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Lecture note on STRUCTURAL MECHANICS
(3rd Semester)

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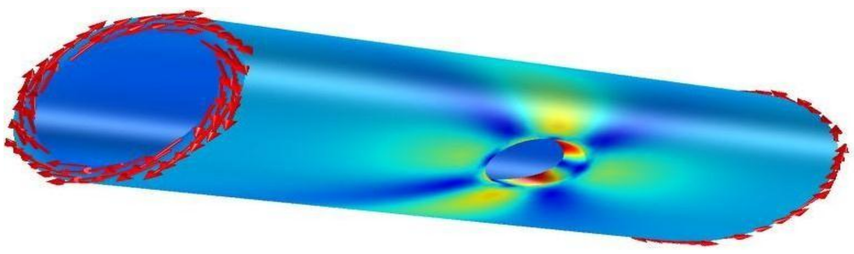
DEPARTMENT OF CIVIL ENGINEERING

INTRODUCTION

Structural *mechanics*, or *solid mechanics*, is a field of applied mechanics in which you compute deformations, stresses, and strains in solid materials. Often, the purpose is to determine the strength of a structure, such as a bridge, in order to prevent damage or accidents. Other common goals of structural mechanics analyses include determining the flexibility of a structure and computing dynamic properties, such as natural frequencies and responses to time-dependent loads.

The study of solid mechanics closely relates to material sciences, since one of the fundamentals is to have appropriate models for the mechanical behavior of the material being used. Different types of solid materials require vastly different mathematical descriptions. Some examples are metals, rubbers, soils, concrete, and biological tissues.



STRUCTURAL MECHANICS

CHAPTER-1

Force:- The external energy required to move the body from one place to the other is a force.

- ⇨ It is Vector quantity.

unit-Network (N)

Or

Kilonewton (KN)

- ⇨ **There are two type of force**
  (1) Pulling forces
  (2) Pushing forces.
- ⇨ A force that changes the direction an object towards you, would be a pull.
- ⇨ On the other handif it moves away it is a push.
- ⇨ Some-times, force is simply defined as apush orpull upon an object resulting from the object's interaction with another object. Hence, any kind of force is basically a push or pull.

MOMENT:-

- ⇨ The product of force and perpendicular distance from the point to the line of action of the force is called moment of a force about that axis.

Or,

- ⇨ The tendency of force is not only to move a body also to rotate the body.This rotational tendency of a force is called moment.

ScalarQuantities :-

- ⇨ The Scalar quantities (or sometimes known as scalars) are those quantities which have magnitude onlysuch as length, mass, time, distance, volume, density, temperature, speed etc.

Vector Quantities:-

- ⇨ The vector quantities (or sometimes known as vectors) are those quantitieswhich have both magnitude and direction such as force, displacement, velocity, acceleration, momentum etc.

Representation of a vector:-

A vector is represented by a directed line as shown in fig.

- ⇨ It may be noted that the length OA representsthe magnitude of the VectorOA. The direction of the vector is 0A is from O to A, It is also known as vector P.

Unit Vector :-

- ⇨ A vector, whose magnitude is unity is known as unit Vector.

Equal vectors:-

- ⇨ The Vectors, which are parallel to each other and have same direction and equal magnitude are known as equalvectors.

Like vectors :-

- ⇨ The vectors which are parallel to each other and have same sense but unequal magnitude, are known as like vectors.

CENTRE OF GRAVITY (C.G) :-

- ⇨ Every particle of a body is attracted by the earth towards its Centre.
- ⇨ The force of attraction, which is proportioned to the mass of the particles, acts vertically downwards and is known as weight of the body.
- ⇨ As the distance between the different particles of a body and the centre of the earth is same, therefore these forces may be taken to acts along parallel axis.
- ⇨ That a point may be found out in a body , through which the resultant of all such parallel forces acts.
- ⇨ This point, through which the whole weight of the body acts , irrespective of its position is known as centre of gravity.
- ⇨ It may be noted that everybody has one and only one centre of gravity.

CENTROID :-

- ⇨ The Plane figure like (triangle, quadrilateral, circle etc.) have only areas but no mass.
- ⇨ The centre of area of such figures is known as centroid.
- ⇨ The method of finding out the centroid of a figure Is the same as that of finding out the centre of gravity of a body.

CENTRE OF GRAVITY OF PLANE FIGURES :-

- ⇨ The plane geometrical figure (such as T-section ,I-section, L-section) have only areas but no mass.
- ⇨ If a section is split with respect to y-axis, such that the section on one side of y axis is the exact image of the remaining portion of the section that lie on the opposite face of y-axis then this section is said to be asymmetrical section with respect to y-axis.
- ⇨ **There are two type of section**

  (1) Symmetrical section.

  (2) Unsymmetrical sections.

Symmetrical section: -

- If a section like 'T','I' and 'C' is cut around axesuch that one portion is the exact imageof the other.
- The axis with respect to which, section divided is known as axis of symmetry.

Unsymmetrical Section :-

- A section will be unsymmetrical Section, if given section is cut around axis such that one portion of a cut section is not the exact image of the part of section that lie on the opposite face of axis like 'L' section.

9.8. Centre of Gravity of Symmetrical Sections

Sometimes, the given section, whose centre of gravity is required to be found out, is symmetrical about *X-X* axis or *Y-Y* axis. In such cases, the procedure for calculating the centre of gravity of the body is very much simplified; as we have only to calculate either $\bar{x}$ or $\bar{y}$. This is due to the reason that the centre of gravity of the body will lie on the axis of symmetry.

EXAMPLE 9.1. *Find the centre of gravity of a 100 mm × 150 mm × 30 mm T-section.*

SOLUTION. As the section is symmetrical about *Y-Y* axis, bisecting the web, therefore its centre of gravity will lie on this axis. Split up the section into two rectangles *ABCH* and *DEFG* as shown in Fig. 9.10.

Let bottom of the web *FE* be the axis of reference.

(*i*) *Rectangle ABCH*

$$a_1 = 100 \times 30 = 3000 \text{ mm}^2$$

and $$y_1 = \left(150 - \frac{30}{2}\right) = 135 \text{ mm}$$

(*ii*) *Rectangle DEFG*

$$a_2 = 120 \times 30 = 3600 \text{ mm}^2$$

and $$y_2 = \frac{120}{2} = 60 \text{ mm}$$

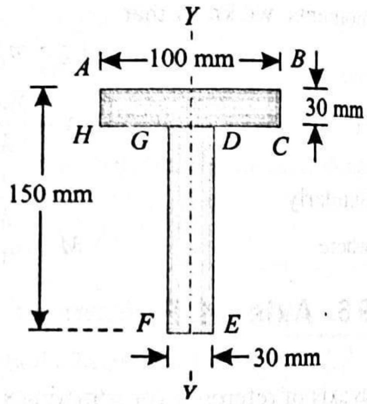


Fig. 9.10

We know that distance between centre of gravity of the section and bottom of the flange *FE*,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(3000 \times 135) + (3600 \times 60)}{3000 + 3600} \text{ mm}$$

$$= 94.1 \text{ mm} \quad \textbf{Ans.}$$

EXAMPLE 9.2. *Find the centre of gravity of a channel section 100 mm × 50 mm × 15 mm.*

SOLUTION. As the section is symmetrical about *X-X* axis, therefore its centre of gravity will lie on this axis. Now split up the whole section into three rectangles *ABFJ*, *EGKJ* and *CDHK* as shown in Fig. 9.11.

Let the face *AC* be the axis of reference.

(*i*) *Rectangle ABFJ*

$$a_1 = 50 \times 15 = 750 \text{ mm}^2$$

and $$x_1 = \frac{50}{2} = 25 \text{ mm}$$

(*ii*) *Rectangle EGKJ*

$$a_2 = (100 - 30) \times 15 = 1050 \text{ mm}^2$$

and $$x_2 = \frac{15}{2} = 7.5 \text{ mm}$$

(*iii*) *Rectangle CDHK*

$$a_3 = 50 \times 15 = 750 \text{ mm}^2$$

and $$x_3 = \frac{50}{2} = 25 \text{ mm}$$

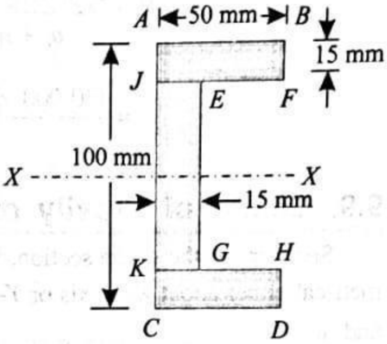


Fig. 9.11

We know that distance between the centre of gravity of the section and left face of the section *AC*,

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3}$$

$$= \frac{(750 \times 25) + (1050 \times 7.5) + (750 \times 25)}{750 + 1050 + 750} = 17.8 \text{ mm} \quad \textbf{Ans.}$$

EXAMPLE 9.3. *An I-section has the following dimensions in mm units :*

Bottom flange = *300 × 100*

Top flange = *150 × 50*

Web = *300 × 50*

Determine mathematically the position of centre of gravity of the section.

SOLUTION. As the section is symmetrical about *Y-Y* axis, bisecting the web, therefore its centre of gravity will lie on this axis. Now split up the section into three rectangles as shown in Fig. 9.12.

Let bottom of the bottom flange be the axis of reference.

(*i*) *Bottom flange*

$$a_1 = 300 \times 100 = 30\,000 \text{ mm}^2$$

and $$y_1 = \frac{100}{2} = 50 \text{ mm}$$

(*ii*) *Web*

$$a_2 = 300 \times 50 = 15\,000 \text{ mm}^2$$

and $$y_2 = 100 + \frac{300}{2} = 250 \text{ mm}$$

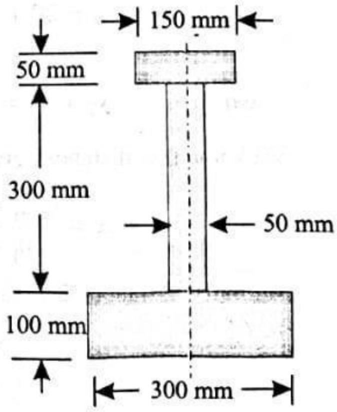


Fig. 9.12

(iii) *Top flange*

$$a_3 = 150 \times 50 = 7500 \text{ mm}^2$$

and $$y_3 = 100 + 300 + \frac{50}{2} = 425 \text{ mm}$$

We know that distance between centre of gravity of the section and bottom of the flange,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

$$= \frac{(30\,000 \times 50) + (15\,000 \times 250) + (7500 \times 425)}{30\,000 + 15\,000 + 7500} = 160.7 \text{ mm} \quad \textbf{Ans.}$$

9.9. Centre of Gravity of Unsymmetrical Sections

Sometimes, the given section, whose centre of gravity is required to be found out, is not symmetrical either about *X-X* axis or *Y-Y* axis. In such cases, we have to find out both the values of $\bar{x}$ and $\bar{y}$.

Example 9.4. *Find the centroid of an unequal angle section 100 mm × 80 mm × 20 mm.*

Solution. As the section is not symmetrical about any axis, therefore we have to find out the values of $\bar{x}$ and $\bar{y}$ for the angle section. Split up the section into two rectangles as shown in Fig. 9.13.

Let left face of the vertical section and bottom face of the horizontal section be axes of reference.

(i) *Rectangle* 1

$$a_1 = 100 \times 20 = 2000 \text{ mm}^2$$

$$x_1 = \frac{20}{2} = 10 \text{ mm}$$

and $$y_1 = \frac{100}{2} = 50 \text{ mm}$$

(ii) *Rectangle* 2

$$a_2 = (80 - 20) \times 20 = 1200 \text{ mm}^2$$

$$x_2 = 20 + \frac{60}{2} = 50 \text{ mm}.$$

and $$y_2 = \frac{20}{2} = 10 \text{ mm}$$

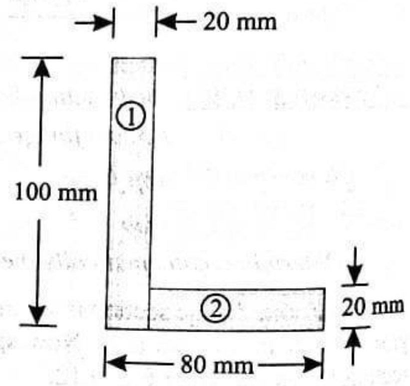


Fig. 9.13

We know that distance between centre of gravity of the section and left face,

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = \frac{(2000 \times 10) + (1200 \times 50)}{2000 + 1200} = 25 \text{ mm} \quad \textbf{Ans.}$$

Similarly, distance between centre of gravity of the section and bottom face,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(2000 \times 50) + (1200 \times 10)}{2000 + 1200} = 35 \text{ mm} \quad \textbf{Ans.}$$

MOMENT OF INERTIA (M.I)

- We known the moment of force is defined as the product of force and perpendicular distance(x) between the point and the line of the action of force is called moment.
- If this moment is again multiplied by the perpendicular distance(x) between the point and the line of action of the force ,this quantity is called moment of the moment of aforce or second moment of force or moment of inertia.
- So M.I = $p*x*x = px^2$

UNITSOF MOMENT OF INERTIA:-

- As a matter of fact the units of moment of inertia of a plane area depend upon the units of the area and the length.
- If area is in m^2 and the length also in m, the moment of inertia is expressed in m^4.
- If area in mm^2 and the length is also in mm, then moment of inertia expressed in mm^4.

DEFINATION OF EQUILLIBRIUM:-

- Any systemof forces which keep the bodyat rest is said to be in equilibrium or when the condition of the body is unaffected even though it is acted upon by number of forces.
- It is said to be in equilibrium.

10.6. Moment of Inertia by Integration

The moment of inertia of an area may also be found out by the method of integration as discussed below:

Consider a plane figure, whose moment of inertia is required to be found out about X-X axis and Y-Y axis as shown in Fig 10.1. Let us divide the whole area into a no. of strips. Consider one of these strips.

Let dA = Area of the strip

x = Distance of the centre of gravity of the strip on X-X axis and

y = Distance of the centre of gravity of the strip on Y-Y axis.

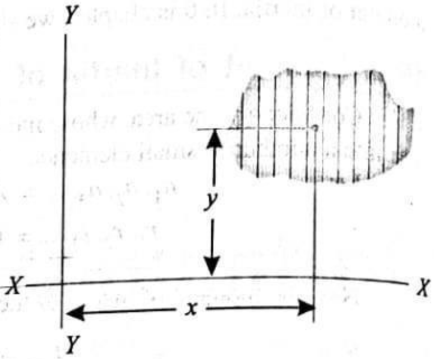


Fig. 10.1. Moment of inertia by integration.

We know that the moment of inertia of the strip about Y-Y axis

$$= dA \cdot x^2$$

Now the moment of inertia of the whole area may be found out by integrating above equation. *i.e.*,

$$I_{YY} = \Sigma\, dA \cdot x^2$$

Similarly $\quad I_{XX} = \Sigma\, dA \cdot y^2$

In the following pages, we shall discuss the applications of this method for finding out the moment of inertia of various cross-sections.

10.7. Moment of Inertia of a Rectangular Section

Consider a rectangular section $ABCD$ as shown in Fig. 10.2 whose moment of inertia is required to be found out.

Let $\quad b$ = Width of the section and

d = Depth of the section.

Now consider a strip PQ of thickness dy parallel to X-X axis and at a distance y from it as shown in the figure

∴ Area of the strip

$$= b.dy$$

We know that moment of inertia of the strip about X-X axis,

$$= \text{Area} \times y^2 = (b.\ dy)\ y^2 = b.\ y^2.\ dy$$

Now *moment of inertia of the whole section may be found out by integrating the above equation for the whole length of the lamina *i.e.* from $-\frac{d}{2}$ to $+\frac{d}{2}$,

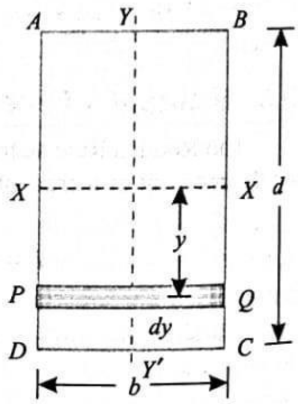


Fig. 10.2. Rectangular section.

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* This may also be obtained by Routh's rule as discussed below :

$$I_{xx} = \frac{AS}{3} \quad \text{...(for rectangular section)}$$

where area, $A = b \times d$ and sum of the square of semi axes Y-Y and Z-Z,

$$S = \left(\frac{d}{2}\right)^2 + 0 = \frac{d^2}{4}$$

$$\therefore \quad I_{xx} = \frac{AS}{3} = \frac{(b \times d) \times \frac{d^2}{4}}{3} = \frac{bd^3}{12}$$

•

$$I_{xx} = \int_{-\frac{d}{2}}^{+\frac{d}{2}} b.y^2.dy = b\int_{-\frac{d}{2}}^{+\frac{d}{2}} y^2 . dy$$

$$= b\left[\frac{y^3}{3}\right]_{-\frac{d}{2}}^{+\frac{d}{2}} = b\left[\frac{(d/2)^3}{3} - \frac{(-d/2)^3}{3}\right] = \frac{bd^3}{12}$$

Similarly, $I_{YY} = \frac{db^3}{12}$

Note. Cube is to be taken of the side, which is at right angles to the line of reference.

Example 10.1. *Find the moment of inertia of a rectangular section 30 mm wide and 40 mm deep about X-X axis and Y-Y axis.*

Solution. Given: Width of the section (b) = 30 mm and depth of the section (d) = 40 mm.

We know that moment of inertia of the section about an axis passing through its centre of gravity and parallel to X-X axis,

$$I_{XX} = \frac{bd^3}{12} = \frac{30 \times (40)^3}{12} = 160 \times 10^3 \text{ mm}^4 \quad \textbf{Ans.}$$

Similarly $$I_{YY} = \frac{db^3}{12} = \frac{40 \times (30)^3}{12} = 90 \times 10^3 \text{ mm}^4 \quad \textbf{Ans.}$$

10.8. Moment of Inertia of a Hollow Rectangular Section

Consider a hollow rectangular section, in which $ABCD$ is the main section and $EFGH$ is the cut out section as shown in Fig 10.3

Let b = Breadth of the outer rectangle,

d = Depth of the outer rectangle and

b_1, d_1 = Corresponding values for the cut out rectangle.

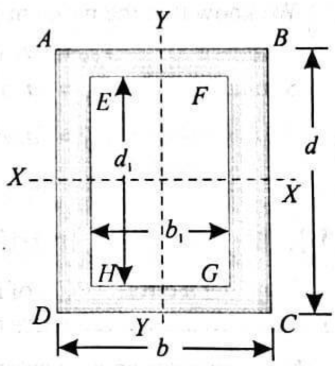


Fig. 10.3. Hollow rectangular section.

We know that the moment of inertia, of the outer rectangle $ABCD$ about X-X axis

$$= \frac{bd^3}{12} \quad ...(i)$$

and moment of inertia of the cut out rectangle $EFGH$ about X-X axis

$$= \frac{b_1 d_1^3}{12} \quad ...(ii)$$

∴ M.I. of the hollow rectangular section about X-X axis,

$$I_{XX} = \text{M.I. of rectangle } ABCD - \text{M.I. of rectangle } EFGH$$

$$= \frac{bd^3}{12} - \frac{b_1 d_1^3}{12}$$

Similarly, $$I_{yy} = \frac{db^3}{12} - \frac{d_1 b_1^3}{12}$$

EXAMPLE 10.10. *Find the moment of inertia of a T-section with flange as 150 mm × 50 mm and web as 150 mm × 50 mm about X-X and Y-Y axes through the centre of gravity of the section.*

SOLUTION. The given *T*-section is shown in Fig. 10.14.

