	800 sin 225° = –565.69
4	$F_4 = 1000$	360 – 30 = 330°	1000 cos 330° = 866.03	1000 sin 330° = –500.00
		Algebraic Sum	$\Sigma H = +600.34 \rightarrow$ kN	$\Sigma V = -565.69 \downarrow$ kN

Step-4: Find magnitude of resultant force by equation:

$$R^2 = \Sigma H^2 + \Sigma V^2 = (600.34)^2 + (-565.69)^2 = 680413.2917$$

R = 824.87 kN **(Answer)**

Step-5: Find the angle α of resultant force R from ΣH sign to ΣV sign

$$\tan\alpha = \frac{\Sigma V}{\Sigma H} = \frac{-565.69}{600.34} = -0.9423$$

$\alpha = \tan^{-1}(-0.9423)$ $\therefore$ $\alpha = 43.30^{o}$ from $\Sigma H \rightarrow$ (East) towards $\Sigma V \downarrow$ (South)

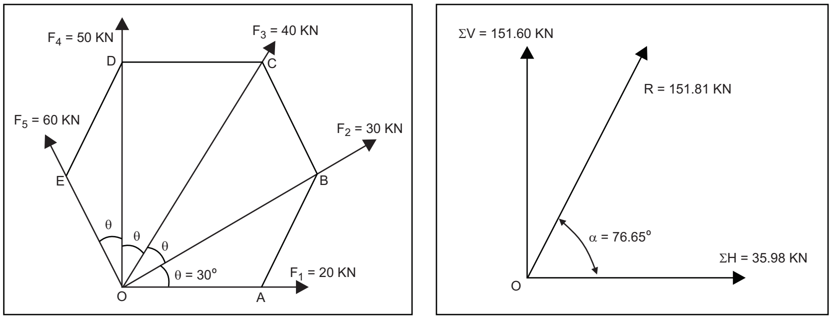
or $\alpha = (360^{o} - 43.30^{\circ}) = 316.70^{\circ}$ from +X axis in anticlockwise manner **(Answer)**

Fig. (c) shows magnitude and direction of resultant force (R) along with their components ΣH and ΣV.

Example 6. *The Forces 20 kN, 30 kN, 40 kN, 50 kN and 60 kN are acting on one of the angular points of a regular hexagon towards the other five angular points taken in order. Find magnitude and direction of resultant force.*

Solution:

First draw the figure with hexagonal & forces at angular points as below,



(a) Data (b) Resultant force

Fig. 1.16

For any Polygon, sum of interior angles = [(2n – 4) × 90°] where n = no of sides of regular polygon.
For hexagonal; n= 6, sum of interior angles = (2 × 6 – 4) × 90 = 720°

$\therefore$ Value of each interior angle = $\frac{720}{6} = 120^\circ$

Hence angle between two adjacent forces = $\theta = \frac{120}{4} = 30^\circ$

Step-1: Here all the forces acting at a point O are of pull sense as shown in fig. (a).

Step 2 & 3: Find Horizontal & vertical components of each force in tabular form.

Sr. No.	Magnitude of Forces (kN)	Angle θ with respect to +X axis	Horizontal Component $F_x = F \cos\theta$ (kN)	Vertical Component $F_y = F \sin\theta$ (kN)
1	$F_1 = 20$	θ = 0°	20 cos 0° = 20.00	20 sin 0° = 0.00
2	$F_2 = 30$	θ = 30°	30 cos 30° = 25.98	30 sin 30° = 15.00
3	$F_3 = 40$	2θ = 60°	40 cos 60° = 20.00	40 sin 60° = 34.64
4	$F_4 = 50$	3θ = 90°	50 cos 90° = 00.00	50 sin 90° = 50.00
5	$F_5 = 60$	4θ = 120°	60 cos 120° = –30.00	60 sin 120° = 51.96
		Algebraic Sum	ΣH = +35.98 → kN	ΣV = +151.60 □ kN

Step-4: Find magnitude of resultant force by equation:

$R^2 = \Sigma H^2 + \Sigma V^2 = (35.98)^2 + (151.60)^2 = 24277.58$

$\therefore$ R = 151.81 kN **(Answer)**

Step-5: Find the angle α of resultant force R from ΣH sign to ΣV sign

$\tan\alpha = \frac{\Sigma V}{\Sigma H} = \frac{151.60}{35.98} = 4.21345$

$\alpha = \tan^{-1}(4.21345)$ $\therefore$ α = 76.65° from ΣH → (East) towards ΣV □ (North)

or α = 76.65° from +X axis in anticlockwise manner **(Answer)**

Example 7. *Calculate the tensile force in the strings AB and AC as shown in figure below. Presume all the pulleys to be frictionless.*

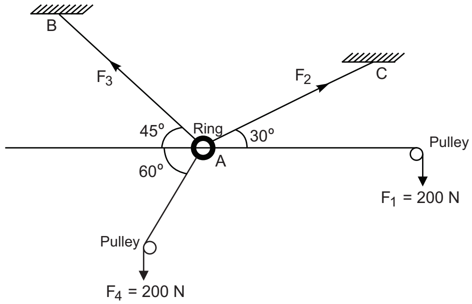


Fig.1.17

Solution:

Since the body is in equilibrium due to force system, we can say that R = 0 means ΣH = 0 and ΣV = 0.

Sr. No.	Magnitude of Forces (N)	Angle θ with respect to +X axis	Horizontal Component $F_x = F \cos\theta$ (N)	Vertical Component $F_y = F \sin\theta$ (N)
1	$F_1 = 200$	0°	200 cos 0° = 200.00	20 sin 0° = 0.00
2	$F_2 = ?$	30°	F_2 cos 30° = 0.866 F_2	F_2 sin 30° = 0.50 F_2
3	$F_3 = ?$	180° – 45° = 135°	F_3 cos 135° = –0.707 F_3	F_3 sin 135° = 0.707 F_3
4	$F_4 = 200$	180° + 60° = 240°	200 cos 240° = –100.00	50 sin 240° = –173.20
		Algebraic Sum	ΣH = 0.0 N	ΣV = 0.0 N

Using these two conditions; ΣH = 0 and ΣV = 0 from table

(a) ΣH = 0

$\therefore\ 200 + 0.866\ F_2 - 0.707\ F_3 - 100.00 = 0.0$

$\therefore\ 0.866\ F_2 - 0.707\ F_3 + 100.00 = 0.0$ ...(a)

(b) ΣV = 0

$\therefore\ 0 + 0.5\ F_2 + 0.707\ F_3 - 173.20 = 0.0$

$\therefore\ 0.5\ F_2 + 0.707\ F_3 - 173.20 = 0.0$ ...(b)

(c) Adding equation (a) & (b), we get

$\therefore\ (0.866 + 0.5)\ F_2 + (-0.707 + 0.707)\ F_3 + (100.00 - 173.20) = 0.0$

$\therefore\ 1.366\ F_2 - 73.20 = 0.0$

$$\therefore\ F_2 = \frac{73.20}{1.366}$$

$\therefore$ Force in string AC = F_2 = 53.44 N **(Answer)**

(d) Put Value of F_2 in equation (a), we will get

$(0.866 \times 53.44) - 0.707\ F_3 + 100.00 = 0.0$

$\therefore$ 0.707 F_3 = 46.28 + 100.00 = 146.28

$\therefore F_3 = \dfrac{146.28}{0.707}$

$\therefore$ Force in string AB = F_3 = 206.90 N **(Answer)**

1.8 COPLANNER NON-CONCURRENT FORCES

1.8.1 Moment of force

When a force acts on a body, the body moves or tends to move in the direction of the force. However, if the force acts on the body at some distance through an arm; it produces moment on the body resulting in rotation. For example, the spanner used to tighten or open the Bolt as shown in fig. 1.18.

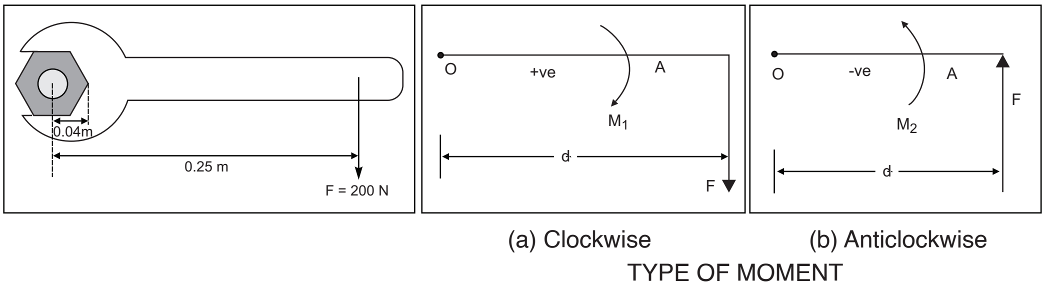


Fig.1.18: Moment of force

Here force F is applied at a distance d from center point O of bolt which producing moment & hence rotation of the bolt. The moment of force about the point is given by product of force F and perpendicular distance (Arm) of the line of action of the force from the given point O as d.

Mathematically; Moment = M = Force (F)X Perpendicular distance (d)

Therefore, $M = F \cdot d$

Unit of moment of force involves two quantities; namely Force and Distance. Depending upon unit of force and distance, the moment can be expressed in Newton · metre (N·m) or kilonewton · meter (kN·m)

Types of moment : Force F on spanner has a turning effect on Bolt in clockwise direction as shown in figure (a). Now if you change the sense of force as upward, it will have turning effect in anticlockwise direction as shown in figure (b). Hence moments are of the two types and gives sign convention as follow:

(i) clockwise moment as positive (+ve) and (ii) anticlockwise moment as negative (–ve)

Do this : *Try to open or close the bolt with the help of spanner of different length. Can you explain, why it is easy to open or close the bolt with long spanner?*

Try this : *Try to open or close the door of your room by pushing it from mid of the door and from handle of the door. Can you explain, which one is easy and why?*

1.8.2 Varignon's principle or Principle of moments

It states : *The moment of force about any point is equal to the sum of moments of the components of the force about the same point.*

This principle is applicable for non-concurrent and parallel force system to find position of resultant force. To understand the Varignon's principle refer fig. 1.19.

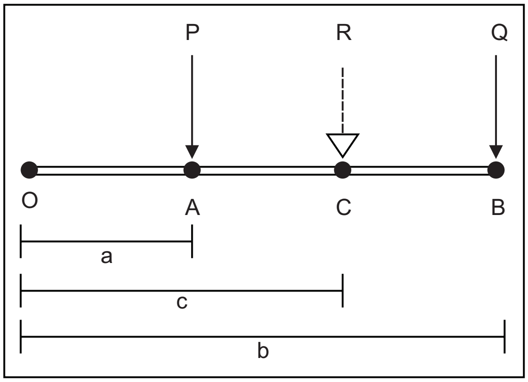


Fig. 1.19: Varignon's principle

Let forces P and Q are acting at point A and B in the plane as shown in fig. R is the resultant of these forces. Let distance of forces P, Q and R from point O be a, b & c respectively.

Using Varignon's Principle:

Moment of R about point O = Moment of force P about point O + Moment of force Q about point O

$$R \cdot c = P \cdot a + Q \cdot b$$

Varignon's Principle is very useful to find out the position of resultant force for Parallel force system as well as for non-concurrent non-parallel force system.

1.8.3 Analytical method for parallel force system

We have already studied the parallel force system in topic force system. We can say that parallel forces are a special class of non-concurrent force system. The line of action in this system are parallel to each other but it is sub divided in two group.

(i) Like parallel force system in which all the forces have same sense either pull or push and

(ii) Unlike parallel force system in which sense of forces are mixed means some are pull or remaining are push.

Resultant of parallel force system is obtain as follows:

(a) Magnitude: By algebraic sum of forces with sign +ve as upward (pull) and –ve as downward (push)

(b) Direction: line of action & sense as per sign of algebraic sum.

(c) Point of application: It can be obtain using Varignon's principle.

Let us take some examples to understand above points.

Example 8. *Find the resultant force of parallel force system as shown in figure below.*

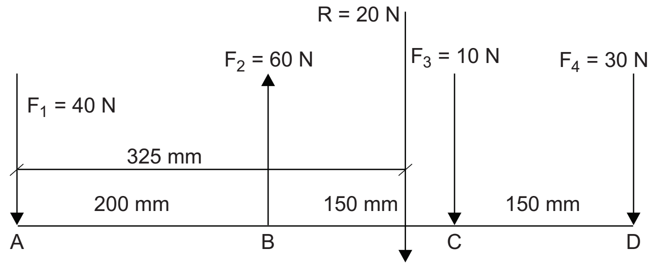


Fig. 1.20

Parallel Force System

Solution:

Consider Sign convention: +ve for ↓ (downward) and –ve for ↑ (upward)

(i) Magnitude of resultant force = R = ΣF

∴ R = 40 – 60 + 10 + 30 =20 N ↓

∴ Magnitude : R = 20 N ↓ **(Answer)**

(ii) Direction line of action & sense as sign +ve means ↓ (downward) **(Answer)**

(iii) Point of Application, using Varignon's principle:

Take movement at point A : +ve as a clockwise

$(40 \times 0) - (60 \times 200) + (10 \times 350) + (30 \times 500) = (R \times X)$

∴ $0 - 12000 + 3500 + 15000 = (20 \times X)$

∴ $6500 = (20 \times X)$

∴ Position : X = 325 mm from point A **(Answer)**

Example 9. *A Uniform wooden plank AB of length 3m has a weight of 40 N. It is supported at end A and at point D which is 1m from other end B. Determine the maximum weight W that can be places at end B, so that Plank does not topple.*

Solution:

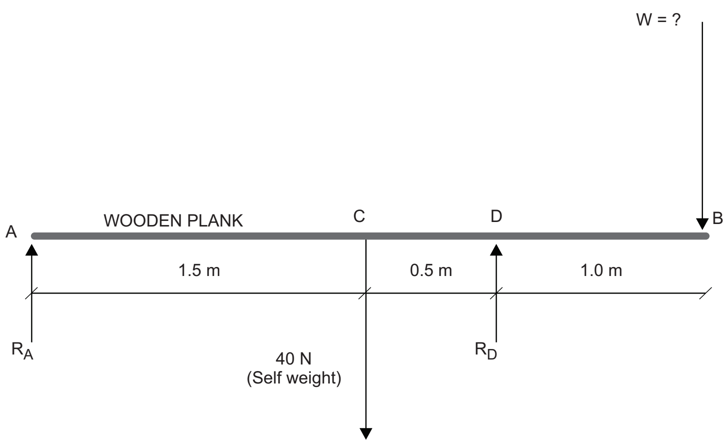


Fig. 1.21

Self-weight act at mid-point of plank. Now when, plank is at state of just being toppled, the reaction R_A at point A is zero.

Hence taking moment about point D, we get

$(\circlearrowright) W \times 1 = 40 \times 0.5 (\circlearrowleft)$

$\therefore$ W = 20 N **(Answer)**

Example 10. *A Uniform rod of 10m length has a self-weight of 5 N. The rod carries a weight of 30 N hung from one of its ends. From what point the rod be suspended so that it remains horizontal?*

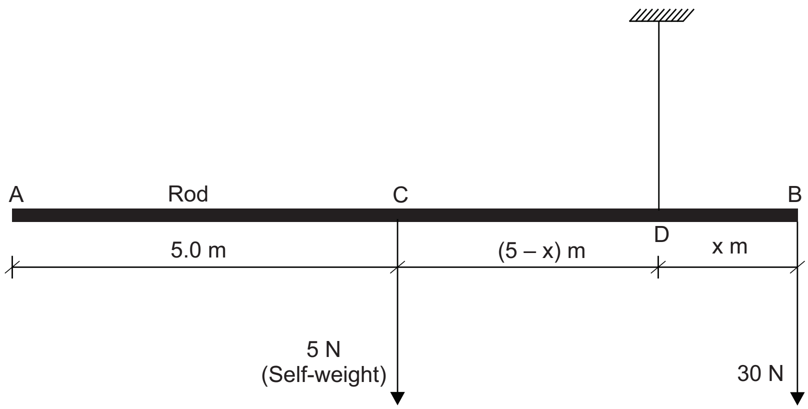


Fig. 1.22

Let us take the rod be suspended at point D, which is at distance x from its end B where weight of 30 N is hung. To remains rod to horizontal, moment at point D should be zero. By considering $\circlearrowright$ clockwise moment as +ve, take moment about point D, we get

$(30 \times x) - [5 \times (5 - x)] = 0$

$\therefore (30 \times x) - 25 + (5 \times x) = 0$

$\therefore (35\,x) = 25$

$\therefore X = 25/35 = 0.714$ m **(Answer)**

The rod is suspended to remain horizontal at point D at distance 0.714m from end B where weight of 30 N is hung.

1.8.4 Analytical method for non-concurrent force system

For a particular force system, if all the forces are not acting on the same line nor parallel to each other nor acting act common point, then that system known as non-concurrent non-parallel or simply non-concurrent force system. To determine resultant force of non-concurrent force system following steps (method) should be follows:

Step-1: Give notation to each force and find angle of direction with respect to +ve x axis in anti-clockwise manner.

Step-2: Find Horizontal components and vertical components of each force (in table)

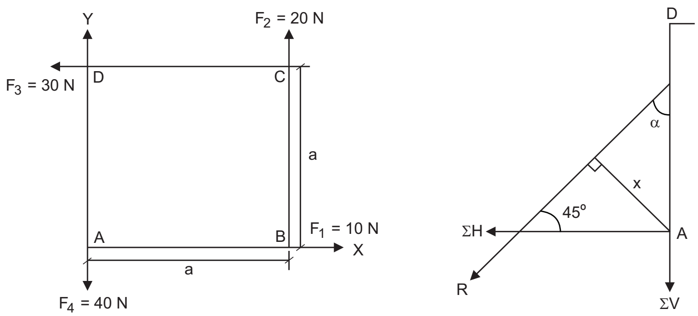
Step-3: Calculate magnitude of resultant force by equation $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

Step-4: Calculate angle α the direction of resultant force by equation $\tan\alpha = \dfrac{\Sigma V}{\Sigma H}$

Step-5: Find position of the point of application of resultant force by using Varignon's Principle.

We will understand these steps by some problems :

Example 11. *Four Forces of magnitude 10 N, 20 N, 30 N & 40 N are acting on a square of side 'a' along four sides AB, BC, CD, DA respectively. Find resultant force & locate point of application with Point A of square.*



(a) Data (b) Resultant force

Fig. 1.23

Non-concurrent force system

Step-1: Draw a figure square ABCD of size 'a' and apply forces 10 N, 20 N, 30 N & 40 N along AB, BC, CD, DA respectively as shown in fig. (a).

Step-2: Find Horizontal components and vertical components of each force (in table)

Sr. No.	Force Magnitude (N)	Angle θ with respect to +X axis	Horizontal Component $F_x = F\cos\theta$ (N)	Vertical Component $F_y = F\sin\theta$ (N)
1	10	0°	10 cos 0° = 10.00	10 sin 0° = 00.00
2	20	90°	20 cos 90° = 0.00	20 sin 90° = 20.00
3	30	180°	30 cos 180° = –30.00	30 sin 180° = 0.00
4	40	270°	40 cos 270° = 0.00	40 sin 270° = –40.00
		Algebraic Sum	ΣH = –20.0 N ←	ΣV = –20.0 N ↓

Step-3: Calculate magnitude of resultant force by equation $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$$R = \sqrt{(-20)^2 + (-20)^2} = \sqrt{800}$$

$\therefore$ R = 28.28 N **(Answer)**

Step-4: Calculate angle α the direction of resultant force by equation $\tan\alpha = \dfrac{\Sigma V}{\Sigma H}$

$$\tan\alpha = \frac{-20}{-20} = 1.0$$

$\therefore\ \alpha = 45^o$ from W – S **(Answer)**

Step-5: Find position of the point of application of resultant force by using Varignon's Principle Consider point A for moment with +ve as ↻ (clockwise)

$$\Sigma M_A = 0$$

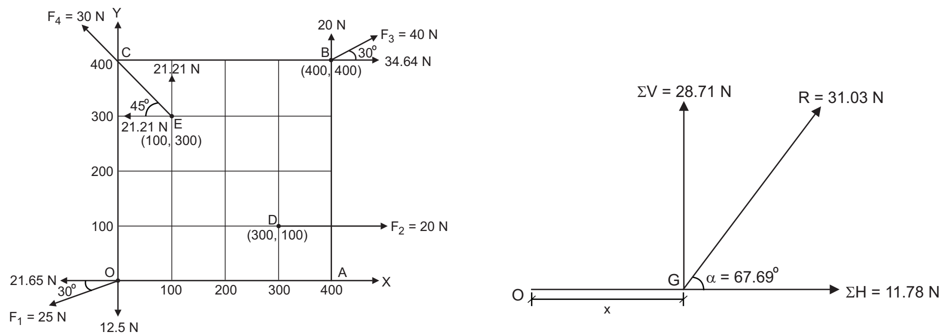
$$[(10 \times 0) - (20 \times a) - (30 \times a) + (40 \times 0)] = -(R \times x)$$

$$\therefore\ -50a = -28.28 \times x$$

$$\therefore\ x = \frac{50a}{28.28} = 1.768\ a \ \textbf{(Answer)}$$

Answer : Resultant force R = 28.28 N at angle α = 45° act at a perpendicular distance of 1.768 a from point A as shown in figure (b).

Example 12. *Four forces are acting on square mesh size 100 mm × 100 mm as shown in Fig. Find resultant force & also point of application with point O of square mesh.*



(a) Data (b) Resultant force

Fig. 1.24

Step-1: Give Notation to all forces as shown in square mesh.

Step-2: Find Horizontal components and vertical components of each force (in table)

Sr. No.	Force Magnitude (N)	Angle θ with respect to +X axis	Horizontal Component $F_x = F\cos\theta$ (N)	Vertical Component $F_y = F\sin\theta$ (N)
1	$F_1 = 25$	180° + 30° = 210°	–21.65	–12.5
2	$F_2 = 20$	0°	20.00	00.00
3	$F_3 = 40$	30°	34.64	20.00
4	$F_4 = 30$	180° – 45° = 135°	–21.21	21.21
		Algebraic Sum	ΣH = 11.78 N □	ΣV = 28.71 N □

Step-3: Calculate magnitude of resultant force by equation $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$R = \sqrt{(11.78)^2 + (28.71)^2} = \sqrt{963.03}$

$\therefore$ R = 31.03 N **(Answer)**

Step-4: Calculate angle α the direction of resultant force by equation $\text{Tan}\,\alpha = \dfrac{\Sigma V}{\Sigma H}$

$\tan\alpha = \dfrac{28.71}{11.78} = 2.4372$

$\therefore$ α = 67.69° from E – N **(Answer)**

Step-5: Find position of the point of application of resultant force by using Varignon's Principle.

In this case, if we take moment of components of each force, then it is easy to calculate as arm can find directly from co-ordinates of point where force is acting.

Consider point O for moment with +ve as ↻ (clockwise)

(a) due to all force:

$\Sigma M_O = (-21.65 \times 0) - (12.50 \times 0) + (20.0 \times 100.0) + (34.64 \times 400) - (20 \times 400) - (21.21 \times 300) - (21.21 \times 100)$

$= 0 + 0 + 2000 + 13856 - 8000 - 6363 - 2121$

$= -628$ N · mm ↺ (Anticlockwise) ...(a)

(b) due to resultant force:

Due to anticlockwise sum of all the forces, R should provide same type of moment,
Hence R acts right to point O, then only moment at point O can be anticlockwise.

Let line of action of R cut x axis at point G at a distance x from point O as shown in figure.
Resolving R in ΣH & ΣV at point G and taking moments at point O.

$\Sigma M_O = (\Sigma H \times 0) + (\Sigma V \times x) = (11.78 \times 0) - (28.71 \times x)$

$= (-28.71 \times x)$ ...(b)

Equating moment calculated in (a) and (b) we get

$28.71 \times x = 628$

$\therefore$ x = 21.87 mm from point O on x-axis **(Answer)**

Answer : The resultant force R = 31.03 N act an angle α = 67.69° with +ve x-axis on point G at a distance x = 21.87 mm from point O.

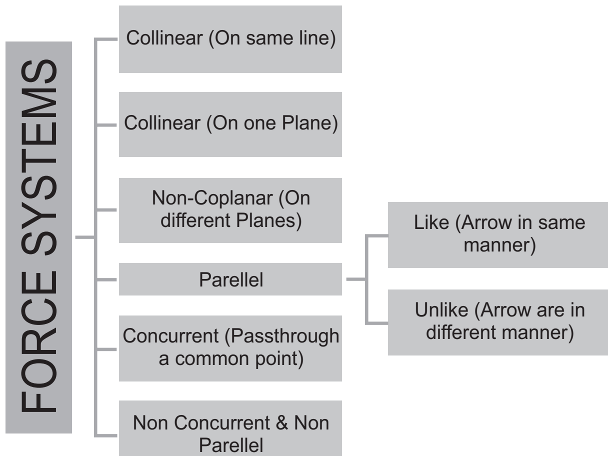
UNIT SUMMARY

- **Scalar Quantity :** A quantity which can be completely specified by magnitude only is known as scalar quantity.
- **Vector Quantity :** A quantity which can be completely specified by both magnitude and direction is known as vector quantity.
- **Fundamental units :** Length, Mass and Time are the basic fundamental quantities and unit of these quantities are known as fundamental units.
- **Derived units :** Units of other than fundamental quantities may be derived from the basic units referred as derived units.

- **SI units :** By international agreement in in 1960, the international system of units known as S.I. Unit is accepted and used all over the world.
- **Force :** A force is an external agent which tends to change the state of body at rest or in motion.

  Characteristics of force : (A) Magnitude (B) Direction (C) Sense (Type of force) & (D) Point of application.

  Effects of force : (A) Change its state of rest OR motion (B)Accelerate OR retard its motion (C)Change its shape and size (D)Turn OR rotate it (E) Keep it in equilibrium.
- **Force system and Classification :**



- **Principles of static for force :**

  Equilibrium law to force : Two forces can be in equilibrium only, if they are equal in magnitude, opposite indirection and collinear in action.

  Principle of superposition of force : The action of a given system of forces on a body will not change, if we add or subtract from it another system of the forces in equilibrium.

  Principle of transmissibility of force : The point of application of force may be transmitted along its line of action without changing the effect of Forces on the body.
- **Resolution of force :** A force can be split up into two given direction such that the resultant of these two forces is a given force. Such procedure is known as resolution of force.
- **Orthogonal components :** Generally, force is split up into two mutually perpendicular co-ordinate Axes X and Y known as horizontal and vertical components respectively.

  $P_x = P \cdot \cos\theta$ and $P_y = P \cdot \sin\theta$
- **Moment of force :** A force acts on the body at some distance through an arm; it produces moment on the body resulting in rotation.

  Unit of moment can be Express in Newton · meter (N·m) or kilonewton meter (kN·m)

  Types of moment (i) Clockwise moment as positive (+ve) and (ii) Anticlockwise moment as negative (–ve).
- **Varignon's principle or Principle of moments :** The moments of force about any point is equal to the sum of moments of components of the force about the same point.

- **Composition of force (Resultant Force) :** If number of forces in force system are applied on a body, then we can replace it in a single force, which produce the same effect as force system, then this replaced single force is known as resultant force and the process by which the resultant force is found out is known as composition of forces.
- **Analytical methods for concurrent force system :**

  Law of parallelogram of forces : Two forces acting simultaneously on a body, if represent in magnitude and direction by two adjacent sides of a parallelogram, then diagonal of parallelogram from the point of intersection of two forces represent the resultant force in magnitude as well as in direction.

  Method of Resolution : When in a force system more than two forces acts at a point, it becomes very lengthy and tedious process to find the resultant force by law of parallelogram, in such case method of resolution is useful.
- **Analytical method for parallel force system :**
  (a) Magnitude : By algebraic sum of forces with sign +ve as upward (pull) and –ve as downward (push).
  (b) Direction line of action & sense as per sign of algebraic sum.
  (c) Point of application can be obtained by Varignon's principle.
- **Analytical method for non-concurrent force system :**

  To find out resultant force of non-concurrent force system, follow the following steps :

  Step-1: Give notation to each force and find angle of direction with respect to +ve x axis in anticlockwise manner.

  Step-2: Find Horizontal components and vertical components of each force (in table).

  Step-3: Calculate magnitude of resultant force by equation $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

  Step-4: Calculate angle α the direction of resultant force by equation $\tan \alpha = \dfrac{\Sigma V}{\Sigma H}$

  Step-5: Find position of the point of application of resultant force by using varignon's principle.

Graphical method to find resultant force for concurrent force system

(A) Law of triangle of force : If two forces acting at a point are represented in magnitude and direction by two sides of a triangle taken in order; than the third side of the triangle taken in opposite order represent the resultant force of two forces in magnitude and direction.

(B) Law of parallelogram of force : If two forces acting at a point are represent in magnitude and direction by adjacent sides of parallelogram, then the diagonal passing through the point represents the resultant of two forces in magnitude & direction.

(C) Law of polygon of forces : If nos. of coplanar concurrent forces acting on a body be represented in magnitude and direction by the sides of a polygon taken in order, then the line joining the start point of first force to end point of last force [closing line] represent the resultant force in magnitude and direction.

EXERCISE

(A) Objective Questions

1.1 The unit of Force in S.I. unit is
(a) kilogram (b) newton (c) watt (d) joule

1.2 Forces are called concurrent when their lines of action meet in
(a) one point (b) two points (c) one plane (d) different planes

1.3 Which is the correct statement about Law of polygon of forces
(a) if any number of forces acting at a point can be represented by the sides of a polygon taken in order then the forces are in equilibrium
(b) if any number of forces acting at a point can be represented in direction & magnitude by the sides of a polygon then the forces are in equilibrium
(c) if any number of forces acting at a point can be represented in direction & magnitude by the sides of a polygon taken in order then the forces are in equilibrium
(d) if a polygon representing forces acting at a point is closed then forces are in equilibrium

1.4 Effect of a force on a body depends upon
(a) magnitude (b) direction (c) sense (type of force) (d) all of the above

1.5 If two equal forces of magnitude P act at an angle 90^o their resultant will be
(a) 2P (b) $\sqrt{2}$ P (c) $\frac{P}{2}$ (d) $2\sqrt{P}$

1.6 If two equal forces of magnitude P act at an angle 180^o their resultant will be
(a) 2P (b) $\sqrt{2}$ P (c) 0 (d) $2\sqrt{P}$

1.7 Which of the following is not a scalar quantity
(a) acceleration (b) time (c) mass (d) density

1.8 Which of the following is a vector quantity
(a) energy (b) mass (c) momentum (d) speed

1.9 The weight of a body is due to
(a) centripetal force of earth
(b) gravitational pull exerted by the earth
(c) force of attraction experienced by the particles
(d) gravitational force of attraction towards the center of the earth

1.10 A number of forces acting at a point will be in equilibrium if
(a) their total sum is zero
(b) two resolved parts in two directions at right angles are equal
(c) sum of resolved parts in any two perpendicular directions are both zero
(d) all of them are inclined equally

1.11 Two non-collinear parallel equal forces acting in opposite direction
(a) balance each other (b) constitute a couple
(c) constitute a moment (d) constitute a resultant couple

[***Ans :*** (1-b), (2-a), (3-d), (4-d), (5-b), (6-c), (7-a), (8-c), (9-d), (10-c), (11-d)]

(B) Subjective Questions

1.1 State the importance of S.I. system for units.

1.2 Define Engineering Mechanics & Explain its branches

1.3 Write the unit of following quantities in SI system.
(i) Force (ii) Velocity (iii) Acceleration (iv) Moment (v) Work

1.4 Justify that Weight of body is a vector quantity.

1.5 Define : (i) Force (ii) Rigid body (iii) Flexible body (iv) Scalar quantity (v) Vector quantity (vi) Fundamental units (vii) Derived Units (viii) Resolution of force (ix) Composition of force (x) Moment of force

1.6 List out & Explain characteristics of force.

1.7 List the systems of force & explain each one with drawing.

1.8 Define : (i) Collinear force system (ii) Concurrent force system (iii) parallel force system (iv) Coplanar force system (v) Non-coplanar force system (vi) Non-concurrent force system

1.9 A stone moves down a hill without physical application of force. Is there any force acting on stone? If yes, name it.

1.10 Explain following : (i) Equilibrium law of force (ii) Principle of superposition of force (iii) Principle of transmissibility of force (iv) Varignon's principle (v) Law of parallelogram of force (vi) Law of triangle of force (vii) Law of polygon of force.

1.11 What is moment of force? Explain its types with drawing.

1.12 Write equation to find resultant force by law of parallelogram of force.

1.13 List out the steps to find resultant force by method of resolution.

1.14 Write the steps to find resultant force for non-concurrent coplanar force system.

1.15 How the Varignon's principle is useful to find the position of point of application?

1.16 Two forces of 240 N and 200 N act at point with angle of 60^o with each other. Find resultant force. [*Ans :* R = 381.57 N at 27^o with P]

1.17 Resultant force of two equal forces acting with an angle 60^o between them is $30\sqrt{3}$ N. Find magnitude of force. [*Ans :* F = 30 N]

1.18 Two forces 100 kN each acting at an angle 45^o between them. Find magnitude and direction of the resultant. [*Ans :* R = 184.77 kN at 22.5^o with each force]

1.19 Two forces 1500 N and 800 N acting at point with angle of 75^o with each other. Find resultant force. [*Ans :* R = 1873.81 N at 24.35^o with P]

1.20 Three forces 2P, 3P and 4P act along three sides of an equilateral triangle taken in order. Find the resultant force. [*Ans :* 1.732 P and 210^o]

1.21 A string ABC of length 50 cm is tied to two points A & C at same level. A weight 500 N is applied by ring at B, 30 cm away from A along the string and Horizontal pull force P is also acts at B. If point B is 15 cm below the level of AC, find magnitude of force P. Consider tensions in the string on both sides of B are same. [*Ans :* P = 82 N]

1.22 Find magnitude and direction of resultant force for following force system.
(i) 30 N force due South
(ii) 30N force inclined at 30^o towards North of East
(iii) 10 N push force inclined at 60^o South of West
(iv) 20 N force is acting due West. [*Ans :* R = 13.68 N at 150^o with + X axis]

2 Equilibrium

UNIT SPECIFICS

I have discussed the following topics in this unit:

- Equilibrium and equilibrant
- Condition of equilibrium
- Free body and free body diagram
- Lami's theorem and its application to solve various engineering problems
- Types of Supports, Loading as well as Beams
- Beam reaction for various type of beam by analytical method
- Beam reaction for parallel force system by graphical method (Funicular Polygon)

Some fundamental concepts discussed here were very important to the students/learners for their future courses. Concepts of beams, loads, types of beams, types of loads, etc. are very important for their future courses also. These concepts were explain here for better clarity of the courses.

Examples were discuss based on quality, instead of quantity. Hence based on the restriction of pages, importance has given to the quality of content.

The topics were discuss in such a manner that it can generate creativity, curiosity and improving problem solving capacity of the student. Couples, inclined loading etc. are some important concepts, discussed for increasing the clarity of the concept. After the related practical, based on the unit content, there is a "KNOW MORE" section, subjective and MCQ exercises, which were design for supplementary information and betterment of the clarity of the users of this book.

RATIONALE

In this unit, we are going to discuss about equilibrium and equilibrant due to application of forces on the body. Lami's theorem is helpful to solve various engineering problems of equilibrium. We are also study the various type of supports, loads & beams. We are also concentrating on beam support reactions to be find by analytical method & graphically method.

PRE-REQUISITE

Knowledge of unit-I from this book for Engineering Mechanics.

UNIT OUTCOMES

After completing this unit, you will be able to :

1. Associate equilibrium & equilibrant
2. Use Lami's theorem for engineering problems
3. Interpret different types of supports, loads & beams
4. Calculate the beam reactions by analytical method
5. Demonstrate the beam reactions by graphical method

MAPPING UNIT OUTCOMES WITH COURSE OUTCOMES

Unit-2 Outcome	Expected Mapping with Programme Outcomes 1- Weak Correlation; 2- Medium correlation; 3- Strong Correlation				
	CO-1	CO-2	CO-3	CO-4	CO-5
U2-O1	2	3	-	-	-
U2-O2	1	3	-	-	-
U2-O3	1	2	-	-	-
U2-O4	1	3	-	-	-
U2-O5	1	3	-	-	-

2.1 EQUILIBRIUM & EQUILIBRANT

If the resultant of all the forces and resultant moments of all the forces on the body is zero, then the body is say to be in equilibrium. In such condition, the body may at rest or moving with constant velocity. Now if resultant force on the body is not zero, then to bring the body in equilibrium, we have to apply the force, which is known as equilibrant force. The equilibrant force is equal, opposite & collinear with the resultant force of force system acting on the body.

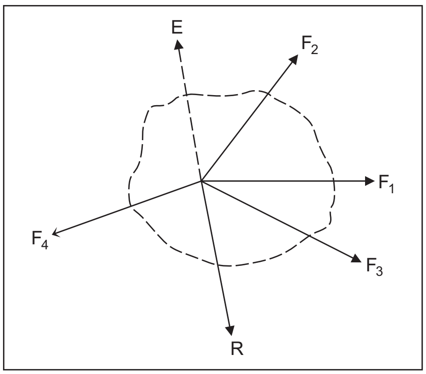


Fig. 2.1: Equilibrium and Equilibrant

Let consider the body is subjected by forces as shown in fig. 2.1. If resultant force R of force system is zero, then body is in equilibrium. In this case, $\Sigma F_x = 0$ and $\Sigma F_y = 0$. But if R is not zero, then to get equilibrium of the body, we have to apply the force E which is equal & opposite and collinear with resultant force R. This force E is known as equilibrant force. Thus equilibrant force give the equilibrium condition, when R is not zero.

2.1.1 Condition of Equilibrium

Coplanar force system as we have already study in unit-I are following.

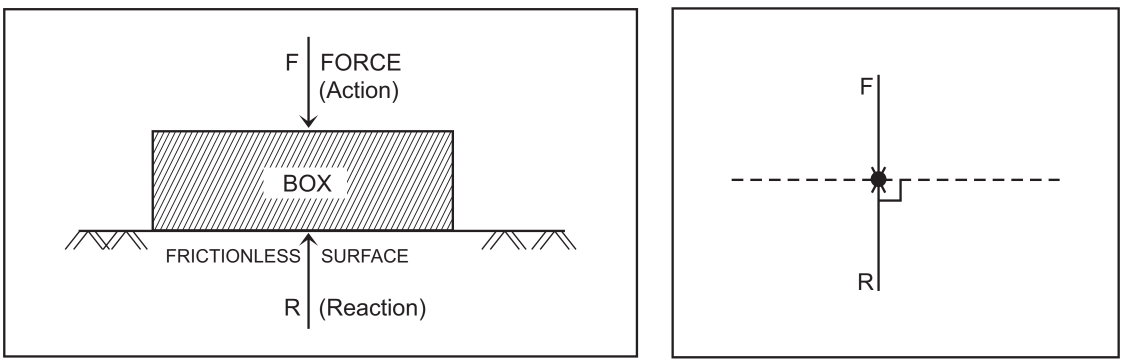
(a) Collinear (b) Concurrent (c) Parallel (d) Non concurrent non parallel

For all above force system, we can say that the body is in equilibrium, when total effect on body is zero. Mathematically (i) $\Sigma H = 0$ (ii) $\Sigma V = 0$ and (iii) $\Sigma M = 0$. These are analytical condition for equilibrium.

Graphical condition of equilibrium is: Force polygon must be close means closing side of polygon is zero.

2.1.2 Free body and Free body diagram

For equilibrium of the body or structure, a diagram of the body is drawn as isolated from its surrounding, removing its supports & holding devices. The forces acting on it are shown clearly showing magnitude, direction and location of all external forces including weight, applied forces, reactions and dimensions & angles. The body may be shown as a point when the forces acting on it are concurrent. The diagram so created is known as the free body diagram & said body as free body. In constructing the free body diagrams, it is necessary to know the kind of the forces offered by the supports.



(a) Space diagram (b) Free-body diagram

Fig. 2.2: Free body diagram

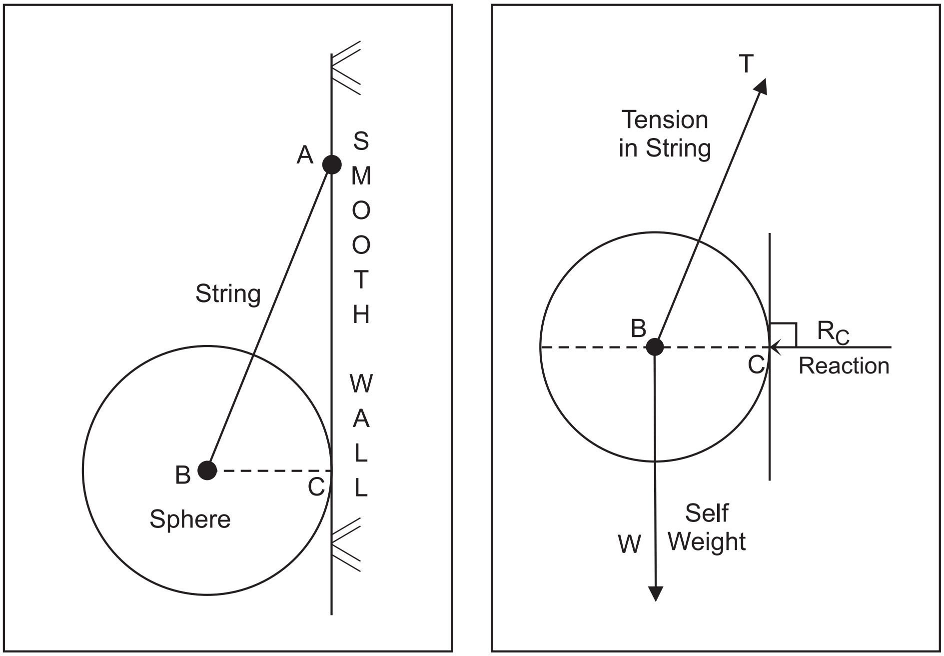
In fig. (a), for the box subjected with force F and resting on a frictionless surface, the action & reaction are shown. Note that these forces are acting on the box & weight of box is neglected. The reaction provided by a frictionless (smooth) surface is in the direction perpendicular to the plane of the surface. The surface may be either horizontal or inclined. The free body diagram makes it easy to apply the conditions of equilibrium to forces acting on the body & is of considerable help in solving the complicated engineering problems. The free body diagram of the body of fig. (a) is shown in fig. (b). The box is now assumed as a point & the forces are indicated. One must have to consideration of internal forces & external forces for drawing the free body diagram.

Internal forces which hold to gather the particles of the body & help it to be rigid i.e. not to deform. If more than one body is involved, internal forces hold the bodies together.

External forces which act on the body externally i.e. applied from outside. The forces essentially denote the action of other bodies (floors, walls, supports) on the rigid body being analyze.

Let us consider some examples to clear above points.

(i) A sphere of weight W hangs by a string & rests against a smooth vertical wall as shown in fig. (a). The forces acting in this systems are : (a) Self-weight W of the sphere acting as gravitational force in vertically downward direction through its centre C. (b) Wall reaction R_C at point of contact C with the wall. The reaction will be normal (perpendicular) to the wall surface. (c) Tension T in the string along BA.



(a) Space diagram (b) Free body diagram

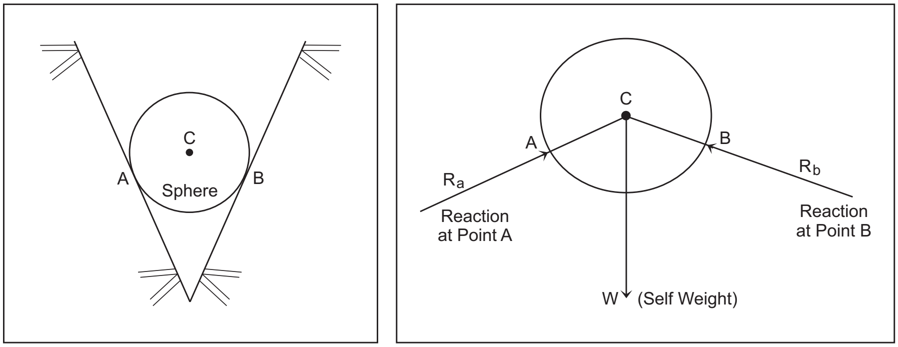
Fig. 2.3: A sphere hang by string & rest on vertical smooth wall

Equilibrium

As sphere is in equilibrium, all the forces will be concurrent & free body diagram will be shown in fig. (b)

(ii) A sphere resting in a V- shaped groove as shown in fig. (a). The forces acting in this systems are: (a) Self-weight W of the sphere acting as gravitational forcein vertically downward direction through its center C (b) Wall reaction R_A acting normal to inclined plane OA at contact point A (c) Wall reaction R_B acting normal to inclined plane OB at contact point B.

As sphere is in equilibrium, all the forces meet at point C & free body diagram will be as shown in fig. (b).



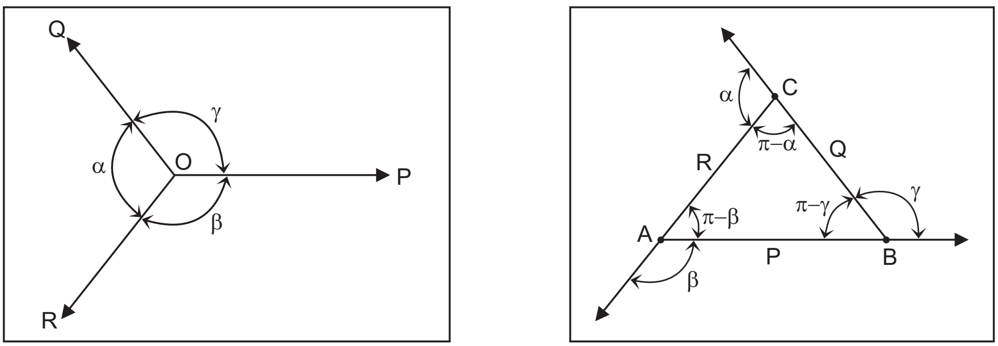
(a) Space diagram (b) Free body diagram

Fig. 2.4: A sphere resting in V-shaped groove

2.2 LAMI'S THEOREM

Let us now consider a special case of three forces acting on body & body is in equilibrium. In such case lami's theorem is very useful to find the unknowns either magnitude of force or direction of force. It states : *If three coplanar concurrent forces acting on the body are in equilibrium then each force is proportional to the sine of angle between the other two forces.*

Let P, Q & R be three forces acting on the body as shown in fig. (a). Since body is in equilibrium, they can be represented by sides of triangle ABC as shown in fig. (b). Applying sine rule for triangle ABC;



(a) Space diagram (b)Vector diagram

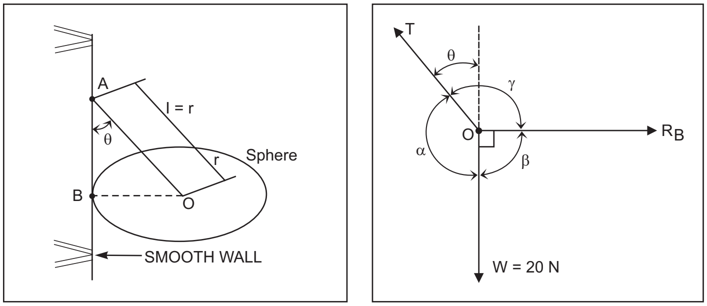
Fig. 2.5: Lami's theorem

$$\frac{AB}{\sin(\pi-\alpha)} = \frac{BC}{\sin(\pi-\beta)} = \frac{CA}{\sin(\pi-\gamma)}$$

$$\therefore \frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

From this equation, we can find any two unknowns out of total six quantities. Let to explain lami's theorem, take some examples.

Example 1. *A smooth sphere of radius r 150 mm and weight W 20 N is hung by string whose length equal the radius of sphere with contact to smooth vertical wall. Find inclination and tension in string as well as reaction of wall.*



(a) Space diagram (b) Free body diagram

Fig. 2.6

Solution:

(i) Draw space diagram as shown in fig. (a), from given data.

In triangle ABO,

$$\sin\theta = \frac{OB}{OA} = \frac{r}{2r} = 0.5$$

$\therefore\ \theta = 30°$ **(Answer)**

(ii) Now apply lami's theorem for free body diagram as shown in fig. (b), We get

$$\frac{R_B}{\sin\alpha} = \frac{T}{\sin\beta} = \frac{W}{\sin\gamma}$$

Here $\alpha = 180 - \theta = 150$; $\beta = 90$ and $\gamma = 90 + \theta = 120$ and W= 20N.

Putting values in Lami's equation, we get

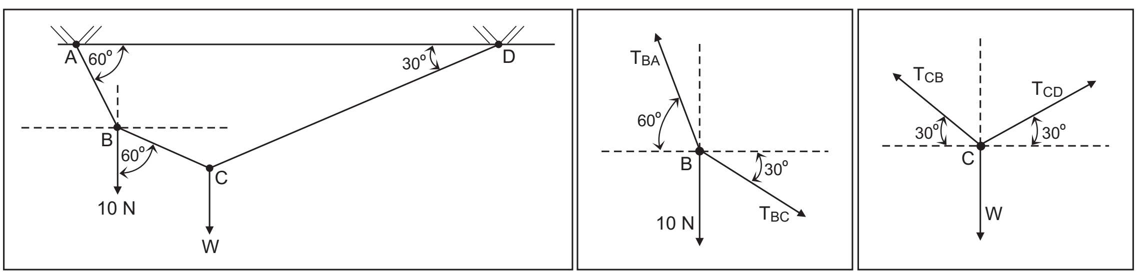
$$\frac{R_B}{\sin 150^o} = \frac{T}{\sin 90^o} = \frac{W}{\sin 120^o}$$

$\therefore\ R_B = 11.55$ N **(Answer)**

and $\therefore$ T = 23.10 N **(Answer)**

Lami's Theorem

Example 2. *Find the value of W if a light weight chain ABCD is suspended as shown in below fig. 2.7(a).*



(a) Data (b) FBD for point B (c) FBD for point C

Fig. 2.7

Solution:

Free body diagram for point B and Point C are shown in fig. (b) and (c) respectively.

(a) Applying Lami's theorem at point B [fig. (b)]

$$\frac{T_{BC}}{\sin(90+60)^o} = \frac{10}{\sin(180-60+30)^o}$$

$$\therefore T_{BC} = \frac{10 \times \sin 150^o}{\sin 150^o} = 10 \text{ N}$$

(b) Applying Lami's theorem at point C [fig. (c)]

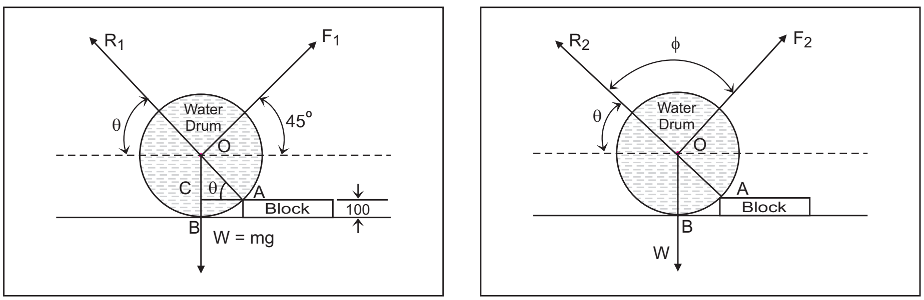
Considering $T_{BC} = T_{CB}$

$$\frac{W}{\sin(150-30-30)^o} = \frac{T_{CB}}{\sin(90+30)^o} = \frac{T_{BC}}{\sin(120)^o}$$

$$\therefore W = \frac{T_{BC} \times \sin(120)^o}{\sin(120)^o}$$

$\therefore$ W = 10 N **(Answer)**

Example 3. *A cylindrical water drum of 500 mm diameter and 1.5 m long is required to be rolled over a block of 100 mm height as shown in fig. Calculate (1) pull force F_1 required to be applied at center at angle 45^o with the horizontal, (2) minimum pull force F_2 required at the center of drum and its direction and (3) the reaction at the block in each case. Take mass density of water as 1000 kg/m^3 and neglect Weight of drum.*



(a) Case (i) (b) Case (ii)

Fig. 2.8

Solution:

First find primary parameters W (Weight of water drum) & angle θ (Direction of reaction).

(i) Weight of water drum W [radius = r = 250 mm & length = h = 1.5m]

(a) Volume = $V = \pi r^2 h = \pi \cdot \left[\frac{250}{1000}\right] \cdot [1.5]$ m^3

(b) Mass of Water = m = V · [1000] kg

(c) Weight of water drum = W = m · g

$$\therefore W = \pi \cdot [0.250]^2 \cdot [1.5] \cdot [1000] \cdot 9.8 \text{ N}$$

$$\therefore W = 2886.34 \text{ N}$$

(ii) Direction of reaction with Horizontal : θ [fig. (a)]

From Δ OAC; $\sin\theta = \dfrac{OC}{OA} = \dfrac{OB - BC}{OA} = \dfrac{250 - 100}{250}$

$$\therefore \sin\theta = \frac{150}{250} = 0.6$$

$$\therefore\ \theta = 36.87 \text{ with Horizontal.}$$

Case (1) Forces acting at center of drum are: [fig. (a)]

(a) Self-Weight of water drum = W = 2886.34 N (↓)

(b) Pull force F_1 at 45° with horizontal (↗)

(c) Reaction at block on drum R_1 at θ with horizontal (↖)

(i) Applying Lami's theorem;

$$\frac{W}{\sin\left(180^o - 45^o - \theta\right)} = \frac{F_1}{\sin\left(90^o + \theta\right)} = \frac{R_1}{\sin\left(90^o + 45^o\right)}$$

(ii) Putting value of W = 2886.34 N & θ = 36.87, we get

$$\frac{2886.34}{\sin 98.13^o} = \frac{F_1}{\sin 126.87^o} = \frac{R_1}{\sin 135^o}$$

(iii) ∴ F_1 = 2332.51 N **(Answer)**

(iv) ∴ R_1= 2061.67N **(Answer)**

Case (2) The forces acting at center of drum are same but pull force F_2 is acting at an angle ϕ with reaction R_2 [fig. (b)].

(1) Applying lami's Theorem;

$$\frac{W}{\sin\phi} = \frac{F_2}{\sin\left(90^o + \theta\right)} = \frac{R_2}{\sin\left(270^o - \theta - \phi\right)}$$

(ii) Putting value of W= 2886.34 N & θ = 36.87, We get

$$F_2 = \frac{2886.34 \times \sin\left(126.87^o\right)}{\sin\phi} = \frac{2309.07}{\sin\phi}$$

(iii) For F_2 to be minimum, sin ϕ should be maximum i.e., 1

Hence ϕ = 90 & F_2= 2309.07 N **(Answer)**

(iv) $R_2 = \dfrac{2886.34 \times \sin\left(143.13^o\right)}{\sin 90^o}$ = 1731.81 N **(Answer)**

Equilibrium-I

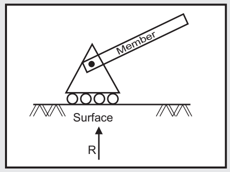
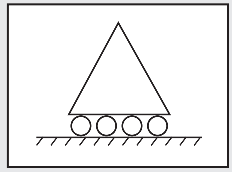
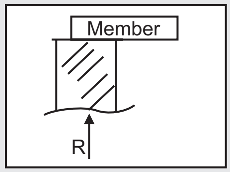
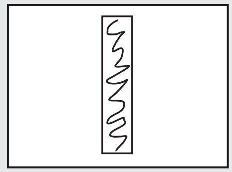
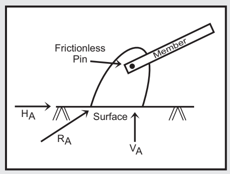
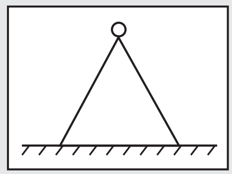
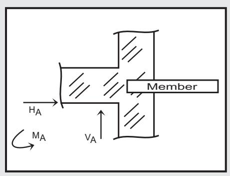
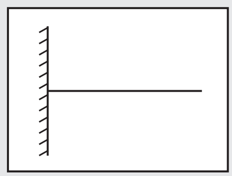
2.3 TYPES OF SUPPORTS, LOADING & BEAM

Beam is a structural element which is taken as specimen for studying the effects of loads, on the structure. It's carrying transverse load. The function of beam is to carry loads. It rest on supports which can offer reaction to keep system in equilibrium. Beam are classified according to their type of supports.

2.3.1 Types of supports

Structure or their components can be supported on different types of supports which can be classified depending upon the reaction offered by them as following.

Table 2.1: Types of Supports

Sr. No.	Name of Support	Description for reaction	Diagram with reaction	Symbol & Nos. of reaction
1	Roller	It provides the resistance to movement in the direction perpendicular to supporting surface. Ex. Skating roller		 (01)
2	Simple	It Supports without any type of joint or connection & hence reaction is always acting along the direction of support.		 (01)
3	Hinge	It provides resistance to movement in any direction by offering inclined reaction. Ex. Door hinge		 (02)
4	Fixed	It provides resistance to rotation & it effectively held in position & restrained against rotation. Ex. Nail in the wall		 (03)

2.3.2 Types of loading

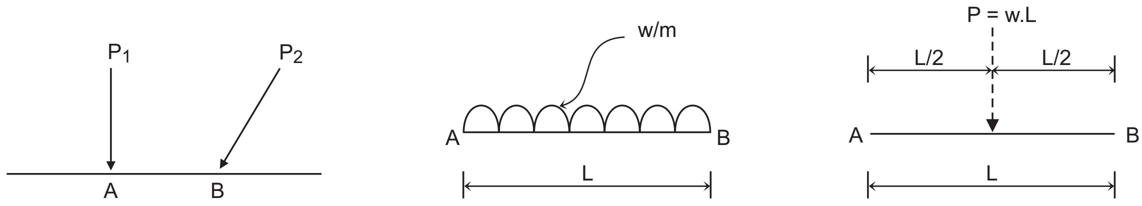
Loads which act on structural components can be external or due to self-weight of body. These load act as forces on structure. Following are important types of loading. (A) Concentrated or Point load (B) Uniformly distributed load (C) Uniformly varying load (D) Moment (E) Couple.

(A) Concentrated or Point load [fig. 2.9 (a)]

Load concentrated on a very small length compared to length of beam is known as concentrated or point load. It is practically assumed to be acting through a point. Example of point load is car standing on ground. In this contact area of wheel on ground is very small and hence load on ground is a point load. A person standing on a beam is also an example of the point load.

(B) Uniformly distributed load (UDL) [fig. 2.9(b)]

Load uniformly spread over the length of a beam is known as uniformly distributed load (UDL). In this type of loading, Weight of load per unit length is known as intensity of load; and is same along the length which denoted by w with units N/cm or kN/cm or N/m or KN/m. A truck loaded with sand of equal height & compound wall transferring load on the ground & person sleeping on bed are examples of UDL. For analysis, total load is taken as $(w \times l)$ acting as point load P at mid-point of length of UDL as equivalent value of UDL to find support reaction of beam as shown in fig. b(ii).



(i) UDL (ii) Equivalent Point Load

(a) Point Load (b) Uniformly distributed load

Fig. 2.9: Types of load

(C) Uniformly varying load (UVL) [fig. 2.9 (c)]

If the intensity of load is not same along the length but if uniformly increasing or decreasing from one end to another is known as uniformly varying load. If intensity increase from 0 to any value w_1 at the other end, then UVL known as triangular load and if intensity increase or decrease from w_1 value at one end to w_2 value at other end then UVL known as trapezoidal load as shown in fig (c).

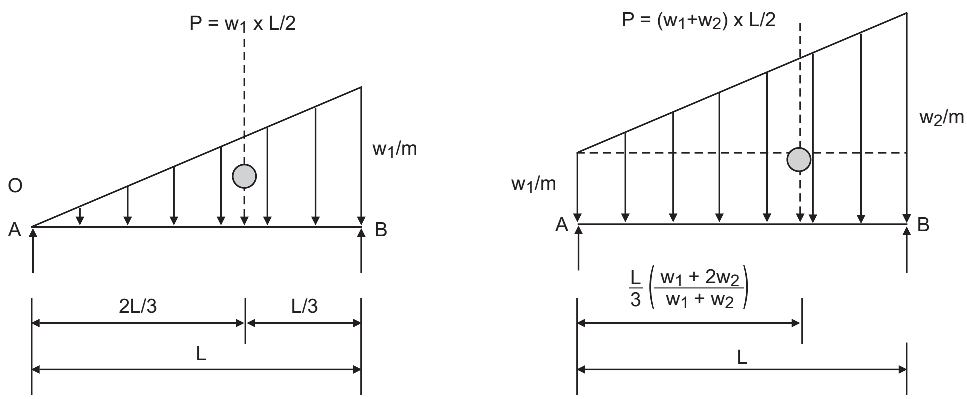


Fig. 2.9: (c) Types of Load - Uniformly Varying Load (UVL)

Equilibrium-II

A truck loaded with sand with top surface as inclined is the Example of UVL. In this type of load, total load is to be acting at C.G. of load diagram & value is total area of load diagram as shown in fig. (c)

(D) Moment : Already study in topic 1.8.1 of unit-I

(E) Couple : A couple is defined as two parallel forces that have the same magnitude, opposite direction & are separated by a perpendicular distance d as shown in fig.(d). The resultant force in this case will be zero but body will not be in equilibrium as these forces will tend to rotate the body. Hence, we can say that effect of couple is to produce a pure moment or tendency of rotation in specified direction. Examples are (i) to open or close a water tap; (ii) rotating steering wheel of vehicle (iii) to wind the spring of the clock.

The plane in which the forces constituting the couple act is called plane of the couple & perpendicular distance between the lines of action of force constituting couple is called arm 'd' of the couple as shown in fig.(d). Moment of couple is multiplication of force F and arm d.