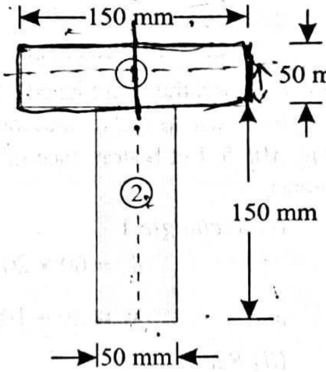


Fig. 10.14

First of all, let us find out centre of gravity of the section. As the section is symmetrical about *Y-Y* axis, therefore its centre of gravity will lie on this axis. Split up the whole section into two rectangles *viz.*, 1 and 2 as shown in figure. Let bottom of the web be the axis of reference.

(*i*) *Rectangle* (1)

$$a_1 = 150 \times 50 = 7500 \text{ mm}^2$$

and $$y_1 = 150 + \frac{50}{2} = 175 \text{ mm}$$

(*ii*) *Rectangle* (2)

$$a_2 = 150 \times 50 = 7500 \text{ mm}^2$$

and $$y_2 = \frac{150}{2} = 75 \text{ mm}$$

We know that distance between centre of gravity of the section and bottom of the web,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(7500 \times 175) + (7500 \times 75)}{7500 + 7500} = 125 \text{ mm}$$

Moment of inertia about X-X axis

We also know that M.I. of rectangle (1) about an axis through its centre of gravity and parallel to *X-X* axis.

$$I_{G1} = \frac{150\,(50)^3}{12} = 1.5625 \times 10^6 \text{ mm}^4$$

and distance between centre of gravity of rectangle (1) and *X-X* axis,

$$h_1 = 175 - 125 = 50 \text{ mm}$$

∴ Moment of inertia of rectangle (1) about X-X axis

$$I_{G1} + a_1 h_1^2 = (1.5625 \times 10^6) + [7500 \times (50)^2] = 20.3125 \times 10^6 \text{ mm}^4$$

Similarly, moment of inertia of rectangle (2) about an axis through its centre of gravity and parallel to X-X axis,

$$I_{G2} = \frac{50\ (150)^3}{12} = 14.0625 \times 10^6 \text{ mm}^4$$

and distance between centre of gravity of rectangle (2) and X-X axis,

$$h_2 = 125 - 75 = 50 \text{ mm}$$

∴ Moment of inertia of rectangle (2) about X-X axis

$$= I_{G2} + a_2 h_2^2 = (14.0625 \times 10^6) + [7500 \times (50)^2] = 32.8125 \times 10^6 \text{ mm}^4$$

Now moment of inertia of the whole section about X-X axis,

$$I_{XX} = (20.3125 \times 10^6) + (32.8125 \times 10^6) = 53.125 \times 10^6 \text{ mm}^4$$ **Ans.**

Moment of inertia about Y-Y axis

We know that M.I. of rectangle (1) about Y-Y axis

$$= \frac{50\ (150)^3}{12} = 14.0625 \times 10^6 \text{ mm}^4$$

and moment of inertia of rectangle (2) about Y-Y axis,

$$= \frac{150\ (50)^3}{12} = 1.5625 \times 10^6 \text{ mm}^4$$

Now moment of inertia of the whole section about Y-Y axis,

$$I_{YY} = (14.0625 \times 10^6) + (1.5625 \times 10^6) = 15.625 \times 10^6 \text{ mm}^4$$ **Ans.**

EXAMPLE 10.11. *An I-section is made up of three rectangles as shown in Fig. 10.15. Find the moment of inertia of the section about the horizontal axis passing through the centre of gravity of the section.*

SOLUTION. First of all, let us find out centre of gravity of the section. As the section is symmetrical about Y-Y axis, therefore its centre of gravity will lie on this axis.

Split up the whole section into three rectangles 1, 2 and 3 as shown in Fig. 10.15. Let bottom face of the bottom flange be the axis of reference.

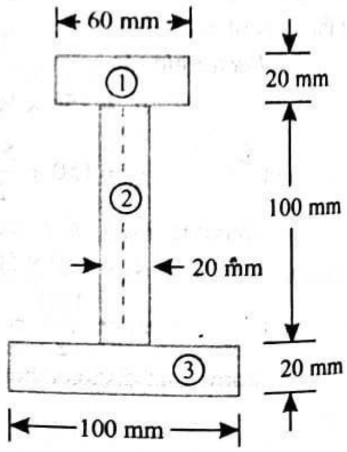


Fig. 10.15

(*i*) *Rectangle* 1

$$a_1 = 60 \times 20 = 1200 \text{ mm}$$

and $$y_1 = 20 + 100 + \frac{20}{2} = 130 \text{ mm}$$

(*ii*) *Rectangle* 2

$$a_2 = 100 \times 20 = 2000 \text{ mm}^2$$

and $$y_2 = 20 + \frac{100}{2} = 70 \text{ mm}$$

(*iii*) *Rectangle* 3

$$a_3 = 100 \times 20 = 2000 \text{ mm}^2$$

and $$y_3 = \frac{20}{2} = 10 \text{ mm}$$

We know that the distance between centre of gravity of the section and bottom face,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3} = \frac{(1200 \times 130) + (2000 \times 70) + (2000 \times 10)}{1200 + 2000 + 2000} \text{ mm}$$

$$= 60.8 \text{ mm}$$

We know that moment of inertia of rectangle (1) about an axis through its centre of gravity and parallel to X-X axis,

$$I_{G1} = \frac{60 \times (20)^3}{12} = 40 \times 10^3 \text{ mm}^4$$

and distance between centre of gravity of rectangle (1) and X-X axis,

$$h_1 = 130 - 60.8 = 69.2 \text{ mm}$$

∴ Moment of inertia of rectangle (1) about X-X axis,

$$= I_{G1} + a_1 h_1^2 = (40 \times 10^3) + [1200 \times (69.2)^2] = 5786 \times 10^3 \text{ mm}^4$$

Similarly, moment of inertia of rectangle (2) about an axis through its centre of grávity and parallel to X-X axis,

$$I_{G2} = \frac{20 \times (100)^3}{12} = 1666.7 \times 10^3 \text{ mm}^4$$

and distance between centre of gravity of rectangle (2) and X-X axis,

$$h_2 = 70 - 60.8 = 9.2 \text{ mm}$$

∴ Moment of inertia of rectangle (2) about X-X axis,

$$= I_{G2} + a_2 h_2^2 = (1666.7 \times 10^3) + [2000 \times (9.2)^2] = 1836 \times 10^3 \text{ mm}^4$$

Now moment of inertia of rectangle (3) about an axis through its centre of gravity and parallel to X-X axis,

$$I_{G3} = \frac{100 \times (20)^3}{12} = 66.7 \times 10^3 \text{ mm}^4$$

and distance between centre of gravity of rectangle (3) and X-X axis,

$$h_3 = 60.8 - 10 = 50.8 \text{ mm}$$

∴ Moment of inertia of rectangle (3) about X-X axis,

$$= I_{G3} + a_3 h_3^2 = (66.7 \times 10^3) + [2000 \times (50.8)^2] = 5228 \times 10^3 \text{ mm}^4$$

Now moment of inertia of the whole section about X-X axis,

$$I_{XX} = (5786 \times 10^3) + (1836 \times 10^3) + (5228 \times 10^3) = 12\,850 \times 10^3 \text{ mm}^4$$ **Ans.**

EXAMPLE 10.12. *Find the moment of inertia about the centroidal X-X and Y-Y axes of the angle section shown in Fig. 10.16.*

SOLUTION. First of all, let us find the centre of gravity of the section. As the section is not symmetrical about any section, therefore we have to find out the values of $\bar{x}$ and $\bar{y}$ for the angle section. Split up the section into two rectangles (1) and (2) as shown in Fig. 10.16.

Moment of inertia about centroidal X-X axis

Let bottom face of the angle section be the axis of reference.

Rectangle (1)

$$a_1 = 100 \times 20 = 2000 \text{ mm}^2$$

and $$y_1 = \frac{100}{2} = 50 \text{ mm}$$

Rectangle (2)

$$a_2 = (80 - 20) \times 20 = 1200 \text{ mm}^2$$

and $$y_2 = \frac{20}{2} = 10 \text{ mm}$$

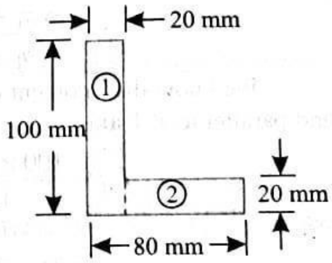


Fig. 10.16

We know that distance between the centre of gravity of the section and bottom face,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(2000 \times 50) + (1200 \times 10)}{2000 + 1200} = 35 \text{ mm}$$

We know that moment of inertia of rectangle (1) about an axis through its centre of gravity and parallel to X-X axis,

$$I_{G1} = \frac{20 \times (100)^3}{12} = 1.667 \times 10^6 \text{ mm}^4$$

and distance of centre of gravity of rectangle (1) from X-X axis,

$$h_1 = 50 - 35 = 15 \text{ mm}$$

∴ Moment of inertia of rectangle (1) about X-X axis

$$= I_{G1} + a h_1^2 = (1.667 \times 10^6) + [2000 \times (15)^2] = 2.117 \times 10^6 \text{ mm}^4$$

Similarly, moment of inertia of rectangle (2) about an axis through its centre of gravity and parallel to X-X axis,

$$I_{G2} = \frac{60 \times (20)^3}{12} \; 0.04 \times 10^6 \text{ mm}^4$$

and distance of centre of gravity of rectangle (2) from X-X axis,

$$h_2 = 35 - 10 = 25 \text{ mm}$$

∴ Moment of inertia of rectangle (2) about X-X axis

$$= I_{G2} + a h_2^2 = (0.04 \times 10^6) + [1200 \times (25)^2] = 0.79 \times 10^6 \text{ mm}^4$$

Now moment of inertia of the whole section about X-X axis,

$$I_{XX} = (2.117 \times 10^6) + (0.79 \times 10^6) = 2.907 \times 10^6 \text{ mm}^4 \quad \textbf{Ans.}$$

Moment of inertia about centroidal Y-Y axis

Let left face of the angle section be the axis of reference.

Rectangle (1)

$a_1 = 2000 \text{ mm}^2$ ...(As before)

and $x_1 = \frac{20}{2} = 10 \text{ mm}$

Rectangle (2)

$a_2 = 1200 \text{ mm}^2$ ...(As before)

and $x_2 = 20 + \frac{60}{2} = 50 \text{ mm}$

We know that distance between the centre of gravity of the section and left face,

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = \frac{(2000 \times 10) + (1200 \times 50)}{2000 + 1200} = 25 \text{ mm}$$

We know that moment of inertia of rectangle (1) about an axis through its centre of gravity and parallel to Y-Y axis,

$$I_{G1} = \frac{100 \times (20)^3}{12} = 0.067 \times 10^6 \text{ mm}^4$$

and distance of centre of gravity of rectangle (1) from Y-Y axis,

$$h_1 = 25 - 10 = 15 \text{ mm}$$

∴ Moment of inertia of rectangle (1) about Y-Y axis

$$= I_{G1} + a_1 h_1^2 = (0.067 \times 10^6) + [2000 \times (15)^2] = 0.517 \times 10^6 \text{ mm}^4$$

Similarly, moment of inertia of rectangle (2) about an axis through its centre of gravity and parallel to *Y-Y* axis,

$$I_{G2} = \frac{20 \times (60)^3}{12} = 0.36 \times 10^6 \text{ mm}^4$$

and distance of centre of gravity of rectangle (2) from *Y-Y* axis,

$$h_2 = 50 - 25 = 25 \text{ mm},$$

∴ Moment of inertia of rectangle (2) about *Y-Y* axis

$$= I_{G2} + a_2 h_2^2 = 0.36 \times 10^6 + [1200 \times (25)^2] = 1.11 \times 10^6 \text{ mm}^4$$

Now moment of inertia of the whole section about *Y-Y* axis,

$$I_{YY} = (0.517 \times 10^6) + (1.11 \times 10^6) = 1.627 \times 10^6 \text{ mm}^4 \quad \textbf{Ans.}$$

QUESTIONS:-

1) **What is force**? [2]
- ⇨ The external energy required to move the body from one place to the other is a force.
- ⇨ It vectorquantity. It's unit isKN,N

2) **What is moment** ? [2]
- ⇨ The Tendency of force is not only to move a body also to rotate the body.This rotational tendency of a force is called moment.

3) **What is scalar quantities** ?[2]
- ⇨ The scalar quantities are those quantities which have magnitude only such as length, mass, time, distance, volume, density, temperature, speed etc.

4) **What is vector quantities** ? [2]
- ⇨ The vector quantities are those quantities which have both magnitude and direction such as force, displacement, velocity, acceleration, momentum etc.

5) **What is moment ofInertia** ? [2]
- ⇨ We knowthe moment of force is defined as the product of force and Perpendiculardistance (x) between the point and the line of the action of force is called moment.
- ⇨ If this moment is again multiplied by the perpendicular distance (x) between the point and the line of action of the force,this quantity is called moment of Inertia.

6) **What is centre of gravity** ? [2]
- ⇨ Every particles of a body is attracted by the earth towardsits centre.
- ⇨ Centre of gravity, in Physics, an imaginary point in a body of matter where, forconvenience in certain calculations, the total weight of the body may be thought to be concentrated.

Long questions:

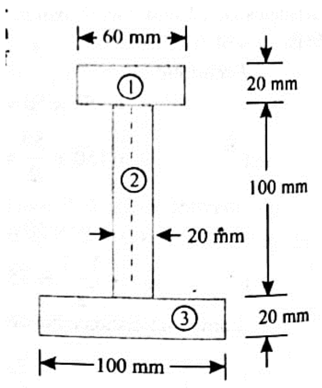
1) Find the Centre of gravity of a 100mm x 150mm x 30mm of ‘T’ section ? [6]

2)Find the centre of gravity of channel section‘C’ 100mm x 50mm x 15mm ? [6]

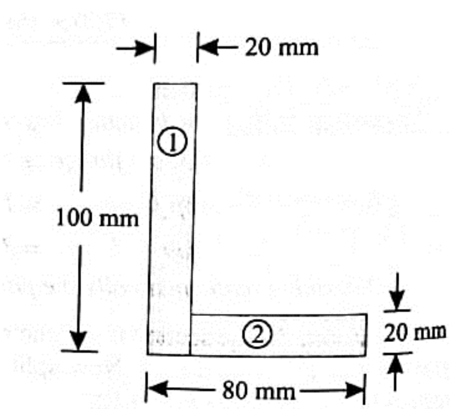
3) Find the centroid of an unequal angle section 100 mm x 80 mm x 20mm ? [6]

Problem :-

1) An I-section is made up three rectangle as shown in figure. find the moment of Inertiaof the sectionabout the horizontal axis passing through the centre of gravity of the section. [10]



2) Find the moment of inertia about the centroidx-x and y-y axes of the angle section shown in figure.[10]



SIMPLE STRESSES AND STRAINS (chapter-2)

INTRODUCTION

Within elastic stage, the resisting force equals applied load. This resisting force perunit area is called stress or intensity of stress.

STRESS:-

The force of resistance per unit area, offered by a body against deformation is known as stress. The external force acting on the body is called the load or force. The load is applied onthe body while the stress is induced in the material of the body. A loaded member remains in equilibrium when the resistance offered by the member against the deformation and the applied load are equal.

Mathematically stress is written as, σ =

Where σ = Stress (also called intensity of stress),

P = Pressure or load, and
A = Cross-Sectional area.

In the S.I. Units, the force is expressed in Newton (Written as N) and area is expressed as m^2. Hence, unit stress becomes as N/m^2. The area is also expressed in millimeter square then unit of force becomes as N/mm^2.

Strain:-

So far, we've focused on the stress within structural elements. When you apply stress to an object, it **deforms**.

Think of a rubber band: you pull on it, and it gets longer – it **stretches**.

Deformation is a measure of how much an object is stretched,

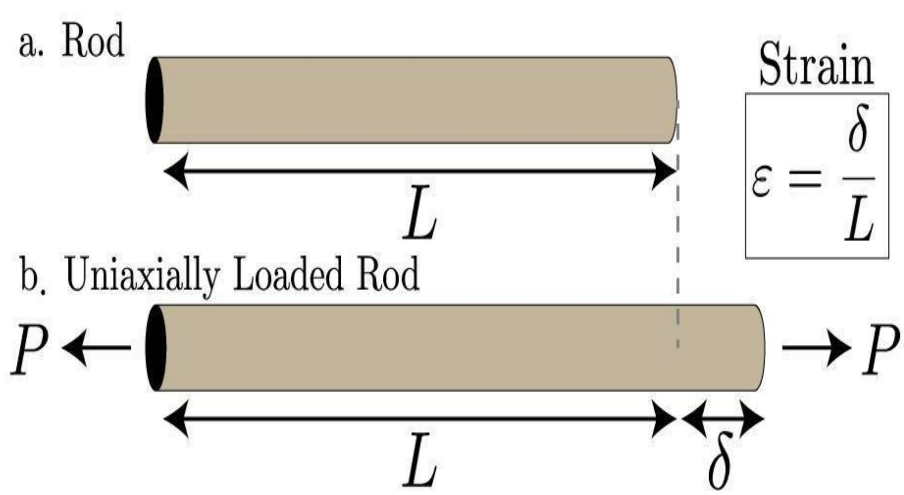
and **strain** is the ratio between the deformation and the original length.

Think of strain as **percent elongation** – how much bigger (or smaller) is the object upon loading it.

Just like stress, there are two types of strain that a structure can experience:

1. **Normal Strain** and 2. **Shear Strain**. When a force acts perpendicular (or "normal") To the surface of an object,
2. It exerts a normal stress. When a force acts parallel to the surface of an object, It exerts a shear stress.

Let's consider a rod under uniaxial tension. The rod elongates under this tension to a new length, and the **normal strain** is a ratio of this small deformation to the rod's original length.



Strain is a **unitless** measure of how much an object gets bigger or smaller from an applied load.

Normal strain occurs when the elongation of an object is in response to a normal stress (i.e. perpendicular to a surface),

and is denoted by the Greek letter **epsilon.** A positive value corresponds to a **tensile** strain, While negative is **compressive**.

Shear strain occurs when the deformation of an object is response to a shear stress (i.e. parallel to a surface), and is denoted by the Greek letter **gamma**.

Mechanics Property of Material

Young's Modulus:-

Within the limits for which Hooke's law is obeyed, the ratio of the direct stress to the strain

Produced are called young's modules or the modules of Elasticity. It is denoted by E

For a bar of uniform cross-section A and length L this can be written as E **= P.L / A.E**

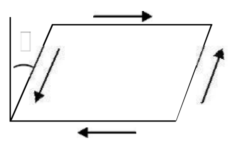
Tangential Stress:-

If the restoring force or deforming force acts perpendicular to the area the the stress is known as normal stresses. Shear stress is tangential to the area over which it acts.

It acts on a line perpendicular to the "longitudinal "and the "radial stress;"

.

Shear Strain



The shear strain or slide can be defined as the change in the right angle .It is measured In radians.

It is the ratio of change in length to its original length .

Modules of rigidity :-

The modulus of rigidity is defined as the ratio of shear stress to shear strain in a body.

$$\text{Modulus of rigidity} = \frac{\textit{Shear Stress}}{\textit{Shear Strain}} \text{ Or } \left(\frac{\tau}{\Phi}\right)$$

The unit of modulus of rigidity in SI is termed as Pascal (Pa). Generally, it is taken as mega Pascal (MPa) or Giga Pascal (GPa). The dimensional term is [$M^1L^{-1}T^{-2}$]. Hence we can understand that the modulus of rigidity is the ratio of shear stress to shear strain in the term of GPa mostly. Here GPa is considered as 10^9 pascal or (1e+9) Pa.