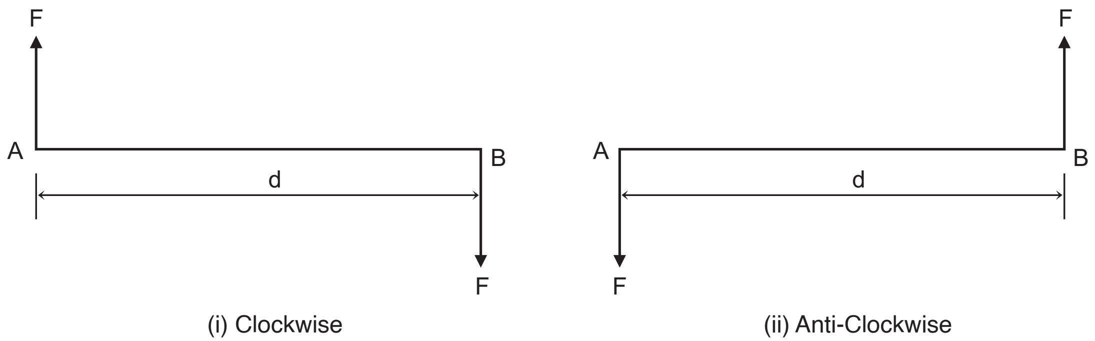


Fig. 2.9: (d) Types of Load - Couple

Type of couple : According to rotation of the body due to couple, it classified as clockwise couple & anticlockwise couple as shown in fig. (d) (i) & (ii) respectively.

2.3.3 Types of Beam

Beam are broadly classified in to two groups.

(A) Statically determinate beam & (B) Statically indeterminate beam.

Analysis of statically indeterminate beams is not in scope for you at this stage.

(A) Statically determinate beams

A beam is said to be statically determinate beam if the number of unknown reactions are not more than the number of equilibrium conditions. There are three equations from equilibrium condition which are (i) $\sum H = 0$ (ii) $\sum V = 0$ (iii) $\sum M = 0$. Hence according to types of supports of beam maximum three unknown reaction can be solved.

Following are statically determinate beams.

(i) Simply supported beam : It is supported on two simple supports at each end of the beam. In this case supports offers only reaction force and not moment. Usually one support is hinge & other is roller or both support is simple support. Nos. of unknown supports reaction are not more than 3 in any case as shown in below fig.

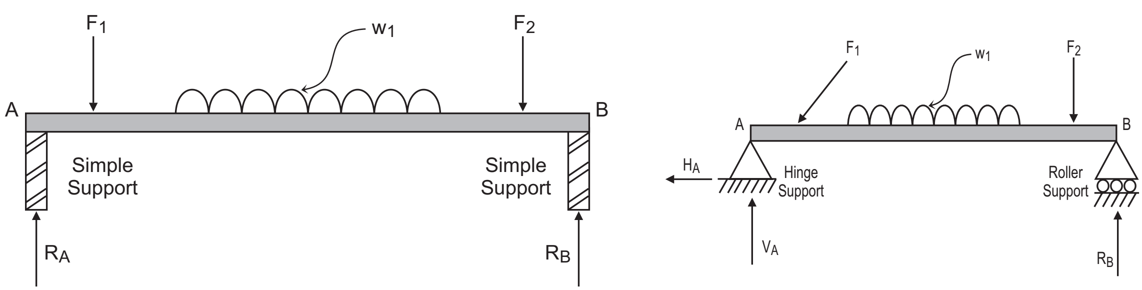
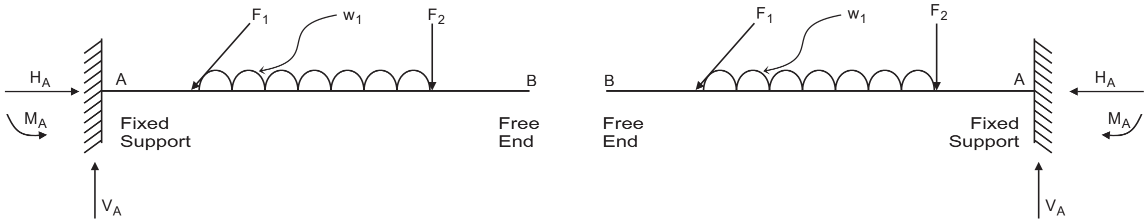


Fig. 2.10: Simply supported beam

(ii) Cantilever beam : In this beam, one end is fixed support and other end is free i.e., no support. In practice such beams are used when it is not possible to provide support at one end of the ends of the beam. Nos. of unknowns are not more than 3 in this beams as shown in below fig. Fixed support may be on left end or right end, as shown in fig. (a) & (b) respectively.



(a) Left hand end fixed support (b) Right hand end fixed support

Fig. 2.11: Cantilever beam

(iii) Overhang beam : If the one portion or two portions of the simply supported beam are extended beyond the support, then it's known as overhang beam. Depending upon the overhang, they are classified as single overhanging beam or double overhanging beam as shown in below fig. We can say that its special type of simply supported beam.

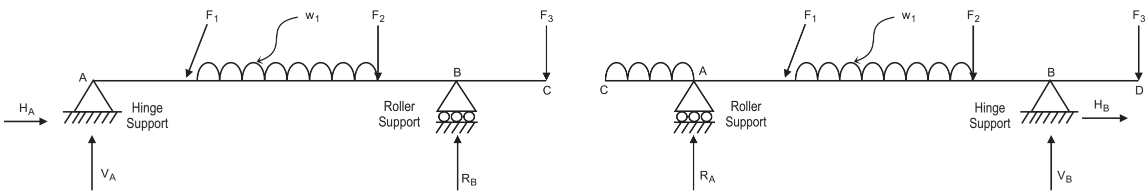
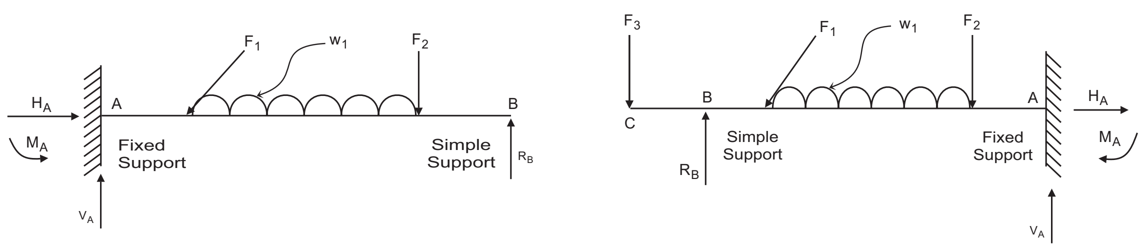


Fig. 2.12: Overhang beam

(B) Statically indeterminate beams

If nos. of unknown reaction are more than the equilibrium condition then such type of beam is known as statically indeterminate beam. Following are different statically indeterminate beams.

(i) Propped cantilever beam : In this beam one end is fixed support and other end is simple support with overhang or no overhang as shown in fig.



(a) Without overhang (b) With overhang

Fig. 2.13: Propped cantilever

(ii) Continuous beam : In this beam nos. of support are more than two as shown in below fig.

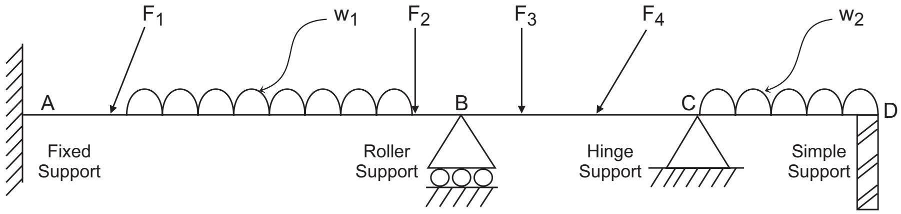


Fig. 2.14: Continuous beam

(iii) Fixed beam : In this beam both ends are with fixed support as shown in below fig.

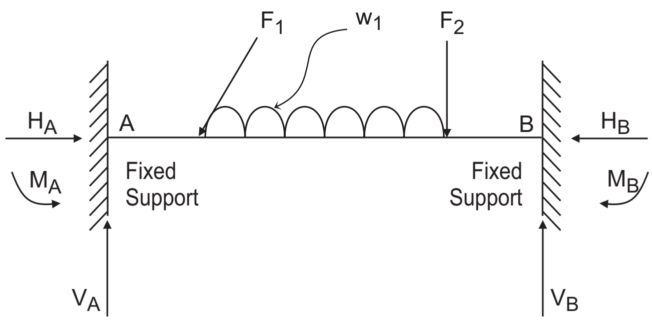


Fig. 2.15: Fixed beam

Beam-I Advanced

2.4 BEAM REACTIONS

Beam supports reaction can be finding by two methods. (I) Analytical method (II) Graphical Method. In analytical method, we have to use conditions of equilibrium to solve unknown support reaction as discuss in this topic. Graphical method will discuss in next topic 2.5. The beam we have to consider are: (A) Cantilever beam (B) Simply supported beam & (C) Overhang beam.

2.4.1. Beam reaction for cantilever beam

As we know the cantilever beam have one end fixed support and other end as free i.e., no support. We can find beam support reactions by using conditions of equilibrium by taking some examples.

Example 4. *A cantilever beam of 4m length having fixed support on left hand is carrying point loads of 10 kN, 5 kN, 20 kN and 15 kN at an interval of 1m from free end respectively find the support reaction for the beam.*

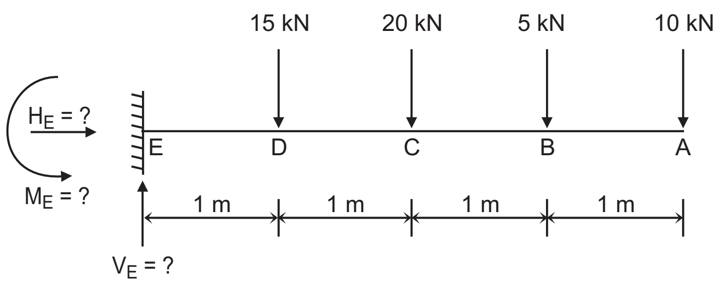


Fig. 2.16

Solution:

First draw the beam and loading on the beam as per given data as shown in fig.

(a) Using equilibrium condition $\Sigma V = 0$ with +ve sign for ↑ upward force. Assume vertical reaction at E V_E as upward load.

$\therefore V_E - 15 - 20 - 5 - 10 = 0$

$\therefore V_E = 50$ kN ↑ **(Answer)**

(b) Using equilibrium condition $\Sigma H = 0$ with +ve sign for → eastward force. As no horizontal load is acting on beam, $H_E = 0$ kN → **(Answer)**

(c) Using equilibrium condition $\Sigma M = 0$ with +ve sign for clockwise. ↻

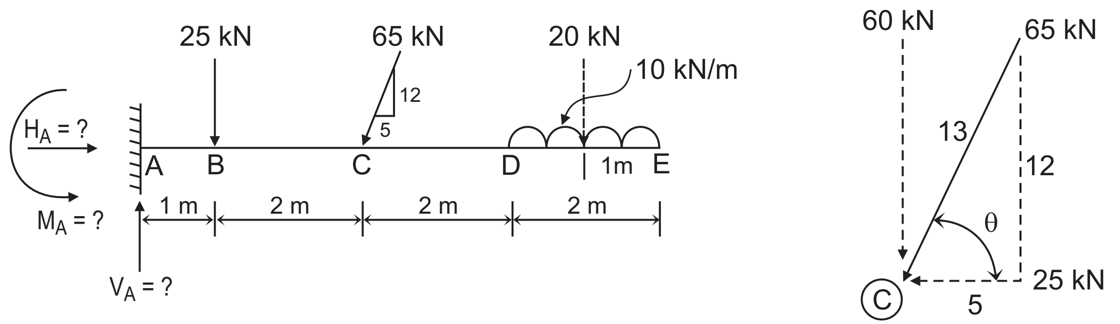
Consider moment at fixed support E, we get

$\Sigma M_E = (15 \times 1) + (20 \times 2) + (5 \times 3) + (10 \times 4) - M_E = 0$

$\therefore M_E = 15 + 40 + 15 + 40 = 110$ kN·m ↺ anticlockwise **(Answer)**

Note : If calculation get +ve sign our assumed sign is perfect otherwise we have to revers it.

Example 5. *Determine the support reaction of a cantilever beam as shown in fig.*



(a) Data (b) components of inclined force

Fig. 2.17

Solution:

(i) First, we have to find horizontal and vertical components of inclined force 65 kN acting at point C as shown in fig. (b).

Here $\tan\theta = \dfrac{12}{5}$; $\sin\theta = \dfrac{12}{13}$ & $\cos\theta = \dfrac{5}{13}$

$\therefore$ Horizontal component at point C $= 65 \times \cos\theta = 65 \times \dfrac{5}{13} = 25$ kN □

& Vertical component at point C $= 65 \times \sin\theta = 65 \times \dfrac{12}{13} = 60$ kN ↓

(ii) Also find equivalent load for UDL on DE portion of the beam.

(I) Total load as point load $= P = w \times l = 10 \times 2 = 20$ kN ↓

(II) Point of application of point load is at mid-point of DE i.e. 1m from point E as shown by dotted line in fig.(a).

Now applying three equilibrium conditions one by one, to get reactions.

(a) $\Sigma V = 0$ with + ve sign as □ upward and assuming V_A as upward.

$\therefore$ $V_A - 25 - 60 - (10 \times 2) = 0$

$\therefore$ $V_A = 25 + 60 + 20 = 105$ kN ↑ **(Answer)**

(b) $\Sigma H = 0$ with + ve sign as → eastward and assuming H_A as eastward.

$\therefore$ $H_A - 25 = 0$

$\therefore$ $H_A = 25$ kN → **(Answer)**

(c) $\Sigma M_A = 0$ with + ve sign as ↻ clockwise and assuming M_A as anticlockwise.

Note : Horizontal component of 60 kN force will pass from point A, hence moment due to this force will be zero.

$\therefore$ $25 \times 1 + 60 \times 3 + 25 \times 0 + (10 \times 2) \times 6 - M_A = 0$

$\therefore$ $M_A = 25 + 180 + 0 + 120 = 325$ kN·m ↺ anticlockwise **(Answer)**

Example 6. *Determine the support reaction of cantilever beam as shown in fig.*

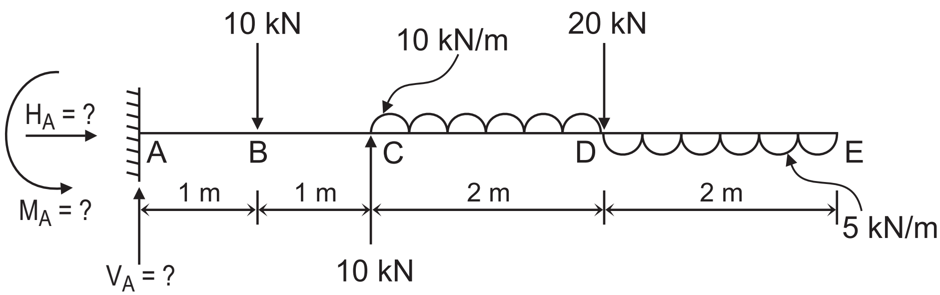


Fig. 2.18

Solution:

Applying three equilibrium conditions one by one to get reaction at support.

(a) $\Sigma H = 0$. As no Horizontal force acting on beam, $H_A = 0$. **(Answer)**

(b) $\Sigma V = 0$ with + ve sign as ↑ upward and assuming V_A as upward.

$\therefore$ $V_A - 10 + 10 - (10 \times 2) - 20 + (5 \times 2) = 0$

$\therefore$ $V_A = 10 - 10 + 20 + 20 - 10 = 30$ kN ↑ **(Answer)**

(c) $\Sigma M_A = 0$ with + ve sign as ↻ clockwise and assuming M_A as anticlockwise.

$\therefore$ $(10 \times 1) - (20 \times 2) + (\{10 \times 2\} \times 3) + (20 \times 4) - (\{5 \times 2\} \times 5) - M_A = 0$

$\therefore$ $M_A = 10 - 40 + 60 + 80 - 50 = 60$ kN·m anticlockwise ↺ **(Answer)**

2.4.2 Beam reaction for simply supported beam

As we know the simply supported beam have simple support at both ends or one end with roller support and other end with hinge support. We have also studied the support reaction for each support. By using equilibrium condition, we get unknown support reaction. Let to explain above point, take some examples.

Example 7. *A simply supported beam of span 10m carries three points loads of 40 kN, 30 kN and 20 kN from left hinge support at the distance 2 m, 5 m and 8 m respectively in downward direction. The right-hand support is roller. Find support reaction for the beam.*

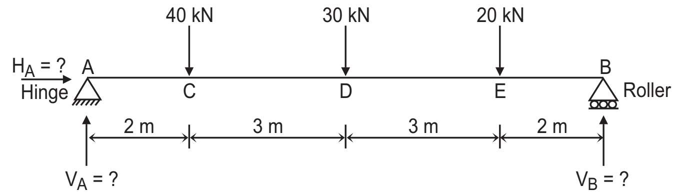


Fig. 2.19

Solution:

First draw space diagram of the beam from given data as shown in fig. Now applying three equilibrium condition for the beam.

(a) $\Sigma H = 0$ since there is no horizontal load on the beam. $\therefore$ $H_A = 0$ kN **(Answer)**

(b) $\Sigma V = 0$ with + ve sign ↑ upward and assume V_A and V_B both upward.

$\therefore$ $V_A - 40 - 30 - 20 + V_B = 0$.

$\therefore$ $V_A + V_B = 90$ kN. We have to use this equation as check point of our calculation.

(c) $\Sigma M = 0$ with + ve sign as ↻ clockwise moment.

(i) Consider moment at support point A.

$\Sigma M_A = (40 \times 2) + (30 \times 5) + (20 \times 8) - (V_B \times 10) + (V_A \times 0) = 0$

$\therefore$ $10\ V_B = 80 + 150 + 160 = 390$

$\therefore \quad V_B = \dfrac{390}{10} = 39$ kN ↑ **(Answer)**

(ii) Consider moment at other support point B.

$\Sigma M_B = (V_A \times 10) - (40 \times 8) - (30 \times 5) - (20 \times 2) + (V_B \times 0) = 0$

$\therefore$ $10V_A = 320 + 150 + 40 = 510$

$\therefore$ $V_A = \dfrac{510}{10} = 51$ kN ↑ **(Answer)**

(d) Now we can check our calculation for perfectness in equation obtain in (b).

If it fulfills, our calculation has no error.

$\therefore$ $V_A + V_B = 90$ put values of V_A and V_B, we get

LHS = 51 + 39 = 90 = RHS. **OK.** Means our answer is perfect.

If not satisfied in any case, you have done some mistake in calculation, recalculate it until it's satisfied the check point equation.

Example 8. *Find the reaction for the beam shown below.*

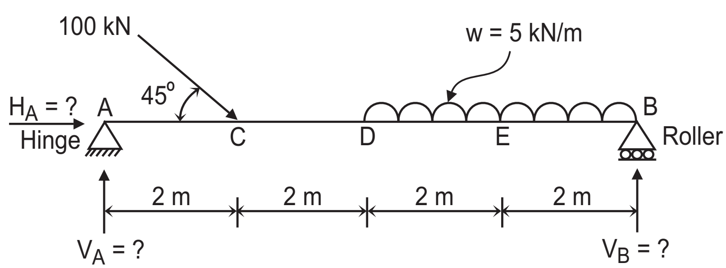


Fig. 2.20

Solution:

First, we have to find horizontal & vertical components of inclined force 100 kN acting at point C at angle 45° with beam alignment.

∴ Horizontal components of force 100 kN at C = 100 × cos 45° = 70.71 kN → eastward

& Vertical components of force 100 kN at C = 100 × sin 45° = 70.71 kN ↓ downward

Secondly, we have to find equivalent load for UDL as point load at mid-point of DB i.e., acting at point E as shown in fig. with dotted line.

Equivalent point load of UDL = P = (5 × 4) = 20 kN at point E.

Now applying conditions of equilibrium for the given beam.

(a) ΣH = 0 with + ve sign as → eastward and assuming H_A as eastward.

∴ $H_A + 70.71 = 0$

∴ $H_A = -70.71$ kN ← westward. As we have get – Ve value, we have to reverse the assumed direction. **(Answer)**

(b) ΣV = 0 with + ve sign ↑ upward and assume V_A and V_B both upward.

∴ $V_A - 70.71 - (5 \times 4) + V_B = 0$

∴ $V_A + V_B = 90.71$ kN. We have to use this equation as check point of our calculation.

(c) ΣM = 0 with + ve sign as ↻ clockwise moment.

(i) Consider moment at support point A.

$\Sigma M_A = V_A \times 0 + H_A \times 0 + 70.71 \times 2 + 70.71 \times 0 + (5 \times 4) \times 6 - V_B \times 8 = 0$

∴ $8\,V_B = 0 + 0 + 141.42 + 0 + 120 = 261.42$

∴ $V_B = \dfrac{261.42}{8} = 32.68$ kN ↑ **(Answer)**

(ii) Consider moment at other support point B.

$\Sigma M_B = V_B \times 0 - (5 \times 4) \times 2 - 70.71 \times 6 + 70.71 \times 0 + H_A \times 0 + V_A \times 8 = 0$

∴ $8\,V_A = 0 + 40 + 424.26 - 0 - 0 = 464.26$

∴ $V_A = 58.03$ kN ↑ **(Answer)**

(d) Now we can get self-check of our calculation for perfectness in equation obtain in (b).

If it fulfills, our calculation has no error.

∴ $V_A + V_B = 90.71$ kN

Put values of V_A and V_B, we get

LHS = 58.03 + 32.68 = 90.71 kN = RHS. **OK.** Means our answer is perfect.

If not satisfied in any case, you have done some mistake in calculation, recalculate it until it's satisfied the check point equation.

Example 9. *A beam AB 10m long is hinge at left hand support A and supported on roller support surface inclined at 30° with horizontal at right hand support B. The beam is carrying load as shown in fig. Find the reactions at supports of the beam.*

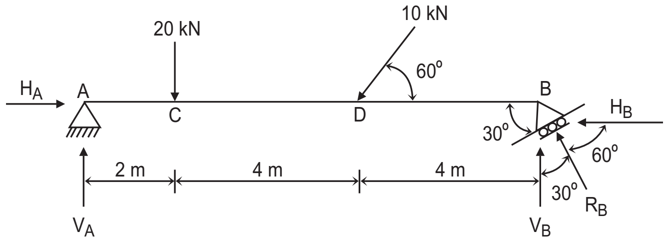


Fig. 2.21

Solution:

(i) In this case, the beam is supported on roller surface inclined at 30° to horizontal. Since roller support provides reaction perpendicular to the surface, the reaction R_B makes an angle of 60° with horizontal as shown in fig. Horizontal & vertical components of R_B, can found as

$\therefore H_B = R_B \times \cos 60^\circ = 0.5\ RB \leftarrow$ westward

& $V_B = R_B \times \sin 60^\circ = 0.866\ RB \uparrow$ upward

(ii) We have to find horizontal & vertical components of inclined force 10 kN acting at point D at angle 60° with beam alignment.

Horizontal components of force 10 kN at D = $10 \times \cos 60^\circ = 5.0$ kN $\leftarrow$ westward

&Vertical components of force 10 kN at D = $10 \times \sin 60^\circ = 8.66$ kN $\downarrow$ downward

Now applying conditions of equilibrium for the given beam.

(a) $\Sigma H = 0$ with + ve sign as $\rightarrow$ eastward and assuming H_A as eastward.

$\therefore H_A - 5.0 - 0.5R_B = 0$

$\therefore H_A - 0.5\ R_B = 5.0$ kN

(b) $\Sigma V = 0$ with + ve sign $\uparrow$ upward and assume V_A and R_B both upward.

$\therefore V_A - 20 - 8.66 + 0.866\ R_B = 0$

$\therefore V_A + 0.866\ R_B = 28.66$ kN

(c) $\Sigma M = 0$ with + ve sign as ↻ clockwise moment & Consider moment at support point A.

$\Sigma M_A = V_A \times 0 + H_A \times 0 + 20 \times 2 + 8.66 \times 6 + 5 \times 0 - V_B \times 10 + H_B \times 0 = 0$

$\therefore 0 + 0 + 40 + 51.96 + 0 - 10 \times 0.866\ R_B + 0 = 0$ (As $V_B = 0.866\ R_B$)

$\therefore 8.66\ R_B = 91.96$

$\therefore R_B = \dfrac{91.96}{8.66} = 10.62$kN ↖ **(Answer)**

(d) Put value of R_B in equation of (a), we get H_A

$H_A - 0.5\ R_B = 5.0$

$\therefore H_A = 5.0 + 0.5 \times 10.62 = 10.31$ kN $\rightarrow$ eastward **(Answer)**

(e) Put value of RB in equation of (b), we get V_A

$V_A + 0.866\ R_B = 28.66$

$\therefore V_A = 28.66 - 0.866 \times 10.62 = 19.46$ kN $\uparrow$ upward **(Answer)**

Advance Beam-II

2.4.3 Beam reaction for simply supported with overhang

As we know that if the one or two portions of the simply supported beam are extended beyond the support, then it's known as overhang beam. Let us take some examples.

Example 10. *Calculate the reactions at supports of the beam shown in figure.*

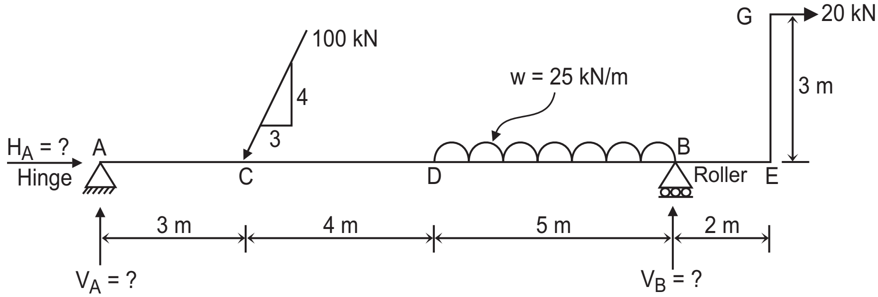


Fig. 2.22

Solution:

(i) We have to find horizontal & vertical components of inclined force 100 kN acting at point C.

$\therefore$ Horizontal components of force 100 kN at C = $\dfrac{100 \times 3}{5}$ = 60 kN ← westward

& Vertical components of force 100 kN at C = $\dfrac{100 \times 4}{5}$ = 80 kN ↓ downward

(ii) We have to find equivalent load for UDL as point load at mid-point of DB portion.
Equivalent point load of UDL = P = (25 × 5) = 125 kN act at distance of 2.5 m from point D.

(iii) Here one horizontal load on bracket EG is act as 20 kN → eastward, which create also moment that should be consider while applying moment condition.

Now applying conditions of equilibrium for the given beam.

(a) $\Sigma H = 0$ with + ve sign as → eastward and assuming H_A as eastward.

$\therefore H_A - 60 + 20 = 0$

$\therefore H_A = 40$ kN → eastward. **(Answer)**

(b) $\Sigma V = 0$ with + ve sign ↑ upward and assume V_A and V_B both upward.

$\therefore V_A - 80 - (25 \text{ x } 5) + VB = 0$

$\therefore V_A + V_B = 205$ kN We have to use this equation as check point of our calculation.

(c) $\Sigma M = 0$ with + ve sign as ↻ clockwise moment.

(i) Consider moment at support point A, we get

$\Sigma M_A = V_A \times 0 + H_A \times 0 + 60 \times 0 + 80 \times 3 + 125 \times 9.5 - V_B \times 12 + 20 \times 3 = 0$

$\therefore 0 + 0 + 0 + 240 + 1187.50 - 12VB + 60 = 0$

$\therefore 12\ V_B = 1247.5$

$\therefore V_B = 103.96$ kN ↑ upward **(Answer)**

(ii) Consider moment at other support point B.

$\Sigma M_B = 20 \times 3 + V_B \times 0 - 125 \times 2.5 - 80 \times 9 + 60 \times 0 + H_A \times 0 + V_A \times 12 = 0$

$\therefore 12\ V_A = -60 + 312.5 + 720 = 972.5$

$\therefore V_A = \dfrac{972.5}{12}$ = 81.04 kN ↑ upward **(Answer)**

(d) Now we can get self-check of our calculation for perfectness in equation obtain in (b) as $V_A + V_B = 205$ kN. If it fulfills, our calculation has no error.

We have LHS as $V_A + V_B = 81.04 + 103.96 = 205$ kN = RHS. **OK.** Means our answer is perfect.

If not satisfied in any case, you have done some mistake in calculation, recalculate it until its satisfied the check point equation.

Example 11. *Find reaction at supports for a double hanging beam shown in below figure.*

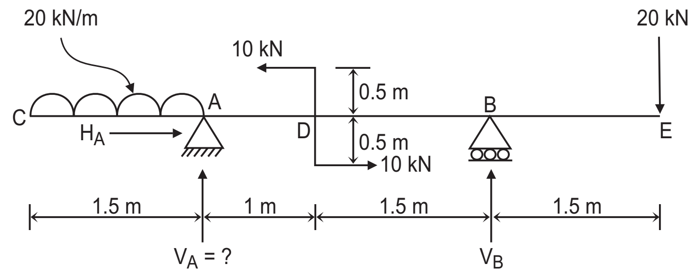


Fig. 2.23

Solution:

(i) As per loading, UDL on AC may have equivalent point load of P as (20 × 1.5) = 30 kN ↓ at mid-point of AC.

(ii) On beam CABE at point D, there are two equal & opposite force of 10 kN, resulting in to a couple of magnitude of 10 × (0.5 + 0.5) = 10 kN·m ↺ anticlockwise with net horizontal force as zero.

Now applying conditions of equilibrium for the given beam.

(a) $\Sigma_H = 0$ with + ve sign as → eastward and assuming H_A as eastward.

$\therefore H_A - 10 + 10 = 0$

$\therefore H_A = 0$ kN **(Answer)**

(b) $\Sigma V = 0$ with + ve sign ↑ upward and assume V_A and V_B both upward.

$\therefore V_A - (20 \times 1.5) - 20 + V_B = 0$

$\therefore V_A + V_B = 50$ kN. We have to use this equation as check point of our calculation.

(c) $\Sigma M = 0$ with + ve sign as ↻ clockwise moment.

(i) Consider moment at support point A, we get

$\Sigma M_A = 20 \times 4 - V_B \times 2.5 - 10 + V_A \times 0 + H_A \times 0 - (20 \times 1.5) \times 0.75 = 0$

$\therefore 2.5\, V_B = 80 - 10 + 0 + 0 - 22.5 = 47.5$

$\therefore V_B = \dfrac{47.5}{2.5} = 19.0$ kN ↑ upward **(Answer)**

(ii) Consider moment at other support point B, we get

$\Sigma M_B = 20 \times 1.5 + V_B \times 0 - 10 + H_A \times 0 + V_A \times 2.5 - 30 \times 3.25 = 0$

$\therefore 2.5\, V_A = -30 + 10 + 97.5 = 77.5$

$\therefore V_A = \dfrac{77.5}{2.5} = 31.0$ kN ↑ upward **(Answer)**

(d) Now we can get self-check of our calculation for perfectness in equation obtain in (b) as $V_A + V_B = 50$ kN. If it fulfills, our calculation has no error.

We have LHS = $V_A + V_B$ = 31.0 + 19.0 = 50 kN = RHS. **OK.** Means our answer is perfect.

If not satisfied in any case, you have done some mistake in calculation, recalculate it until it's satisfied the check point equation.

Example 12. *A beam is loaded as shown in below fig. If R_A = 5.6 kN, find the intensity of UDL w in kN/m on length AC and reaction R_B.*

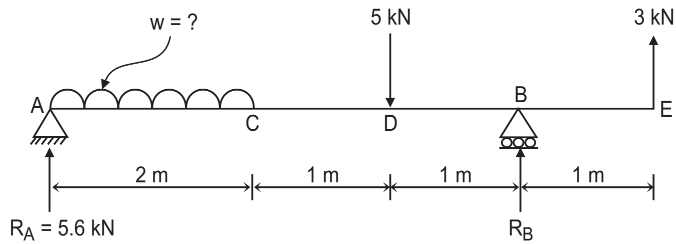


Fig. 2.24

Solution:

Here one reaction R_A at support A is given, but the value of UDL (kN/m) is unknown with other reaction R_B. We have to apply two equilibrium conditions to solve these two unknowns.

(i) $\Sigma M = 0$ with + ve sign as ↻ clockwise moment. Consider moment at support point B, we get

$\Sigma M_B = R_A \times 4 - (w \times 2) \times 3 - 5 \times 1 + R_B \times 0 - 3 \times 1 = 0$

$\therefore\ 5.6 \times 4 - 6w - 5 + 0 - 3 = 0$

$\therefore\ 6w = 22.4 - 5 - 3 = 14.4$

$\therefore\ w = \dfrac{14.4}{6} = 2.4$ kN/m **(Answer)**

(b) $\Sigma V = 0$ with + ve sign ↑upward and assume R_B upward.

$\therefore\ R_A + R_B + 3 - (w \times 2) - 5 = 0$

$\therefore\ 5.6 + R_B + 3 - (2.4\ x2) - 5 = 0$

$\therefore\ R_B = 1.2$ kN ↑ upward **(Answer)**

(c) We can take check point as $\Sigma M_A = 0$ with + ve sign as ↻ clockwise moment.

$\Sigma M_A = (w \times 2) \times 1 + 5 \times 3 - R_B \times 4 - 3 \times 5 = 2.4 \times 2 \times 1 + 15 - 1.2 \times 4 - 15$

$= 4.8 + 15 - 4.8 - 15 = 0$

So, we obtained that $\Sigma M_A = 0$ means our calculation is error free. Thus, we get self-assessment for TRUE value.

2.5 BEAM REACTION BY GRAPHICAL METHOD

We have discussed analytical method in previous topic 2.4 for different types of beams. Now we are discussing the second method i.e., graphical method to find beam support reactions. In this method, we have to study only for simply supporting beam carrying only point load as per your syllabus.

2.5.1 Funicular Polygon graphical method

It is necessary to understand, certain technical terminologies for graphical method. This graphical method is also known as Funicular Polygon Method.

(i) Bow's notation : Forces are identified by two different identical capital letters, placed on both side (in space) of the force, as shown in figure.

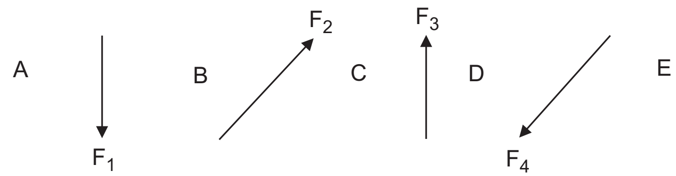


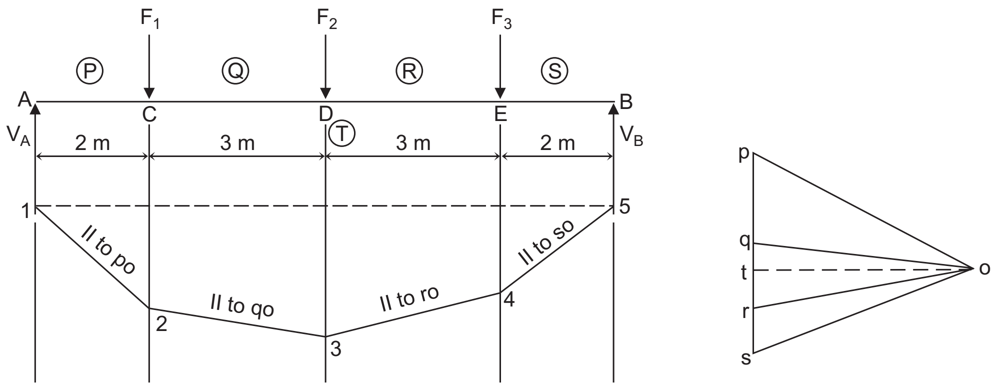
Fig. 2.25 : Bow's notation

In this case, four forces F_1, F_2, F_3 and F_4 are acting on the body. Now placed capital letters of alphabets on either sides of the force direction, we have to put A & B for force F_1. Now as per bow's notation, this force F_1 can identify as F_{AB}. Similarly for other forces F_2, F_3 and F_4 fill the space on either sides. For force F_2 on one side B letter is already place on LHS, so on other side (RHS) put another letter as C. So F_2 represent by bow' notation as F_{BC}. Thus force F_3 and F_4 can identify as F_{CD} & F_{DE} respectively by bow's notation.

(ii) Space diagram : A diagram showing all the forces in position along with their magnitude and direction acting on a body is known as space diagram. Span & position of load shown as per suitable linear scale i.e., 1 cm = _______ m.

(iii) Vector diagram : All the forces on the body is represented one by one in vectorial form by magnitude and direction. The direction is represented by its original direction, while magnitude is represented by some suitable force scale i.e., 1 cm = _____ N or kN.

Now, we can understand the steps of drawing funicular polygon. The graphical method to determine support reaction is given in following.



(a) Space diagram with Length scale (b) Vector diagram with Force scale

Fig. 2.26: Funicular polygon graphical method

Step-1: Draw the space diagram which shows position, direction & magnitude of all the forces acting on the body (beam) as shown if fig.(a). The distance between forces may be drawn with some length scale i.e. 1 cm = _____ m.

Step-2: Give bow's notations to all forces by placing alphabets on both sides of arrow. Reaction is considered as force acting on the body.

Step-3: Draw vector diagram for the given forces with some suitable force scale i.e. 1 cm = ___ N or kN which represents the magnitude of each force. All the forces were drawn; one by one taken in order; in a vector diagram as shown in fig.(b).

Step-4: Take some convenient point O in front of vectorial form of forces & join all the points of vector diagram with this point O.

Step-5: Now select a point 1 on the line of action of first force R_A & through it draw a line parallel to "op" which cross at point 2 on force F_1. Now through point 2 draw line 2-3 parallel to "oq". Similarly draw lines 3-4 & 4-5 parallel to vector diagram lines "or" and "os" respectively on space diagram.

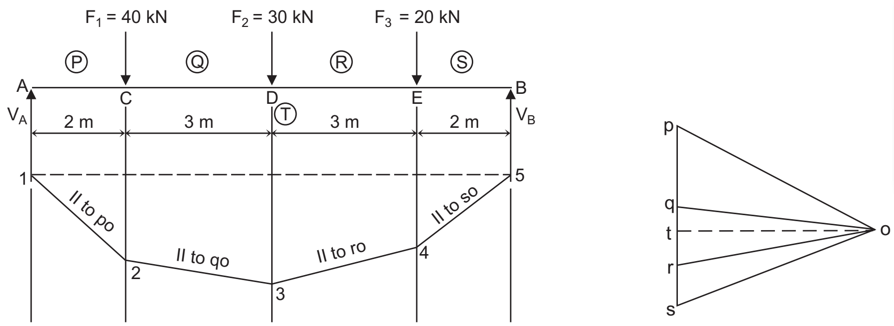
Step-6: Now join first start point 1 & last end point 5 obtained on line of force R_B as dotted line 1-5 in space diagram as shown in fig. (a). Draw a parallel line to 1-5 on vector diagram passing through point O as "ot" as dotted line as shown in fig. (b).

Step-7: Now measure "pt" & "ts" length on vector diagram & convert it by force scale as reaction R_A and R_B respectively.

Let to explain steps of graphical method (Funicular Polygon), take one example.

Example 13. *Solve example 7 as shown in below fig. by graphical method.*

Graphical method for Parallel Force System



(a) Space diagram with Length scale (b) Vector diagram with Force scale

Fig. 2.27

Solution:

Step-1: Draw space diagram from given data with length scale as 1 cm = 1 m as shown in fig.(a).

Step-2: Gives bow's notations to all forces including reactions V_A & V_B by placing alphabets on either side space on line of direction. Here for force F_1 = 40 kN, we have placed "P" & "Q" on either sides of space on line of direction of force F_1 means force F_1 is now F_{pq} as per bow's notation. Similarly placed "R", "S" & "T" as bow's notation as shown in fig.(a).

Step-3: Draw vector diagram for the forces on the beam with force scale as 1 cm = 20 kN as shown in fig.(b). Select start point "p" & draw parallel line as line of action of force F_{pq} (in this case vertically downward) and get point "q" on it by converting magnitude of force F_{pq} as 40 kN as per scale as 2 cm from point "p". Thus line "pq" represent vectorial form of force F_{pq}. Similarly draw all the forces taken in order in vector form in vector diagram as "qr" & "rs" for force F_{qr} (F_2) & F_{rs} (F_3) respectively.

Step-4: Take some convenient point "o" in front of vectorial form of forces. Join all points of vector diagram p, q, r & s with o and obtained line po, qo, ro & so as shown in fig.(b).

Step-5: Now on space diagram extend all line of action for all the forces. Select start point "1" on line of reaction R_A. Through this point "1", draw line parallel to line "po" of vector diagram to get point "2" on line of action of force F_1. Thus line "1-2" is parallel to line "po". Similarly draw "2-3", "3-4" & "4-5" parallel to line "qo", "ro" & "so" respectively.

Step-6: Now join first start point "1" with last end point "5" in this case, as line "1-5" as shown dotted line in space diagram fig.(a). Draw parallel line to "1-5" on vector diagram through point "o" & get line "ot" as shown in dotted line in fig.(b).

Step-7: Now measure length of line "pt" & "ts" and convert it in force magnitude by using force scale, we get

V_A = pt × force scale = 2.6 cm × 20 = 52 kN ↓ **(Answer)** &

V_B = ts × force scale = 1.9 cm × 20 = 38 kN ↓ **(Answer)**

Here, the value of reaction by graphical method, may vary by 5 to 10 %, as compared to analytical method and is permitted.

UNIT SUMMARY

- **Equilibrium :** If the resultant of all the forces and resultant moments of all the forces on the body is zero, then the body is said to be in equilibrium.
- **Equilibrant force :** The equilibrant force is equal, opposite & collinear with the resultant force of force system acting on the body.
- **Analytical condition of equilibrium :** (i) ΣH = 0 (ii) ΣV = 0 and (iii) ΣM = 0.
- **Graphical condition of equilibrium :** Force polygon must be close means closing side of polygon is zero.
- **Free body & Free body diagram :** The forces acting on body are shown clearly showing magnitude, direction and location of all external forces including weight, applied forces, reactions and dimensions & angles. The body may be shown as a point when the forces acting on it are concurrent. The diagram so created is known as the free body diagram & said body as free body.
- **Lami's theorem :** If three coplanar concurrent forces acting on the body are in equilibrium then each force is proportional to the sine of angle between the other two forces.
- **Types of supports :** Structure or their components can be supported on different types of supports which can be classified depending upon the reaction offered by them as following.

  (a) Roller support (b) Simple support (c) Hinge support (d) Fixed support
- **Types of loading :** Loads which act on structural components can be external or due to self-weight of body. Following are important types of loading. (A) Concentrated or Point load (B) Uniformly distributed load (C) Uniformly varying load (D) Moment (E) Couple.
- **Types of Beam :** (A)Statically determinate beam & (B) Statically indeterminate beam

  Statically determinate beam : A beam is said to be statically determinate beam if the number of unknown reactions are not more than the number of equilibrium conditions.

  Following are statically determinate beams :

  (a) Simply supported beam (b) Cantilever beam (c) Overhang beam.

  Statically indeterminate beam : If nos. of unknown reaction are more than the equilibrium condition then such type of beam is known as statically indeterminate beam.

  Following are statically indeterminate beams :

  (a) Propped cantilever beam (b) Continuous beam (c) Fixed beam.

- **Beam reactions :** Beam supports reaction can be finding by following two methods.
  (I) Analytical method (II) Graphical Method.
  Analytical method : We have to use conditions of equilibrium to solve unknown support reaction.
  Graphical method : This graphical method is also known as Funicular Polygon Method.
- **Bow's notation :** Forces are identified by two different identical capital letters, placed on both side (in space) of the force.
- **Space diagram :** A diagram showing all the forces in position along with their magnitude and direction acting on a body is known as space diagram. Span & position of load shown as per suitable linear scale i.e., 1 cm = _____ m.
- **Vector diagram :** All the forces on the body is represented one by one in vectorial form by magnitude and direction. The direction is represented by its original direction, while magnitude is represented by some suitable force scale i.e., 1 cm = ___ N or kN.
- **Graphical method (Funicular Polygon Method) :** To find out beam supports reactions, follow the following steps: [Refer fig. 2.26]

  Step-1: Draw the space diagram which shows position, direction & magnitude of all the forces acting on the body (beam). The distance between forces may be drawn with some length scale i.e. 1 cm = ______ m.

  Step-2: Give bow's notations to all forces by placing alphabets on both sides of arrow. Reaction is considered as force acting on the body.

  Step-3: Draw vector diagram for the given forces with some suitable force scale i.e. 1 cm = ___ N or kN which represents the magnitude of each force. All the forces were drawn; one by one taken in order; in a vector diagram.

  Step-4: Take some convenient point O in front of vectorial form of forces & join all the points of vector diagram with this point O.

  Step-5: Now select a point 1 on the line of action of first force R_A & through it draw a line parallel to "op" which cross on point 2 on force F_1. Now through point 2 draw line 2-3 parallel to "oq". Similarly draw lines 3-4 & 4-5 parallel to vector diagram lines "or" & "os" respectively on space diagram.

  Step-6: Now join first start point 1 & last end point 5 obtained on line of force RB as dotted line 1-5 in space diagram. Draw a parallel line to 1-5 on vector diagram passing through point O as "ot" as dotted line.

  Step-7: Now measure "pt" & "ts" length on vector diagram & convert it by force scale as reaction R_A & R_B respectively.

EXERCISE

(A) Objective Questions

2.1 A number of forces acting at a point will be in equilibrium if
  (a) their total sum is zero
  (b) two resolved parts in two directions at right angles are equal
  (c) sum of resolved parts in any two perpendicular directions are both zero
  (d) all of them are inclined equally

2.2 Two non-collinear parallel equal forces acting in opposite direction
(a) balance each other (b) constitute a couple
(c) constitute a moment (d) constitute a resultant couple

2.3 According to Lami's theorem
(a) three forces acting at a point will be in equilibrium.
(b) three forces acting at a point can be represented by a triangle, each side being proportional to force.
(c) if three forces acting upon a particle are represented in magnitude and direction by the sides of a triangle, taken in order, they will be in equilibrium.
(d) if three forces acting at a point are in equilibrium each force is proportional to the sine of the angle between the other two

2.4 Necessary & sufficient conditions for equilibrium of a system of coplanar non-concurrent forces are
(a) $\Sigma H = 0$ (b) $\Sigma V = 0$ (c) $\Sigma H = 0$ and $\Sigma V = 0$ (d) $\Sigma H = 0$, $\Sigma V = 0$ and $\Sigma M = 0$

2.5 A couple consist of
(a) Two like parallel forces of same magnitude
(b) Two like parallel forces of different magnitude
(c) Two unlike parallel forces of same magnitude
(d) Two unlike parallel forces of different magnitude

2.6 Which of the following is an example of a couple
(a) Turning the cap of ink bottle (b) Twisting a screw driver
(c) Steering the car (d) All of the above

2.7 If a beam has one end fixed and other end free then it is a
(a) Simply supported beam (b) Overhanging beam (c) Cantilever beam (d) Fixed beam

2.8 If a load is distributed evenly over an entire length of beam then the type of load is
(a) Uniformly varying load (b) Point load
(c) Concentrated load (d) Uniformly distributed load

2.9 If the reactions at support are horizontal and vertical then the type of support is
(a) Simple support (b) Hinged support (c) Roller support (d) Fixed support

2.10 If simply supported beam of span 5 m carries a point load of 5 kN at 2 m and 2 kN at 4 m from left support then reaction at left support is
(a) 3.4 kN (b) 3.6 kN (c) 4.5 kN (d) 2.5 kN

2.11 The reaction at the roller support of a beam is always
(a) horizontal (b) inclined (c) vertical (d) normal to support surface

2.12 If the reaction of a beam, at one of its supports is the resultant of horizontal & vertical forces, then it is a
(a) simply supported end (b) roller supported end (c) hinged supported end (d) fixed supported end

2.13 Lami's theorem is applicable only for
(a) coplanar forces (b) concurrent forces
(c) coplanar and concurrent forces (d) any type of forces

2.14 If body is in equilibrium, we may conclude that

(a) no force is acting on the body (b) the resultant of all the forces acting on it is zero

(c) the moments of the forces about any point is zero (d) both (b) & (c)

2.15 The intensity of UDL is 2 kN/m acting on 4 m span of beam, what is value of equivalent point load

(a) 2 kN (b) 4kN (c) 6 kN (d) 8 kN

[*Ans :* (1-c), (2-d), (3-d), (4-d), (5-c), (6-d), (7-c), (8-d), (9-b), (10-a), (11-d), (12-c), (13-c), (14-d), (15-d)]

(B) Subjective Questions

2.1 Explain the terms : (a) Space diagram (b) Free body diagram (c) Vector diagram.

2.2 What will be the value of resultant force for system of forces which is in equilibrium?

2.3 Distinguish between equilibrium force & equilibrant force.

2.4 List out the condition of equilibrium of coplanar concurrent forces.

2.5 State the lamis theorem.

2.6 Explain the conditions for equilibrium with sketch.

2.7 Explain different types of beam with sketches.

2.8 Distinguish between Moment and Couple.

2.9 Explain different types of supports & beam with neat sketch.

2.10 Explain different types of beam and different types of load on beam.

2.11 Two men carry a weight of 2 kN by means of two ropes fixed to the weight. One rope is inclined at 45° and other at 30° with their vertices. Find the tension in each rope. [*Ans :* 1.04 kN & 1.46 kN]

2.12 A smooth sphere of weight W is supported by a string fastened to a point A on smooth vertical wall, the other end is in contact with point B on the wall. If the length of string AC is equal to the radius of sphere, find the tension in the string & reaction of the wall. [*Ans :* 1.155 W & 0.577 W]

2.13 A sphere ball of weight 50 N is suspended vertically by a string 500 mm long. Find the magnitude & direction of the least force, which can hold the ball 100 mm above to lowest point. Also find the tension in the string at that point. [*Ans :* 30 N at angle of 90° with the string, 40 N]

2.14 A spherical ball of weight W, rest in a triangular groove whose sides are inclined at angles α & β to the horizontal. Find the reactions at the surface of contact.

[*Ans :* $W \sin \alpha / \sin(\alpha + \beta)$, $W \sin \beta / \sin(\alpha + \beta)$]

2.15 An electric light fixture weighting 20 N hangs from point C by two strings AC and BC. The string AC is inclined at 60° to the horizontal and BC at 45° to the horizontal as shown in fig.-1. Using Lami's theorem, determine the forces in the strings AC and BC. [*Ans :* 14.641 N & 10.352 N]

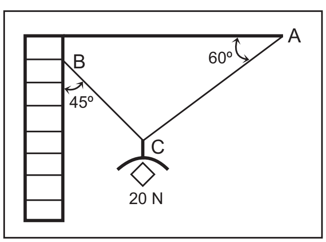


Fig. 1

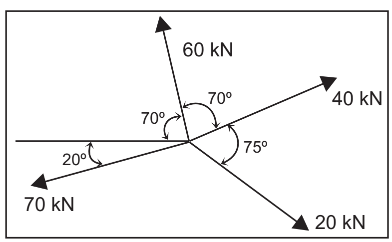


Fig. 2

2.16 System of forces is shown in above fig. 2 which is in equilibrium. Find the magnitude and direction of force that make the system in equilibrium condition with neat sketch.
[*Ans :* 61 kN at 14.92° anticlockwise from + X axis]

2.17 A cantilever beam ACB of 5 m long carries UDL of 10 kN/m on part AC 3m long from left hand fixed support A, point load of 50 kN at point C and clockwise moment of 50 kN·m at free end B. Find support reactions. [*Ans :* Ha = o, Va = 80 kN, Ma = 245 kN·m (Anticlockwise)]

2.18 A simply supported beam AB of span 4 m is carrying a point of 5, 2 and 3 kN at 1, 2 and 3 m respectively from left hand support A. Find the supports reactions at A & B. [*Ans :* 5.5 kN & 4.5 kN]

2.19 A simply supported beam of span 6 m carrying a UDL of 2 kN/m over a length 3 m from right hand end B. Find the supports reactions at A & B. [*Ans :* 1.5 kN & 4.5 kN]

2.20 A beam AB 6 m long rest on two simple supports 4 m apart, the right hand end is overhanging by 2 m. The beam carries a UDL of 1 kN/m over entire length of the beam. Find the supports reactions at A & B. [*Ans :* 1.5 kN & 4.5 kN]

2.21 A simply supported beam 8 m span, subjected to two point loads 50 kN and 100 kN at 2 m from each support. It is also subjected to uniformly distributed load of 20 kN/ m on full length. Find reaction at the support. [*Ans :* 142.5 kN & 167.5 kN]

2.22 Determine the reaction of the beam with overhang as shown in fig. 3.
[*Ans :* 4.33 kN, 24.5 kN &138 kN]

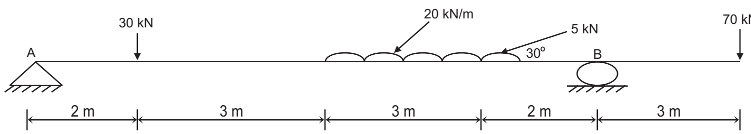


Fig. 3

2.23 Find the support reaction for a beam shown in fig. 4. [*Ans :* 0.695 kN, 5.12 kN & 6.32 kN]

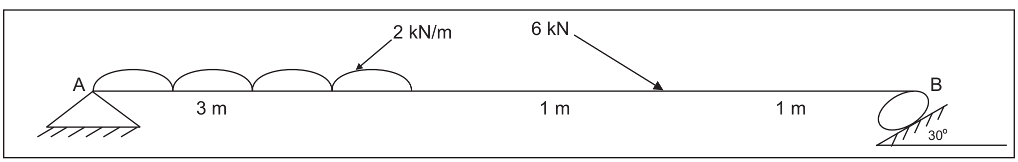


Fig. 4

2.24 Find the support reaction for a beam shown in fig. 5. [*Ans :* 2.49 kN, 8.13 kN & 12.46 kN]

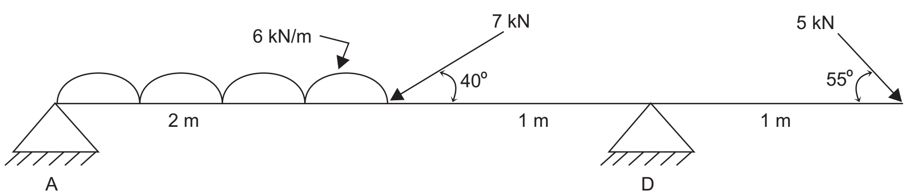


Fig. 5

3 Friction

UNIT SPECIFICS

In this unit, we are discusses the following topics:

- Friction and its related technical terms
- Types of friction
- Law of friction
- Equilibrium of a body on horizontal plane surface
- Equilibrium of a body on inclined plane surface

Some activities, taken under “Activity part”; are taken in such a manner that students understand the theory in better way. Large number of multiple-choice questions (MCQ) in “Objective Question” category as well as questions of short and long answer types as per Bloom’s taxonomy with number of numerical problems are covered under “EXERCICES” section for further work out on the unit.

As this unit is very important for future application in many branches of engineering; the practical applications of the unit topics are discussed for generating further curiosity and creativity. Some advanced problems limiting to the curriculum requirements, were discuss for improving problem solving capacity of the student.

A list of references and suggested reading given in the unit, so that one can go through for more information. It is important to note that for getting more information on various topics some QR code have been provide, which can be scan for relevant supportive knowledge. Again, QR code references were select in such a manner that students were encouraged to take out the courses of SWAYAM/NPTEL.

RATIONALE

Have you ever thought that the athletes are wearing rigged shoes for faster running for top speed can’t they run fast by wearing simple canvas shoe, why? Have you ever tried taking out water from the well with the help of pulley, what happens? Athletes are required to run at top speed their shoes have specially designed pattern underneath their shoe sole so that the grip of shoes with the ground will be stronger. In simple canvas shoe where rubber sole is used has smooth surface in comparison to rigged sole, so the grip of shoe with ground is less and there are chances of sliding. Likewise, when we take out the water from the well the force required to bring the water up in the bucket should be more then weight of bucket with water otherwise the rope will slide down from pulley into the well. The concept behind these activities is friction due to which we are able to do the above activities without sliding. We are going to discuss this concept along with their laws in detail.

PRE-REQUISITES

Basic knowledge of Physics and Math from Secondary Education [Standard 8 to standard 10] and previous unit I & II of this book.

UNIT OUTCOMES

After completing this unit, you will be able to :

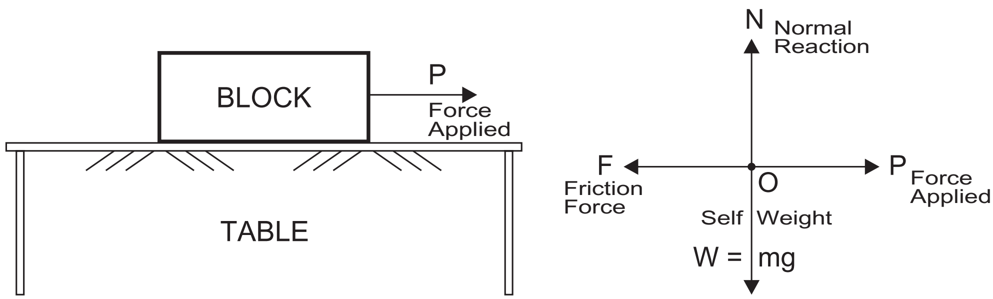
1. Explain terminologies associated with friction.
2. Illustrate laws of friction to the practical problems.
3. Analyze the problems of friction of the body on horizontal surface and inclined surface.

MAPPING UNIT OUTCOMES WITH COURSE OUTCOMES

Unit-3 Outcome	Expected Mapping with Programme Outcomes 1- Weak Correlation; 2- Medium correlation; 3- Strong Correlation				
	CO-1	CO-2	CO-3	CO-4	CO-5
U3-O1	-	-	3	-	-
U3-O2	1	-	3	-	-
U3-O3	1	2	3	-	-

3.1 FRICTION

Consider a block of weight W resting on a table. An external horizontal variable force P is apply on the block as shown in fig. 3.1(a). The block is in equilibrium and therefore, $\Sigma V = 0$ and the weight W of the block is resisted by normal reaction N offered by the top of table. $\therefore$ N = W.



(a) Block rest on table (b) Free body diagram

Fig. 3.1: Friction

Now, try to move this block on the top of the table maintain contact surface of the block and top of the table. The rough surface of the table top will offer internal resistance to the motion. This resistance to the motion, which always opposes motion, is force of friction or simply friction, denoted by F in a free body diagram as shown in fig. 3.1(b). As per the condition of equilibrium $\Sigma H = 0$, this frictional force F should be equal to the applied external force P, when block just to get motion. $\therefore$ F = P.

Friction is a force that resists sliding it is described in terms of a coefficient, is almost always assumed to be constant and specific to each materials /surface. Characteristics of friction is that it is parallel to the contact surface between two surfaces and always in a direction that opposes motion or attempted motion of the systems relative to each other.

When you push to get the block moving, you must raise the block until it can skip along with just the tips of the surface hitting, break off the points, or do both. A considerable force can be resist by friction with no apparent motion. This is due to the interlocking of these irregularities on the surface, which resist the motion. The harder the surfaces are pushed together (Putting another block over the block) the more force is required to move them because interlocking of irregularities take place. Thus, a force is required just to set the object in motion. Some of the peaks will be broken off, also requiring a force to maintain the motion. This is due to adhesive force between the surface molecules of the two objects. Even perfectly smooth surfaces also consist some force of friction. Adhesive forces or force of friction depends on the material of surface. For example, rubber-soled shoes slip less than those with leather-soled. At small but nonzero speeds, friction is nearly independent of speed.

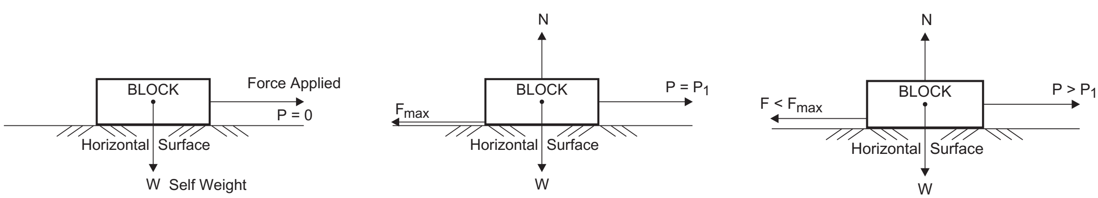
Activity-1:

Take a small plastic object (such as a food container) and slide it on a table by giving it a gentle tap see what happens. Now spray a light shower water on the table. What happens now when you give the same-sized tap to the same object? Now add a few drops of vegetable oil on the surface of the water and give the same tap again. What happens now ? Write your observations after each step.

3.1.1 Limiting friction

Consider again the block of weight W place on the rough horizontal surface as shown in fig. 3.2(a). At this stage, we have not applied any external force on the block i.e., P = 0. Therefore, Internal resistive frictional force F will not develop i.e., F = 0 and the block will remain at rest position.

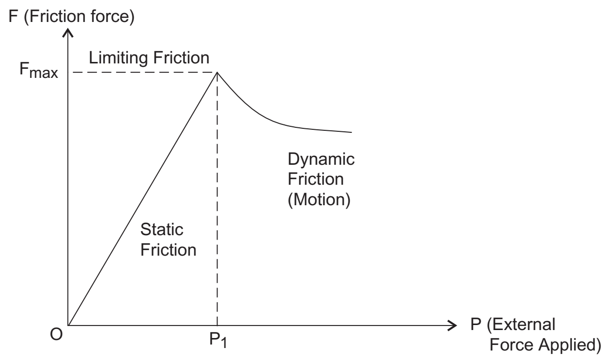
Let us apply; gradually more & more external force P on the block. As we increase P, resistive frictional force F should also increase. A stage will reach, when the block will just start to move or we can call it about to move, (impending motion) $P = P_1$ as shown in fig. 3.2(b). Note that in this case, motion has not occurred, but when negligible force is applied i.e., if tapping with finger or pen is made, the block will start moving. In this case, we say that motion is impending or just going to begin, frictional force F has reached its maximum value or limiting frictional force (F_{max}). We have seen that, the frictional force F is self-adjusting i.e., as P increases F also increases or self-adjusts from zero to F_{max}. If external force P is further increase from P_1, the block will now move. It has been observe that during motion, frictional force F decreases as P increases.