The modulus of rigidity has different values for different materials as we discussed earlier but the modulus of rigidity is zero for ideal gas. An ideal gas particle is having sufficient space and it does not have any frictional resistance, due to not presenting any frictional resistance tangential stress is also zero.

Modulus of Elasticity:-

As per Hooke's law, up to the proportional limit, "for small deformation, stress is directly proportional to strain."

Mathematically, Hooke's Law expressed as:

Stress α Strain

σ = E ε

In the formula as mentioned above, "E" is termed as Modulus of Elasticity.

σ is the Stress, and ε denotes strain.

We can write the expression for Modulus of Elasticity using the above equation as,

E = (F*L) / (A * δL)

So we can define modulus of Elasticity as the ratio of normal stress to longitudinal strain.

Units of modulus elasticity:-

The unit of normal Stress is Pascal, and longitudinal strain has no unit. Because longitudinal strain is the Ratio of change in length to the original length. So the unit of Modulus of Elasticity is same as of Stress, and it is Pascal (Pa). We use most commonly Megapascals (MPa) and Gigapascals (GPa)

To measure the modulus of Elasticity.

1 MPa = Pa

1 GPa = Pa

Compressibility:-

Compressibility is a measure of the relative volume change of a solid or a fluid in response to a pressurechange. For a given mass of fluid, an increase in pressure, Δp> 0, will cause a decrease in volume, ΔV < 0.

Hardness:-

Hardness refers to the property of a material to resist pressing-in or scratch of a sharp object. The materials of different kinds of hardness need various testing methods. The hardness of steel, wood and concrete is tested by pressing-in method.

Stiffness:-

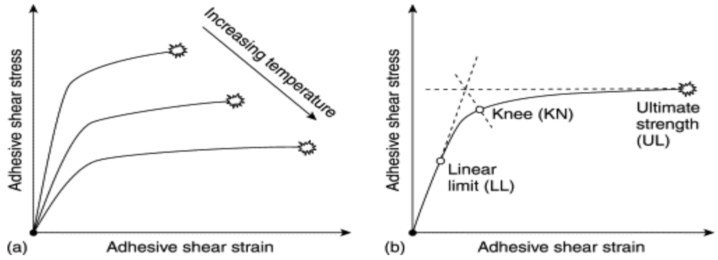
A measure of the material stiffness, described as the ratio of shear stress to shear strain When the material is deformed by a force parallel to its surface.

Brittleness:-

Brittleness describes the property of a material that fractures when subjected to stress but has a little tendency to deform before rupture. Brittle materials are characterized by little deformation, Poor capacity to resist impact and vibration of load, high compressive strength, and low tensile strength.

Ductility:-

Ductility is the ability of a material to be drawn or plastically deformed without fracture. It is therefore an indication of how 'soft' or malleable the material is. The ductility of steels varies depending on the types and levels of alloying elements present.



Malleability:-

Malleability describes the property of a metal's ability to be distorted below compression.
It is a physical property of metals by which they can be hammered, shaped and rolled into a very thin sheet without rupturing. A malleable fabric could be planate by blow or rolling.

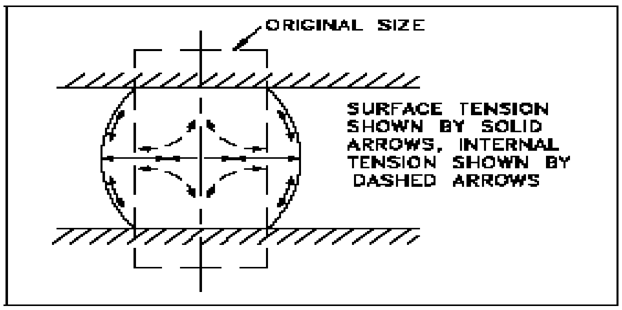


Figure 7 Malleable Deformation of a Cylinder Under Uniform Axial Compression

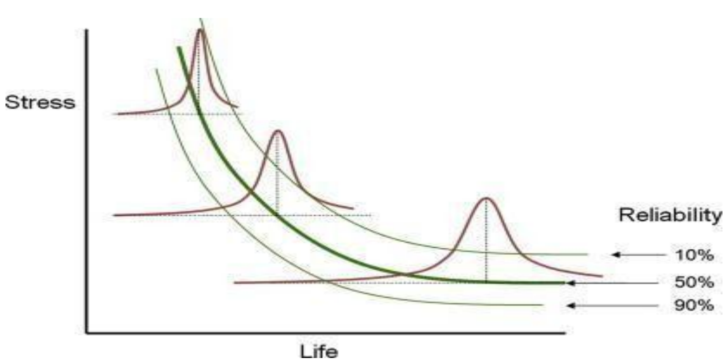
Creep:-

It is the tendency of a solid material to move slowly or deform permanently under the influence Of persistent mechanical stresses. It can occur as a result of long-term exposure to high levels of stress that are still below the yield strength of the material.

- Creep is *the time-dependent deformation below the strength of the material yield of a Material under constant stress*.

Fatigue:-

Fatigue is defined as a process of progressive localized plastic deformation occurring in a material Subjected to cyclic stresses and strains at high stress concentration locations that may culminate in Cracks or complete fracture after a sufficient number of fluctuations.



Tenacity :

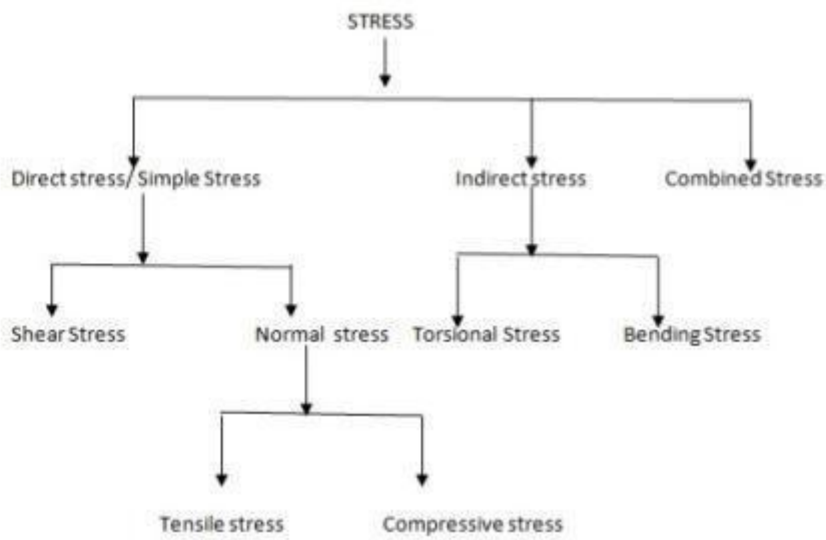
Tenacity refers to a mineral's toughness or resistance to breaking or being deformed.
Minerals such as halite, calcite and fluorite are brittle, as their molecules are held together By weak ionic bonds.

Durability:-

Durability is defined as the ability of a material to remain serviceable in the
Surrounding environment during the useful life without damage or unexpected maintenance.

Types of Stresses:-

There are following type of stresses as displayed in figure here and we will discuss each type of stress
In detail with the help of this post. Basically stresses are classified in to three types.



1. Direct stress or simple stress
2. Indirect stress
3. Combined stress

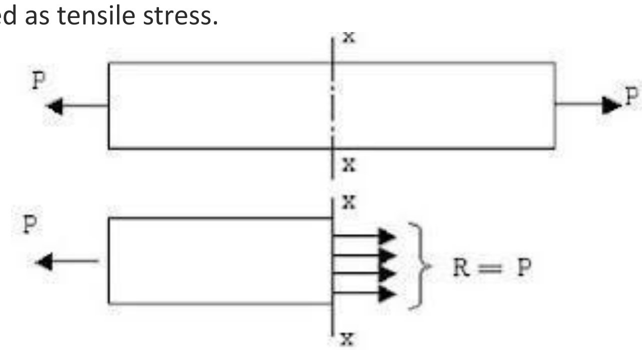
Further direct stress or simple stress is classified in two type i.e. normal stress and shear stress.
As it is also displayed in figure, normal stress will be divided in two type i.e. tensile stress and compressive stress.
Similarly, indirect stress will also be divided in two type i.e. torsion stress and bending stress.
Above figure displayed here indicates the brief introduction for the classification of stress in strength of material.

Normal stress:-

Normal stress is basically defined as the stress acting in a direction perpendicular to the area.
Normal stress will be further divided, as we have seen above, in two types of stresses
i.e. tensile stress and compressive stress.

Tensile stress:-

Let us see here the following figure; we have one bar of length L. There are two equal and opposite pulling type of forces P, acting axially and trying to pull the bar and this pulling action will be termed as tensile and stress developed in material of the bar will be termed as tensile stress.



Therefore we can define the tensile stress as the stress developed in a member due to the pulling action of two equal and opposite direction of forces. σ_t is the symbol which is used to represent the tensile stress in a member.

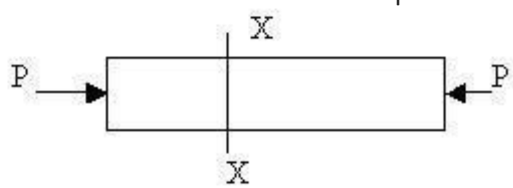
There will be increase in the length of the bar under the action of tensile loading, but diameter of the bar will be reduced under the action of tensile loading. Tensile stress will be determined with the help of following formula.

Tensile stress (σ_t) = Resisting force (R) / Cross sectional area (A)
Tensile stress (σ_t) =Tensile load or applied load (P) / Cross sectional area (A)

$$\sigma_t = P/A$$

Compressive stress:-

Let us see here the following figure; we have one bar of length L. There are two equal and opposite push type of loading P, acting axially and trying to push the bar and this pushing action will be termed as compression and stress developed in material of the bar will be termed as compressive stress.



Therefore we can define the compressive stress as the stress developed in a member due to the pushing action of two equal and opposite direction of forces. σ_c is the symbol which is used to represent the compressive stress in a member.

There will be decrease in the length of the bar under the action of compressive loading, but diameter of the bar will be increased under the action of compressive loading. Compressive stress will be determined with the help of following formula.

Compressive stress (σ_c) = Resisting force (R) / Cross sectional area (A)
Compressive stress (σ_c) = Compressive load or applied load (P) / Cross sectional area (A)

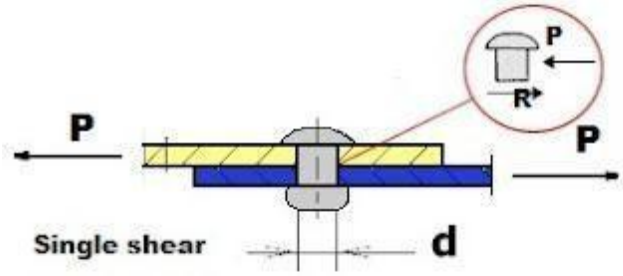
$$\sigma_c = P/A$$

Shear stress:-

Let us see here the following figure; we have two plates and these two plates are connected with each other with the help of a pin or a rivet as shown in figure. There are two equal and opposite forces (P) acting tangentially across the resisting section.

One force is acting on top plate towards left direction and second force is acting towards right side as shown in figure and hence such type of loading will try to shear off the body across the resisting section.

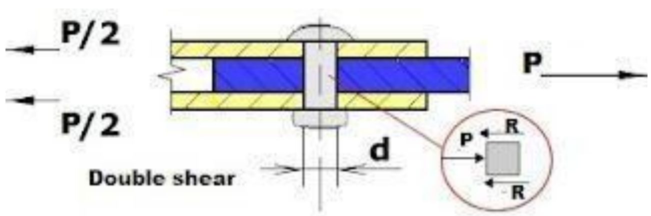
This type of loading action will be termed as shear loading and stress developed in material of the body will be termed as shear stress.



Therefore we can define the shear stress as the stress developed in a member, when member will be subjected
With two equal and opposite forces acting tangentially across the resisting section. τ is the symbol which is used
to represent the shear stress in a member. Shear resistance R will be equivalent to P here.
Shear stress (τ) = P/A

As we have seen above in figure, two plates are connected with each other with the help of a pin and therefore we can also say such type of loading as single shear. Let we have three plates and these three plates are connected with a pin or rivet, such type of loading will be termed as double shear.
Shear resistance R will be P/2.

Shear stress (τ) = P/2A



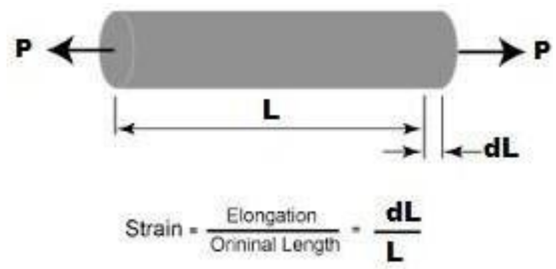
Strain:-
When an external force is applied on a body, there is some change occur in the dimension of the body. The ratio of this change of dimension in the body to its actual dimension is called strain.
Types of Strain:-
Strain in mechanics is of four types and these are:

1. **Tensile strain:**

The strain produced in a body due to tensile force is called the tensile strain. The tensile force always results in the increment of the length and decrease in the cross-section area of the body. In this case, the ratio of the increase in length to the original length is called tensile strain.
Therefore we can define the tensile strain as the strain developed in a member due to the pulling action of two equal and opposite direction of forces. $\mathcal{E}_t$ is the symbol which is used to represent the tensile strain in a member.

Tensile strain will be determined with the help of following formula.
Tensile strain ($\mathcal{E}_t$) = Increase in length (dL) / Original length (L)

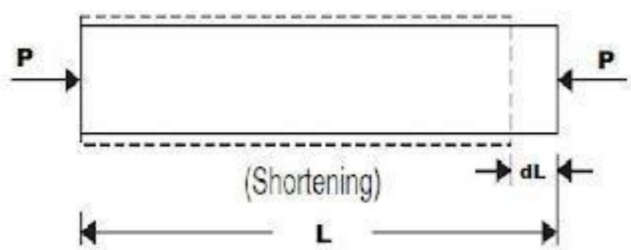
$\mathcal{E}_t = dL / L$



2. Compressive strain:

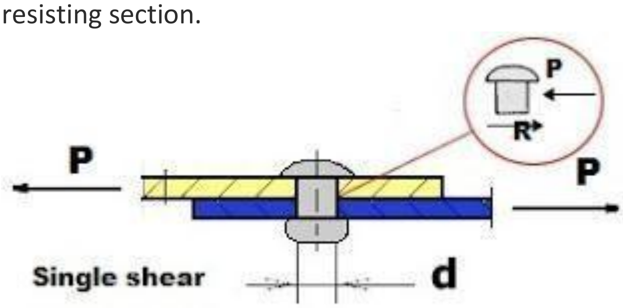
The strain appears due to the compressive force is called compressive strain. In compressive force there is a Decrease in the dimension of the body. So the ratio of the decrease in the length of the body to the original length is called compressive strain.
Compressive strain will be determined with the help of following formula.
Compressive strain ($\mathcal{E}_c$) = Decrease in length (dL) / Original length (L)

$\mathcal{E}_c = dL / L$

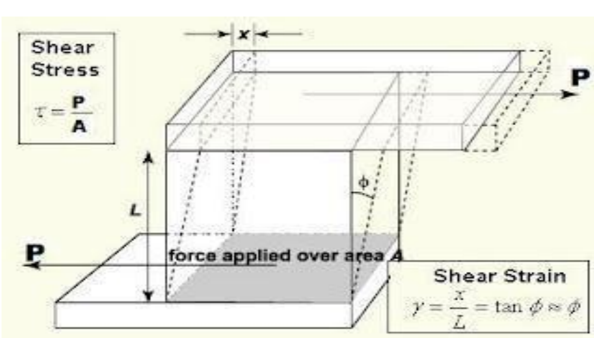


3- Shear strain:-

Let us see here the following figure; we have two plates and these two plates are connected with each other with the help of a pin or a rivet as shown in figure. There are two equal and opposite forces (P) acting tangentially across the resisting section.



One force is acting on top plate towards left direction and second force is acting towards right side as shown in figure and hence such type of loading will try to shear off the body across the resisting section.
This type of loading action will be termed as shear loading and stress developed in material of the body will be termed as shear stress and corresponding strain will be termed as shear strain.
Shear strain will be determined as displayed in following figure

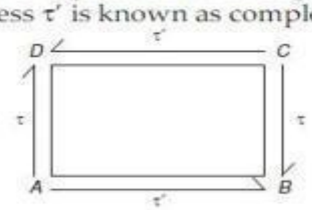


Complementary shear stress:-

Complementary shear stress is *that assumed (pseudo) force which is equal in magnitude and opposite in direction to the given stresses like friction etc*.

Explain complementary shear stress and shear strain.

Complementary shear Stress: When ever a shear stress τ is applied on parallel surface of body then to keep the body in equilibrium a shear stress 'τ'' is induced on remaining surface of body. These stresses form a couple. The couple form due to shear stress τ produces clockwise moment. For equilibrium this couple is balanced by couple developed by τ'. This resisting shear stress τ' is known as complementary shear stress.



Couple produced by τ

$$(\tau\, BC) \times AB$$

Couple produced by τ'

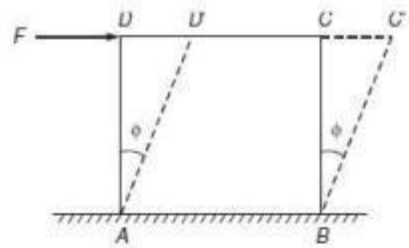
$$\tau'(CD) \times BC$$

For equilibrium

$$(\tau\, BC) \cdot AB = (\tau'\, CD) \cdot BC$$

$$\Rightarrow \quad \tau = \tau' \; \{AB = CD\}$$

SHEAR STRAIN



The measure of small distortion of block caused by shear force is called as shear strain. The block ABCD gets distorted under action of applied tangential force on the face CD.

$$\Rightarrow \quad \text{Shear Strain } e_s = \frac{CC'}{BC} = \tan\phi$$

if ϕ very small

$$e_s = \phi$$

$\Rightarrow$ Shear Strain is ϕ in radian.

Longitudinal strain:-

A longitudinal strain is defined as Change in the length to the original length of an object. It is caused due to longitudinal stress and is denoted by the Greek letter epsilon s.

$$Longitudinal\ strain\ (\varepsilon_l) = \frac{Change\ in\ Length}{Original\ Length}$$

This type of strain is produced when the deforming force causes a change in the length of the body. It is defined as the ratio of the change in length to the original length of the body.

Consider a rod of length (L) having an area of cross-section (A). Weight is added at the free end of the rod that applies force (F) on the rod. Initially, the rod is hinged from the roof and the weight is not added. After the weight is added to the rod, the length of the rod gets extended.

Original length = L

Area of cross-section = A

Applied force due to weight = F

Final length = L + ΔL

Change in length = Final length - Original length = $L + \Delta L - L = \Delta L$

Lateral Strain:-

Whenever the bar is subjected to the axial load, there will be decrease in the dimensions of the bar in the perpendicular direction of loading. Therefore lateral strain is defined as ratio of decrease in the length of the bar in the perpendicular direction of applied load to that of the original length (gauge length).
i.e. e = dB/B or dD/D

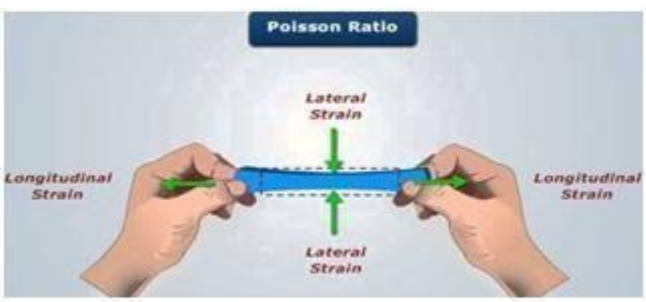
Where
e = lateral strain
dd= decrease in depth
D= gauge or original depth
db= decrease in breadth
B = gauge or original breadth

Poisson's ratio:

The ratio of lateral strain to that of the longitudinal strain is termed as poison's ratio and it is represented by ᴍ or 1/m.

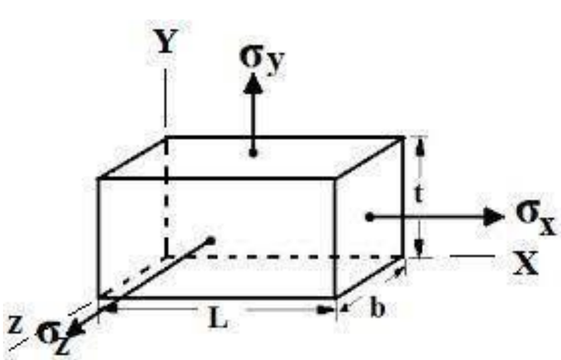
i.e., **ᴍ or 1/m = lateral strain / longitudinal strain**

Value of the Poisson's ratio for most materials lies between 0.25 and 0.33



Volumetric Strain:-

Volumetric strain is a ratio of change in volume of a body to its original volume.
SI unit for both change in volume and volume will be m^3.
So, the volumetric strain will be a **unitless quantity**.
The volumetric strain is generally denoted by E_V.
Therefore, the equation for volumetric strain will be $E_V = \Delta V/V$.

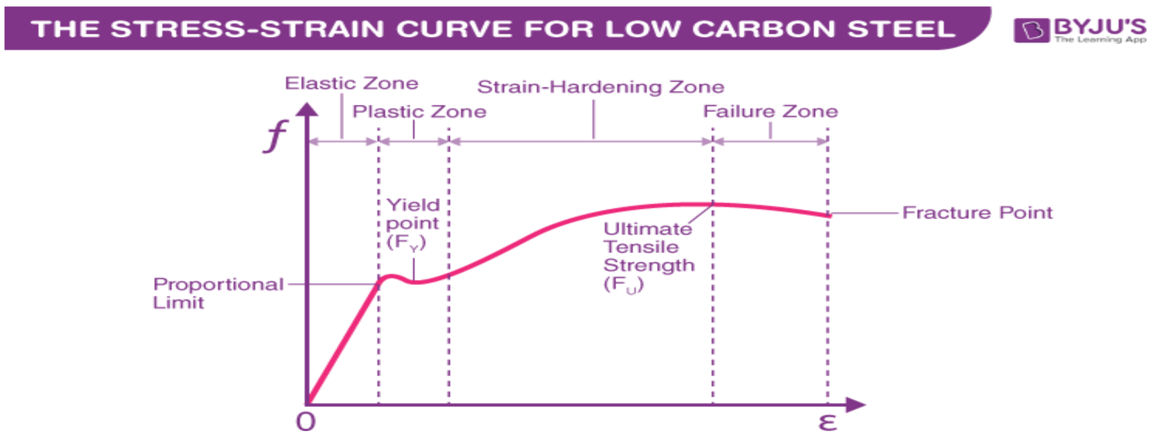


Hooke's Law:-

Hooke's law states that the strain of the material is proportional to the applied stress within the elastic limit of that material.

Hooke's Law Graph:-

The figure below shows the stress-strain curve for low carbon steel.



The material exhibits elastic behavior up to the yield strength point, after which the material loses elasticity and exhibits plasticity.

From the origin till the proportional limit nearing yield strength, the straight line implies that the material follows Hooke's law. Beyond the elastic limit between proportional limit and yield strength,

the material loses its elasticity and exhibits plasticity. The area under the curve from origin to the proportional limit falls under the elastic range. The area under the curve from a proportional limit

to the rupture/fracture point falls under the plastic range.

The material's ultimate strength is defined based on the maximum ordinate value given by the stress-strain curve (from origin to rupture). The value provides the rupture with strength at a point of rupture.

Hooke's Law Applications:-

Following are some of the applications of Hooke's Law:

- It is used as a fundamental principle behind the manometer, spring scale, and the balance wheel of the clock.
- Hooke's law sets the foundation for seismology, acoustics and molecular mechanics.

Hooke's Law Disadvantages:-

Following are some of the disadvantages of Hooke's Law:

- Hooke's law ceases to apply past the elastic limit of a material.

- Hooke's law is accurate only for solid bodies if the forces and deformations are small.
- Hooke's law isn't a universal principle and only applies to the materials as long as they aren't stretched way past their capacity.

STRESS & STRAIN DIAGRAM FOR MILD STEEL MATERIAL:-

When steel is curved, it is important to keep the stress-strain curve ratio for mild steel in mind.

Below is a stress-strain graph that reviews the properties of steel in detail.

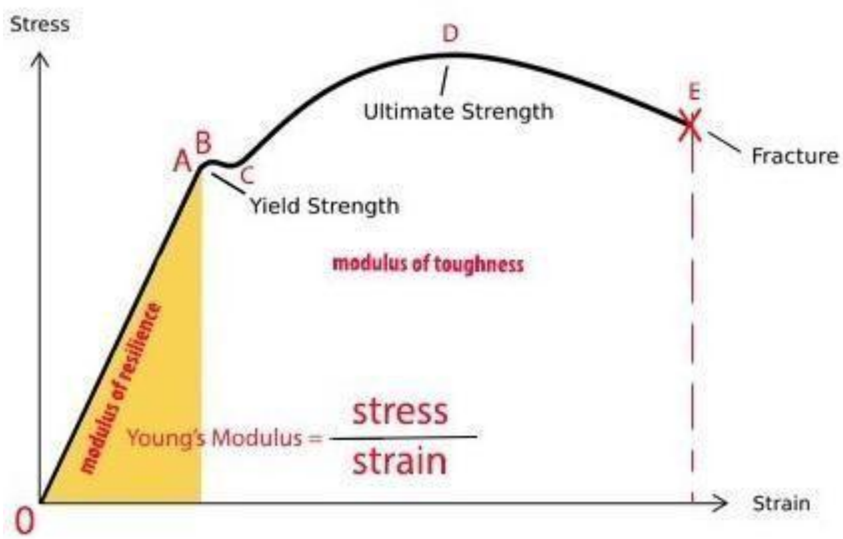
If tensile force is applied to a steel bar, it will have some elongation. If the force is small enough,

the ratio of the stress and strain will remain proportional.

This can be seen in the graph as a straight line between zero and point A – also called the

Limit of proportionality.

If the force is greater, the material will experience elastic deformation, but the ratio of stress and strain will not be proportional. This is between points A and B, known as the elastic limit.



Beyond the elastic limit, the mild steel will experience plastic deformation.

This starts the yield point – or the rolling point – which is point B, or the upper yield point.

As seen in the graph, from this point on the correlation between the stress and strain is no longer

on a straight trajectory. It curves from point C (lower yield point), to D (maximum ultimate stress),

Ending at E (fracture stress).

QUESTION & ANSWER

SHORT TYPE QUESTION……

1-(a) what is Stress?

Ans: - The force of resistance per unit area, offered by a body against deformation is known as stress. The external force acting on the body is called the load or force. The load is applied onthe body while the stress is induced in the material of the body. A loaded member remains in equilibrium when the resistance offered by the member against the deformation and the applied load are equal.

(b) What is strain?

Ans: - When an external force is applied on a body, there is some change occur in the dimension Of the body.

The ratio of this change of dimension in the body to its actual dimension is called strain.

(c) What is Hooke' s law ?

Ans: - Hooke's law states that the strain of the material is proportional to the applied stress within the elastic limit of that material.

(d) What are the advantages of Hooke's law?

Ans: - It is used as a fundamental principle behind the manometer, spring scale, and

the balance wheel of the clock.

Hooke's law sets the foundation for seismology, acoustics and molecular mechanics.

(e) What are the disadvantages of Hooke's law?

Ans:-

- Hooke's law ceases to apply past the elastic limit of a material.
- Hooke's law is accurate only for solid bodies if the forces and deformations are small.
- Hooke's law isn't a universal principle and only applies to the materials as long as they aren't stretched way past their capacity.

(f) What is Volumetric strain?

Ans: Volumetric strain is a ratio of change in volume of a body to its original volume.

SI unit for both change in volume and volume will be m^3.

So, the volumetric strain will be a **unitless quantity**.

The volumetric strain is generally denoted by E_V.

Therefore, the equation for volumetric strain will be $E_V=\Delta V/V$.

(g) What is longitudinal strain?

Ans:-

A longitudinal strain is defined as Change in the length to the original length of an object.

It is caused due to longitudinal stress and is denoted by the Greek letter epsilon ꜱ.

(h) What is Poisson's ratio?

Ans :-

The ratio of lateral strain to that of the longitudinal strain is termed as poison's ratio and it is represented by ᴍ or 1/m.

i.e., **ᴍ or 1/m = lateral strain / longitudinal strain**

Value of the Poisson's ratio for most materials lies between 0.25 and 0.33

(i) What is tensile strain?

Ans:-

The strain produced in a body due to tensile force is called the tensile strain. The tensile Force always results in the increment of the length and decrease in the cross-section area of the body. In this case, the ratio of the increase in length to the original length is called tensile strain.

(j) What is Fatigue?

Ans: - Fatigue is defined as a process of progressive localized plastic deformation occurring in a material Subjected to cyclic stresses and strains at high stress concentration locations that may culminate in Cracks or complete fracture after a sufficient number of fluctuations.

LONG TYPE QUESTIONS…..

1- Draw stress-strain diagram for ductile material (mild steel)?

2- Derive Hooke's law?

3- Describe different types of stress?

4- Describe different types of strain?

5- Describe shear stress & shear strain?

COLUMN AND STRUTS(ch-4)

INTRODUCTION :

Column or strut is defined as a member of a structure, which is subjected to axial compressive load. If the member of the structure is vertical and both of its ends are fixed rigidly while subjected to axial compressive load, the member is known as column, for example a vertical pillar between the roof and floor. If the member of the structure is not vertical and one or both of its ends are hinged or pin joined, the bar is known as strut i.e. connecting rods, piston rods etc.

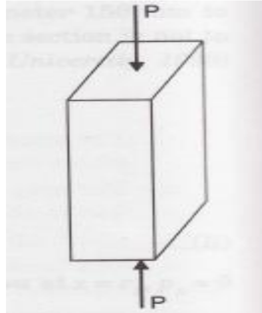
FAILURE OF A COLUMN:

The failure of a column takes place due to the anyone of the following stresses set up in thecolumns:

a) Direct compressive stresses.
b) Buckling stresses.
c) Combined of direct compressive and buckling stresses.

Failure of a Short Column:

A short column of uniform cross-sectional area A, subjected to an axial compressive load P, asshown in Fig. The compressive stress induced is given by; **p=P/A**



If the compressive load on the short column is gradually increased, a stage will reach when the column will be on the point of failure by crushing. The stress induced in the column corresponding to this load is known as crushing stress and the load is called crushing load.

Let, P_c = Crushing load,
σ_c= Crushing stress, and A = Area of cross-section

$$\sigma_c=P_c/A$$

All short columns fail due to crushing.

Failure of a Long Column:

A long column of uniform cross-sectional area A and of length l, subjected to an axial compressive load P, is shown in Fig. A column is known as long column, if the length of the column in comparison to its lateral dimensions, is very large Such columns do not fail bycrushing alone, but also by bending (also known buckling) as shown in figure. The buckling load at which the column just buckles, is known as buckling or crippling load. The buckling load isless than the crushing load for a long column. Actually the value of buckling load for long columns is low whereas for short columns the value of buckling load is relatively high.

Let l = Length of a long column
P = Load (compressive) at which the column has just buckledA = Cross-sectional area of the column
e = Maximum bending of the column at the centreσ_o = Stress due to direct load = P/A
σ_b = Stress due to bending at the centre of the column = (P x e) /Z

Where,
Z = Section modulus about the axis of bending. The extreme stresses on the mid-section are given by:

Maximum stress = $\sigma_o + \sigma_b$ and
Minimum stress = $\sigma_o - \sigma_b$

The column will fail when maximum stress (i.e., $\sigma_o + \sigma_b$) is more than the crushing stress σ_c. But in case of long columns, the direct compressive stresses are negligible as compared to buckling stresses. Hence very long columns are subjected to buckling stresses only.

Assumptions made in the Euler's Column theory:

The following assumptions are made in the Euier's column theory:

1. The column is initially perfectly straight and the load is applied axially.
2. The cross-section of the column is uniform throughout its length.
3. The column material is perfectly elastic, homogeneous and isotropic and obeys Hooke'slaw.
4. The length of the column is very large as compared to its lateral dimensions.
5. The direct stress is very small as compared to the bending stress.
6. The column will fail by buckling alone.

7. The self-weight of column is negligible.

End conditions for Long Columns:

In case of long columns, the stress due to direct load is very small in comparison with the stress due to buckling. Hence the failure of long columns

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End conditions for Long Columns: In case of long columns, the stress due to direct loadis very small in comparison with the stress due to buckling. Hence the failure of long columns.

takes place entirely due to buckling (or bending). The following four types of end conditions ofthe columns are important:

1. Both the ends of the column are hinged (or pinned).
2. One end is frxed and the other end is free.
3. Both the ends of the column are frxed.
4. One end is frxed and the other is pinned.

For a hinged end, the deflection is zero. For a fixed end the deflection and slope are zero.For a free end the deflection is not zero.

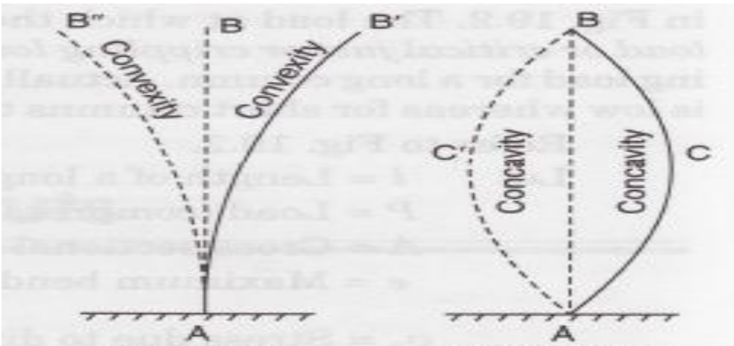
Sign Conventions:

The following sign conventions for the bending of the columns will be used :

1. A moment which will bend the column with its convexity towards its initial central line as shown in Fig.

(a) is taken as positive. In Fig (a), AB represents the initial centre line of a column. Whether the column bends taking the shape AB' or AB", the moment producing this type of curvature is positive.

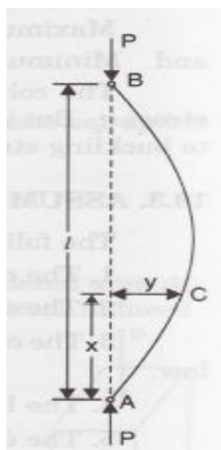
2. A moment which will tend to bend the column with its concavity towards its initial centre line as shown in Fig. (b) is taken as negative.



(a) Positive (b) Negative

Expression for crippling load when both the ends of the Column are hinged:

The load at which the column just buckles (or bends) is called crippling load. Consider a columnAB of length l and uniform cross-sectional area, hinged at both of its ends A and B. Let P be thecrippling load at which the column has just buckled. Due to the crippling load, the column willdeflect into a curved form ACB as shown in Fig.



Consider any section at a distance r from the end A.

Let y = Deflection (lateral displacement) at the section.

The moment due to the crippling load at the section = -P x y (- ve sign is taken due to signconvention)

But moment $= EI \dfrac{d^2y}{dx^2}$.

Equating the two moments, we have

$$EI \frac{d^2y}{dx^2} = -P \cdot y \quad \text{or} \quad EI \frac{d^2y}{dx^2} + P \cdot y = 0$$

or

$$\frac{d^2y}{dx^2} + \frac{P}{EI} \cdot y = 0$$

The solution* of the above differential equation is

$$y = C_1 \cdot \cos\left(x\sqrt{\frac{P}{EI}}\right) + C_2 \cdot \sin\left(x\sqrt{\frac{P}{EI}}\right)$$

Where C_1 and C_2 are the constants of integration and the values are obtained as follows:At A, x = 0 and y = 0

Substituting these values in equation (*i*), we get

$$0 = C_1 \cdot \cos 0° + C_2 \sin 0$$

$$= C_1 \times 1 + C_2 \times 0 \qquad (\because \cos 0 = 1 \text{ and } \sin 0 = 0)$$

$$= C_1$$

$$\therefore \quad C_1 = 0. \qquad ...(ii)$$

(*ii*) At B, $x = l$ and $y = 0$ (See Fig. 19.4).

Substituting these values in equation (*i*), we get

$$0 = C_1 \cdot \cos\left(l \times \sqrt{\frac{P}{EI}}\right) + C_2 \cdot \sin\left(l \times \sqrt{\frac{P}{EI}}\right)$$

$$= 0 + C_2 \cdot \sin\left(l \times \sqrt{\frac{P}{EI}}\right) \qquad [\because C_1 = 0 \text{ from equation } (ii)]$$

$$= C_2 \sin\left(l\sqrt{\frac{P}{EI}}\right) \qquad ...(iii)$$

From equation (*iii*), it is clear that either $C_2 = 0$

$$\sin\left(l\sqrt{\frac{P}{EI}}\right) = 0.$$

As if $C_1 = 0$, then if C_2 is also equals to zero, then from Eq^n no. (i), we will find that y = 0. This means that the bending of the column will be zero or the column will not bend at all, which is not true.

$$\therefore \quad \sin\left(l\sqrt{\frac{P}{EI}}\right) = 0$$

$$= \sin 0 \text{ or } \sin \pi \text{ or } \sin 2\pi \text{ or } \sin 3\pi \text{ or } ...$$

$$\text{or} \quad l\sqrt{\frac{P}{EI}} = 0 \text{ or } \pi \text{ or } 2\pi \text{ or } 3\pi \text{ or } ...$$

Taking the least practical value.

Expression for crippling load when one end of the column is fixed and the other end is free :

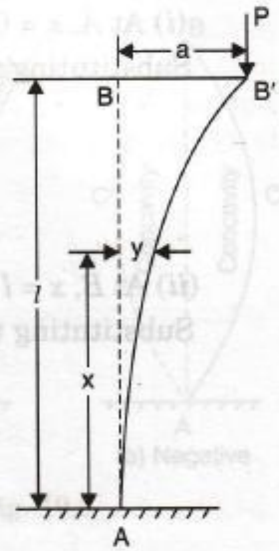
Consider a column AB, of length l and uniform cross-sectional area, fixed at the end A and freeat the end B. The free end will sway sideways when load is applied at free end and curvature in the length I will be similar to that of upper half of the column whose both ends are hinged.

Let P is the crippling load at which the column has just buckled. Due to the crippling load P, the column will deflect as shown in Fig., in which AB is the original position of the column and AB', is the deflected position due to crippling load P.

Consider any section at a distance x from the fixed end A.

Let y = Deflection (or lateral displacement) at the section a = Deflection at the free end B'

Then moment at the section due to the crippling load = P (a - y) (+ve. sign is taken due to sign convention)



But moment is also $= EI \dfrac{d^2y}{dx^2}$

$\therefore$ Equating the two moments, we get

$$EI \frac{d^2y}{dx^2} = P(a - y) = P \cdot a - P \cdot y$$

or
$$EI \frac{d^2y}{dx^2} + P \cdot y = P \cdot a$$

or
$$\frac{d^2y}{dx^2} + \frac{P}{EI} \cdot y = \frac{P}{EI} \cdot a. \quad \text{...(A)}$$

1- At fixed end, the deflection as well as slope will be zero.