(a) Block at rest (b) Block in pending (just) to move (c) Block in motion

Fig. 3.2: Limiting friction

Fig. 3.3 shows graphically, the relationship between external force P and Frictional force F. You can observe this, as P increases F also increases, until P becomes P1. At this stage, motion is just impending. This is the case of limiting friction or maximum friction. Nature of friction also becomes clear from this diagram, we notice that when P = 0, F = 0; but as P increase, F self-adjusts itself to become equal to P. If P increases further i.e., $P > P_1$, motion occurs. It is observed, that in this case as P increase, there may be a slight decrease in frictional force F as shown in fig. 3.3.



Basic Friction

Fig. 3.3: Variation of F with P

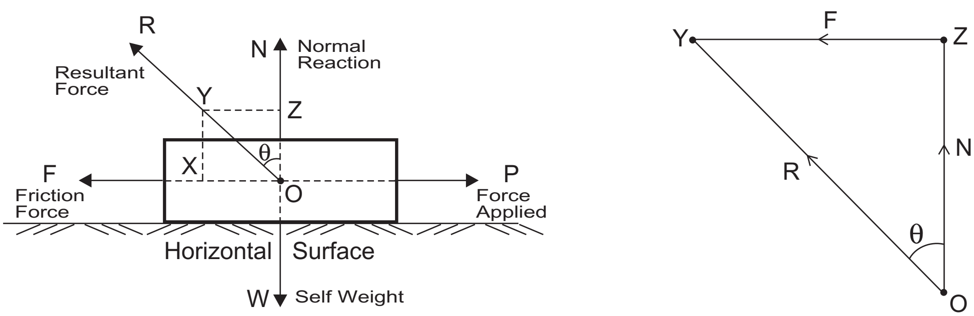
3.1.2 Coefficient of friction (λ)

Coefficient of friction is define as ratio of the maximum frictional force $F_{max.}$, which resist the motion of two surfaces in contact, to the normal reaction force N, which pressing the two surfaces together. It is usually symbolize by the Greek letter mu (μ). Mathematically, $\mu = \frac{F_{max}}{N}$, where $F_{max.}$ is the maximum frictional force and N is the normal force (reaction).

3.1.3 Angle of Friction (μ)

Consider a block is resting on a horizontal surface and subjected to horizontal pull P as shown in the fig 3.4. Let R is resultant force of two forces, frictional force F and normal reaction force N, which acts at angle θ to normal reaction, then angle θ is call the angle of friction as shown in fig. 3.4. From triangle OZY,

$$\tan\theta = \frac{ZY}{OZ} = \frac{\text{Friction force}}{\text{Normal reaction}} = \frac{F}{N}$$



(a) Body on horizontal plane

(b) Free body

Fig. 3.4: Angle of friction

As P increases, F increases and θ also increases. It can reach maximum value of α, when F reach limiting frictional force $F_{max.}$, at this stage, angle θ known as angle of friction α. Mathematically,

$$\tan \alpha = \frac{F_{max}}{N} = \mu \text{ (Coefficient of friction)}$$

$$\alpha = \tan^{-1}(\mu)$$

What is Friction ?

3.1.4 Angle of repose (ε)

Angle of repose can be define as the minimum angle of the inclined plane such that an object placed on it just begins to slide. Consider the block of weight W resting on inclined plane, which makes an angle θ with the horizontal as shown in fig 3.5. When θ be less the block will rest on the plane. Now, θ is increase gradually; a stage will reach, at which the block starts sliding down the plane. The angle θ, at which motion is impending call the angle of repose. Thus, the maximum inclination of the plane at which the body can repose, without any external force, is call Angle of Repose. It is usually symbolize by the Greek letter phai (ϕ).

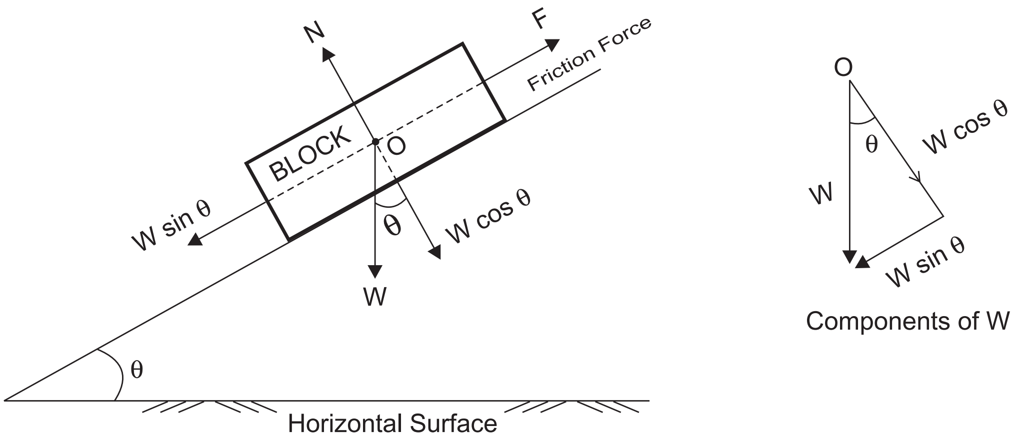


Fig. 3.5: Angle of repose

Since, block is at rest and hence in equilibrium, conditions of equilibrium can be apply.

(i) Resolving weight of block W, normal to the inclined plane, we get N = W cos θ …(i)

(ii) Resolving weight of block W, along to the inclined plane, we get F = W sin θ …(ii)

At just start of block sliding, θ reach at ϕ, F becomes F_{max}

But, from topic no 3.1.3, $F_{max} = \mu N$

Put the values from equation (i) & (ii), we get,

$W \sin \phi = \mu W \cos \phi$

$\tan \phi = \mu = \tan \alpha$

So, Angle of repose ϕ = Angle of friction α

3.1.5 Types of friction

There are two types of friction. (a) Static friction & (b) Dynamic friction.

(a) Static Friction

Static friction can act between two objects, when objects are stationary. The maximum frictional force present in the body, when it is in rest position (i.e., limiting friction is a kind of static friction). When body just tends to move on surface of another body is call static friction. It is denote as F_s. Magnitude of static friction is $F_s \leq \mu_s$ N.

Here μ_s is the coefficient of static friction and N is the magnitude of the normal force (the force perpendicular to the surface).

(b) Kinetic friction / Dynamic friction

If two surfaces are in contact and moving relative to one another, then the friction between them called kinetic friction. This happens, when the value of applied force exceeds the limiting friction and body is moving. Kinetic friction is less then limiting friction. It is denote by F_k. Once the applied external force exceeds P_1 the body will move, the magnitude of kinetic friction F_k is given by $F_k = \mu_k$ N. Where μ_k is the coefficient of kinetic friction and N is the magnitude of the normal force.

Dynamic friction can be sub divided in to two types. (i) Sliding friction & (ii) Rolling friction.

(i) **Sliding friction :** It is the frictional force, which comes into play, when one body sides over the other under action of external force.

(ii) **Rolling friction :** It is the frictional force, which comes into play, when one body rolls over the other under the action of external force.

Table 3.1: APPROXIMATE RANGE OF COEFFICIENT OF FRICTION

Sr.	System	Static Friction (μ_s)	Kinetic Friction (μ_k)
1	Wood on wood	0.4 – 0.7	0.3
2	Wood on metal	0.25 – 0.65	0.3
3	Wood on leather	0.5 – 0.6	0.3 – 0.5
4	Steel on steel (dry)	0.6	0.3
5	Steel on steel (oiled)	0.05	0.03
6	Steel on ice	0.4	0.02
7	Steel on concrete	0.3 – 0.6	0.4

3.1.6 Laws of friction

We have learnt about frictional phenomenon. Important points for it are list below as the laws of friction.

1. Force of friction is proportional to the normal reaction between the two surfaces of contact, acting parallel to the surface in contact and always act opposite to the relative motion of the two surfaces.

2. Force of friction depends upon the material of contact surfaces and the roughness of contact surfaces.
3. Force of friction is independent of the area of contact surfaces.
4. Force of friction is independent of the relative velocity of contact surfaces.
5. Ratio of friction force and normal reaction is known as the coefficient of friction and its value for the given two surfaces will always constant.
6. Coefficient of static friction is greater than coefficient kinetic friction.

3.2 EQUILIBRIUM OF A BODY ON A HORIZONTAL PLANE SURFACE

Now, it's clear that, a body lying on a rough horizontal plane surface is always in equilibrium. But body will start moving in the direction of the force, when an external force P is applied on it. This external force P can be apply in the two different ways. (i) Parallel to plane (i.e., horizontal) & (ii) Inclined to horizontal plane.

Let us discuss these cases one by one.

3.2.1 Equilibrium of a body on a horizontal plane with horizontal external force (Fig. 3.1)

For such case, equilibrium equations were applied on the body as horizontally (parallel to plane) & vertically (normal to plane).

(i) $\Sigma H = 0 \therefore F_{max.} = P$, where $F_{max.}$ = Frictional force & P = External force applied.

(ii) $\Sigma V = 0 \therefore N = W$, where N = normal reaction & W = Self-weight of the body.

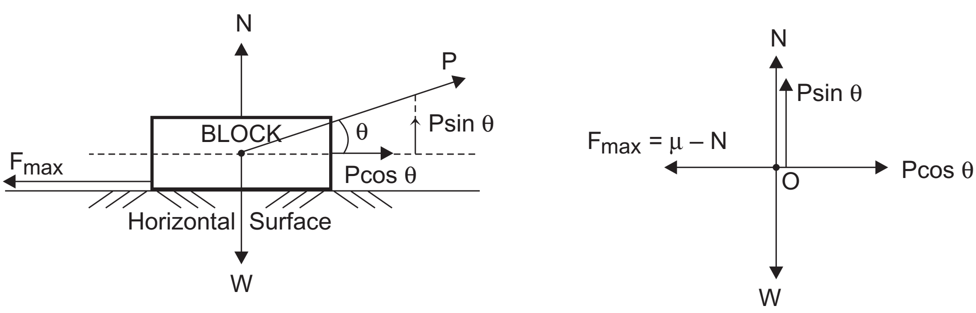
Now the value of the frictional force F_{max} is obtained from the relation:

(iii) $F_{max.} = \mu N$, where μ = Coefficient of friction & N = Normal force of reaction.

Using above three equations, we can solve the problems.

3.2.2 Equilibrium of a body on a horizontal plane with inclined external force (Fig. 3.6)

For such case, inclined force was resolved (parallel to the plane and perpendicular to the plane) as explained in unit 1 of this book. Now equilibrium equations were applied on the body as horizontally (parallel to plane) & vertically (normal to plane); as shown in fig. 3.6.



(a) Body (b) Free body diagram

Fig. 3.6: Equilibrium of body on horizontal plane

(i) $\Sigma H = 0 \therefore F_{max.} = P \cos \theta$, where F = Frictional force & P = External force applied.

(ii) $\Sigma V = 0 \therefore W = N + P \sin \theta$, where N = normal reaction & W = Self-weight of the body.

Now the value of the force of friction F_{max} is obtained from the relation:

(iii) $F_{max} = \mu N$, where μ = Coefficient of friction & N = Normal force of reaction.

Using above three equations, we can solve the problems. We calculate some examples to understand all above points.

Example 1. *A block of weight 150 N is resting on the horizontal surface. The coefficient of friction between the block and horizontal surface is 0.25.*

(a) Explain what happens to the block, if horizontal external force of P = 30 N is apply, as shown in fig.

(b) Now determine the external force required to just start the motion of the block, when P is a pull horizontal force, (c) P is a pull force inclined at 30° with horizontal and (d) P is push force inclined at 30° with horizontal.

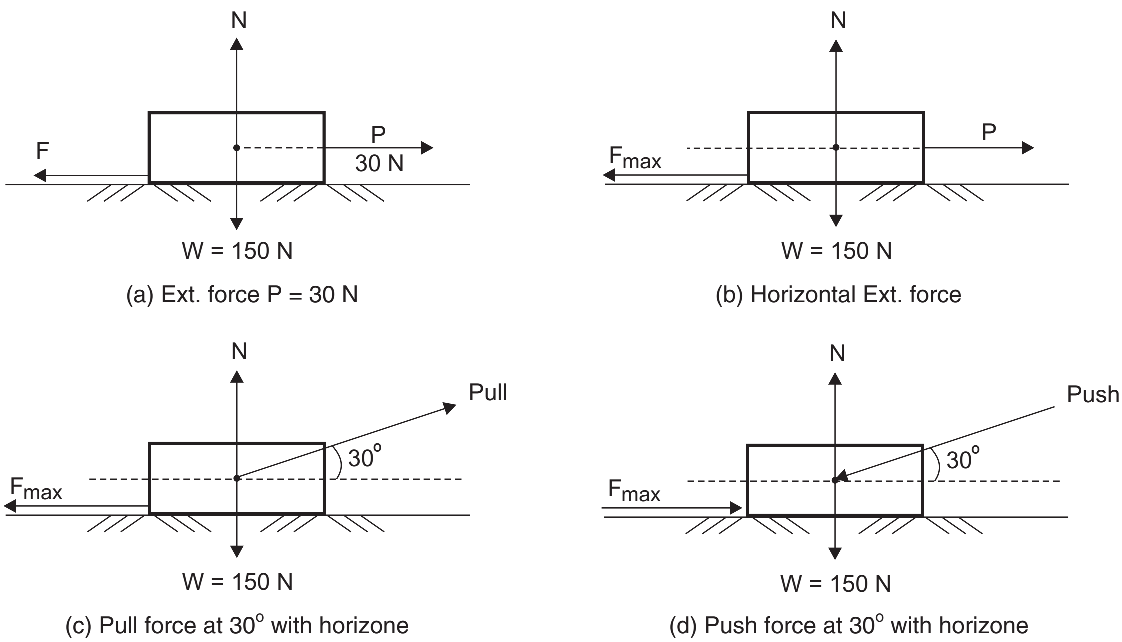


Fig. 3.7

Solution:

(a) $W = 150$ N, $\mu = 0.25$ & $P = 30$ N. [fig. 3.7(a)]

Apply equilibrium condition equations,

(i) $\Sigma V = 0, \therefore N = W = 150$ N

(ii) $\Sigma H = 0, \therefore F = P = 30$ N

(iii) $F_{max.} = \mu N = 0.25 \times 150 = 37.5$ N

Since, $F < F_{max}$, the block will be in equilibrium. It will not move as applied force is less than F_{max}. **(Answer)**

(b) $W = 150$ N, $\mu = 0.25$ [Fig. 3.7(b)]

(i) $\Sigma V = 0, \therefore N = W = 150$ N

(ii) We know that $F_{max.} = \mu N = 0.25 \times 150 = 37.5$ N

(iii) From $\Sigma H = 0$, $P = F_{max} = 37.5$ N **(Answer)**

(c) $W = 150$ N, $\mu = 0.25$ [fig. 3.7(c)]

(i) From $\Sigma V = 0$, $N = 150 - P \sin 30$

$N = 150 - 0.5 P$

(ii) As we know that $F_{max.} = \mu N$

$F_{max} = 0.25\,(150 - 0.5P)$

$F_{max} = 37.5 - 0.125 P$

(iii) From $\Sigma H = 0$, $P \cos 30 = F_{max}$

Put F_{max} from (ii),

$P \cos 30 = 37.5 - 0.125 P$

$\therefore 0.866 P + 0.125 P = 37.5$

$\therefore P = 37.84$ N **(Answer)**

(d) $W = 150$ N, $\mu = 0.25$ [Fig. 3.7(d)]

Here it should be clear that Push type of externally applied force (P) is apply on the body. Hence force of friction (Fmax) will be in opposite direction to that of probable motion or applied external force.

(i) From $\Sigma V = 0$, $N = 150 + P \sin 30$

$N = 150 + 0.5 P$

(ii) As we know that, $F_{max} = \mu N$

$F_{max} = 0.25\,(150 + 0.5P)$

$F_{max} = 37.5 + 0.125 P$

(iii) From $\Sigma H = 0$, $P \cos 30 = F_{max}$

Now put value of F_{max} from (ii), we get

$0.866 P = 37.5 + 0.125 P$

$\therefore 0.741 P = 37.5$

$\therefore P = 50.61$ N **(Answer)**

Friction on horizontal plane

Example 2. *A body is resting on a rough horizontal plane. The coefficient of friction between the body and the plane is 0.2 and the limiting friction force that is acting on the body is 80 N. Given that R is the resultant of the force of friction and the normal reaction force, find the magnitude of R.*

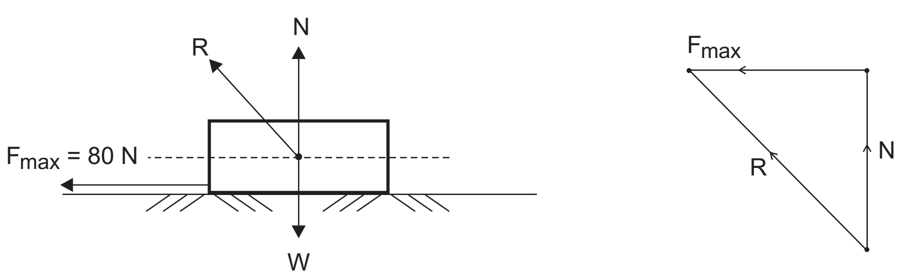


Fig. 3.8

Solution:

The forces acting on the body shown in fig. 3.8. Here, $F_{max.}$ = 80 N & μ = 0.2

(i) Now, limiting friction = F_{max} = μ N,

$\therefore$ 80 = 0.2 N

$\therefore N = \dfrac{80}{0.2}$

$\therefore$ N = 400 Newton

(ii) Force R is the resultant of the normal reaction force N and the limiting friction F_{max},

$R^2 = N^2 + F^2_{max}$ (From Pythagoras Theorem)

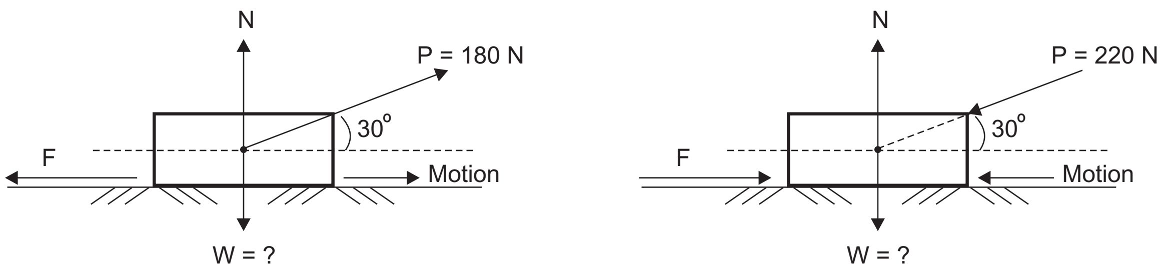
$\therefore R^2 = 400^2 + 80^2 = 166400$

$\therefore$ R = 407.92 N **(Answer)**

Try this:

Can you determine this resultant force R (of Example 2) with any other method ? Discuss this solution with your class teacher. (Hint : Can you use any method of Unit 1 or Unit 2)

Example 3. *A body is resting on a rough horizontal plane. It requires an external force of 180 N (pull type), inclined at 30° to the horizontal plane, just to start the motion. Furthermore, it is also observed that an external force of 220 N (push type) inclined at 30° to the horizontal plane, can also just start the motion. Find the weight the body and coefficient of friction.*



(a) Pull of 180 N at 30°

(b) Push of 220 N at 30°

Fig. 3.9

Solution:

The forces acting on the body shown in fig. 3.9 (a) & (b) for the external force pull & push respectively.

Here, θ = 30° & pull P = 180 N and push = P = 220 N

Resolving all forces horizontally & vertically for both cases, we get;

(A) Case I as external force is apply as pull of 180 N at inclination of 30° with horizontal.

(i) ΣH = 0 $\therefore$ F = P cos θ

$\therefore$ F = 180 × cos 30

$\therefore$ F = 155.9 N

(ii) ΣV = 0 $\therefore$ W = N + P sin θ

$\therefore$ W = N + 180 × sin 30

$\therefore$ W = N + 90

$\therefore$ N = W – 90

(iii) We know that frictional force = F = μ N = μ (W – 90)

$\therefore$ 155.9 = μ (W – 90) … (i)

(B) Case II as external force is apply as push of 220 N at inclination of 30° with horizontal.

(i) $\Sigma H = 0$ $\therefore F = P \cos \theta$

$\therefore F = 220 \times \cos 30$

$\therefore F = 190.5$ N

(ii) $\Sigma V = 0$ $\therefore N = W + P \sin \theta$

$\therefore N = W + 220 \cdot \sin 30$

$\therefore N = W + 110$

(iii) We know that frictional force = F = μ N = μ (W + 110)

$\therefore$ 190.5 = μ (W + 110) …(ii)

(C) Dividing equation (i) by (ii), we get;

$$\frac{155.9}{190.5} = \frac{\mu(W-90)}{\mu(W+110)} = \frac{(W-90)}{(W+110)}$$

$\therefore$ 155.9 (W + 110) = 190.5 (W – 90)

$\therefore$ 155.9 W + 17149 = 190.5 W – 17145

$\therefore$ 34.6 W = 34294

$\therefore$ W = 991.16N **(Answer)**

(D) Substituting the value of W in equation (i), we get;

$\therefore$ 155.9 = μ (W – 90) = μ (991.16 – 90) = μ (901.16)

$$\therefore \mu = \frac{155.9}{901.16}$$

$\therefore$ μ = 0.173 **(Answer)**

3.3 EQUILIBRIUM OF A BODY ON AN INCLINED PLANE SURFACE

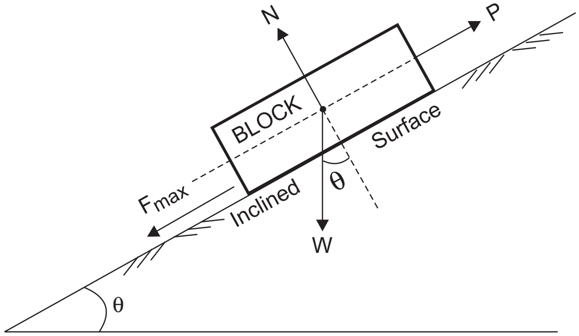
We have observed that, if the inclination of rough inclined plane surface is less than the angle of repose ϕ (angle of friction α) and no external force was applied on the body; the body will remain at rest (in equilibrium). The application of external force, in the direction of motion, is necessary for motion of the body in upward or downward direction. Now opposite to this case, if the inclination of the inclined plane surface is more than the angle of repose (angle of friction), the body will not be in equilibrium (at rest) due to downward motion. In such case, an upward external force is necessary to keep the body in equilibrium. This external force is applied at an angle to inclined plane surface or parallel to plane surface or in the horizontal direction.

Here we are limiting our discussion (from curriculum perspective), in which body is pulled up on inclined plane surface by an external force applied parallel to inclined plane surface.

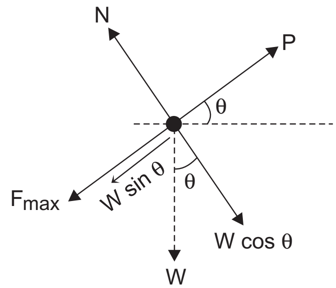
3.3.1 Equilibrium of a body on an inclined plane with parallel external force to plane (Fig. 3.10)

In such case, equilibrium of the body is study by resolving all the forces acting on the body as parallel (along) to plane surface & normal (perpendicular) to plane surface with external force to be apply as (i) Pull for upward motion & (ii) Push for downward motion of the body.

(a) External force is applied as pull for upward motion of the body on inclined plane surface:
The forces acting on the body for this case are shown in fig. 3.10.



(a) Body on inclined plane



(b) Free body diagram

Fig. 3.10: Equilibrium of body on inclined plane surface

(i) Resolving all the forces parallel (along) to given an inclined plane surface, we get,

$P = F_{max} + W \sin\theta$

$P = \mu N + W \sin\theta$ [as $F_{max} = \mu N$] ...(i)

(ii) Resolving all the forces perpendicular (normal) to given an inclined plane surface, we get,

$N = W \cos\theta$ ...(ii)

Substituting equation (ii) in equation (i), we get, $P = \mu W \cos\theta + W \sin\theta$ ...(iii)

Now μ = Coefficient of friction = $\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha}$, Substituting in equation (iii), we get,

$$P = \frac{\sin\alpha}{\cos\alpha} W \cos\theta + W \sin\theta$$

$\therefore P \cos\alpha = W \sin\alpha \cos\theta + W \cos\alpha \sin\theta$

$\therefore P \cos\alpha = W \sin(\alpha + \theta)$

$$\therefore P = \frac{W \sin(\alpha+\theta)}{\cos\alpha} \quad ...(iv)$$

Thus, an externally applied force P can be obtained by using equation (iv) for upward motion of body.

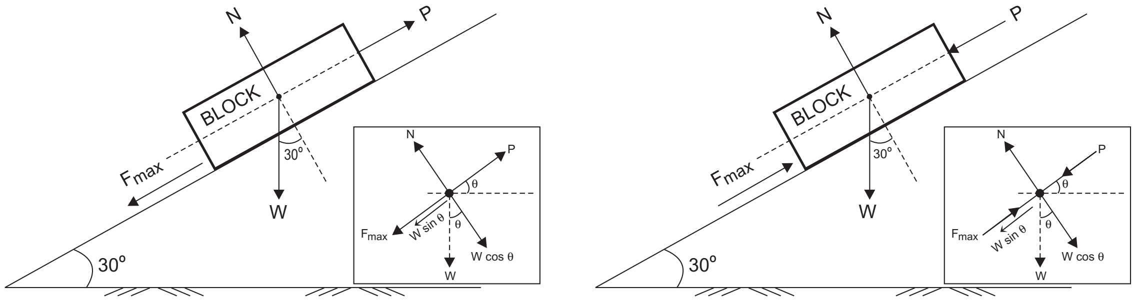
(b) External force is applied as push of force for downward motion of the body on inclined plane surface:

In the similar way as obtained in (a), we get,

$$P = \frac{W \sin(\alpha-\theta)}{\cos\alpha} \quad ...(v)$$

Thus, the external force to be apply P for downward motion of body can be find by using equation (v). We can solve some examples to understand above points.

Example 4. *A block of mass 10 kg is resting on the rough inclined plane surface with inclination of 30° to horizontal. If coefficient of friction is 0.25 between two contact surfaces, find the external force to be apply parallel to inclined plane to move the block (i) upward and (ii) downward.*



(a) Upward motion of clock on inclined plane (b) Downward motion of clock on inclined plane

Fig. 3.11

Solution:

Given data : $\theta = 30^o$, $\mu = 0.25$ & mass = m = 10 kg., W = mg = 10 × 9.8 N = 98 N.

(A) Block motion (move) upward on inclined plane surface : [fig. 3.11(a)]

(i) Resolving all the forces perpendicular (normal) to given an inclined plane surface, we get,

$N = W \cos\theta$

$\therefore N = 98 \times \cos 30^o$

$\therefore N = 84.87$ N …(i)

(ii) Resolving all the forces parallel (along) to given an inclined plane surface, we get,

$P = F_{max} + W \sin\theta$

$\therefore P = \mu N + W \sin\theta$ …(ii)

Putting values of μ, θ, N & W, we get;

$P = (0.25 \times 84.87) + (98 \times \sin 30)$

$\therefore P = 70.22$ N **(Answer)**

(B) Block motion (move) downward on inclined plane surface : [fig. 3.11(b)]

(i) Resolving all the forces perpendicular (normal) to given an inclined plane surface, we get,

$N = W \cos\theta$

$\therefore N = 98 \times \cos 30^o$

$\therefore N = 84.87$ N …(i)

(ii) Resolving all the forces parallel (along) to given an inclined plane surface, we get,

$P = F_{max} - W \sin\theta$

$\therefore P = \mu N - W \sin\theta$ …(ii)

Putting values of μ, θ, N & W, we get;

$P = (0.25 \times 84.87) - (98 \times \sin 30)$

$\therefore P = -27.78$ N (Push) **(Answer)**

Try this:

Find the external force to be apply as pull and push for upward and downward motion of the body, directly from Equation (iv) and (v) respectively as obtained in topic 3.3.1.

Example 5. *A block weighing of 500 N just start its motion in downward direction, on rough inclined plane surface, when a pull force of 200 N is apply parallel to the inclined plane surface. The same block is at the point of moving upward, when a pull force of 300 N is apply parallel to the inclined plane surface. Find the inclination of the plane and the coefficient of friction between block and inclined plane surface.*

Solution:

Free body diagrams of block to move downward & upward as shown in fig. 3.12 (a) & (b) respectively. We may note that frictional force F_{max} opposing the motion & acting on opposite direction of motion of the body. In both cases F_{max}. = μ N [as body is about (just) to move]

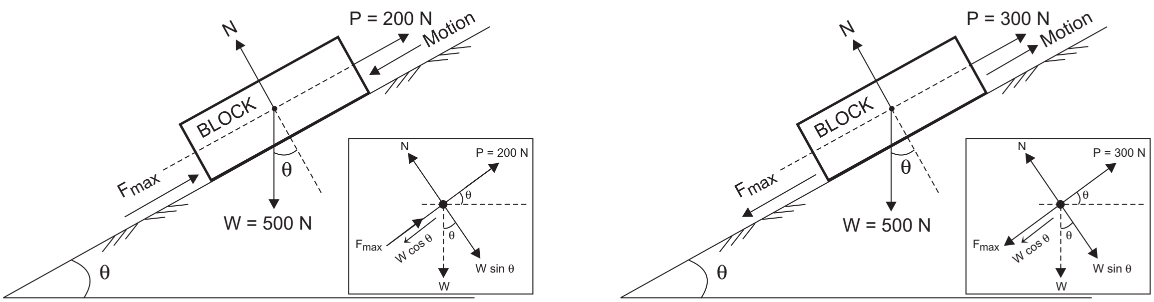
Given data: W = 500 N, External force (i) 200 N & (ii) 300 N.