Hence at end A (which is fixed), the deflection $y = 0$ and also slope $\frac{dy}{dx} = 0$.

Hence at A, $x = 0$ and $y = 0$

Substituting these values in equation (*i*), we get

$$0 = C_1 \cdot \cos 0 + C_2 \sin 0 + a$$
$$= C_1 \times 1 + C_2 \times 0 + a \qquad (\because \cos 0 = 1, \sin 0 = 0)$$
$$= C_1 + a$$

$\therefore$ $$C_1 = -a \qquad \text{...(ii)}$$

At A, $x = 0$ and $\frac{dy}{dx} = 0$.

Differentiating the equation (*i*) w.r.t. x, we get

$$\frac{dy}{dx} = C_1 \cdot (-1) \sin\left(x\sqrt{\frac{P}{EI}}\right) \cdot \sqrt{\frac{P}{EI}} + C_2 \cos\left(x\sqrt{\frac{P}{EI}}\right) \cdot \sqrt{\frac{P}{EI}} + 0$$

$$= -C_1 \cdot \sqrt{\frac{P}{EI}} \sin\left(x\sqrt{\frac{P}{EI}}\right) + C_2 \cdot \sqrt{\frac{P}{EI}} \cos\left(x\sqrt{\frac{P}{EI}}\right)$$

But at A, $x = 0$ and $\frac{dy}{dx} = 0$.

$\therefore$ The above equation becomes as

$$0 = -C_1 \cdot \sqrt{\frac{P}{EI}} \sin 0 + C_2 \sqrt{\frac{P}{EI}} \cos 0$$

$$= -C_1 \sqrt{\frac{P}{EI}} \times 0 + C_2 \cdot \sqrt{\frac{P}{EI}} \times 1 = C_2 \sqrt{\frac{P}{EI}}.$$

From the above equation it is clear that either $C_2 = 0$.

$$\sqrt{\frac{P}{EI}} = 0.$$

But for the crippling load P, the value of $\sqrt{\frac{P}{EI}}$ cannot be equal to zero.

$\therefore$ $C_2 = 0$.

Substituting the values of $C_1 = -a$ and $C_2 = 0$ in equation (*i*), we get

$$y = -a \cdot \cos\left(x\sqrt{\frac{P}{EI}}\right) + a. \qquad \text{...(iii)}$$

But at the free end of the column, x = l and y = a,

Substituting these values in equation (*iii*), we get

$$a = -a \cdot \cos\left(l \cdot \sqrt{\frac{P}{EI}}\right) + a$$

or $$0 = -a \cdot \cos\left(l \cdot \sqrt{\frac{P}{EI}}\right) \text{ or } a \cos\left(l \cdot \sqrt{\frac{P}{EI}}\right) = 0$$

But 'a' cannot be equal to zero

$$\therefore \quad \cos\left(l.\sqrt{\frac{P}{EI}}\right) = 0 = \cos\frac{\pi}{2} \text{ or } \cos\frac{3\pi}{2} \text{ or } \cos\frac{5\pi}{2} \text{ or } \ldots\ldots$$

$$\therefore \quad l.\sqrt{\frac{P}{EI}} = \frac{\pi}{2} \text{ or } \frac{3\pi}{2} \text{ or } \frac{5\pi}{2} \ldots\ldots$$

Taking the least practical value,

$$l.\sqrt{\frac{P}{EI}} = \frac{\pi}{2} \quad \text{or} \quad \sqrt{\frac{P}{EI}} = \frac{\pi}{2l}$$

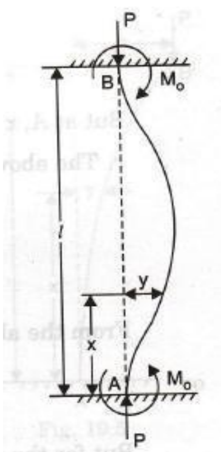
or

$$P = \frac{\pi^2 EI}{4l^2}$$

Expression for the crippling load when both ends of the column are fixed:

Consider a column AB of length l and uniform cross-sectional area fixed at both ends A and B asshown in Fig. Let P is the crippling load at which the column has buckled. Due to the cripplingload P, the column will deflect as shown. Due to fixed ends, there will be fixed end moments sayM_0 at the ends A and B. The fixed end moments will be acting in such direction so that slope atthe fixed ends becomes zero.

Consider a section at a distance x from the end A. Let the deflection of the column at the section is y. As both the ends of the column are fixed and the column carries a crippling load, there will be some fixed end moments at A and B.



Let M_0 = Fixed end moments at A and B.

Then moment at the section = $M_0 - P.y$

But moment at the section is also = $EI\,\frac{d^2y}{dx^2}$

$\therefore$ Equating the two moments, we get

$$EI\,\frac{d^2y}{dx^2} = M_0 - P.y$$

or

$$EI\,\frac{d^2y}{dx^2} + P.y = M_0$$

or $$\frac{d^2y}{dx^2} + \frac{P}{EI} \cdot y = \frac{M_0}{EI} \qquad ...(A)$$

$$= \frac{M_0}{EI} \times \frac{P}{P} = \frac{P}{EI} \cdot \frac{M_0}{P}$$

The solution* of the above differential equation is

$$y = C_1 \cdot \cos\left(x \cdot \sqrt{\frac{P}{EI}}\right) + C_2 \cdot \sin\left(x \cdot \sqrt{\frac{P}{EI}}\right) + \frac{M_0}{P} \qquad ...(i)$$

where C_1 and C_2 are constant of integration and their values are obtained from boundary conditions. Boundary conditions are :

(i) At A, $x = 0$, $y = 0$ and also $\frac{dy}{dx} = 0$ as A is a fixed end.

(ii) At B, $x = l$, $y = 0$ and also $\frac{dy}{dx} = 0$ as B is also a fixed end.

Substituting the value x = 0 and y = 0 in equation no (i), we get.

$$0 = C_1 \times 1 + C_2 \times 0 + \frac{M_0}{P}$$

$$= C_1 + \frac{M_0}{P}$$

$$C_1 = -\frac{M_0}{P}$$

Differentiating equation (i), with respect to x, we get.

$$\frac{dy}{dx} = C_1 \cdot (-1) \sin\left(x \cdot \sqrt{\frac{P}{EI}}\right) \cdot \sqrt{\frac{P}{EI}} + C_2 \cos\left(x \cdot \sqrt{\frac{P}{EI}}\right) \cdot \sqrt{\frac{P}{EI}} + 0$$

$$= -C_1 \sin\left(x \cdot \sqrt{\frac{P}{EI}}\right) \cdot \sqrt{\frac{P}{EI}} + C_2 \cos\left(x \cdot \sqrt{\frac{P}{EI}}\right) \cdot \sqrt{\frac{P}{EI}}$$

Substituting the value $x = 0$ and $\frac{dy}{dx} = 0$, the above equation becomes

$$0 = -C_1 \times 0 + C_2 \times 1 \times \sqrt{\frac{P}{EI}} \qquad (\because \sin 0 = 0 \text{ and } \cos 0 = 1)$$

$$= C_2 \sqrt{\frac{P}{EI}}$$

From the above equation, it is clear that either $C_2 = 0$ or $\sqrt{\frac{P}{EI}} = 0$. But for a given crippling load P, the value of $\sqrt{\frac{P}{EI}}$ cannot be equal to zero.

$\therefore$ $C_2 = 0.$

Now substituting the values of $C_1 = -\frac{M_0}{P}$ and $C_2 = 0$ in equation (i), we get

$$y = -\frac{M_0}{P} \cos\left(x\sqrt{\frac{P}{EI}}\right) + 0 + \frac{M_0}{P}$$

$$= -\frac{M_0}{P} \cos\left(x\sqrt{\frac{P}{EI}}\right) + \frac{M_0}{P} \qquad ...(iii)$$

At the end B of the column, $x = l$ and $y = 0$.

Substituting these values in equation (iii), we get

$$0 = -\frac{M_0}{P} \cos\left(l \cdot \sqrt{\frac{P}{EI}}\right) + \frac{M_0}{P}$$

or $$\frac{M_0}{P} \cos\left(l \cdot \sqrt{\frac{P}{EI}}\right) = \frac{M_0}{P}$$

or $$\cos\left(l \cdot \sqrt{\frac{P}{EI}}\right) = \frac{M_0}{P} \times \frac{P}{M_0} = 1 = \cos 0, \cos 2\pi, \cos 4\pi, \cos 6\pi, \ldots\ldots$$

$$\therefore \quad l \cdot \sqrt{\frac{P}{EI}} = 0, 2\pi, 4\pi, 6\pi, ...$$

Taking the least practical value,

$$l \cdot \sqrt{\frac{P}{EI}} = 2\pi \quad \text{or} \quad P = \frac{\pi^2 EI}{l^2}.$$

Expression for the crippling load when one end of the column is fixed and the other end is hinged....

Consider a column AB of length l and uniform cross-sectional area, fixed at the end A and hinged at the end B as shown in Fig. Let P is the crippling load at which the column has buckled. Due to the crippling load P, the column will deflect as shown in Fig. There will be fixed end moment (M_0) at the fixed end A. This will try to bring back the slope of deflected column zero at

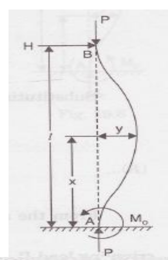
A. Hence it will be acting anticlockwise at A. The fixed end moment M_0 at A is to be balanced. This will be balanced by a horizontal reaction (H) at the top end B as shown in Fig.

Consider a section at a distance x from the end A Let y =

Deflection of the column at the section,

M_0 = Fixed end moment at A, and

H = Horizontal reaction at B.



The moment at the section = Moment due to crippling load at B + Moment due to horizontal reaction at B

$$= -P \cdot y + H \cdot (l - x)$$

But the moment at the section is also

$$= EI \frac{d^2y}{dx^2}$$

Equating the two moments, we get

$$EI \frac{d^2y}{dx^2} = -P \cdot y + H(l - x)$$

or

$$EI \frac{d^2y}{dx^2} + P \cdot y = H(l - x)$$

or

$$\frac{d^2y}{dx^2} + \frac{P}{EI} \cdot y = \frac{H}{EI}(l - x) \quad \text{(Dividing by } EI\text{)} \quad ...(A)$$

$$= \frac{H}{EI}(l - x) \times \frac{P}{P} = \frac{P}{EI} \cdot \frac{H(l - x)}{P}$$

The solution* of the above differential equation is

$$y = C_1 \cos\left(x\sqrt{\frac{P}{EI}}\right) + C_2 \sin\left(x\sqrt{\frac{P}{EI}}\right) + \frac{H}{P}(l-x) \qquad \ldots(i)$$

where C_1 and C_2 are constants of integration and their values are obtained from boundary conditions. Boundary conditions are :

(*i*) At the fixed end A, $x = 0$, $y = 0$ and also $\frac{dy}{dx} = 0$

(*ii*) At the hinged end B, $x = l$ and $y = 0$.

Substituting the value $x = 0$ and $y = 0$ in equation (*i*), we get

$$0 = C_1 \times 1 + C_2 \times 0 + \frac{H}{P}(l-0) = C_1 + \frac{H \cdot l}{P}$$

$$\therefore \qquad C_1 = -\frac{H}{P} \cdot l \qquad \ldots(ii)$$

Differentiating the equation (*i*) w.r.t. x, we get

$$\frac{dy}{dx} = C_1(-1)\sin\left(x \cdot \sqrt{\frac{P}{EI}}\right) \cdot \sqrt{\frac{P}{EI}} + C_2 \cos\left(x \cdot \sqrt{\frac{P}{EI}}\right) \cdot \sqrt{\frac{P}{EI}} - \frac{H}{P}$$

$$= -C_1 \sin\left(x \cdot \sqrt{\frac{P}{EI}}\right) \cdot \sqrt{\frac{P}{EI}} + C_2 \cos\left(x \cdot \sqrt{\frac{P}{EI}}\right) \cdot \sqrt{\frac{P}{EI}} - \frac{H}{P}$$

At A, $x = 0$ and $\frac{dy}{dx} = 0$.

$$\therefore \qquad 0 = -C_1 \times 0 + C_2 \cdot 1 \cdot \sqrt{\frac{P}{EI}} - \frac{H}{P} \qquad (\because \ \sin 0 = 0, \cos 0 = 1)$$

$$= C_2\sqrt{\frac{P}{EI}} - \frac{H}{P} \quad \text{or} \quad C_2 = \frac{H}{P}\sqrt{\frac{EI}{P}}$$

Substituting the values of $C_1 = -\frac{H}{P} \cdot l$ and $C_2 = \frac{H}{P}\sqrt{\frac{EI}{P}}$ in equation (*i*), we get

$$y = -\frac{H}{P} \cdot l \cos\left(x\sqrt{\frac{P}{EI}}\right) + \frac{H}{P}\sqrt{\frac{EI}{P}} \sin\left(x\sqrt{\frac{P}{EI}}\right) + \frac{H}{P}(l-x)$$

At the end B, $x = l$ and $y = 0$.

Hence the above equation becomes as

$$0 = -\frac{H}{P}\, l \cos\left(l\sqrt{\frac{P}{EI}}\right) + \frac{H}{P}\sqrt{\frac{EI}{P}} \sin\left(l\sqrt{\frac{P}{EI}}\right) + \frac{H}{P}(l-l)$$

$$= -\frac{H}{P}\, l \cos\left(l\sqrt{\frac{P}{EI}}\right) + \frac{H}{P}\sqrt{\frac{EI}{P}} \sin\left(l\sqrt{\frac{P}{EI}}\right) + 0$$

or $$\frac{H}{P}\sqrt{\frac{EI}{P}} \sin\left(l\sqrt{\frac{P}{EI}}\right) = \frac{H}{P}\, l \cos\left(l\sqrt{\frac{P}{EI}}\right)$$

or $$\sin\left(l\sqrt{\frac{P}{EI}}\right) = \frac{H}{P} \cdot l \times \frac{P}{H} \times \sqrt{\frac{P}{EI}} \cdot \cos\left(l\sqrt{\frac{P}{EI}}\right)$$

$$= l \cdot \sqrt{\frac{P}{EI}} \cdot \cos\left(l \cdot \sqrt{\frac{P}{EI}}\right)$$

or $$\tan\left(l\sqrt{\frac{P}{EI}}\right) = l \cdot \sqrt{\frac{P}{EI}}.$$

or

The solution to the above equation is, $l\sqrt{\frac{P}{EI}} = 4.5$ radians

Squaring both sides, we get

$$l^2 \cdot \frac{P}{EI} = 4.5^2 = 20.25$$

$$\therefore \qquad P = 20.25\,\frac{EI}{l^2}$$

But approximately $20.25 = 2\pi^2$

$$\therefore \qquad P = \frac{2\pi^2 EI}{l^2}$$

Effective length (or equivalent length) of a column:

The effective length of a given column with given end conditions is the length of an equivalentcolumn of the same material and cross-section with hinged ends and having the value of thecrippling load equal to that of the given column. Effective length is also called equivalent length. Let L_e = Effective length of a column

l = Actual length of the column and

P = Crippling load for the column

Then the crippling load for any type of end condition is given by

$$P = \frac{\pi^2 EI}{L_e^2}.$$

The crippling load (P) in terms of actual length and effective length and also the relation betweeneffective length and actual length are given in Table below.

S.No.	End conditions of column	Crippling load in terms of		Relation between effective length and actual length
		Actual length	Effective length	
1.	Both ends hinged	$\frac{\pi^2 EI}{l^2}$	$\frac{\pi^2 EI}{L_e^2}$	$L_e = l$
2.	One end is fixed and other is free	$\frac{\pi^2 EI}{4l^2}$	$\frac{\pi^2 EI}{L_e^2}$	$L_e = 2l$
3.	Both ends fixed	$\frac{4\pi^2 EI}{l^2}$	$\frac{\pi^2 EI}{L_e^2}$	$L_e = \frac{l}{2}$
4.	One end fixed and other is hinged	$\frac{4\pi^2 EI}{l^2}$	$\frac{\pi^2 EI}{L_e^2}$	$L_e = \frac{l}{\sqrt{2}}$

There are two values of moment of inertia i.e., I_{xx} and I_{yy}.

The value of I (moment of inertia) in the above expressions should be taken as the least value of the two moments of inertia as the column will tend to bend in the direction of least moment of inertia.

Crippling stress in Terms of Effective Length and Radius of Gyration,

The moment of inertia (I) can be expressed in terms of radius of gyration (k) as

$I = Ak^2$ where A = Area of cross-section.

As I is the least value of moment of inertia, then

k = Least radius of gyration of the column section.

Now crippling load P in terms of effective length is given by

$$P = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 E \times Ak^2}{L_e^2} \qquad (\because \quad I = Ak^2)$$

$$= \frac{\pi^2 E \times A}{\frac{L_e^2}{k^2}} = \frac{\pi^2 E \times A}{\left(\frac{L_e}{k}\right)^2} \qquad \text{...(19.6)}$$

Slenderness Ratio:

The ratio of the actual length of the column to the least radius of gyration of the column is known as slenderness ratio.

$$\text{Slenderness ratio} = \frac{\text{Actual length}}{\text{Least radius of gyration}} = \frac{l}{k}$$

And the stress corresponding to crippling load is given by

$$\text{Crippling stress} = \frac{\text{Crippling load}}{\text{Area}} = \frac{P}{A}$$

$$= \frac{\pi^2 E \times A}{A\left(\frac{L_e}{k}\right)^2} \qquad \text{(Substituting the value of } P\text{)}$$

$$= \frac{\pi^2 E}{\left(\frac{L_e}{k}\right)^2} \qquad \text{...(19.7)}$$

Limitations of the Euler's Formula:

From equation (19.6), we have

$$\text{Crippling stress} = \frac{\pi^2 E}{\left(\frac{L_e}{k}\right)^2}$$

For a column with both ends hinged, $L_e = l$. Hence Crippling stress becomes as $= \frac{\pi^2 E}{\left(\frac{l}{k}\right)^2}$,

where $\frac{l}{k}$ is slenderness ratio.

if the slenderness ratio i.e. (l/k) is small the crippling stress (or the stress at failure) will be high. But for the column material the crippling-stress cannot be greater than the crushing stress. Hence when the slenderness ratio is less than a certain limit Euler's formula gives a value of crippling stress greater than the crushing stress. In the limiting case we can find the value of l/k, for which the crippling stress is equal to crushing stress.

For example, for a mid steel column with both ends hinged.Crushing stress
= 330 N/mm^2
Young's modulus, E = 2.1 x 10^5 N/mm^2
Equating the crippling stress to the crushing stress corresponding to the minimum value ofslenderness ratio, we get

Crippling stress = Crushing stress

or $\frac{\pi^2 E}{\left(\frac{l}{k}\right)^2} = 330$ or $\frac{\pi^2 \times 2.1 \times 10^5}{\left(\frac{l}{k}\right)^2} = 330$

$\therefore$ $\left(\frac{l}{k}\right)^2 = \frac{\pi^2 \times 2.1 \times 10^5}{330} = 6282$

$\therefore$ $\frac{l}{k} = \sqrt{6282} = 79.27$, say 80.

Hence if the slenderness ratio is less than 80 for mild steel column with both ends hinged, the Euler's formula will not hold good.