(A) Block motion (move) downward on inclined plane surface:

(i) Resolving all the forces perpendicular (normal) to given an inclined plane surface, we get,

$N = W \cos \theta$

$\therefore\ N = 500 \times \cos \theta$ …(i)



(a) Block just move downwards

(b) Block just move upwards

Fig. 3.12

(ii) Resolving all the forces parallel (along) to the inclined plane surface, we get,

$P + F_{max} = W \sin \theta$, where $F_{max} = \mu N$ …(ii)

Substituting values of P, N & W in equation (ii), we get;

$200 + (\mu \times 500 \times \cos \theta) = (500 \times \sin \theta)$

$\therefore\ 200 = 500 \times \sin \theta - \mu \times 500 \times \cos \theta$ …(iii)

(B) Block motion (move) upward on inclined plane surface:

(i) Resolving all the forces perpendicular (normal) to given an inclined plane surface, we get,

$N = W \cos \theta$

$\therefore\ N = 500 \times \cos \theta$ same as case (A)

(ii) Resolving all the forces parallel (along) to given an inclined plane surface, we get,

$P = W \sin \theta + F_{max}$, where $F_{max} = \mu N$

$\therefore\ 300 = (500 \times \sin \theta) + (\mu \times 500 \times \cos \theta)$ …(iv)

(C) Now, Adding equation (iii) & (iv), we get;

$500 = 1000 \times \sin \theta$

$\therefore\ \sin \theta = 0.5$

$\therefore\quad \theta = 30^{o}$ **(Answer)**

(D) Substituting value of θ in equation (iv), we get;

$300 = (500 \times \sin\theta) + (\mu \times 500 \times \cos\theta)$

$300 = (500 \times 0.5) + (\mu \times 500 \times 0.866)$

$\therefore 300 = 250 + (\mu \times 433.01)$

$\therefore \quad \mu = 0.1155$ **(Answer)**

UNIT SUMMARY

- **Friction** is the force which ensure that you don't slip.
- **Friction** is a force that resists sliding, it is described in terms of a coefficient and is almost always assumed to be constant and specific to each material.
- **Coefficient of friction** (λ) is define as ratio of the maximum frictional force F max. , which resist the motion of two surfaces in contact, to the normal reaction force N, which pressing the two surfaces together.
- **Angle of repose** (ε) is the maximum inclination of the plane at which the body can repose, without any external force.
- **Angle of friction**(μ) **:** R is resultant force of two forces, frictional force F and normal reaction force N, which acts at angle θ to normal reaction, then angle θ is called the angle of friction for F_{max}.
- **Limiting friction :** Limiting friction is just at the point of motion
- **Types of friction :**

  Static friction can act between two objects, when objects are stationary. When body just tends to move on surface of another body is call static friction.

  Kinetic friction / Dynamic friction : If two surfaces are in contact and moving relative to one another, then the friction between them call kinetic friction.

  (i) Sliding friction : It is the frictional force, which comes into play, when one body sides over the other under action of external force.

  (ii) Rolling friction : It is the frictional force, which comes into play, when one body rolls over the other under the action of external force.
- **Laws of friction :**
  1. Force of friction is proportional to the normal reaction between the two surfaces of contact, acting parallel to the surface in contact and always act opposite to the relative motion of the two surfaces.
  2. Force of friction depends upon the material of contact surfaces and the roughness of contact surfaces.
  3. Force of friction is independent of the area of contact surfaces.
  4. Force of friction is independent of the relative velocity of contact surfaces.
  5. Ratio of friction force and normal reaction known as the coefficient of friction and its value for the given two surfaces will always constant.
  6. Coefficient of static friction is greater than coefficient kinetic friction.
- **Equilibrium of a body on a horizontal plane with horizontal external force:**

  (i) $\Sigma H = 0 \quad \therefore F_{max} = P$, where F_{max} = Frictional force & P = External force applied.

  (ii) $\Sigma V = 0 \quad \therefore N = W$, where N = normal reaction & W = Self-weight of the body.

(iii) $F_{max} = \mu N$, where μ = Coefficient of friction & N = Normal force of reaction.

- **Equilibrium of a body on a horizontal plane with inclined external force:**
  - (i) $\Sigma H = 0 \quad \therefore F_{max} = P \cos \theta$, where F = Frictional force & P = External force applied.
  - (ii) $\Sigma V = 0 \quad \therefore W = N + P \sin \theta$, where N = normal reaction & W = Self-weight of the body.
  - (iii) $F_{max} = \mu N$, where μ = Coefficient of friction & N = Normal force of reaction.
- **Equilibrium of a body on an inclined plane with parallel external force to plane:**
  - (a) External force is applied as pull for upward motion of the body on inclined plane surface:

$$P = \frac{W \sin(\alpha + \theta)}{\cos \alpha}$$

  - (b) External force is applied as push for downward motion of the body on inclined plane surface:

$$P = \frac{W \sin(\alpha - \theta)}{\cos \alpha}$$

EXERCISE

(A) Objective Questions

3.1 Coefficient of friction depends on

(a) Area of contact only (b) Nature of surfaces only (c) Both (a) & (b) (d) None of the above

3.2 The coefficient of friction is:

(a) The ratio of the friction and the normal reaction
(b) The force of friction when the body is in motion
(c) The angle between the normal reaction and the resultant of normal reaction and limiting friction
(d) The force of friction at which the body is just about to move

3.3 The angle which an inclined surface makes with the horizontal when a body placed on it is on the point of moving down, is called

(a) Angle of repose (b) Angle of friction (c) Angle of inclination (d) None of these

3.4 Which one of the following statements is true?

(a) The tangent of the angle of friction is equal to coefficient of friction
(b) The angle of repose is equal to angle of friction
(c) The tangent of the angle of repose is equal to coefficient of friction
(d) All the above

3.5 The maximum frictional force which comes into play, when a body just begins to slide over the surface of another body, is known

(a) Sliding friction (b) Rolling friction (c) Limiting friction (d) None of these

3.6 The friction experienced by a body, when at rest, is known as

(a) static friction (b) dynamic friction (c) limiting friction (d) coefficient of friction

3.7 The friction experienced by a body, when in motion, is known as

(a) rolling friction (b) dynamic friction (c) limiting friction (d) static friction

3.8 Which of the following statement is correct?
(a) The force of friction does not depend upon the area of contact.
(b) The magnitude of limiting friction bears a constant ratio to the normal reaction between the two surfaces.
(c) The static friction is slightly less than the limiting friction.
(d) Both (a) & (c) (e) All (a), (b) & (c).

3.9 The magnitude of the force of friction between two bodies one lying above the other, depends upon the roughness of the
(a) upper body (b) lower body
(c) both the bodies (d) the body having more roughness

3.10 The force of friction always acts in a direction opposite to that
(a) in which the body tends to move (b) in which the body is moving
(c) both (a) & (b) (d) none of (a) & (b)

[*Answer :* (1-b), (2-c), (3-a), (4-d), (5-c), (6-a), (7-b), (8-e), (9-c), (10-c)]

(B) Subjective Questions

3.1 Define the following terms:
(a) Friction (c) Angle of friction (e) Limiting friction (g) Dynamic friction
(b) Coefficient of friction (d) Angle of repose (f) Static friction

3.2 List the few examples in which friction is helpful to us.

3.3 List the few examples in which friction is not helpful to us.

3.4 Explain the difference between coefficient of friction & angle of friction.

3.5 What do you understand by angle of repose?

3.6 Derived that the angle of repose is equal to the angle of friction.

3.7 If the area of contact surfaces is increase, what will be its effect on friction?

3.8 State factors on which coefficient of friction depends.

3.9 What happens to the body when resultant force at contacting surfaces lies inside the angle of friction?

3.10 State the law of friction.

3.11 A body of weight 60 N is place on rough horizontal plane surface. Application of push force of 18 N acting at 20° to horizontal requires just move the body. Find the coefficient of friction. [*Ans. :* 0.255]

3.12 A pull force of 60N acting at 25° to horizontal plane, is requires just to move the body placed on rough horizontal plane surface. For the same body a push force of 75N acting at 25° to horizontal plane is requires just move the body. Find the weight of the body & coefficient of friction between the body and the rough horizontal plane surface. [*Ans. :* 253.83N, 0.238]

3.13 For a body of weight placed on rough horizontal plane surface, show that the least pull force requires to be apply at an angle θ with the horizontal is $W\sin\theta$.

3.14 Find the horizontal force require to move a body of weight 100N along a horizontal plane rough surface. If the plane, when gradually raised up to 15° the body will begin to slide. [*Ans. :* 26.79N]

3.15 A block of weight 50N is move along a rough horizontal plane surface by a pull of 18N acting at an angle of 14° with horizontal. Find the coefficient of friction. [*Ans. :* 0.383]

3.16 A force of 250N pull up a body of weight 500N on an inclined plane surface at angle of 15° with horizontal, force of pull apply parallel to the inclined plane. Find coefficient of friction. [*Ans. :* 0.25]

4 Centroid and Center of Gravity

UNIT SPECIFICS

In this unit, we are discusses the following topics:

- Definition of centre of gravity (CG) and centroid
- Comparison between centroid and centre of gravity (CG)
- Various technical terms related to CG
- Centroids of standard shapes for 1D & 2D elements
- Centroids of composite figure (lamina)
- CG of simple standard solids [3D elements]
- CG of composite solids

As unit is important for future unit (moment of Inertia) in other courses; easy tabular method, to solve example was discussed. Some other way to solve the examples were given under the heading "Try This". Some activities, taken under "Activity" section; are taken in such a manner that students understand the theory in better way. Multiple-choice questions (MCQ) in "Objective Question" category as well as questions of short and long answer types as per Bloom's taxonomy with number of numerical problems are covered under "EXERCICES" section for further work out on the unit.

A list of references and suggested reading given in the unit, so that one can go through for more information. It is important to note that for getting more information on various topics some QR code have been provide, which can be scan for relevant supportive knowledge.

RATIONALE

Have you ever thought that why a ship is floating on the water, why a bus or a car running on the road and does not topple? Buses do not tip over even if the bottom deck is empty and the top deck is full of people? Here the center of gravity plays an important role. The position of the center of gravity of an object affects its stability. In this unit, you will able to appreciate the importance of center of gravity and centroid. The determination of centroid and center of gravity will be of great importance in engineering designing like structural design, machine design and strength of material.

PRE-REQUISITES

Basic ideas of area of standard shapes and volume of standard solids.

UNIT OUTCOMES

After completing this unit, you will be able to:

1. Distinguish between centroid and center of gravity.
2. Identify the point of centroid and center of gravity of the symmetrical objects.
3. Calculate centroid and center of gravity of a given object.

MAPPING UNIT OUTCOMES WITH COURSE OUTCOMES

Unit-4 Outcome	Expected Mapping with Programme Outcomes 1- Weak Correlation; 2- Medium correlation; 3- Strong Correlation				
	CO-1	CO-2	CO-3	CO-4	CO-5
U4-O1	-	-	-	3	-
U4-O2	-	-	-	3	-
U4-O3	-	--	-	3	-

4.1 INTRODUCTION

The shape of the body influences the way the body behaves. To study this behavior information is required about centroid, centre of gravity as well as moment of area & mass. In this unit we are studying, how to find out the centroid and centre of gravity for a given body which may be combination of different standard shapes.

4.1.1 Center of Gravity (CG)

It has been established, since long, that every particle of a body is attract by the earth towards its centre. The force of attraction, which is proportional to the mass of the particles of the body, acts vertically downwards and known a weight of the body. As the distance between the different particles of a body & the centre of the earth is taken to be same (because of very small size of the body as compared to the earth), these forces may be taken to act along parallel lines as shown in fig. 4.1. The resultant R of all such parallel forces acts on one point G. This point through which the whole weight of the body acts, is known as center of gravity (CG), irrespective of the position of the body. It may be note that everybody has only one & one CG. If you balance the body on this point of CG, it will balance.

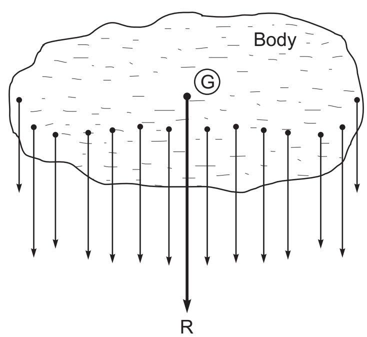


Fig. 4.1: Center of gravity (CG)

4.1.2 Centroid

The plane figures (like triangle, circle etc.) have only areas, but no mass. The center of area of two-dimension figures known as centroid. Centroid is the point in a plane section such that for any axis through that point moment of area is zero.

4.1.3 Comparison between Center of Gravity and Centroid

Parameter of Comparison	Center of Gravity	Centroid
Perception	Center of gravity is the point where total mass of the object acts.	Centroid is the geometric center of the object where whole area is assume to be concentrate.
Object density	Centre of Gravity is applicable to objects with any density.	Centroid is the central point of objects with uniform density.
Dealing with structure	Generally, deals with 3D structure.	Generally, deals with 2D structure.
Subject association	Centre of Gravity a term often found in Physics.	Centroid a term often used in Mathematics, in relation to any figure.
Examples	Cube, Cone, Cylinder, Sphere, Hemi sphere…. etc.	Square, Rectangle, Triangle, Circle, Semi-circle, Quarter circle… etc.

4.1.4 Axis of Reference

The C.G. or centroid of a body is always calculate with reference to the assumed axis. This assumed axis known as axis of reference. The axis of reference, of plane figure generally taken as the left most line (OY) of the figure for calculating $\bar{x}$ and the lowest line (OX) of the figure for calculating $\bar{y}$, as shown in fig.4.2.

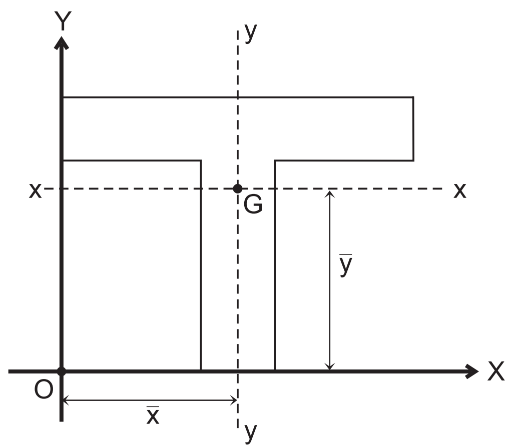


Fig. 4.2: Axis of reference

Basic for Center of Gravity

4.1.5 Axis of Symmetry

The axis (x-x axis or y-y axis) which divide the figure into two identical parts is call axis of symmetry.

If figure is symmetrical about y-y axis, $\bar{x}$ is directly available & $\bar{y}$ needs to be calculated.

If figure is symmetrical about x-x axis, $\bar{y}$ is directly available & $\bar{x}$ needs to be calculated.

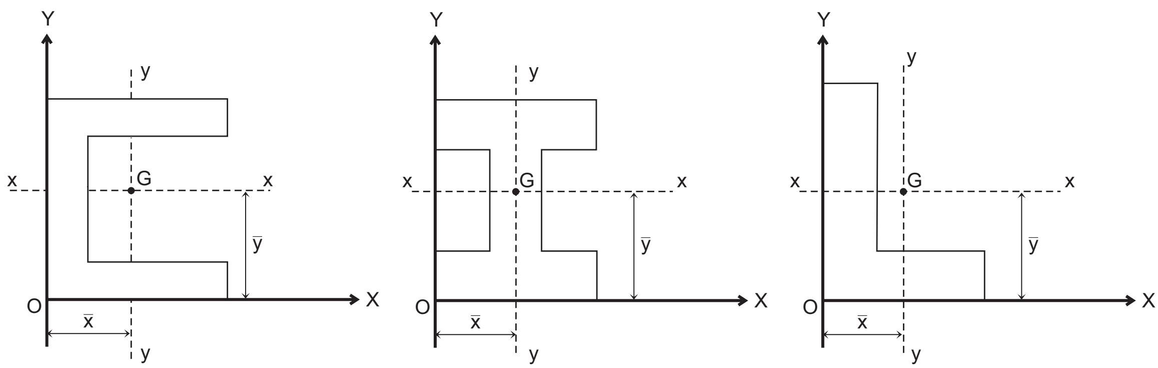
Take some examples to observe different type of symmetry.

(a) T-Section [fig. 4.2] Section is symmetrical about y-y axis. So $\bar{x}$ is directly available and $\bar{y}$ is to be calculated.

(b) C- Section (Channel) [fig. 4.3(a)] Section is symmetrical about x-x axis. So $\bar{y}$ is directly available and $\bar{x}$ is to be calculated.

(c) I-Section [fig. 4.3(b)] Section is symmetrical about both x-x and y-y axis. So $\bar{x}$ and $\bar{y}$ both are directly available.

(d) L-Section (Angle Section) [fig. 4.3(c)] Section is not symmetrical about any axis. So $\bar{x}$ and $\bar{y}$ both are required to be calculated.

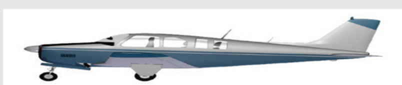


(a) C – Section (b) I – Section (c) L – Section

Fig. 4.3: Axis of symmetry

Activity-1:

The position of the centre of gravity depends on the shape and composition of an object. In many engineer design ballast weight can also be add to shift the centre of gravity to a more desirable position. Look at the image of an aero plane. Discuss where you migh tideally want the centre of gravity.

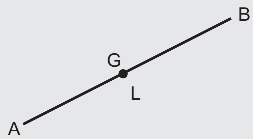
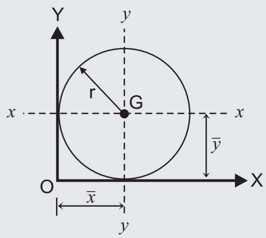
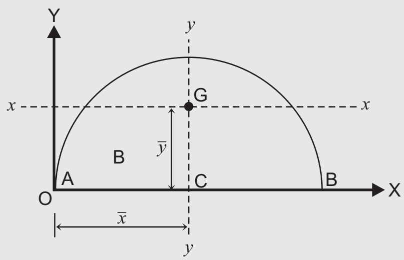
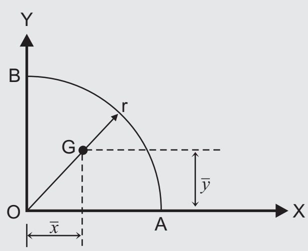


4.2 CENTROIDS OF STANDARD SHAPES

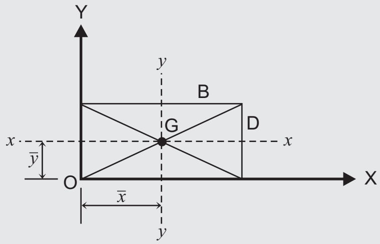
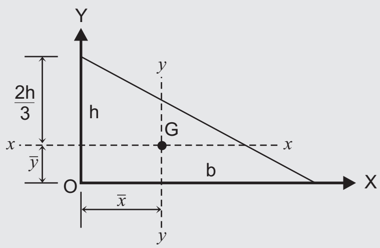
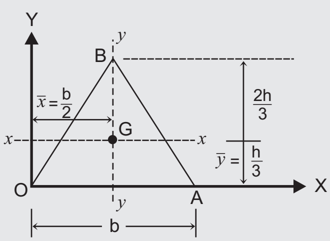
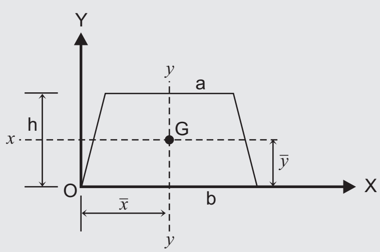
For standard 1D (wire) and 2D (plane figure) elements the centroid are shown in Table 4.1.

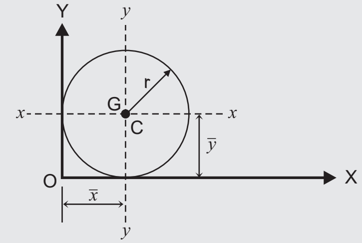
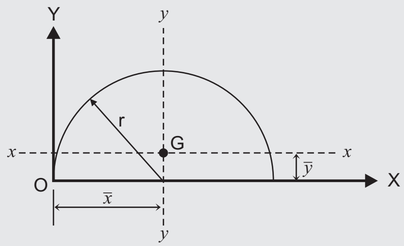
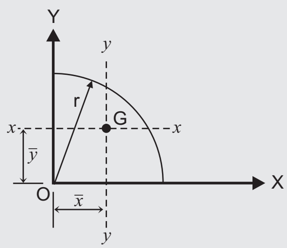
Table 4.1: Centroid of standard shapes [1D & 2D elements]

(A) One Dimensional shape (Wires)

Sr. No.	Geometrical Shape	Length	$\bar{x}$	$\bar{y}$
1.	 Straight wire AB	L	Centre of Length $\left(\frac{L}{2}\right)$	
2.	 Wire ring	$2\pi r$	Centre of Circle (r) $\bar{x} = r$	$\bar{y} = r$
3.	 Semicircular wire AB	πr	r	$\frac{2r}{\pi}$
4.	 Quarter-circular wire AB	$\frac{\pi r}{2}$	$\frac{2r}{\pi}$	$\frac{2r}{\pi}$

(B) Two-Dimensional shape (plane figures)

Sr. No.	Geometrical Shape	Area	$\bar{x}$	$\bar{y}$
1.	 Rectangular or square	$A = B \cdot D$	$\frac{B}{2}$	$\frac{D}{2}$
2.	 Right angle triangle	$A = \frac{1}{2}\ b \cdot h$	$\frac{1}{3}\ b$	$\frac{1}{3}\ h$
3.	 Symmetrical triangle	$A = \frac{1}{2} \cdot b \cdot h$	$\frac{b}{2}$	$\frac{h}{3}$
4.	 Trapezium	$A = (a+b)\frac{h}{2}$	$\frac{b}{2}$	$\frac{h}{3}\left(\frac{b+2a}{b+a}\right)$

5.	 Circle	$A = \pi r^2$ or $A = \frac{\pi}{4} d^2$	r	r
6.	 Semicircle	$A = \frac{\pi r^2}{2}$	r	$\frac{4r}{3\pi}$
7.	 Quarter circle	$A = \frac{\pi r^2}{2}$	$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$

4.3 CENTRIOD OF COMPOSITE FIGURES

When more than one standard plane figures combine to gather, it forms a composite plane figures. To find the centroid (CG) of composite figure, we have to break up in to the standard plane figures and follow the steps as explain in next sub-topic 4.3.1. We have to study only the composite figure, which are composed of not more than three geometrical figures in this book.

4.3.1 Steps for finding centriod of Composite figures

To find CG of composite figures, we have to follow following steps.

Step-1: Divide the given composite (compound) shape into various standard figures. These standard figures include square, rectangles, circles, semicircles, triangles and many more. In dividing the composite figure, include parts with holes (cut out) are to treat as components with negative values. There is also possibility of rotation (90°, 180°, 270° & 360°) of standard figure to adjust in composite section. Make sure that you break down every part of the compound shape in to various components with designate name (Component-1, Component-2 & so on) before proceeding to the next step.

Step-2: Calculate the area of each component as per standard shape from table 4.1 (B). Make the area negative for designated areas that act as holes (cut out).

Step-3: The given figure should have an X-axis and Y-axis as reference line. Draw the X-axis as the horizontal line passing through bottom most point of the given composite figure, while the Y-axis as the vertical line passing through left most point of the given composite figure.

Step-4: Get the distance of the centroid of each component, as divided into standard figure in step-1, from the X-axis and Y-axis as reference lines.

Step-5: Make a calculation in tableas shown below.

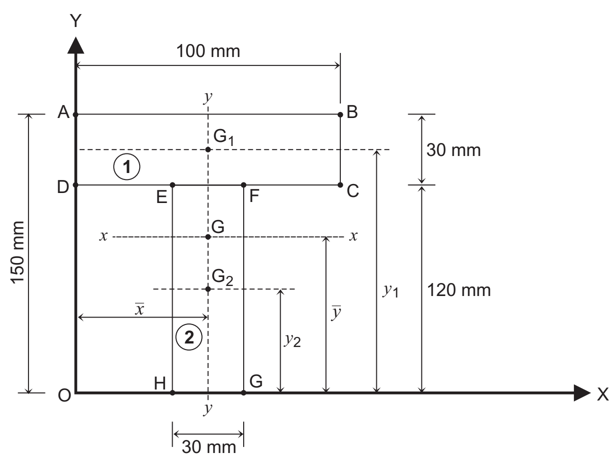
Sr. No.	Component Name	Area of Component A in mm²	Distance of CG of component from Reference lines		A·x	A·y
			x	y		
1	Component 1	A_1	x_1	y_1	A_1x_1	A_1y_1
2	Component 2	A_2	x_2	y_2	A_2x_2	A_2y_2
n	Component n	A_n	x_n	y_n	A_nx_n	A_ny_n
	Summation	$\Sigma A =$	---	---	$\Sigma A{\cdot}x =$	$\Sigma A{\cdot}y =$

Step-6: Use the equations to find the coordinates ($\bar{x}, \bar{y}$) of centroid (CG) from reference lines.

(a) $\bar{x} = \dfrac{\Sigma A \cdot x}{\Sigma A}$ and (b) $\bar{y} = \dfrac{\Sigma A \cdot y}{\Sigma A}$

Let to explain above point, take some examples of composite figure (section), as per syllabus composite figure must be composed of not more than three geometrical figures.

Example 1. *Find the centroid (CG) of a 100 mm × 150 mm × 30 mm T-section as shown in the figure.*



Centroid of plane figure

Fig. 4.4

Solution:

Step-1: Divide the given composite (compound) shape into various standard figures. In this case, the T-shape is combination of two rectangles. Name the two components as component-1 for top rectangle (Flange) ABCD & component-2 for vertical rectangle (Web) EFGH as shown in Table below.

Step-2: Calculate the area of each component as per standard shape (Rectangles).

Step-3: The given figure should have an X-axis (OX line) and Y-axis (OY line) as reference line.

Step-4: Get the distance of the centroid (*x* & *y*) of each component as per standard figure from reference lines (X-axis & Y-axis).

Step-5: Put all the value obtained from step-2 to step-4 in the table as follow.

Sr. No.	Component Name	Area of Component A in mm^2	Distance of CG of component from Reference lines		A·x	A·y
			x	y		
1	Top Rectangle- 1 ABCD (100 x 30 mm)	100 x 30 = 3000	$\frac{100}{2} = 50$	$120 + \frac{30}{2} = 135$	150000	405000
2	Vertical Rectangle-2 EFGH (30 x 120 mm)	30 x 120 = 3600	50 from symmetry	$\frac{120}{2} = 60$	180000	216000
	Summation	ΣA = 6600	---	---	ΣA·x = 330000	ΣA·y = 6210000

Step-6: Use the equations, to calculate Centroid (CG) of composite plane figure by putting the value from table.

(a) $\bar{x} = \frac{\Sigma A \cdot x}{\Sigma A} = \frac{330000}{6600} = 50.00$ mm **(Answer)**

[As per symmetry of composite figure about yy axis (vertical), we can directly,

Find $\bar{x} = \frac{\text{Total width}}{2} = \frac{100}{2} = 50.00$ mm; As we obtained by calculations.]

(b) $\bar{y} = \frac{\Sigma A \cdot y}{\Sigma A} = \frac{621000}{6600} = 94.09$ **mm (Answer)**

Example 2. *Find the centroid of given C-section as shown in figure.*

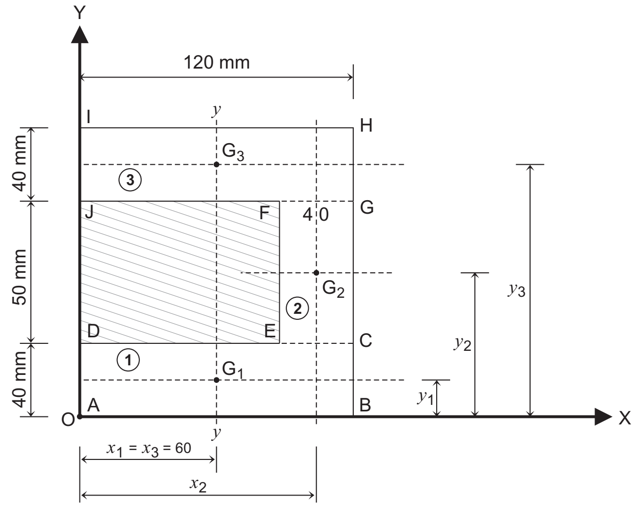


Fig. 4.5

Solution:

Step-1: Divide the given composite (compound) shape into various standard figures. In this case, the C-shape is combination of three rectangles. Name the three components as component-1, component-2 and component-3 as shown in Table below.

Step-2: Calculate the area of each components as per standard shape (all rectangles).

Step-3: The given figure should have an X-axis (OX line) and Y-axis (OY line) as reference line.

Step-4: Get the distance of the centroid (x & y) of each components as per standard figure from reference lines (X-axis & Y-axis).

Step-5: Put all the value obtained in step-2 to step-4 in the table.

Sr. No.	Component Name	Area of Component A in mm²	Distance of CG of component from Reference lines		A·x	A·y
			x	y		
1	Bottom Rect-1 [ABCD]	120 x 40 = 4800	$\frac{120}{2} = 60$	$\frac{40}{2} = 20$	288000	96000
2	Vertical Rect.-2 [CEFG]	40 x 50 = 2000	120 – 20 = 100	$40 + \frac{50}{2} = 65$	200000	130000
3	Top Rect.-3 [GHIJ]	120 x 40 = 4800	$\frac{120}{2} = 60$	$40 + 50 + \frac{40}{2} = 110$	288000	528000
	Summation	ΣA = 11600	---	---	ΣA·x = 776000	ΣA·y = 754000

Step-6: Use the equations, to calculate Centroid (CG) of composite plane figure, put the value from table.

(a) $\bar{x} = \dfrac{\Sigma A \cdot x}{\Sigma A} = \dfrac{776000}{11600} = 66.90$ mm **(Answer)**

(b) $\bar{y} = \dfrac{\Sigma A \cdot y}{\Sigma A} = \dfrac{754000}{11600} = 65.00$ mm **(Answer)**

[As per symmetry of composite figure about *x-x* axis (horizontal), we can directly,

Find $\bar{y} = \dfrac{\text{Total Depth}}{2} = \dfrac{130}{2} = 65.00$ mm; As we obtained by calculations.]

> ***Try this :*** We can solve above example by considering two rectangles. One the bigger one of size (120 × 130) mm (+ ve) and another rectangle (hole) of size (80 × 50) mm (–ve). Can you imagine the answer?

Example 3. *Find the centroid of the given composite figure shown in figure.*

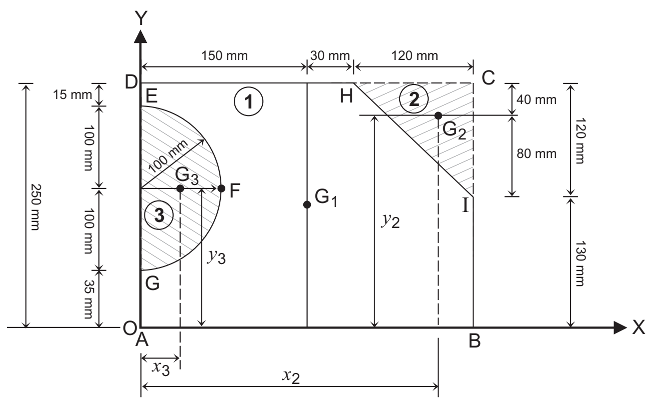


Fig. 4.6

Step-1: Divide the given composite (compound) shape into various standard figures. In this case, its combination of three figures. Name the three components as component-1, cut of component-2, and cut of component-3 as shown in Table below. Here both cutouts are oriented from standard shape shown in table 4.1(B), Hence the position of CG shifted accordingly. For semicircle as it oriented to 90°, $\bar{x}$ coordinator becomes $\bar{y}$ & vice versa. For right angle triangle as it oriented at 180°, base goes at top, CG distance may be measured accordingly.

Step-2: Calculate the area of each component as per standard shape with negative sign for cuts of component- 2 & 3.

Step-3: The given figure should have an X-axis (AB line) and Y-axis (AD line) as reference line.

Step-4: Get the distance of the centroid (*x* & *y*) of each component as per standard figure.

Step-5: Put all the value obtained in step-2 to step-4 in the table.

Sr. No.	Component Name	Area of Component A in mm^2	Distance of CG of component from Reference lines		A·x	A·y
			x	y		
1	Rectangle- 1 [ABCD] (300 x 250 mm)	300 x 250 = 75000	$\frac{300}{2}$ = 150	$\frac{250}{2}$ = 250	11250000	9375000
2	CUT of Triangle- 2 [CHI] (120 mm Base & Height both)	120 x $\frac{120}{2}$ = 7200	300 – 40 = 260	250 – 40 = 210	–1872000	–1512000
3	CUT of Semiciecle-3 [EFG] (Radius = 100 mm)	$\frac{\pi \times 100^2}{2}$ = –15707.96	$\frac{4\times 100}{(3\times\pi)}$ = 42.44	35 + 100 = 135	–666645.82	–2120574.60
	Summation	ΣA = 52092.04	---	---	ΣA·x = 8711354.18	ΣA·y = 5742425.40

Step-6: Use the equations, to calculate Centroid (CG) of composite plane figure, put the value from table.

(a) $\bar{x} = \frac{\Sigma A \cdot x}{\Sigma A} = \frac{8711354.18}{52092.04} = 167.23$ mm **(Answer)**

(b) $\bar{y} = \frac{\Sigma A \cdot y}{\Sigma A} = \frac{5742425.40}{52092.04} = 110.23$ mm **(Answer)**

Activity-2:

Provide each student or group one or more of the real-world objects. Have them find a way to use the gravitational balance method to determine the center(s) of gravity. One way of doing this:

(a) Punch two different holes in the object.

(b) Hang the object and the weighted string from one of the holes.

(c) On the object, draw a line where the string hangs.

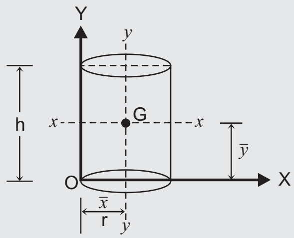
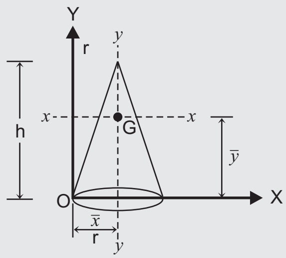
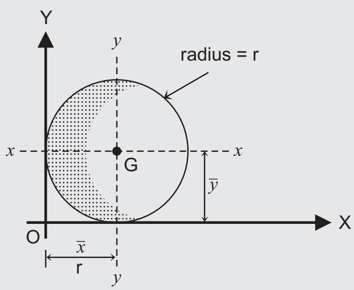
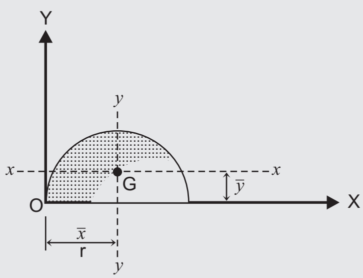
(d) Repeat steps (b) and (c) using the second hole.

Where the two lines intersect is the center of gravity

4.4 CENTER OF GRAVITY OF SIMPLE SOLIDS [3-D ELEMENTS]

For standard 3 D elements (Simple solids), the center of gravity (CG) are shown in Table 4.2.

Table 4.2: Center of Gravity (CG) of Three Dimensional Standard Solid

Sr. No.	Geometrical Shape	Volume	$\bar{x}$	$\bar{y}$
1.	 Cylinder	$V = \pi r^2 h$	r	$\frac{h}{2}$
2.	 Cone	$V = \frac{\pi}{3} r^2 h$	r	$\frac{h}{4}$
3.	 Sphere	$V = \frac{4}{3} \pi r^3$	r	r
4.	 Hemisphere	$V = \frac{2}{3} \pi r^3$	r	$\frac{3r}{8}$

4.5 CENTRE OF GRAVITY (CG) OF COMPOSITE SOLIDS

In this, we have to consider the volume of solids (V) instead of area (A) considered in plane figure. All other process remains same, but for conveyance the step list out as follows.

Step-1: Divide the given composite (compound) solids into various standard solids. These standard solids include Cone, Cylinder, Sphere, Hemi sphere. In dividing the composite solids, include parts with holes (cut out) are to treat as components with negative values. Make sure that you break down every part of the compound solids in to various components with designate name (Component-1, Component-2 & so on) before proceeding to the next step.

Step-2: Calculate the volume of each components as per standard solid from table 4.2. Make the volume negative for designated solid that act as holes (cut out).

Step-3: The given solid should have an X-axis and Y-axis as reference line. Draw the X-axis as the horizontal line passing through bottom most point of the given composite solid, while the Y-axis as the vertical line passing through left most point of the given composite solid.

Step-4: Get the distance of the centroid of each components as divided into standard solid in step-1 from the X-axis and Y-axis as reference lines.

Step-5: Make a calculation in tableas shown below.

Sr. No.	Component Name	Volume of Component V in mm^3	Distance of CG of component from Reference lines		V·x	V·y
			x	y		
1	Component 1	V_1	x_1	y_1	V_1x_1	V_1y_1
2	Component 2	V_2	x_2	y_2	V_2x_2	V_2y_2
n	Component n	V_n	x_n	y_n	V_nx_n	V_ny_n
	Summation	ΣV =	---	---	ΣV·x =	ΣV·y =

Step-6: Use the equations to find the coordinates ($\bar{x}, \bar{y}$) of centroid (CG) from reference lines.

(a) $\bar{x} = \dfrac{\Sigma V \cdot x}{\Sigma V}$ and (b) $\bar{y} = \dfrac{\Sigma A_x}{\Sigma A}$

Let to explain above point, take some examples of composite solid (section), as per syllabus composite solid must be composed of not more than two geometrical solids.

Example 4. *Find the centre of gravity (CG) of the composite solid having cylinder of diameter and height as same with 160 mm which supports a cone of base diameter and height as same with 160 mm. Show the position of CG in the figure.*

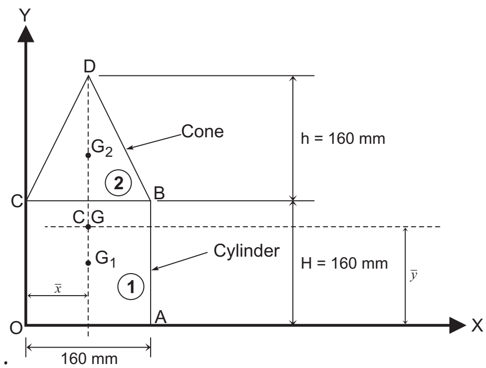


Fig. 4.7

Solution:

Step-1: Divide the given composite (compound) solids into two standard solids. Here bottom part cylinder OABC designate as component-1 and upper part cone BCD designate as component-2 as shown in fig.

Step-2: Calculate the volume of each components as per standard solid from table 4.2.

Step-3: The given solid should have an X-axis and Y-axis as reference line. Draw the X-axis as the horizontal line passing through bottom most point of the given composite solid, while the Y-axis as the vertical line passing through left most point of the given composite solid.

Step-4: Get the distance of the centroid of each components as divided into standard solid in step-1 from the X-axis and Y-axis as reference lines.

Step-5: Make a calculation in table as shown below.

Sr. No.	Component Name	Volume of Component V in mm³	Distance of CG of component from Reference lines		V·x	V·y
			x	y		
1	Cylinder OABC (D = H = 160 mm)	$\pi R^2 H$ $= \pi \times 80^2 \times 160$ $= 321699.88$	$\frac{D}{2} = \frac{160}{2} = 80$	$\frac{H}{2} = \frac{160}{2} = 80$	257359270.40	257359270.40
2	Cone BCD (d = h = 160 mm)	$\frac{\pi r^2 h}{3} = \frac{\pi \times 80^2 \times 160}{3} = 1072330.29$	$\frac{d}{2} = \frac{160}{2} = 80$	$H + \frac{h}{4} = 160 + \frac{160}{4} = 200$	85786423.20	214466058.00
	Summation	$\Sigma V = 4289321.17$	---	---	$\Sigma V \cdot x = 343145693.60$	$\Sigma V \cdot y = 471825328.40$

Step-6: Use the equations to find the coordinates ($\bar{x}, \bar{y}$) of centroid (CG) from reference lines.

(a) $\bar{x} = \dfrac{\Sigma V \cdot x}{\Sigma V} = \dfrac{343145693.60}{4289321.17} = 80.00$ mm **(Answer)**

(b) $\bar{y} = \dfrac{\Sigma V \cdot y}{\Sigma V} = \dfrac{471825328.40}{4289321.17} = 110.00$ mm **(Answer)**

Example 5. *A frustum of cone is having base diameter 100 mm and top diameter 50 mm with height as 100 mm. Find the CG of this frustum.*

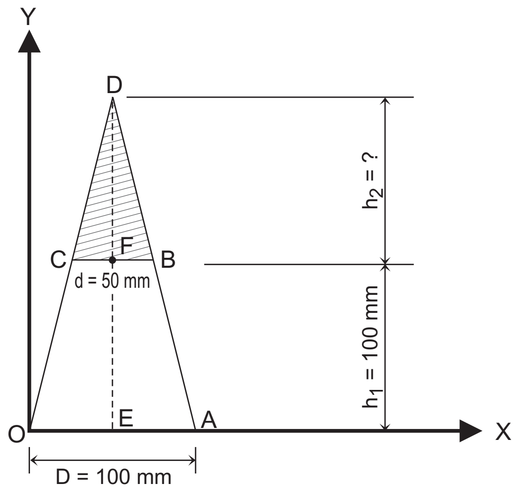


Fig. 4.8

Solution:

Here to get frustum, we have to substrate upper cone BCD from full cone OAD as shown in fig. For full cone OAD, compare triangles DOE & DCF, we get

$$\frac{DE}{OE} = \frac{DF}{CF}$$

$$\therefore \frac{DE}{50} = \frac{(DE - 100)}{25}$$

$\therefore$ 25 DE = 50 DE – 5000

$\therefore$ 25 DE = 5000

$\therefore$ DE = 200 mm

$\therefore$ h_2 = DF

= DE – EF

= 200 – 100

$\therefore$ h_2 = 100 mm

Step-1: To get the given solids, we have to substrate upper cone BCD from full cone OAD as shown in fig. Here full cone OAD designate as component- 1 and upper cone BCD designate as component- 2 as shown in fig.

Step-2: Calculate the volume of each components as per standard solid from table 4.2.

Step-3: The given solid should have an X-axis and Y-axis as reference line. Draw the X-axis as the horizontal line passing through bottom most point of the given composite solid, while the Y-axis as the vertical line passing through left most point of the given composite solid.

Step-4: Get the distance of the centroid of each components as divided into standard solid in step-1 from the X-axis and Y-axis as reference lines.

Step-5: Make a calculation in table as shown below.

Sr. No.	Component Name	Volume of Component V in mm^3	Distance of CG of component from Reference lines		V·x	V·y
			x	y		
1	Full cone OAD (D = 100 mm & H = 200 mm)	$\pi R^2 H$ $= \pi \times 50^2 \times 200$ $= 1570796.33$	$\frac{D}{2}$ $= \frac{100}{2}$ $= 50$	$\frac{H}{4}$ $= \frac{200}{4}$ $= 50$	78539816.50	78539816.50
2	Cut of Upper Cone BCD (d = 50 mm & h_2 = 100 mm)	$-\pi r^2 h$ $= \pi \times 25^2 \times 100$ $= -196349.54$	50 From Symmetry	$h_1 + \frac{h_2}{4}$ $= 160 + \frac{100}{4}$ $= 200$	–9817477.04	–24543692.50
	Summation	$\Sigma V = 1374446.79$	---	---	$\Sigma V.x = 68722339.46$	$\Sigma V \cdot y = 53996124.00$

Step-6: Use the equations to find the coordinates ($\bar{x}, \bar{y}$) of centroid (CG) from reference lines.

(a) $\bar{x} = \dfrac{\Sigma V \cdot x}{\Sigma V} = \dfrac{343145693.60}{4289321.17} = 50.00$ mm **(Answer)**

[As per symmetry of composite figure about YY axis (vertical), we can directly,

Find $\bar{x} = \dfrac{\text{Total width}}{2} = \dfrac{100}{2} = 50.00$ mm; As we obtained by calculations.]

(b) $\bar{y} = \dfrac{\Sigma V \cdot y}{\Sigma V} = \dfrac{53996124.00}{4289321.17} = 39.29$ mm **(Answer)**

<u>Activity-3:</u> Center of Mass Challenge

- Place a chair against the wall so that it cannot slide backward.
- Sit on the chair with your feet flat on the floor in front of you. (Your feet should not be angle or slanted to the side.)
- Have a partner gently place a thumb in the middle of your forehead.

Now try to stand up without forcing your partner's hand back. WHAT HAPPENED ?

UNIT SUMMARY

- **Centre of gravity** of a body is the point through which the resultant weight of the body passes through in whichever position the body is kept
- **Centroid** is the point in a plane section such that for any axis through that point moment of area is zero
- **Axis of Reference :** The C.G. or centroid of a body always calculated with reference to the assumed axis. This assumed axis known as axis of reference.
- **Axis of Symmetry :** The axis (*x-x* axis or y-y axis) which divide the figure into two identical parts call axis of symmetry.
- **The center of gravity of composite section with cut out holes** found out by considering the main section; first as a complete one and then deducting the area of the cut out holes that is taking the area of the cut out hole as negative.
- **The center of gravity of composite solid with cut out holes** found out by considering the main section; first as a complete one and then deducting the volume of the cut out hole that is taking the volume of the cut out hole as negative.

EXERCISE

(A) Objective Questions

4.1 Which statement is correct from following?

(a) The CG of a triangle lies at a point where any two medians meet each other.

(b) An irregular body can have more than one CG.

(c) The CG of a triangle lies at a point where the bisectors of all three angles meet.

(d) all of the above. (e) none of the above.

4.2 The CG of an equilateral triangle with each side 'a' is ……. from any of the three sides.

(a) $\frac{a\sqrt{3}}{2}$ (b) $\frac{a\sqrt{2}}{3}$ (c) $\frac{a}{2\sqrt{3}}$ (d) $\frac{a}{3\sqrt{2}}$

4.3 The CG of a trapezium with parallel sides a & b at distance h apart lies at distance from base

(a) $\frac{h}{3}\times\frac{b+2a}{b+a}$ (b) $\frac{h}{4}\times\frac{b+2a}{b+a}$ (c) $\frac{h}{2}\times\frac{b+2a}{b+a}$ (d) (h) $h\times\frac{b+a}{b+2a}$

4.4 The CG of a semicircle lies at a distance of ……...… from its base measured along the vertical radius r.

(a) $\frac{3r}{4\pi}$ (b) $\frac{4r}{3\pi}$ (c) $\frac{4\pi}{3r}$ (d) $\frac{3\pi}{4r}$

4.5 The CG of a hemisphere lies at a distance of ……...… from its base measured along the vertical radius r.

(a) $\frac{3r}{8}$ (b) $\frac{3}{8r}$ (c) $\frac{8r}{3}$ (d) $\frac{8}{3r}$

4.6 The CG of a right circular cone of diameter d & height h lies at a distance of ...... from the base measured along the vertical radius.

(a) $\frac{h}{2}$ (b) $\frac{h}{3}$ (c) $\frac{h}{4}$ (d) $\frac{h}{6}$

47 The CG of cylinder of diameter d & height h lies at a distance of ...... from the base measured along the vertical radius.

(a) $\frac{h}{2}$ (b) $\frac{h}{3}$ (c) $\frac{h}{4}$ (d) $\frac{h}{6}$

4.8 The CG of a quarter circle lies at a distance of .... from its base measured along the vertical radius r.

(a) $\frac{3r}{4\pi}$ (b) $\frac{4r}{3\pi}$ (c) $\frac{4\pi}{3r}$ (d) $\frac{3\pi}{4r}$

4.9 The CG of an right angle triangle with base 'b' & height 'h' lies at a distance of ...... from the base measured along the vertical line.

(a) $\frac{h}{2}$ (b) $\frac{h}{3}$ (c) $\frac{h}{4}$ (d) $\frac{h}{6}$

4.10 A circle hole of radius r is cut out from a circular disc of radius 2r is such a way that the diameter of the hole is the radius of the disc. The CG of the section lies at

(a) center of the disc (b) center of hole

(c) somewhere in the disc (d) somewhere in the hole

[*Answer* : (1-a), (2-c), (3-a), (4-b), (5-a), (6-c), (7-a), (8-b), (9-b), (10-c)]

(B) Subjective Questions

4.1 Differentiate between centroid and centre of gravity.

4.2 Define: (a) centroid (b) centre of gravity

(c) symmetry axis (d) axis of reference

4.3 Draw neat sketch of the following and show centroid.

(a) Quarter circle (b) Semi circle

(c) Triangle (d) Right circular cone

4.3 Calculate centre of gravity of T-Section having flange 200 x 20 mm and web 300 x 20 mm. Also show position of C.G. on figure. [*Ans.* : $\bar{x}$ = 100 mm, $\bar{y}$ = 214.0 mm]

4.5 Calculate centre of gravity of I-Section having top flange 200 x 20 mm and web 300 x 20 mm and bottom flange 400 x 40 mm. Also show the position of CG on figure. [*Ans.* : ($\bar{x}$,$\bar{y}$) = (200, 110 mm)]

4.6 Calculate centroid of angle section ISA 90 x 60 x 6 mm keeping longer leg vertical.

[*Ans.* : $\bar{x}$ = 14.25 mm, $\bar{y}$ = 29.25 mm]

4.7 Find centroid of dam section with top width 3 m, bottom width 6 m and height 9 m with one face vertical. [*Ans.* : ($\bar{x}$,$\bar{y}$) = (2.33 m, 4.0 m)]

4.8 Calculate centre of gravity on T-Section having flange 150 × 20 mm and web 200 × 20 mm.

[*Ans.* : ($\bar{x}$, $\bar{y}$) = (75 mm, 147.1 mm)]

4.9 A circle of 100 mm diameter is cutout from circle of 500 mm diameter. The distance between centre of circle is 150 mm on left side to centre of big circle. Find centroid of the given lamina.

[*Ans.* : ($\bar{x}$, $\bar{y}$) = (256.25 mm, 250 mm)]

5 Simple Lifting Machines

UNIT SPECIFICS

In this unit, we are discusses the following topics:

- Definitions and some technical terms related to simple lifting machine
- Law of machine
- Velocity ratio (VR) for some simple lifting machine

Unit is a tricky and practical in nature. Some popular day – to – day applications; like screw jack, pulley, etc. are really life saving applications. But their technical concept is very much needed to be understand by the user of the book along with figure and practical. Hence technical understanding of machine along with figures and practical were given in this unit.

Law of machine, their graphical understanding, other graphs for the machines are some of the important concepts discussed here. MA_{max}, VR, η_{max} etc are some important topics need to be discuss for clarity of the unit.

Multiple-choice questions (MCQ) in "Objective Question" category as well as questions of short and long answer types as per Bloom's taxonomy with number of numerical problems are covered under "EXERCICES" section for further work out on the unit.

RATIONALE

Have you ever used a rod to move some heavy load? Have you ever seen to lift water from well with help of pulley? Have you ever used a jack to lift your puncher car for wheel replacement? This long rod to displace heavy load or pulley to lift water or jack for car is nothing but a simple machine. These simple machines are very simple in nature, but very useful for normal day to day working. Such machines will decrease our effort. In this unit, we are going to discuss about simple lifting machine, which are very useful to human life. We will also discuss different types of terminologies and use of law of machine in this unit.

We are also going to discuss velocity ratio of various lifting machine to achieve higher mechanical advantage.

PRE-REQUISITE

Basic knowledge of Physics and Math's from Secondary Education [Standard 8 to standard 10]. Again, knowledge of previous units from this book of Engineering Mechanics.

UNIT OUTCOMES

After completing this unit, you will be able to

1. Associate simple lifting machine and its various terminology.
2. Use of law of machine and its application for maximum mechanical advantage and efficiency.
3. Interpret the velocity ratio for various simple lifting machine.

MAPPING UNIT OUTCOMES WITH COURSE OUTCOMES

Unit-5 Outcome	Expected Mapping with Programme Outcomes 1- Weak Correlation; 2- Medium correlation; 3- Strong Correlation				
	CO-1	CO-2	CO-3	CO-4	CO-5
U5-O1	-	-	-	-	3
U5-O2	-	-	-	-	3
U5-O3	-	--	-	-	3

5.1 DEFINITIONS

We (mankind) have invented various machines to overcome many difficulties or to reduce effort. Basically, all machines are designed with simple principle in our mind "reduce effort". A man alone is able to lift a very heavy weight with the help of the machine. For example, 3 to 4 persons were required to lift an automobile while Screw Jack can easily utilize to lift an automobile. Same way, to fetch a bucket of water directly from the well is inconvenient. This work is made convenient by the application of the simple pulley over the well.

To understand, all these simple machines, it is very essential for us to understand some important terminologies associated with it.

(i) Simple machine : Simple machine is a device in which effort is applied at one place and work is done at some other place. Simple machines are run manually, not by electric power.

e.g. pulley, bicycle, sewing machine & simple screw jack, etc.

(ii) Compound machine : If a machine, consists of many simple machines, it is called compound machine. Such machines are run by electric or mechanical power. Such machines work at higher speed. Using compound machines more work is done at less effort.

e.g. scooter, lathe, crane & grinding machine, etc.

(iii) Lifting machine : Lifting machine is a device in which heavy load can be lifted by less effort.

e.g. lift, crane, etc.

(iv) Simple lifting machine : Simple lifting machine is a device in which heavy load can be lifted by small effort manually.

e.g. simple pulley, simple crew jack, etc.

5.2 TECHNICAL TERMS RELATED TO SIMPLE LIFTING MACHINES

(i) Load (W) : The weight of lifted elements is called load (W).

(ii) Effort (P) : The force apply to lift the load (W) is called effort (P).

(iii) Mechanical advantage (MA) : The ratio of load (W) lifted and effort (P) required to lift the load is called Mechanical advantage. It is always express as pure number mathematically.

$$\text{MA} = \frac{\text{Load lifted}}{\text{Effort required}}$$

$$\therefore \ \text{MA} = \frac{W}{P} \quad \text{Where, W = Load in N OR kN \& P = Effort in N}$$

(iv) Velocity ratio (VR) : The ratio of distance moved by effort (y) and the distance moved by load (x) is called velocity ratio. It is unit less quantity and hence expressed as pure number. Mathematically –

$$\text{VR} = \frac{\text{Distance moved by effort}}{\text{Distance moved by load}}$$

$$\therefore \ \text{VR} = \frac{y}{x}$$

It is also noted here that Velocity ratio is constant for a particular machine. It will not change over period of time.

(v) Efficiency (η) : The ratio of work done by the machine (output) and work done on the machine (input) is called efficiency of the machine. The output and input mathematically express as

(a) Input = Effort × Distance moved by effort

$\therefore$ Input = $P \cdot y$ and

(b) Output = Load × Distance moved by load

$\therefore$ Output = $W \cdot x$

It is express as percentage. Mathematically, Efficiency = $\frac{\text{output}}{\text{input}} \times 100\%$

We know that output = $W \cdot x$ and input = $P \cdot y$

$$\therefore \ \eta = \frac{\text{output}}{\text{input}} \times 100$$

$$= \frac{W \cdot x}{P \cdot y} \times 100 = \frac{W/P}{y/x} \times 100$$

$$\therefore \ \eta = \frac{\text{MA}}{\text{VR}} \times 100\%$$

$$\therefore \ \eta = \frac{\text{output}}{\text{input}} \times 100\% = \frac{\text{MA}}{\text{VR}} \times 100\%$$

(vi) Law of machine : For a particular machine, if we record various values of effort (P) required to lift the corresponding loads (W) and plot a graph between effort and load, we shall get a straight line AB as shown in figure.

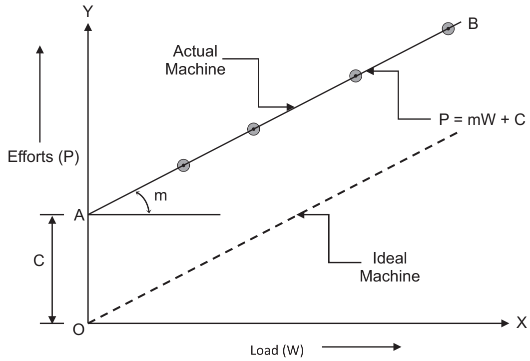


Fig. 5.1: Law of Machine

Mathematically, the law of machine is given by relation :

$P = mW + C$

Where, P = Effort applied, W = Load lifted, m = constant (coefficient of friction) = slope of line AB and C = constant = machine friction

Following observations are made from the graph :

(a) On a machine, if W = 0, effort C is required to run the machine. Hence, effort C is required to overcome machine friction.

(b) If line AB passes through origin, no effort is required to balance friction. Such a graph is for Ideal machine.

(c) If line AB crosses *x-x* axis, without effort (p), some load can be lifted, which is impossible. Hence, line AB never crosses *x-x* axis.

(vii) Maximum mechanical advantage (MA_{max}) : We know that $MA = \frac{W}{P}$.

To get maximum MA, put P from law of machine as, $P = mW + C$

$$\therefore MA_{max} = \frac{W}{mW + C}$$

$$= \frac{1}{m + \frac{C}{W}} \qquad \text{neglecting } \frac{C}{W}\text{, we get}$$

$$\therefore MA_{max} = \frac{1}{m}$$

(viii) Maximum efficiency (η_{max}) : We know that, Velocity Ratio (VR) is constant for a given machine and MA varies.

Now $\eta = \frac{MA}{VR}$ $\quad \because$ Substitute MA as $MA_{max} = \frac{1}{m}$ to get η_{max},

$$\therefore \eta_{max} = \frac{1/m}{VR}$$

$$\therefore \eta_{max} = \frac{1}{m \times VR}$$

(ix) Ideal machine : A machine having 100% efficiency is called an ideal machine. In an ideal machine friction is zero.

For ideal machine, Output = Input or MA = VR

(x) Effort lost in friction (P_f) : In a simple machine, effort required to overcome the friction between various parts of a machine is called effort lost in friction.

Let P = Effort, P_o = Effort for ideal machine, P_f = Effort lost in friction

$\therefore$ Effort lost in friction, $P_f = P - P_o$

For Ideal machine MA = VR

$$\therefore \frac{W}{P_o} = VR$$

$$\therefore P_o = \frac{W}{VR} = \text{Ideal effort}$$

Due to friction, Actual P > Ideal Effort P_o

$$\therefore P_f = P - P_o$$

$$\therefore P_f = P - \frac{W}{VR}$$

(xi) Friction load (W_f) : Total friction force produced, when machine is in motion, is called friction load.

Let W = Load (Actual), W_o = Load for Ideal machine and P = Effort

For ideal machine, MA = VR

$W_o = P \times VR$ = Ideal load

Now, friction load $W_f = W_o - W$

$\therefore W_f = (P \times VR) - W$

(xii) Reversible machine : If a machine is capable of doing some work in the reverse direction, after the effort is removed, is called reversible machine.

For reversible machine, $\eta \geq 50\%$

(xiii) Non-reversible machine or self-locking machine : If a machine is not capable of doing some work in the reverse direction, after the effort is removed, is called non-reversible machine or self-locking machine. Generally all lifting machines are self-locking machines.

For non-reversible machine, $\eta < 50\%$

(xiv) Condition for reversibility of machine :

Let W = Load lifted, P = Effort required, x = Distance moved by load and y = Distance moved by effort and $P \cdot y$ = input & $W \cdot x$ = output

We know that, Machine friction = Input – Output = $P \cdot y - W \cdot x$

For a machine to reverse,

Output $\geq$ Machine friction

$$\therefore W \cdot x \geq P \cdot y - W \cdot x$$

$$\therefore 2\,W \cdot x \geq P \cdot y$$

$$\therefore \frac{W \cdot x}{P \cdot y} \geq \frac{1}{2}$$

Basic Definition

$$\therefore \frac{\text{output}}{\text{input}} \geq 0.5$$

$\therefore \eta \geq 50\%$

For a machine to reverse, $\eta \geq 50\%$

Example 1. *In a lifting machine, an effort of 30 N just lift a load of 720 N. What is the mechanical advantage, if efficiency of machine is 30% at the load ? Calculate velocity ratio of machine.*

Solution :

W = 720 N, P = 30 N and η = 30% = 0.3

(a) $MA = \frac{W}{P} = \frac{720}{30}$

$\therefore$ MA = 24 **(Answer)**

(b) $\eta = \frac{MA}{VR}$

$\therefore 0.30 = \frac{24}{VR}$

$\therefore$ VR = 80 **(Answer)**

Example 2. *The velocity ratio of a machine is 20 and efficiency is 80%. Find how much load will be lifted by an apply effort of 200 N.*

Solution :

VR = 10, η = 80% = 0.80, P = 200 N

(a) $\eta = \frac{MA}{VR}$

$0.80 = \frac{MA}{20}$

$\therefore$ MA = 16

(b) $MA = \frac{W}{P}$

$16 = \frac{W}{200}$

$\therefore$ W = 3200 N **(Answer)**

Example 3. *A single purchase crab which has the following details : It is observed that an effort of 60 N lifts a load of 1800 N and an effort of 120 N lifts a load of 3960 N. The Velocity Ratio VR of machine is 42. (a) Establish the law of machine. (b) Find the efficiency in any one case of above.*

Solution :

(i) When P_1 = 60 N, W_1 = 1800 N (ii) When P_2 = 120 N, W_2 = 3960 N (iii) VR = 42

(A) Law of machine

(a) Put the value of P and W of two observations in Law of machine.