Problem1. A hollow mild steel tube 6 m long 4 cm internal diameter and 6 mm thick is used as a strut with both ends hinged. Find the crippling load and safe load taking factor of safety as 3. Take E = 2 x 10^5 N / mm^2.

Sol. Given :

Length of tube,	$l = 6 \text{ m} = 600 \text{ cm}$
Internal dia.,	$d = 4 \text{ cm}$
Thickness,	$t = 5 \text{ mm} = 0.5 \text{ cm}$
$\therefore$ **External dia.,**	$D = d + 2t = 4 + 2 \times 0.5 = 4 + 1 = 5 \text{ cm}$
Young's modulus,	$E = 2 \times 10^5 \text{ N/mm}^2$
Factor of safety	$= 3.0$

Moment of inertia of section, $I = \frac{\pi}{64}(D^4 - d^4) = \frac{\pi}{64}[5^4 - 4^4] \text{ cm}^4$

$$= \frac{\pi}{64}(625 - 256) = 18.11 \text{ cm}^4 = 18.11 \times 10^4 \text{ mm}^4$$

Since both ends of the strut are hinged,

$\therefore$ **Effective length,** $L_e = l = 600 \text{ cm} = 6000 \text{ mm}$

Let P **= Crippling load**

Using equation (19.5), we get

$$P = \frac{\pi^2 EI}{L_e^2}$$

$$= \frac{\pi^2 \times 2.0 \times 10^5 \times 18.11 \times 10^4}{6000^2} = 9929.9 \text{ say } 9930 \text{ N.} \quad \textbf{Ans.}$$

And safe load $= \dfrac{\text{Crippling load}}{\text{Factor of safety}} = \dfrac{9930}{3.0} = 3310$ **N. Ans.**

Problem2. A simply supported beam of length 4m is subjected to a uniformly distributed load of 30 KN/m over the whole span and deflects 15 mm at the centre. Determine the crippling load the beam is used as a column with the following conditions:

(i) One end fixed and another end hinged

(2)Both the ends pin jointed

Sol. Given :

Length, $L = 4 \text{ m} = 4000 \text{ mm}$

Uniformly distributed load, $w = 30 \text{ kN/m} = 30{,}000 \text{ N/m}$

$$= \frac{30{,}000}{1000} \text{ N/mm} = 30 \text{ N/mm}$$

Deflection at the centre, $\delta = 15$ mm.

For a simply supported beam, carrying U.D.L. over the whole span, the deflection at the centre is given by,

$$\delta = \frac{5}{384} \times \frac{w \times L^4}{EI}$$

or

$$15 = \frac{5}{384} \times \frac{30 \times 4000^4}{EI}$$

$$\therefore \quad EI = \frac{5}{384} \times \frac{30 \times 4000^4}{15}$$

$$= \frac{5}{384} \times \frac{3 \times 256}{15} \times 10^{13} = \frac{2}{3} \times 10^{13} \text{ N mm}^2.$$

(i) Crippling load when the beam is used as a column with one end fixed and other end hinged.

The crippling load P for this case in terms of actual length is given by equation (19.4) as

$$P = \frac{2\pi^2 \times EI}{L_e^{\,2}}, \text{ where } l = \text{actual length} = 4000 \text{ mm}$$

$$= \frac{2 \times \pi^2 \times \frac{2}{3} \times 10^{13}}{4000^2} = \mathbf{8224.5 \text{ kN.} \quad Ans.}$$

(ii) Crippling load when both the ends are pin-jointed

This is given by equation (19.1) in terms of actual length as

$$P = \frac{2\pi^2 \times EI}{l^2} \quad \text{where } l = \text{actual length} = 4000 \text{ mm}$$

$$= \frac{\pi^2 \times \frac{2}{3} \times 10^{13}}{4000^2} = \mathbf{4112.25 \text{ kN.} \quad Ans.}$$

Rankin's Formula:

We have learnt that Euler's formula gives correct results only for very long columns. But what happens when the column is a short or the column is not very long. On the basis of results of experiments performed by Rankine, he established an empirical formula which is applicable to all columns whether they are short or long. The empirical formula given by Rankine is known as Rankin's formula, which is given as

$$\frac{1}{P} = \frac{1}{P_C} + \frac{1}{P_E} \qquad \text{...(i)}$$

where P = Crippling load by Rankine's formula

P_C = Crushing load = $\sigma_c \times A$

σ_c = Ultimate crushing stress

A = Area of cross-section

P_E = Crippling load by Euler's formula

$= \dfrac{\pi^2 EI}{L_e^{\,2}}$, in which L_e = Effective length

For a given column material the crushing stress σ_c is a constant. Hence the crushing load P_c (which is equal to σ_c x A) will also be constant for a given cross-sectional area of the column. In equation (i), P_c is constant and hence value of P depends upon the value of P_E. But for a given column material and given cross-sectional area, the value of P_E depends upon the effective length of the column.

(i) If the column is a short, which means the value of L_e is small, then the value of P_E will be large. Hence the value of $1/P_E$ will be small enough and is negligible as comparedto the value of $1/P_C$. Neglecting the value of $1/P_E$ in equation (i), we get,

$$\frac{1}{P} \to \frac{1}{P_C} \quad \text{or} \quad P \to P_C.$$

Hence the crippling load by Rankine's formula for a short column is approximatelyequal to crushing load. Also we have seen that short columns fail due to crushing.

(ii) If the column is long, which means the value of L_e is large. Then the value of P_E will be small and the value of $1/P_E$ will be large enough compared with $1/P_C$.Hence the value of $1/P_C$ may be neglected in equation (i).

$$\frac{1}{P} = \frac{1}{P_E} \quad \text{or} \quad P \to P_E.$$

(iii) Hence the crippling load by Rankine's formula for long columns is approximately equal to crippling load given by Euler's formula.

Hence the Rankine's formula $\frac{1}{P} = \frac{1}{P_C} + \frac{1}{P_E}$ gives satisfactory results for all lengths of columns, ranging from short to long columns.

Now the Rankine's formula is $\frac{1}{P} = \frac{1}{P_C} + \frac{1}{P_E} = \frac{P_E + P_C}{P_C \cdot P_E}$.

Taking reciprocal to both sides, we have

$$P = \frac{P_C \cdot P_E}{P_E + P_C} = \frac{P_C}{1 + \frac{P_C}{P_E}}$$ (Dividing the numerator and denominator by P_E)

$$= \frac{\sigma_c \times A}{1 + \frac{\sigma_c \cdot A}{\left(\frac{\pi^2 EI}{L_e^2}\right)}} \qquad \left(\because P_c = \sigma_c \cdot A \text{ and } P_E = \frac{\pi^2 EI}{L_e^2}\right)$$

But $I = Ak^2$, where k = least radius of gyration

$\therefore$ The above equation becomes as

$$P = \frac{\sigma_c \cdot A}{1 + \frac{\sigma_c \cdot A \cdot L_e^2}{\pi^2 E \cdot Ak^2}} = \frac{\sigma_c \cdot A}{1 + \frac{\sigma_c}{\pi^2 E} \cdot \left(\frac{L_e}{k}\right)^2}$$

$$= \frac{\sigma_c \cdot A}{1 + a \cdot \left(\frac{L_e}{k}\right)^2} \qquad \text{...(19.9)}$$

where $a = \frac{\sigma_c}{\pi^2 E}$ and is known as Rankine's constant.

The equation (19.9) gives crippling load by Rankine's formula. As the Rankine formula is empirical formula, the value of 'a' is taken from the results of the experiments and is not calculated from the values of σ_c and E.

The values of σ_c and a for different columns material are given below in Table 19.2.

S. No.	Material	σ_c in N/mm^2	a
1.	Wrought Iron	250	$\frac{1}{9000}$
2.	Cast Iron	550	$\frac{1}{1600}$
3.	Mild Steel	320	$\frac{1}{7500}$
4.	Timber	50	$\frac{1}{750}$

Problem: A hollow cylindrical cast iron column is 4 m long with both ends fixed. Determine the minimum diameter of the column if it has to carry a safe load of 250 KN with a factorof safety of 5. Take the internal diameter as 0.8 times the external diameter. Take

σ_C = 550 N /mm^2 and. a = 1/1600 in Rankine's formula.

Sol. Given :

Length of column, l = 4 m = 4000 mm

End conditions = Both ends fixed

$\therefore$ Effective length, $L_e = \frac{l}{2} = \frac{4000}{2} = 2000$ mm

Safe load, = 250 kN

Factor of safety, = 5

Let External dia., = D

Internal dia. = $0.8 \times D$

Crushing stress, $\sigma_c = 550$ N/mm^2

Value of 'a' = $\frac{1}{1600}$ in Rankine's formula

Now factor of safety = $\frac{\text{Cirippling load}}{\text{Safe load}}$ or $5 = \frac{\text{Cirippling load}}{250}$

$\therefore$ Crippling load, $P = 5 \times 250 = 1250$ kN = 1250000 N

Area of column, $A = \frac{\pi}{4}[D^2 - (0.8D)^2]$

$$= \frac{\pi}{4}[D^2 - 0.64D^2] = \frac{\pi}{4} \times 0.36D^2 = \pi \times 0.09D^2$$

Moment of Inertia, $I = \frac{\pi}{64}[D^4 - (0.8D^4)] = \frac{\pi}{64}[D^4 - 0.4096D^4]$

$$= \frac{\pi}{64} \times 0.5904 \times D^4 = 0.009225 \times \pi \times D^4$$

But $I = A \times k^2$, where k is radius of gyration

$$\therefore \quad k = \sqrt{\frac{I}{A}} = \sqrt{\frac{0.009225 \times \pi D^4}{\pi \times 0.09 \times D^2}} = 0.32D$$

Now using equation (19.9), $P = \dfrac{\sigma_c \, . \, A}{1 + a\left(\dfrac{L_e}{k}\right)^2}$

or
$$1250000 = \frac{550 \times \pi \times 0.09\, D^2}{1 + \dfrac{1}{1600} \times \left(\dfrac{2000}{0.32D}\right)^2} \qquad (\because \; A = \pi \times 0.09D^2)$$

$$\frac{1250000}{550 \times \pi \times 0.09} = \frac{D^2}{1 + \dfrac{24414}{D^2}} \quad \text{or} \quad 8038 = \frac{D^2 \times D^2}{D^2 + 24414}$$

or $8038D^2 + 8038 \times 24414 = D^4$ or $D^4 - 8038D^2 - 8038 \times 24414 = 0$

or $D^4 - 8038\,D^2 - 196239700 = 0$.

The above equations is a quadratic equation in D^2. The solution is

$$\therefore \quad D^2 = \frac{8038 \pm \sqrt{8038^2 + 4 \times 1 \times 196239700}}{2}$$

$$\left(\text{Roots} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right)$$

$$= \frac{8038 \pm \sqrt{646094 + 784958800}}{2} = \frac{8038 \pm 29147}{2}$$

$$= \frac{8038 + 29147}{2} \quad \text{(The other root is not possible)}$$

$$= 18592.5 \text{ mm}^2$$

$$\therefore \quad D = \sqrt{18592.5} = 136.3 \text{ mm}$$

$\therefore$ External diameter = **136.3 mm. Ans.**

Internal diameter = 0.8×1363 = **109 mm. Ans.**

QUESTIONS…..

1(a)What is column and strut ?
(b)what is Euler's column theory ?
(c) write assumptions in the Euler's theory ?
(d)Define failures in column ?
(e) write different types of end conditions of column ?

2(a)Explain the failure of long column and short column ?
(b)Describe the assumption in the Euler's column theory ?
(c)Define equivqlent length and its uses ?
(D) Obtain a relation for the Rankine's crippling load for column ?

SLOPE AND DEFLECTION (CH-6)

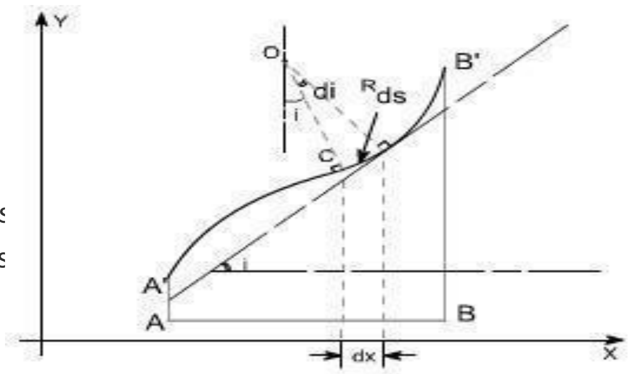
INTRODUCTION

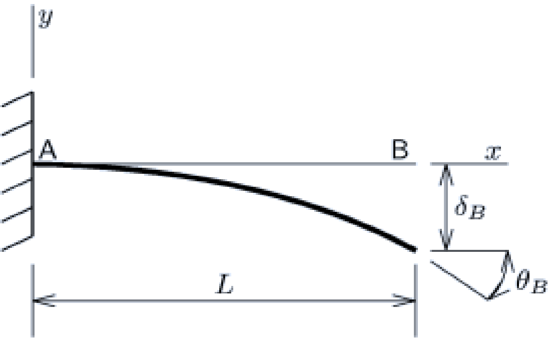
SLOPE OF BEAM :-

- ✓ slope at any section in a deflected beam is defined as the angle in radians which the tangent atthe section makes with the original axis of the beam.
- ✓ slope of that deflection is the angle between the initial position and the deflected position.

DEFLECTION OF A BEAM:

- ✓ The deflection at any point on the axis of the before and after loading.
- ✓ When a structural is loaded may it be Beam or upon it bends from its initial position that is meansthe beam is deflected from its original pos





Consider a beam AB which is initially straight and horizontal when unloaded. If under the actionof loads the beam deflect to a position A'B' under load or infact we say that the axis of the beam bends to a shape A'B'. It is customary to call A'B' the curved axis of the beam as the elastic line or deflection curve.

In the case of a beam bent by transverse loads acting in a plane of symmetry, the bending moment M varies along the length of the beam and we represent the variation of bending moment in B.M diagram. Futher, it is assumed that the simplebending theory equation holdsgood.

$$\boxed{\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R}}$$

If we look at the elastic line or the deflection curve, this is obvious that the curvatureat every point is different; hence the slope is different at different points.

To express the deflected shape of the beam in rectangular co-ordinates let us take two axes x andy, x-axis coincide with the original straight axis of the beam and the y – axis shows the deflection.

Further, let us consider an element ds of the deflected beam. At the ends of this element let us construct the normal which intersect at point O denoting the angle between these two normal be di.

But for the deflected shape of the beam the slope i at any point C is defined,

$\tan i = \frac{dy}{dx}$ ......(1) or $i = \frac{dy}{dx}$ Assuming $\tan i = i$

Futher

$ds = R\,di$

however,

$ds = dx$ [usually for small curvature]

Hence

$ds = dx = R\,di$

or $\boxed{\frac{di}{dx} = \frac{1}{R}}$

substituting the value of i, one get

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{1}{R}$ or $\frac{d^2y}{dx^2} = \frac{1}{R}$

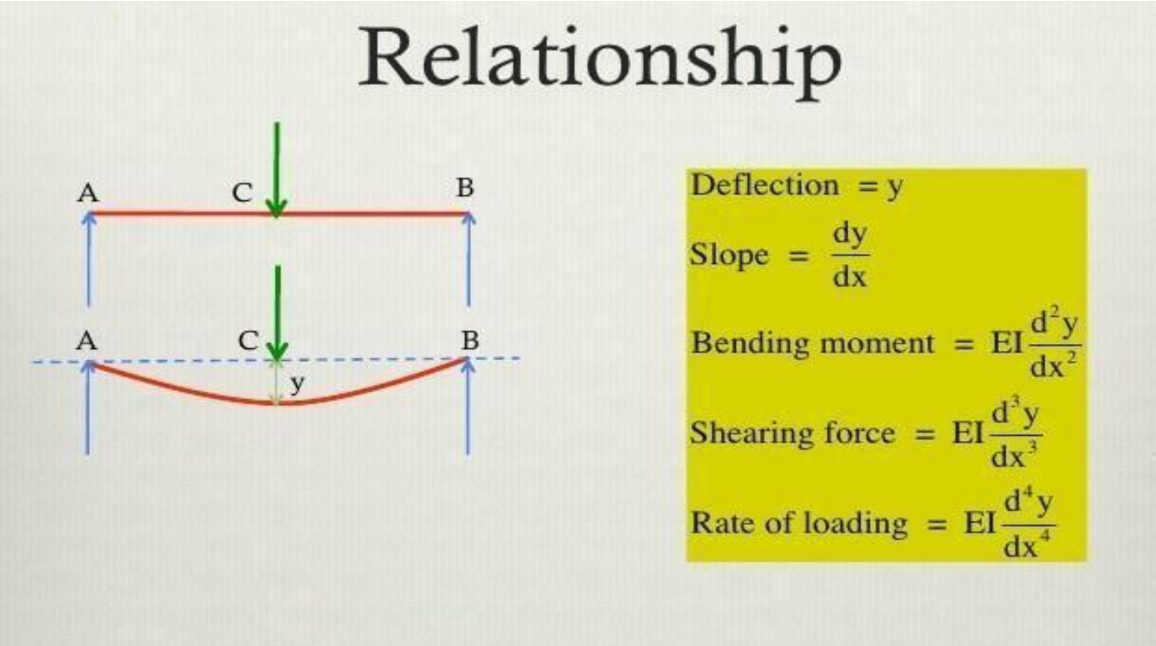
From the simple bending theory

$\frac{M}{I} = \frac{E}{R}$ or $M = \frac{EI}{R}$

so the basic differential equation governing the deflection of beams is

$$\boxed{M = EI\frac{d^2y}{dx^2}}$$

This is the differential equation of the elastic line for a beam subjected to bending in the plane of symmetry.



METHODS FOR FINDING THE SLOPE AND DEFLECTION OF BEAMS:

- ➢ Double integration method
- ➢ Moment area method
- ➢ Macaulay's method
- ➢ Conjugate beam method
- ➢ Strain energy method

DOUBLE INTEGRATION METHOD:

- ✓ The double integration method is a powerful tool in solving deflection and slope of a beam at any point because we will be able to get the equation of the elastic curve.
- ✓ This method entails obtaining the deflection of a beam by integrating the differential equationof the elastic curve of a beam twice and using boundary conditions to determine the constantsof integration.
- ✓ The first integration yields the slope, and the second integration gives the deflection.

CONJUGATE BEAM:

- ✓ Conjugate beam is defined as the imaginary beam with the same dimensions (length) as thatof the original beam but load at any point on the conjugate beam is equal to the bending moment atthat point divided by EI.
- ✓ Slope on real beam = Shear on conjugate beam
- ✓ Deflection on real beam = Moment on conjugate beam
- ✓

PROPERTIES OF CONJUGATE BEAM:

- ✓ The length of a conjugate beam is always equal to the length of the actual beam.
- ✓ The load on the conjugate beam is the M/EI diagram of the loads on the actual beam.
- ✓ A simple support for the real beam remains simple support for the conjugate beam.
- ✓ A fixed end for the real beam becomes free end for the conjugate beam.
- ✓ The point of zero shear for the conjugate beam corresponds to a point of zero slope forthe real beam.
- ✓ The point of maximum moment for the conjugate beam corresponds to a point of maximum deflection for the real beam.

SLOPE AND DEFLECTION FOR A SIMPLY SUPPORTED BEAM WITH CENTRALPOINT LOAD:

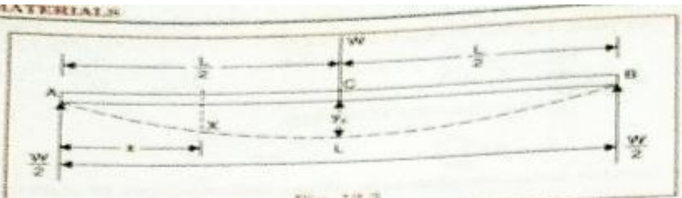



Fig. 12.3

Now $$R_A = R_B = \frac{W}{2}$$

Consider a section X at a distance x from A. The bending moment at this section is given by,

$$M_x = R_A \times x$$

$$= \frac{W}{2} \times x$$

(Plus sign is as B.M. for left portion at X is clockwise)

But B.M. at any section is also given by equation (12.3) as

$$M = EI \frac{d^2y}{dx^2}$$

Equating the two values of B.M., we get

$$EI \frac{d^2y}{dx^2} = \frac{W}{2} \times x \qquad ...(i)$$

On integration, we get

$$EI \frac{dy}{dx} = \frac{W}{2} \times \frac{x^2}{2} + C_1 \qquad ...(ii)$$

where C_1 is the constant of integration. And its value is obtained from boundary conditions. The boundary condition is that at $x = \frac{L}{2}$, slope $\left(\frac{dy}{dx}\right) = 0$ (As the maximum deflection is at the centre, hence slope at the centre will be zero). Substituting this boundary condition in equation (ii), we get

$$0 = \frac{W}{4} \times \left(\frac{L}{2}\right)^2 + C_1$$

or $$C_1 = -\frac{WL^2}{16}$$

Substituting the value of C_1 in equation (ii), we get

$$EI \frac{dy}{dx} = \frac{Wx^2}{4} - \frac{WL^2}{16} \qquad ...(iii)$$

The above equation is known the *slope equation*. We can find the slope at any point on the beam by substituting the values of x. Slope is maximum at A. At A, $x = 0$ and hence slope at A will be obtained by substituting $x = 0$ in equation (iii).

$$\therefore \quad EI\left(\frac{dy}{dx}\right)_{at\ A} = \frac{W}{4} \times 0 - \frac{WL^2}{16}$$

$\left[\left(\frac{dy}{dx}\right)_{at\ A}\right.$ is the slope at A and is represented by $\left.\theta_A\right]$

$$EI \times \theta_A = -\frac{WL^2}{16}$$

or

$$\therefore \quad \theta_A = -\frac{WL^2}{16EI}$$

The slope at point B will be equal to θ_A, since the load is symmetrically applied.

$$\therefore \quad \theta_B = \theta_A = -\frac{WL^2}{16EI} \quad \text{...(12.6)}$$

Equation (12.6) gives the slope in radians.

Deflection at any point

Deflection at any point is obtained by integrating the slope equation (*iii*). Hence integrating equation (*iii*), we get

$$EI \times y = \frac{W}{4} \cdot \frac{x^3}{3} - \frac{WL^2}{16} x + C_2 \quad \text{...(iv)}$$

where C_2 is another constant of integration. At A, $x = 0$ and the deflection (y) is zero.

Hence substituting these values in equation (*iv*), we get

$$EI \times 0 = 0 - 0 + C_2$$

or

$$C_2 = 0$$

Substituting the value of C_2 in equation (*iv*), we get

$$EI \times y = \frac{Wx^3}{12} - \frac{WL^2 . x}{16} \quad \text{...(v)}$$

The above equation is known as *the deflection equation*. We can find the deflection at any point on the beam by substituting the values of x. The deflection is maximum at centre point C, where $x = \frac{L}{2}$. Let y_c represents the deflection at C. Then substituting $x = \frac{L}{2}$ and $y = y_c$ in equation (*v*), we get

$$EI \times y_c = \frac{W}{12}\left(\frac{L}{2}\right)^3 - \frac{WL^2}{16} \times \left(\frac{L}{2}\right)$$

$$= \frac{WL^3}{96} - \frac{WL^3}{32} = \frac{WL^3 - 3WL^3}{96}$$

$$= -\frac{2WL^3}{96} = -\frac{WL^3}{48}$$

$$\therefore \quad y_c = -\frac{WL^3}{48EI}$$

(Negative sign shows that deflection is downwards)

$$\therefore \quad \text{Downward deflection, } y_c = \frac{WL^3}{48EI} \quad \text{...(12.7)}$$

PROBLEMS:

1.A beam 6 m long, simply supported at its ends, is carrying a point load of 50 KN at its centre.The moment of inertia of the beam is 78 x 10^6 mm^4. If E for the material of the beam = 2.1 X 10^5 N/mm^2. calculate deflection at the centre of the beam and slope at the supports.

GIVEN DATA:

L = 6 m

W = 50 KN = 50 X 10^3 N

I = 78 X 10 mm^4

E = 2.1 X 10^5 N/mm^2

SOLUTION:

1. DEFLECTION AT THE CENTRE OF THE BEAM, y_c = WL^3

/ 48 EI

= 50000 X 6000^3 / (48 X 2.1 X 10^5 X 78 X 10^6)

= 13.736 mm.

SLOPE AND DEFLECTION FOR A SIMPLY SUPPORTED WITH A UNIFORMLYDISTRIBUTED LOAD:

- ✓ A simply supported beam AB of length L and carrying a uniformly distributed load of w per unit length over the entire length is shown in fig.
- ✓ The reactions at A and B will be equal.
- ✓ Also, the maximum deflection will be at the centre of the beam.
- ✓ Each vertical reaction = (w X L)/2

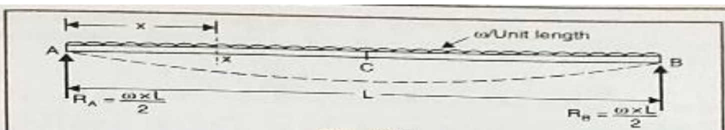


Fig. 12.6

$$\therefore \quad R_A = R_B = \frac{w \times L}{2}$$

Consider a section X at a distance x from A. The bending moment at this section is given by,

$$M_x = R_A \times x - w \times x \times \frac{x}{2} = \frac{w \cdot L}{2} \cdot x - \frac{w \cdot x^2}{2}$$

But B.M. at any section is also given by equation (12.3), as

$$M = EI \frac{d^2y}{dx^2}$$

Equating the two values of B.M., we get

$$EI \frac{d^2y}{dx^2} = \frac{w \cdot L}{2} x - \frac{w \cdot x^2}{2}$$

Integrating the above equation, we get

$$EI \frac{dy}{dx} = \frac{w \cdot L}{2} \cdot \frac{x^2}{2} - \frac{w}{2} \cdot \frac{x^3}{3} + C_1 \qquad \text{...(i)}$$

where C_1 is a constant of integration.

Integrating the above equation again, we get

$$EI.y = \frac{w \cdot L}{4} \cdot \frac{x^3}{3} - \frac{w}{6} \cdot \frac{x^4}{4} + C_1x + C_2 \qquad \text{...(ii)}$$

where C_2 is another constant of integration. Thus two constants of integration (*i.e.*, C_1 and C_2) are obtained from boundary conditions. The boundary conditions are :

(*i*) at $x = 0, y = 0$ and (*ii*) at $x = L, y = 0$

Substituting first boundary condition *i.e.*, $x = 0, y = 0$ in equation (*ii*), we get

$$0 = 0 - 0 + 0 + C_2 \quad \text{or} \quad C_2 = 0$$

Substituting the second boundary condition *i.e.*, at $x = L, y = 0$ in equation (*ii*), we get

$$0 = \frac{w \cdot L}{4} \cdot \frac{L^3}{3} - \frac{w}{6} \cdot \frac{L^4}{4} + C_1 \cdot L \qquad (C_2 \text{ is already zero})$$

$$= \frac{w \cdot L^4}{12} - \frac{w \cdot L^4}{24} + C_1.L$$

or

$$C_1 = -\frac{wL^3}{12} + \frac{wL^3}{24} = -\frac{wL^3}{24}$$

Substituting the value of C_1 in equations (*i*) and (*ii*), we get

$$EI \frac{dy}{dx} = \frac{w \cdot L}{4} \cdot x^2 - \frac{w}{6} x^3 - \frac{wL^3}{24} \qquad \text{...(iii)}$$

and $$EI.y = \frac{w.L}{12}x^3 - \frac{w}{24}.x^4 + \left(-\frac{wL^3}{24}\right)x + 0 \quad (\because C_2 = 0)$$

or $$EIy = \frac{w.L}{12}x^3 - \frac{w}{24}.x^4 - \frac{wL^3}{24}x \quad ...(iv)$$

Equation (*iii*) is known as *slope equation*. We can find the slope $\left(i.e., \text{the value of } \frac{dy}{dx}\right)$ at any point on the beam by substituting the different values of x in this equation.

Equation (*iv*) is known as *deflection equation*. We can find the deflection (*i.e.*, the value of y) at any point on the beam by substituting the different values of x in this equation.

Slope at the Supports

Let θ_A = Slope at support A. This is equal to $\left(\frac{dy}{dx}\right)_{at\ A}$

and θ_B = Slop at support $B = \left(\frac{dy}{dx}\right)_{at\ B}$

At A, $x = 0$ and $\frac{dy}{dx} = \theta_A$.

Substituting these values in equation (*iii*), we get

$$EI.\theta_A = \frac{wL}{4}\times 0 - \frac{w}{6}\times 0 - \frac{wL^3}{24}$$

$$= -\frac{wL^3}{24} = -\frac{WL^2}{24} \quad (\because w.L = W = \text{Total load})$$

$$\therefore \quad \theta_A = -\frac{WL^2}{24EI} \quad ...(12.12)$$

(Negative sign means that tangent at A makes an angle with AB in the anti-clockwise direction)

By symmetry, $$\theta_B = -\frac{WL^2}{24EI} \quad ...(12.13)$$

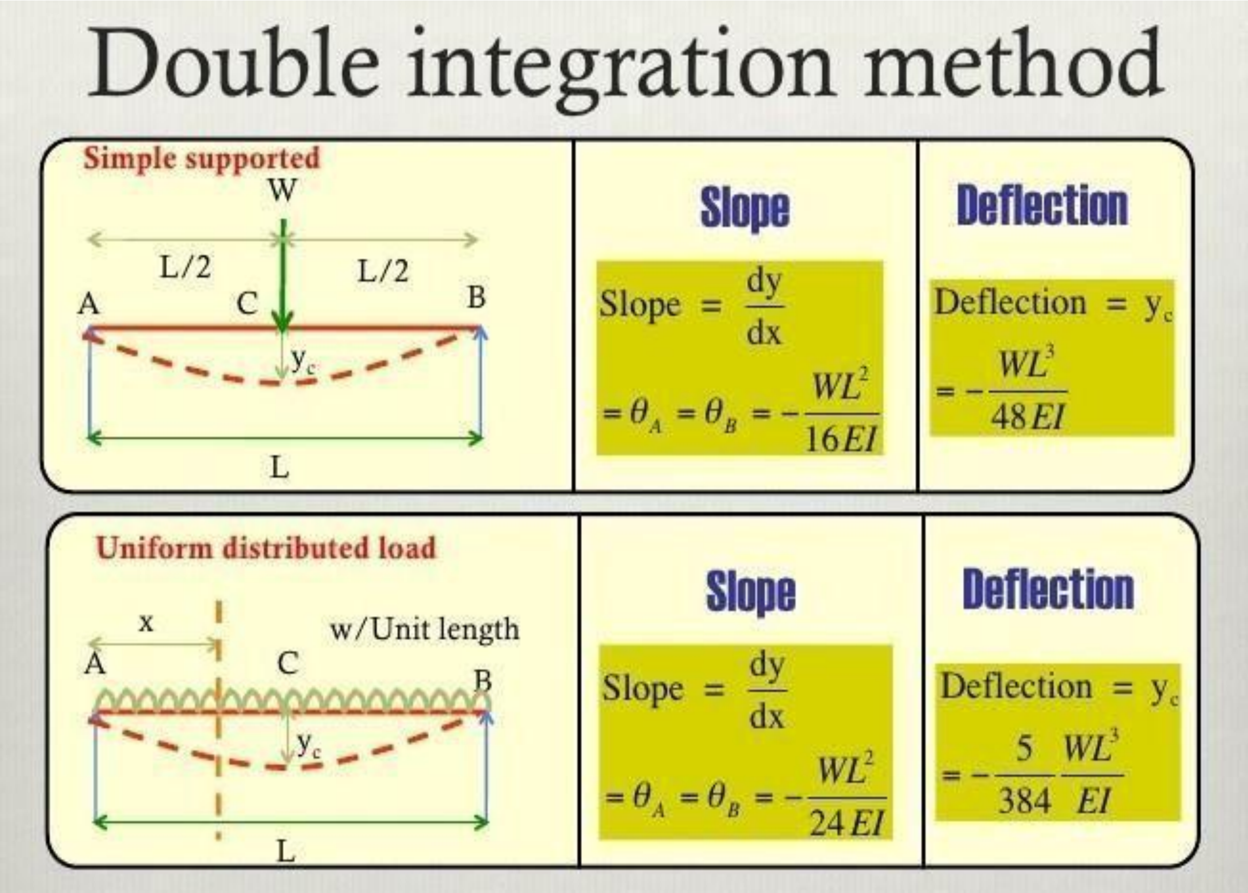
Maximum Deflection

The maximum deflection is at the centre of the beam *i.e.*, at point C, where $x = \frac{L}{2}$. Let y_C = deflection at C which is also maximum deflection. Substituting $y = y_C$ and $x = \frac{L}{2}$ in equation (*iv*), we get

$$EI.y_C = \frac{w.L}{12}.\left(\frac{L}{2}\right)^3 - \frac{w}{24}.\left(\frac{L}{2}\right)^4 - \frac{wL^3}{24}-\left(\frac{L}{2}\right)$$

$$= \frac{w.L^4}{96} - \frac{wL^4}{384} - \frac{wL^4}{48} = -\frac{5w.L^4}{384}$$

$$\therefore \quad y_C = -\frac{5}{384}.\frac{wL^4}{EI} = -\frac{5}{384}.\frac{W.L^3}{EI} \quad (\because w.L = W = \text{Total load})$$



4. A beam of uniform rectangular section 200 mm wide and 300 mm deep is simply supported at its ends. It carries a uniformly distributed load of 9 KN/m run over the entire span of 5 m. if the value of E for the beam material is 1 X 10^4 N/mm^2 , find the slope at the supports and maximum deflection.

GIVEN DATA:

L = 5 m = 5 X 10^3 mm

w = 9 KN/m = 9000 N/m

E = 1 X 10^4 N/mm^2

b = 200 mm

d = 300 mm

SOLUTION:

1. SLOPE AT THE SUPPORTS,

θ_A = - WL^2 / 24 EI

$$= \frac{45000 \times 5000^2}{24 \times 1 \times 10^4 \times 4.5 \times 10^8}$$

W = w.L = 9000 X 5 = 45000 N

I = $bd^3/12$ = 200 X 300^3 / 12

= 4.5 X 10^8 mm^4

= **0.0104 radians.**

2. MAXIMUM DEFLECTION,y =

$$\frac{5\ W\ L^3}{384\ E\ I}$$

$$= \frac{5 \times 45000 \times 5000^3}{384 \times 1 \times 10^4 \times 4.5 \times 10^8}$$

= 16.27 mm.

MOMENT AREA METHOD:

- ✓ **MOHR'S THEOREM – I:**

The change of slope between any two points is equal to the net area of the B.M. diagram betweenthese points divided by EI.

- ✓ **MOHR'S THEOREM – II:**

The total deflection between any two points is equal to the moment of the area of B.M. diagrambetween the two points about the last point divided by EI.

MOHR'S THEOREMS IS USED FOR FOLLOWING CASES:

- ✓ Problems on Cantilevers
- ✓ Simply supported beams carrying symmetrical loading
- ✓ Fixed beams
- ✓

SLOPE AND DEFLECTION FOR A SIMPLY SUPPORTED BEAM WITH CENTRALPOINT LOAD:

Fig. 12.19 (*a*) shows a simply supported AB of length L and carrying a point load W at the centre of the beam *i.e.*, at point C. The B.M. diagram is shown in Fig. 12.19 (*b*). This is a case of symmetrical loading, hence slope is zero at the centre *i.e.*, at point C. But the deflection is maximum at the centre.

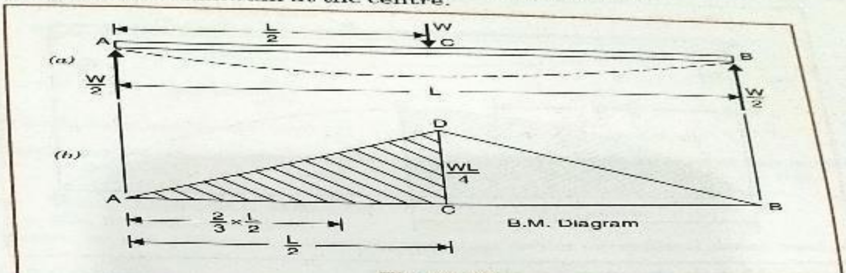


Fig. 12.19

Now using Mohr's theorem for slope, we get

$$\text{Slope at } A = \frac{\text{Area of B.M. diagram between } A \text{ and } C}{EI}$$

But area of B.M. diagram between A and C

$$= \text{Area of triangle } ACD$$

$$= \frac{1}{2} \times \frac{L}{2} \times \frac{WL}{4} = \frac{WL^2}{16}$$

$\therefore$ Slope at A or θ_A $= \dfrac{WL^2}{EI}$

Now using Mohr's theorem for deflection, we get from equation (12.17) as

$$y = \frac{A\bar{x}}{EI}$$

where A = Area of B.M. Diagram between A and C

$$= \frac{WL^2}{16}$$

$\bar{x}$ = Distance of C.G. of area A from A

$$= \frac{2}{3} \times \frac{L}{2} = \frac{L}{3}$$

$$\therefore \quad y = \frac{\frac{WL^2}{16} \times \frac{L}{3}}{EI} = \frac{WL^3}{48EI}.$$

CONJUGATE BEAM METHOD:

- **CONJUGATE BEAM:**
  - ✓ Conjugate beam is an imaginary beam of length equal to that of the original beam but forwhich the load diagram is the M/EI diagram.
  - **NOTE 1 :**
  - ✓ The slope at any section of the given beam is equal to the shear force at the correspondingsection of the conjugate beam.
  - **NOTE 2 :**
  - ✓ The deflection at any section for the given beam is equal to the bending moment at thecorresponding section of the conjugate beam.

SLOPE AND DEFLECTION FOR A SIMPLY SUPPORTED BEAM WITH CENTRALPOINT LOAD:

- ✓ A simply supported beam AB of length L carrying a point load W at the centre C.
- ✓ The B.M at A and B is zero and at the centre B.M will be WL/4.
- ✓ Now the conjugate beam AB can be constructed.

- ✓ The load on the conjugate beam will be obtained by dividing the B.M at that point by EI.
- ✓ The shape of the loading on the conjugate beam will be same as of B.M diagram.
- ✓ The ordinate of loading on conjugate beam will be equal to M/EI = WL/4EI.

$EI \quad EI \quad 4EI$

load diagram for conjugate beam is shown in Fig. 14.1 (c).