$P = mW + C$

$\therefore \quad 60 = m \times 1800 + C$ ... (i)

$\underline{-120 = -m \times 3960 \pm C}$ ...(ii)

$\therefore \quad -60 = -2160\ m$ ...(i) – (ii)

$\therefore \ m = \dfrac{60}{2160}$

$\therefore \ m = 0.0277$

(b) Substitute the value of m in equation (i),

$\therefore \ 60 = 0.0277 \times 1800 + C$

$\therefore \ 60 = 49.86 + C$

$\therefore \ C = 10.14$

(c) Law of machine for the given machine is

$P = 0.0277\ W + 10.14$ **(Answer)**

(B) Efficiency for case-1

(a) $MA = \dfrac{W}{P} = \dfrac{1800}{60} = 30$

(b) $VR = 42$

(c) $\eta = \dfrac{MA}{VR} = \dfrac{30}{42} = 0.7142$

$\therefore \ \eta = 71.42\%$ **(Answer)**

Example 4. *Fill in the blanks given below for a simple lifting machine having velocity ratio VR = 30. Find maximum efficiency the machine can reach stating whether the machine is reversible or not.*

Sr. No.	Load (W) in kN	Effort (P) in kN	Efficiency in %
1	100	9.82	______
2	600	49.82	______
3	1000	______	______

Solution :

(A) For first observation :

W = 100 kN, P = 9.82 kN and VR = 30

(i) $MA = \dfrac{W}{P} = \dfrac{100}{9.82} = 10.18$

(ii) $\eta = \dfrac{MA}{VR} \times 100$

$= \dfrac{10.18}{30} \times 100$

$\therefore \ \eta = 33.93\%$ **(Answer)**

(B) For second observation :

W = 600 kN, P = 49.82 kN and VR = 30

(i) $MA = \frac{W}{P} = \frac{600}{49.82} = 12.04$

(ii) $\eta = \frac{MA}{VR} \times 100$

$= \frac{12.04}{30} \times 100$

$\therefore \eta = 40.13\%$ **(Answer)**

(C) For third observation :

(I) We know the law of machine as P = mW + C

Put the values of two observations

(i) P = 9.82 kN and W = 100 kN

(ii) P = 49.82 kN and W = 600 kN

We get,

$$\therefore \quad 9.82 = m \times 100 + C \qquad \text{...(i)}$$

$$49.82 = m \times 600 + C \qquad \text{...(ii)}$$

$$-40 = -500\,m \qquad \text{...(i) – (ii)}$$

$$\therefore \quad m = 0.08$$

(II) Substitute m = 0.08 in equation (i)

$9.82 = 0.08 \times 100 + C$

$\therefore 9.82 = 8 + C$

$\therefore C = 1.82$

(III) Law of machine is

$P = 0.08\,W + 1.82$

(IV) Now, when W = 1000 kN,

$P = 0.08 \times 1000 + 1.82$

$\therefore P = 81.82$ kN **(Answer)**

(V) (i) $MA = \frac{W}{P} = \frac{1000}{81.82} = 12.22$

(ii) $\eta = \frac{MA}{VR} = \frac{12.22}{30} \times 100 = 40.73\%$ **(Answer)**

(D) $\eta_{max} = \frac{1}{m \times VR} \times 100$

$= \frac{100}{0.08 \times 30}$

$= 41.67\%$

$\therefore \eta_{max} = 41.67\%$ **(Answer)**

(E) In all the above observations, η is less than 50%. Hence, the machine is non-reversible (self-locking machine). **(Answer)**

Example 5. *In a lifting machine an effort of 30 N can lift a load of 350 N and an effort of 40 N can lift a load of 500 N. If velocity of machine is 20, prove that maximum efficiency is 75%.*

Solution :

VR = 20, η_{max} = 75%, P_1 = 30 N and W_1 = 350 N, P_2 = 40 N and W_2 = 500 N

(a) Put the values of two observations in Law of machine,

$P = mW + C$

$\therefore \quad 30 = m \times 350 + C \quad ...(i)$

$\underline{-40 = -m \times 500 - C} \quad ...(ii)$

$-10 = -150\,m \quad ...(i) - (ii)$

$\therefore$ m = 0.067

(b) Substitute, m = 0.067 in equation (i), we get

$30 = 0.067 \times 350 + C$

$\therefore$ C = 6.55

(c) $$\eta_{max} = \frac{1}{m \times VR} = \frac{1}{0.067 \times 20}$$

$\therefore \eta_{max} = 0.746 = 74.6\% \cong 75\%$ **(Answer)**

Example 6. *In a machine whose velocity ratio is 6 and which lifts the load of 100 N with an effort of 20 N. Find (i) Efficiency of machine, (ii) Effort lost in friction, (iii) Frictional load, (iv) Ideal effort and (v) ideal load.*

Solution :

Here VR = 6, W = 100 N & P = 20 N

(i) Mechanical Advantage, $MA = \frac{W}{P} = \frac{100}{20} = 5$

(ii) Efficiency of machine, $\eta = \frac{MA}{VR} = \frac{5}{6} = 0.8333$

$\therefore$ Efficiency of machine, η = 83.33% **(Answer)**

(iii) Effort lost in friction, $P_F = P - \frac{W}{VR} = 20 - \frac{100}{6}$

$\therefore P_F = 3.33$ N **(Answer)**

(iv) Frictional load, $W_F = (P \times VR) - W$

$\therefore W_F = (20 \times 6) - 100$

$\therefore$ Frictional load, W_F = 20 N **(Answer)**

(v) Ideal effort (P_o), $P_o = \frac{W}{VR} = \frac{100}{6}$

$\therefore P_o = 16.67$ N **(Answer)**

(vi) Ideal Load (W_o), $W_o = P \times VR$

$\therefore\ W_o = 20 \times 6 = 120$ N **(Answer)**

5.3 VELOCITY RATIO FOR DIFFERENT SIMPLE LIFTING MACHINES

(a) Simple axle & wheel

In fig. 5.2 is shown a simple axle and wheel in which the wheel A and axle B are keyed to the same shaft.

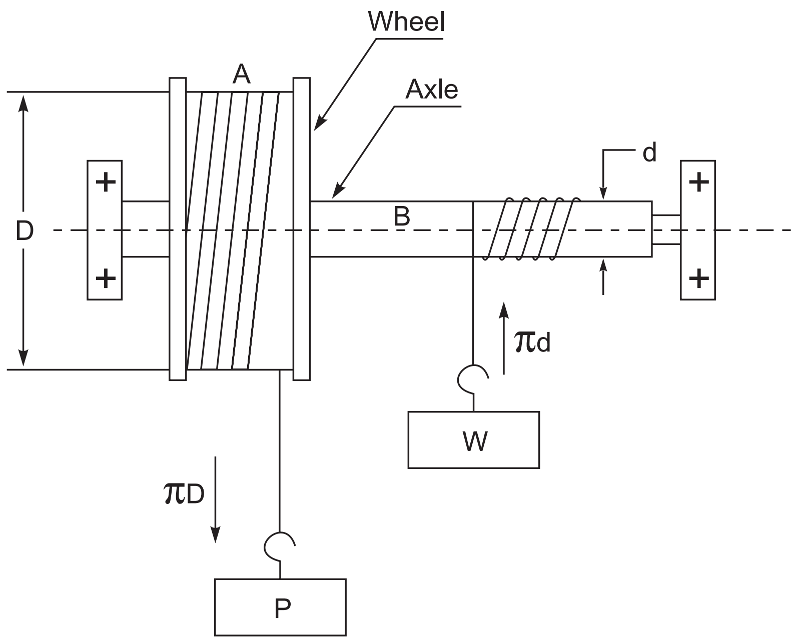


Fig. 5.2: Simple axle & wheel

The string is wound around the axle B, which carries the load W to be lifted. A second string is wound around the wheel A in opposite direction to that of string on axle B, so that downward motion of effort P will lift the load W.

Let D = Diameter of wheel and d = Diameter of axle, then

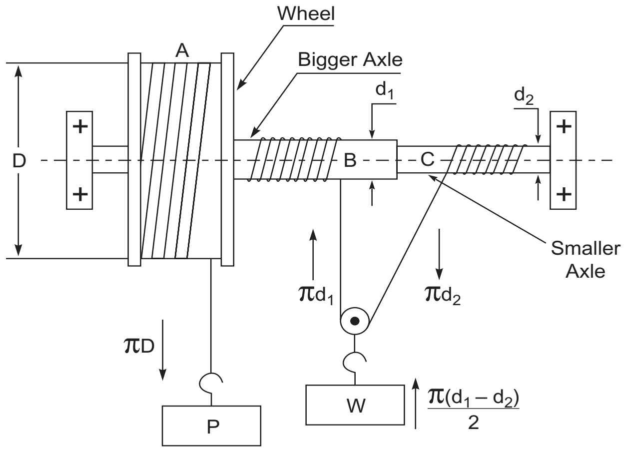
$$VR = \frac{\text{Distance moved by Effort}}{\text{Distance moved by Load}}$$

$$= \frac{y}{x} = \frac{\pi D}{\pi d}$$

$$\therefore\ VR = \frac{D}{d}$$

(b) Differential axle and wheel

In fig. 5.3 is shown a differential axle and wheel. In this case, the load axle BC is made of two parts of different diameters & effort wheel A are key to same shaft.



Differential Axle and Wheel

Fig. 5.3: Differential axle and wheel

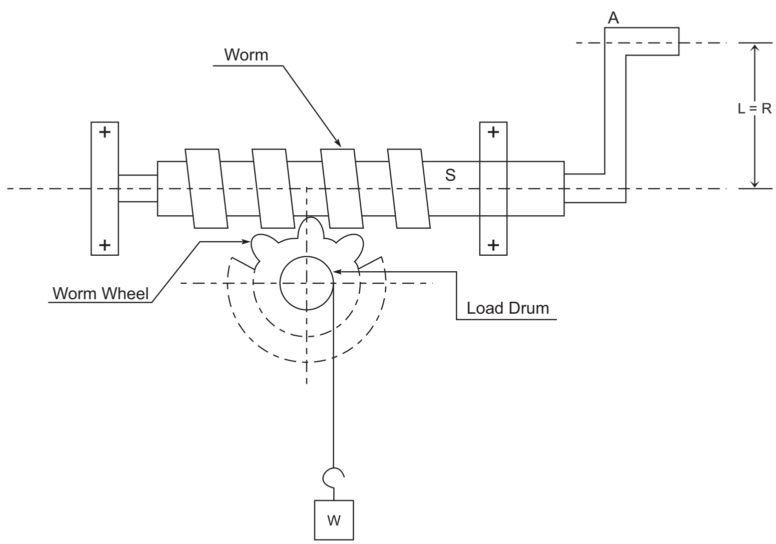
The effort string is wound round the wheel A and another string is wound round the axle B which after passing round the pulley (to which the weight to be lift is attached) is wound round the axle C in opposite direction to that of axle B. So unwinds string from wheel A, other string also unwinds from axle C. But it winds on axle B to lift the load W.

Let D = Diameter of wheel, d_1 = Diameter of bigger axle & d_2 = Diameter of smaller axle, then

$$VR = \frac{2D}{d_1 - d_2}$$

(c) Worm and worm wheel

It consists of a square threaded screw 'S' known as worm and a toothed wheel known as worm wheel geared with each other as shown in fig. 5.4. A wheel or handle A is attach to the worm to apply effort P. A load drum is securely mount on the worm wheel.



Worm and worm wheel

Fig. 5.4: Worm and worm wheel

Let R = Radius of effort wheel = Length of handle, r = Radius of load drum, T = no. of teeth on worm wheel and n = no. of worm thread (single, double etc.), then

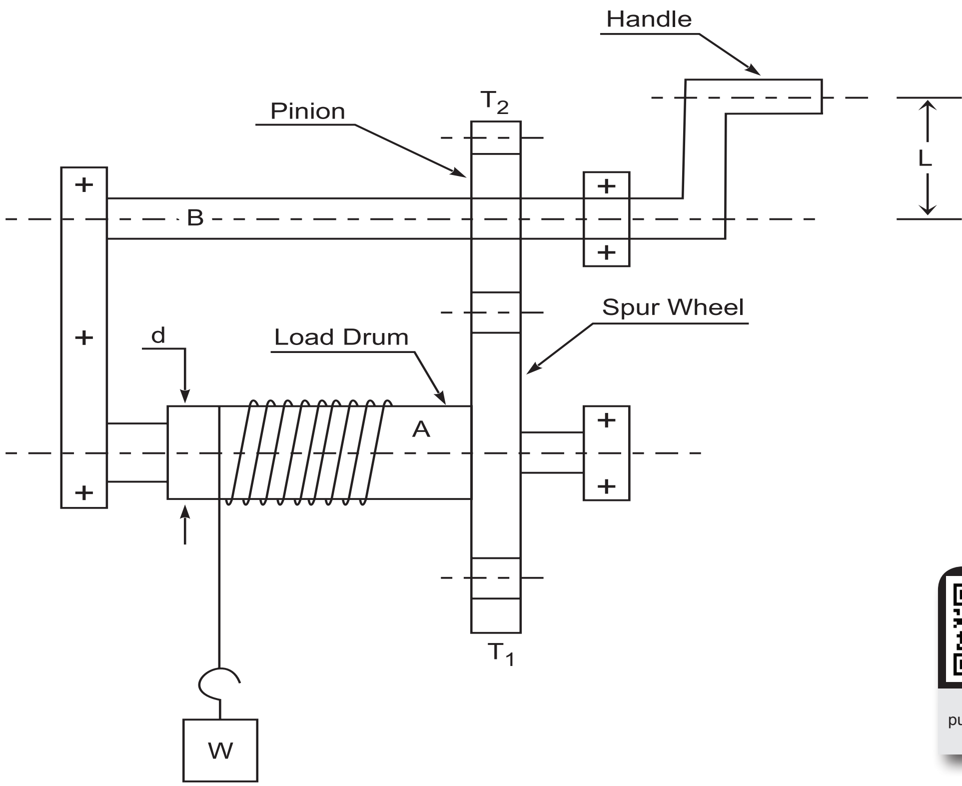
$$VR = \frac{RT}{r} \text{ or } VR = \frac{RT}{nr}$$

(d) Single purchase crab winch

In a single purchase crab winch, a rope is fix to the load drum A and is wound a few turns round it. The free end of the rope lift up the load W. A toothed spur wheel (T_1) is rigidly mount on the load drum A. Another toothed pinion wheel (T_2) is gear with spur wheel as shown in fig. 5.5.

Let l = Length of handle, r = Radius of load drum, T_1 = No. of teeth on main gear (spur wheel) and T_2 = No. of teeth on pinion, then

$$VR = \frac{l}{r} \times \frac{T_1}{T_2}$$



Single purchase crab winch

Fig. 5.5: Single purchase crab winch

(e) Double purchase crab winch

A double purchase crab winch is as intensified design of a single purchase crab winch, to obtain higher value of VR. In this, there are two spur wheel of teeth T_1 and T_3 as well as two pinion teeth T_2 and T_4.

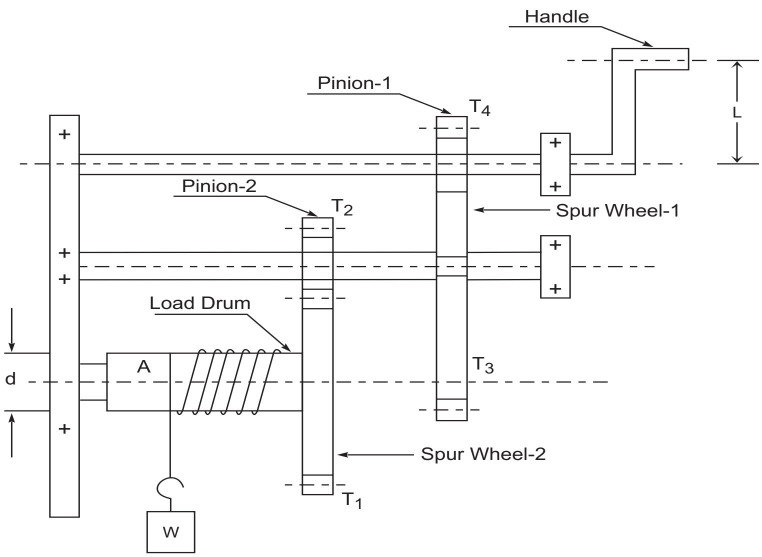


Fig. 5.6: Double purchase crab winch

Let l = Length of handle, r = Radius of load drum, T_1 & T_3 = No. of teeth on main gears (spur wheel), T_2 & T_4 = No. of teeth on pinions, then

$$VR = \frac{l}{r} \times \frac{T_1}{T_2} \times \frac{T_3}{T_4}$$

(f) Simple screw jack

It consists of a screw, fitted in nut, which forms the body of the jack. In which screw is rotate by the application of an effort P, at the end of the lever handle, for lift the load W considering a single threaded simple screw jack.

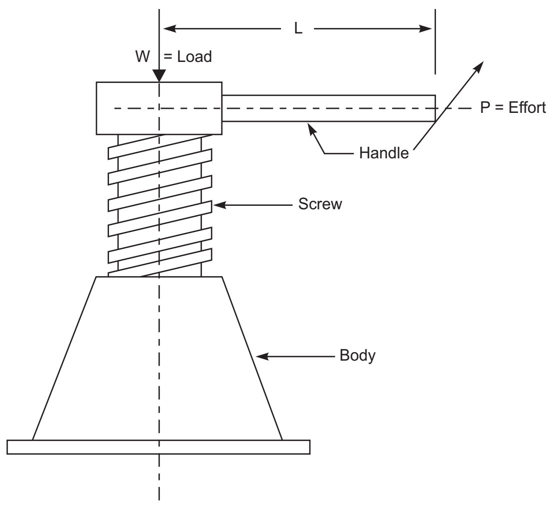


Fig. 5.7: Simple screw jack

Let l = Length of handle & p = Pitch of screw, then

$$VR = \frac{2\pi l}{p}$$

(g) Weston's differential pulley block

It consists of two pulley blocks A and B. The upper block A has two pulleys (P_1 & P_2), one having its diameter a little larger than that of the other. i.e. both of pulley behaves as one pulley with two grooves. The lower block B also carries a pulley, to which the load W is attach to lift up. A continuous chain passes around the pulley P_1 then around the lower block pulley and then finally round the pulley P_2. The effort P is apply to the chain passing over the pulley P_1, so that load W can be lift up as shown in fig.5.8.

Let D = Diameter of bigger pulley and d = Diameter of smaller pulley, then

$$VR = \frac{2D}{D-d}$$

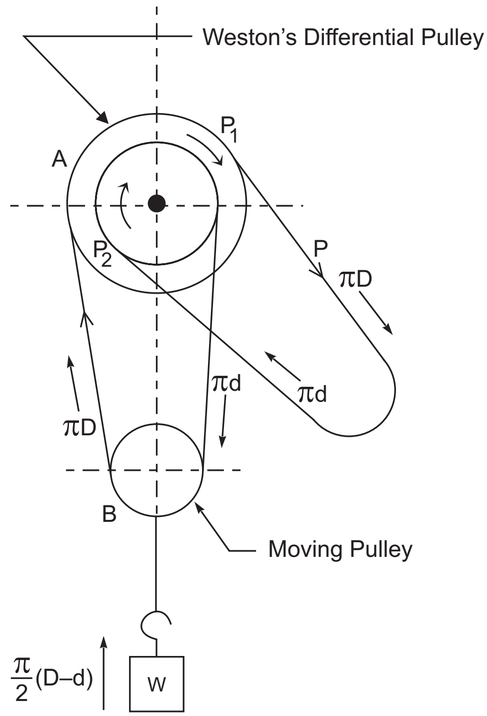


Fig. 5.8: Weston's differential pulley block

(h) Geared pulley block

In consists of a cog wheel A, around which is passed an endless chain. A small gear wheel B known as pinion is key to the same shaft as that of A. The wheel axle B is gear with another bigger wheel C called the spur wheel. A cogwheel D is key to the same shaft as that of spur wheel C.

The load W is attach to a chain that passes over the cogwheel D and the effort P is applied to the endless chain, which passes over the wheel A as shown in fig. 5.9.

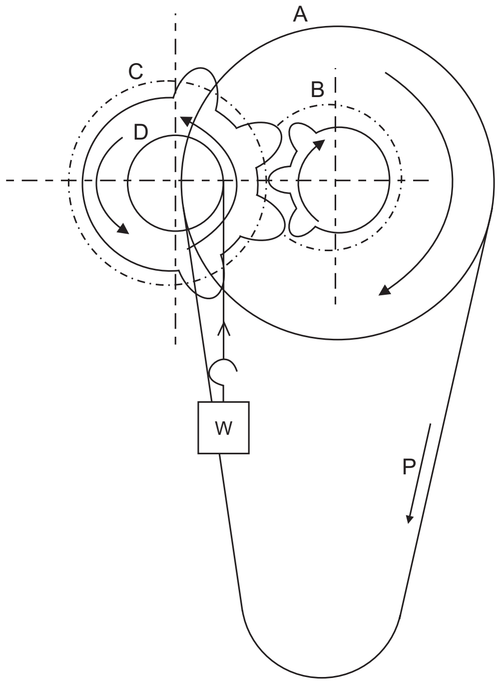


Fig. 5.9: Geared pulley block

Let T_1 = No. of cogs on effort wheel A, T_2 = No. of teeth on pinion wheel B, T_3 = No. of teeth on spur wheel C, T_4 = No. of cogs on load wheel D, then

$$VR = \frac{T_1}{T_2} \times \frac{T_3}{T_4}$$

Activity :

(1) Verify the equations of V.R. for various lifting machines in your laboratory.

(2) If string diameter is heavy or to be considerable, then what correction requires in V.R. equations for listed machine ?

Example 7. *In a double purchase crab winch the pinion has 10 and 20 teeth and spur wheels have 40 and 50 teeth. The handle is 30 cm long and load axle drum is 20 cm diameter. Find the effort required to lift a load of 1500 N when efficiency is 40%.*

Solution :

$$l = 30 \text{ cm}, \; r = \frac{20}{2} = 10 \text{ cm}, \; T_1 = 40, \; T_2 = 10, \; T_3 = 50 \;\&\; T_4 = 20, \; \eta = 0.40$$

(a) $$VR = \frac{l}{r} \times \frac{T_1}{T_2} \times \frac{T_3}{T_4}$$

$$\therefore \; VR = \frac{30}{10} \times \frac{40 \times 50}{10 \times 20} = 30$$

(b) $\eta = \frac{MA}{VR}$

$0.40 = \frac{MA}{30}$

$\therefore$ MA = 12

(c) $MA = \frac{W}{P}$

$12 = \frac{1500}{P}$

$\therefore$ P = 125 N **(Answer)**

Example 8. *In a worm and worm wheel, worm wheel has 120 teeth. Length of handle is 30 cm and diameter of load drum is 10 cm. To lift a load of 1800 N effort of 350 N is required. If maximum efficiency is 40%, find the law of machine. Worm is single threaded.*

Solution :

T = 120, R = 30 cm, $r = \frac{10}{2}$ = 5 cm, n = 1 (screw is single threaded) and η_{max} = 40%

(a) $VR = \frac{RT}{nr} = \frac{30 \times 120}{1 \times 5} = 720$

(b) $\eta_{max} = \frac{1}{m \times VR}$

$\therefore 0.40 = \frac{1}{m \times 720}$

$\therefore$ m = 0.00347

(c) P = mW + C

350 = 0.00347 × 1800 + C

$\therefore$ C = 343.75

(d) Law of machine, P = 0.00347 W + 343.75 **(Answer)**

Example 9. *A single purchase crab which has the following details : (1) Length of lever = 80 cm, (2) Number of teeth on pinion = 20, (3) Number of teeth on spur wheel = 120, (4) Diameter of load drum (axle) = 30 cm. It is observe that an effort of 80 N lifts a load of 2000 N and an effort of 160 N lifts a load of 4200 N (1) Establish the law of machine, (2) Find the efficiency in any one case.*

Solution :

l = 80 cm, $r = \frac{30}{2}$ = 15 cm, T_1 = 120 & T_2 = 20

(a) $VR = \frac{l}{r} \times \frac{T_1}{T_2} = \frac{80}{15} \times \frac{120}{20}$

$\therefore$ VR = 32

(b) $MA = \frac{W}{P} = \frac{2000}{80} = 25$

(c) Efficiency for reading No. 1

$$\eta = \frac{MA}{VR} \times 100 = \frac{25}{32} \times 100$$

$\eta = 78.125\%$ **(Answer)**

(d) Law of machine

(i) Put value of two observation in law of machine P = mW + C, we get

$$80 = m \times 2000 + C \quad \text{...(i)}$$

$$\underline{160 = m \times 4200 + C} \quad \text{...(ii)}$$

$$-80 = -2200\, m \text{ [Subtracting (ii) from (i)]}$$

$\therefore m = 0.036$

(ii) Put value of m in equation (i), we get

$80 = m \times 2000 + C$

$\therefore 80 = 0.036 \times 2000 + C$

$\therefore C = 8$

(ii) Thus Law of machine P = mW + C by putting value of m & C, we get

P = 0.036W + 8 **(Answer)**

Example 10. *In a differential axle and wheel, the diameter of the effort wheel is 400 m. The radii of the axles are 150 mm and 100 mm respectively. The diameter of the rope is 1 cm. Find the load which can be lifted by an effort of 200 N assuming efficiency of machine to be 75%.*

Solution :

$D = 400$ mm, $d_1 = 2 \times 150 = 300$ mm, $d_2 = 2 \times 100 = 200$ mm,

t_1 = diameter of the rope = 1 cm = 10 mm & $\eta = 75\%$

(a) For differential axle and wheel,

$$VR = \frac{2(D + t_1)}{(d_1 + t_1) - (d_2 + t_1)} = \frac{2(D + t_1)}{(d_1 - d_2)}$$

$$\therefore VR = \frac{2 \times (400 + 10)}{300 - 200}$$

$\therefore VR = 8.2$

(b) $\eta = \frac{MA}{VR} \times 100$

$$75 = \frac{MA}{8.2} \times 100$$

$\therefore MA = 6.15$

$$MA = \frac{W}{P}$$

$\therefore\ 6.15 = \dfrac{W}{200}$

$\therefore\ W = 1230$ N **(Answer)**

Example 11. *In a double purchase crab winch number to teeth on pinion are 120 and 150 and that of spur are 300 and 400. Diameter of axle is 20 cm. Find V.R. Also find friction in terms of effort and load when effort of 105 N is required to lift a load of 1.834 kN. Take length of handle as 80 cm.*

Solution :

$l = 80$ cm, $r = \dfrac{20}{2} = 10$ cm, $T_1 = 300$, $T_2 = 120$ & $T_3 = 400$ & $T_4 = 150$

(a) $VR = \dfrac{l}{r} \times \dfrac{T_1}{T_2} \times \dfrac{T_3}{T_4}$

$\therefore\ VR = \dfrac{80}{10} \times \dfrac{300 \times 400}{120 \times 150} = 53.33$ **(Answer)**

(b) Effort lost in friction : $P_f = P - \dfrac{W}{VR}$

Put P = 105 N and W = 1.834 kN = 1834 N

$\therefore\ P_f = 105 - \dfrac{1834}{53.33}$

$\therefore\ P_f = 70.61$ N **(Answer)**

(c) Friction load : $W_f = 105 \times 53.33 - 1834$

$\therefore\ W_f = 3765.65$ N **(Answer)**

UNIT SUMMARY

- **Mechanical advantage :** $MA = \dfrac{W}{P}$, where W = load, P = Effort
- **Velocity ratio (VR) :** $VR = \dfrac{y}{x}$, where y = distance moved by effort and x = distance moved by load
- **Input :** Input = $P \cdot y$
- **Output :** Output = $W \cdot x$
- **Efficiency (η) :** $\eta = \dfrac{MA}{VR} \times 100\%$ or $\eta = \dfrac{\text{output}}{\text{input}} \times 100\%$
- **Ideal machine :** A machine having η = 100% is called an ideal machine.
  For ideal machine, MA = VR or Output = Input.
- **For reversible machine,** $\eta \geq 50\%$ and **for non-reversible machine,** $\eta < 50\%$.
- **Effort lost in Friction (P_f) :** $P_f = P - \dfrac{W}{VR}$
- **Friction load (W_f) :** $W_f = P \times VR - W$

- **Law of machine :** P = mW + C, where m = slope of the curve and C = constant of friction
- **Maximum MA :** $MA_{max} = \dfrac{1}{m}$
- **Maximum efficiency :** $\eta_{max} = \dfrac{1}{m \times VR}$
- **Velocity ratio for different machine :**

(a) Simple axle & wheel : $VR = \dfrac{D}{d}$

(b) Differential axle & wheel : $VR = \dfrac{2D}{d_1 - d_2}$

(c) Worm & worm wheel : $VR = \dfrac{RT}{r}$

(d) Single purchase crab winch : $VR = \dfrac{l}{r} \times \dfrac{T_1}{T_2}$

(e) Double purchase crab winch : $VR = \dfrac{l}{r} \times \left(\dfrac{T_1 \times T_3}{T_2 \times T_4}\right)$

(f) Simple screw jack : $VR = \dfrac{2\pi l}{p}$

(g) Weston's differential pulley block : $VR = \dfrac{2D}{D - d}$

(h) Geared pulley block : $VR = \dfrac{T_1}{T_2} \times \dfrac{T_3}{T_4}$

Where,
D = R = diameter of effort wheel; L = length of handle; d = d_1 = d_2 = diameter of axles; r = radius of load drum; p = pitch of screw; T_1 & T_2 = Nos. of teeth on spur wheel; T_3 & T_4 = Nos. of teeth on pinion wheel

EXERCISE

(A) Objective Questions

5.1 The efficiency of a simple lifting machine is the ratio of
(a) Output to input (b) work done by it to work done on it
(c) mechanical advantage to velocity ratio (d) all of the above

5.2 If efficiency of a simple lifting machine is kept constant, its velocity ratio is directly proportional to its
(a) Mechanical advantage (b) effort applied
(c) machine friction (d) all of the above

5.3 A simple lifting machine having an efficiency greater than 50 % is known as
(a) Self-locking machine (b) non-reversible machine

(c) ideal machine (d) none of the above

5.4 A simple lifting machine having an efficiency less than 50 % is known as
(a) Reversible machine (b) non-reversible machine
(c) ideal machine (d) none of the above

5.5 In an ideal machine, the mechanical advantage is …………… Velocity ratio
(a) Equal to (b) less than (c) greater than (d) none of the above

5.6 A weight W of 1000 N can be lifted by an effort P of 100 N. If VR of the machine is 30, then the machine is
(a) Reversible (b) non-reversible (c) ideal (d) none of the above

5.7 The law of machine of a simple lifting machine is given by the relation
(a) $P = mW - C$ (b) $P = mW + C$ (c) $P = mW \times C$ (d) none of the above
Where P is the effort applied to lift load W and m & C are constants.

5.8 The maximum efficiency of a simple lifting machine is
(a) $\frac{1}{m}$ (b) $\frac{VR}{m}$ (c) $\frac{m}{VR}$ (d) $\frac{1}{(m \times VR)}$

5.9 The maximum mechanical advantage of a simple lifting machine is
(a) $1 - m$ (b) $1 + m$ (c) $\frac{1}{m}$ (d) m

5.10 The velocity ratio of a simple axle & wheel with D & d as the diameter of effort wheel & load axle is
(a) $D + d$ (b) $D - d$ (c) $D \times d$ (d) $\frac{D}{d}$

5.11 The radius of the effort wheel of a worm & worm wheel has nothing to do with its efficiency
(a) agree (b) disagree

5.12 The VR of a single purchase crab winch can be increased by
(a) Increasing the length of the handle (b) increasing the radius of the load drum
(c) increasing the number of teeth on the pinion (d) all of the above

5.13 In a simple screw jack, with l as length of the effort handle and p as pitch of screw, its VR is
(a) $\frac{2\pi l}{p}$ (b) $\frac{\pi l}{2p}$ (c) $\frac{2\pi p}{l}$ (d) $\frac{\pi p}{2l}$

[*Ans :* (1-d), (2-a), (3-d), (4-b), (5-a), (6-b), (7-b), (8-d), (9-c), (10-d), (11-b), (12-a), (13-a)]

(B) Subjective Questions

5.1 Define the following : (i) Mechanical advantage (ii) Velocity ratio (iii) Input (iv) Output (v) Effort lost in friction (vi) Reversible machine (vii) Non-reversible machine (viii) Maximum MA (ix) Maximum efficiency (x) Ideal machine (xi) Load lost in friction.

5.2 Explain law of machine.

5.3 Explain reversible and non-reversible machine.

5.4 Prove that for reversible machine $\eta \geq 50\%$.

5.5 Explain maximum efficiency is $\frac{1}{m \times VR}$.