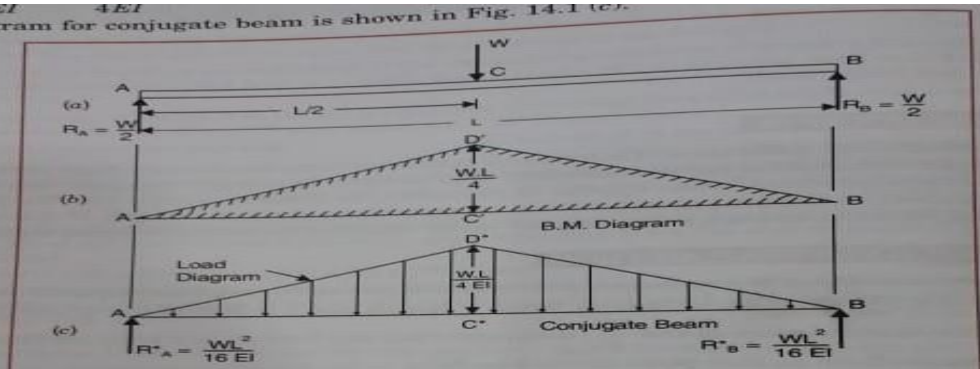


Fig. 14.1

Let R_A^* = Reaction at A for conjugate beam

R_B^* = Reaction at B for conjugate beam

Total load on the conjugate beam [See Fig. 14.1 (c)]

= Area of the load diagram

$$= \frac{1}{2} \times AB \times C^*D^* = \frac{1}{2} \times L \times \frac{WL}{4EI}$$

$$= \frac{WL^2}{8EI}$$

Reaction at each support for the conjugate beam will be half of the total load

$$\therefore \quad R_A^* = R_B^* = \frac{1}{2} \times \frac{WL^2}{8EI} = \frac{WL^2}{16EI}$$

Let θ_A = Slope at A for the given beam *i.e.*, $\left(\frac{dy}{dx}\right)$ at A

y_c = deflection at C for the given beam.

Then according to conjugate beam method,

θ_A = Shear force at A for the conjugate beam

6EIL

Problem 14.1. *A simply supported beam of length 5 m carries a point load of 5 kN at a distance of 3 m from the left end. If $E = 2 \times 10^5$ N/mm² and $I = 10^8$ mm⁴, determine the slope at the left support and deflection under the point load using conjugate beam method.*

Sol. Given :

Length, $L = 5$ m

Point load, $W = 5$ kN

Distance AC, $a = 3$ m

Distance BC, $b = 5 - 3 = 2$ m

Value of $E = 2 \times 10^5 \text{ N/mm}^2 = 2 \times 10^5 \times 10^6 \text{ N/m}^2$

$= 2 \times 10^5 \times 10^3 \text{ kN/m}^2 = 2 \times 10^8 \text{ kN/m}^2$

Value of $I = 1 \times 10^8 \text{ mm}^4 = 10^{-4} \text{ m}^4$

Let R_A = Reaction at A

and R_B = Reaction at B.

Taking moments about A, we get

$$R_B \times 5 = 5 \times 3$$

$$\therefore \quad R_B = \frac{5 \times 3}{5} = 3 \text{ kN}$$

and R_A = Total load $- R_B = 5 - 3 = 2$ kN

The B.M. at $A = 0$

B.M. at $B = 0$

B.M. at $C = R_A \times 3 = 2 \times 3 = 6$ kNm.

Now B.M. diagram is drawn as shown in Fig. 14.3 (*b*).

Now construct the conjugate beam as shown in Fig. 14.3 (*c*). The vertical load at C^* on conjugate beam

$$= \frac{\text{B.M. at } C}{EI} = \frac{6 \text{ kNm}}{EI}$$

Now calculate the reaction at A^* and B^* for conjugate beam

Let R_A^* = Reaction at A^* for conjugate beam

R_B^* = Reaction at B^* for conjugate beam.

Taking moments about A^*, we get

$R_B^* \times 5$ = Load on $A^*C^*D^*$ × distance of C.G. of $A^*C^*D^*$ from A^*

+ Load on $B^*C^*D^*$ × Distance of C.G. of $B^*C^*D^*$ from A^*

$$= \left(\frac{1}{2} \times 3 \times \frac{6}{EI}\right) \times \left(\frac{2}{3} \times 3\right) + \left(\frac{1}{2} \times 2 \times \frac{6}{EI}\right) \times \left(3 + \frac{1}{3} \times 2\right)$$

$$= \frac{18}{EI} + \frac{6}{EI} \times \frac{11}{3} = \frac{8}{EI} + \frac{22}{EI} = \frac{40}{EI}$$

$$\therefore \quad R_B^* = \frac{40}{EI} \times \frac{1}{5} = \frac{8}{EI}$$

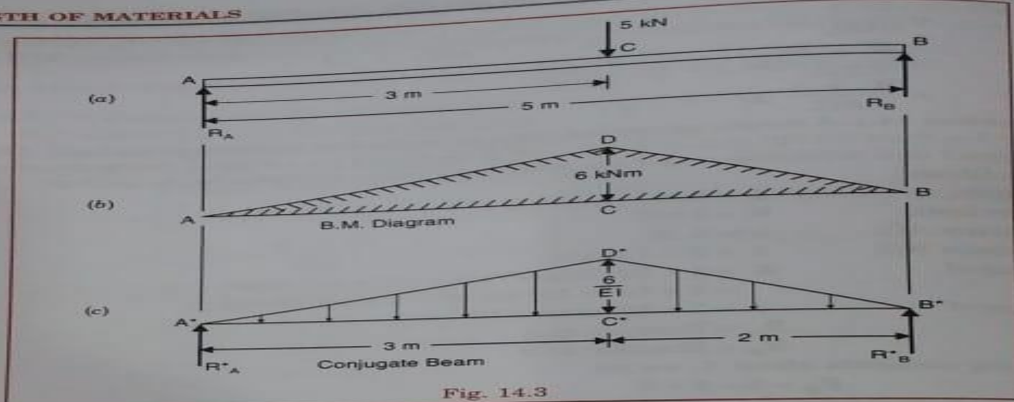


Fig. 14.3

$$\therefore \quad R_A^* = \text{Total load (i.e., load } A^*B^*D^*) - R_B^*$$

$$= \left(\frac{1}{2} \times 5 \times \frac{6}{EI}\right) - \frac{8}{EI}$$

$$= \frac{15}{EI} - \frac{8}{EI} = \frac{7}{EI}$$

Let θ_A = Slope at A for the given beam *i.e.*, $\left(\frac{dy}{dx}\right)$ at A

y_C = Deflection at C for the given beam

Then according to conjugate beam method,

$$\theta_A = \text{Shear force at } A^* \text{ for conjugate beam} = R_A^*$$

$$= \frac{7}{EI} = \frac{7}{2 \times 10^8 \times 10^{-4}} \quad (\because E = 2 \times 10^8 \text{ kN/m}^2 \text{ and } I = 10^{-4} \text{ m}^4)$$

$$= \mathbf{0.00035 \ radians.} \quad \textbf{Ans.}$$

$$y_C = \text{B.M. at } C^* \text{ for conjugate beam}$$

$$= R_A^* \times 3 - \text{Load } A^*C^*D^* \times \text{Distance of C.G. of } A^*C^*D^* \text{ from } C^*$$

$$= \frac{7}{EI} \times 3 - \left(\frac{1}{2} \times 3 \times \frac{6}{EI}\right) \times \left(\frac{1}{3} \times 3\right)$$

$$= \frac{21}{EI} - \frac{9}{EI} - \frac{12}{EI}$$

$$= \frac{12}{2 \times 10^8 \times 10^{-4}} = \frac{6}{10^4} \text{ m} = \frac{6 \times 1000}{10000} \text{ mm} = \mathbf{0.6 \ mm.} \quad \textbf{Ans.}$$

Problem 14.2. *A simply supported beam of length 4 m carries a point load of 3 kN at a distance of 1 m from each end. If E = 2 × 10⁵ N/mm² and I = 10⁸ mm⁴ for the beam, then using conjugate beam method determine : (i) slope at each end and under each load (ii) deflection under each load and at the centre.*

Sol. Given :

Length, $L = 4$ m

Value of $E = 2 \times 10^5 \text{ N/mm}^2 = 2 \times 10^5 \times 10^6 \text{ N/m}^2$

$= 2 \times 10^5 \times 10^3 \text{ kN/m}^2 = 2 \times 10^8 \text{ kN/m}^2$

Value of $I = 10^8 \text{ mm}^4 = \frac{10^8}{10^{12}} \text{ m}^4 = 10^{-4} \text{ m}^4.$

As the load on the beam is symmetrical as shown in Fig. 14.4 (*a*), the reactions R_A and R_B will be equal to 3 kN.

Now B.M. at A and B are zero.

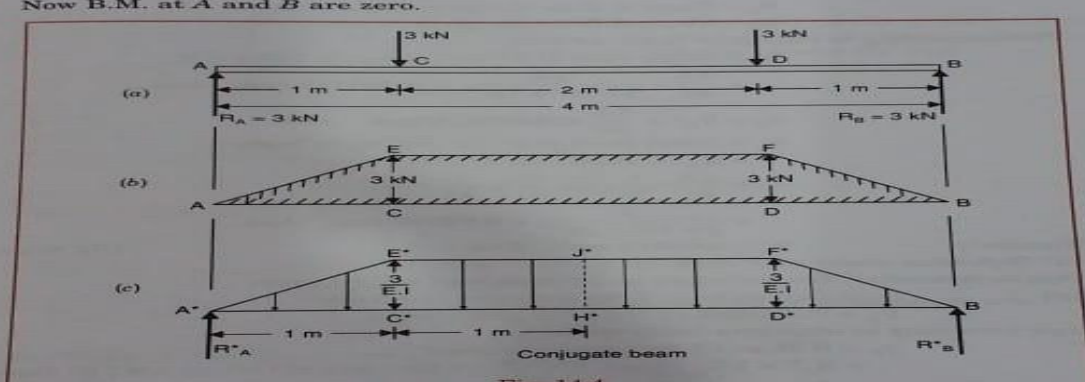


Fig. 14.4

B.M. at $C = R_A \times 1 = 3 \times 1 = 3$ kNm

B.M. at $D = R_B \times 1 = 3 \times 1 = 3$ kNm

Now B.M. diagram can be drawn as shown in Fig. 14.4 (*b*).

Now by dividing the B.M. at any section by EI, we can construct the conjugate beam as shown in Fig. 14.4 (*c*). The loading are shown on the conjugate beam.

Let R_A^* = Reaction at A^* for the conjugate beam and

R_B^* = Reaction at B^* for conjugate beam

The loading on the conjugate beam is symmetrical

$\therefore \quad R_A^* = R_B^* =$ Half of total load on conjugate beam

$= \frac{1}{2}$ [Area of trapezoidal $A^*B^*F^*E^*$]

$= \frac{1}{2}\left[\frac{(E^*F^* + A^*B^*)}{2} \times E^*C^*\right]$

$= \frac{1}{2}\left[\frac{(2+4)}{2} \times \frac{3}{EI}\right] = \frac{4.5}{EI}$

(i) Slope at each end and under each load

Let θ_A = Slope at A for the given beam *i.e.*, $\left(\frac{dy}{dx}\right)$ at A

θ_B = Slope at B for the given beam

θ_C = Slope at C for the given beam and

θ_D = Slope at D for the given beam

Then according to conjugate beam method,

θ_A = Shear force at A^* for conjugate beam $= R_A^*$

$= \frac{4.5}{EI} = \frac{4.5}{2 \times 10^8 \times 10^{-4}}$ = **0.000225 rad. Ans.**

$\theta_B = R_B^* = \frac{4.5}{EI}$ = **0.000225 rad. Ans.**

θ_C = Shear force at C^* for conjugate beam

$= R_A^*$ − Total load $A^*C^*D^*$

$= \frac{4.5}{EI} - \frac{1}{2} \times 1 \times \frac{3}{EI} = \frac{3}{EI}$

$= \frac{3}{2 \times 10^8 \times 10^{-4}}$ = **0.00015 rad. Ans.**

Similarly, θ_D = **0.00015 rad. Ans.** (By symmetr

QUESTION BANK:

1. What are the methods used for determining slope and deflection?
2. What is the slope and deflection equation for simply supported beam carrying UDL through out the length?
3. What is a Macaulay’s method?
4. What is moment area method?
5. Define : Conjugate beam.
6. Find the slope and deflection of a simply supported beam carrying a point load at the centre using moment area method.

7. Distinguish between actual beam and conjugate beam.

8. A beam 4m long, simply supported at its ends, carries a point load W at its centre. If the slope at the ends of the beam is not to exceed 1^o, find thedeflection at the centre of the beam.

9. A cantilever of length 2 m carries a point load of 30 KN at the free end and another load of 30 KN at its centre. If EI = 1013 N.mm2 for the cantilever then determine slope and deflection at the free end by moment area method.

10. Determine slope at the left support, deflection under the load and maximum deflection of a simply supported beam of length 10 m, which is carrying a point load of 10 kN at a distance of 6 m from the left end. Take E = 2 x 105 N/mm2 and I = 1 x 108 mm4.

11. A cantilever of length 3 m is carrying a point load of 25 KN at the free end. If I = 108 mm4 and E = 2.1 X 105 N/mm2, then determine slope and deflection of the cantilever using conjugate beam method.

12. A simply supported beam of length 5 m carries a point load of 5 kN at adistance of 3m from the left end. If E = 2 x 105 N/mm2 and I = 108 mm4, determine the slope at the left support and deflection under the point load using conjugate beam method.

SHEAR FORCE AND BENDING MOMENT (CH-5)

INTRODUCTION :

Beams

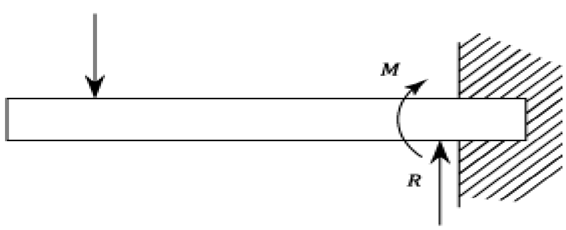
A beam is a structural member resting on supports to carry vertical loads. Beams are generallyplaced horizontally; the amount and extent of external load which a beam can carry depends upon:

a. The distance between supports and the overhanging lengths from supports;
b. The type and intensity of load;
c. The type of supports; and
d. The cross-section and elasticity of the beam.

Classification of beams

1. Cantilever Beam

A **Built-in** or **encastre'** support is frequently met. The effect is to fix the direction of the beam at the support. In order to do this the support must exert a "fixing" moment M and a reaction R on the beam. A beam which is fixed at one end in this way is called a **Cantilever**.If both ends are fixed in this way the reactions are not statically determinate. In practice, it isnot usually possible to obtain perfect fixing and the fixing moment applied will be related tothe angular movement of the support. When in doubt about the rigidity, it is safer to assume that the beam is freely supported.



(CANTILEVER BEAM)

2-Simply Supported Beam

It is a beam having its ends freely resting on supports.

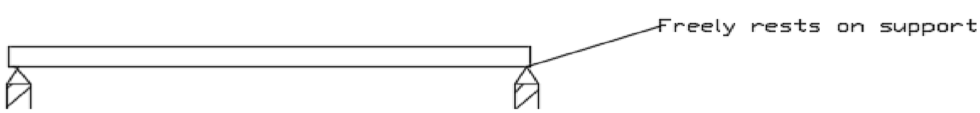


Fig. 9 Simply supported beam

3- Overhanging Beam

A beam having one or both ends extended over supports is known as overhanging beam.

Fig. 10. (i) Overhanging at one end (ii) Overhanging at both ends

4-Propped Cantilever Beam

When a support is provided at some suitable point of a Cantilever beam, in order to resist the deflection of the beam, it is known as propped Cantilever beam.

Fig. 11. Propped Cantilever beam

5-Fixed Beam

A beam having its both ends rigidly fixed or built0in to the supporting walls or colums isknown as fixed beam.

Fig. 12. Fixed beam

2.2 TYPES OF LOADING

1. Point Load or Concentrated Load

These loads are usually considered to be acting at a point. Practically point load cannot be placed on a beam. When a member is placed on a beam it covers some space or width. But forcalculation purpose, we consider the load as transmitting at the central with of the member.

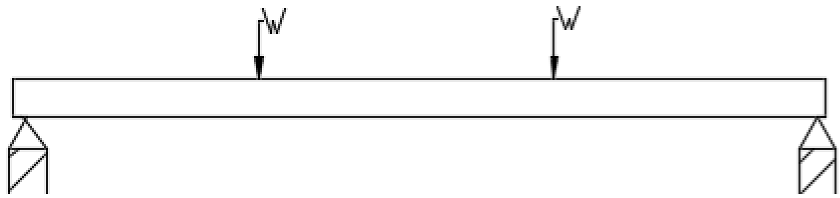


Fig. 13. Concentrated load

2. Uniformly Distributed Load or U.D.L

Uniformly distributed load is one which is spread uniformly over beam so that each unit oflength is loaded with same amount of load, and are denoted by Newton/metre.

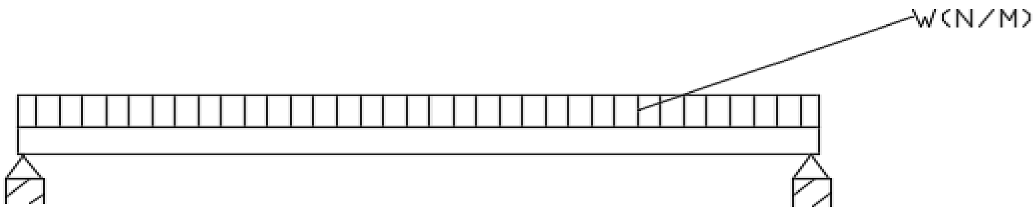


Fig. 14. UDL

3. Gradually Varying Load

If the load is spread, varying uniformly along the length of a beam, then it is called uniformlyvarying load.

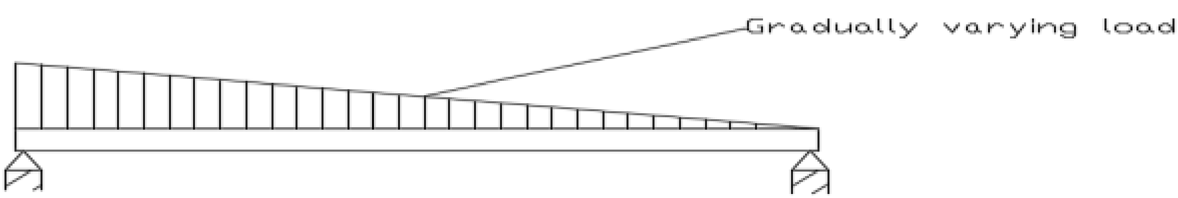


Fig. 15 Gradually varying load

4. Continuous Beam

A beam which rests on more than two supports is known as continuous beam. This may either be overhanging at one or both ends.

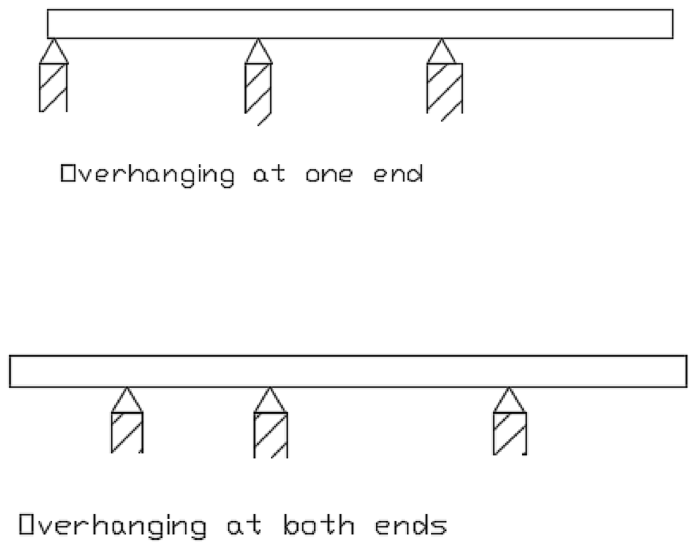


Fig. 16. Overhanging beam

2.4. Span

Clear Span: This is the clear horizontal distance between two supports

Effective Span: This is the horizontal distance between the Centre of end bearings of support.

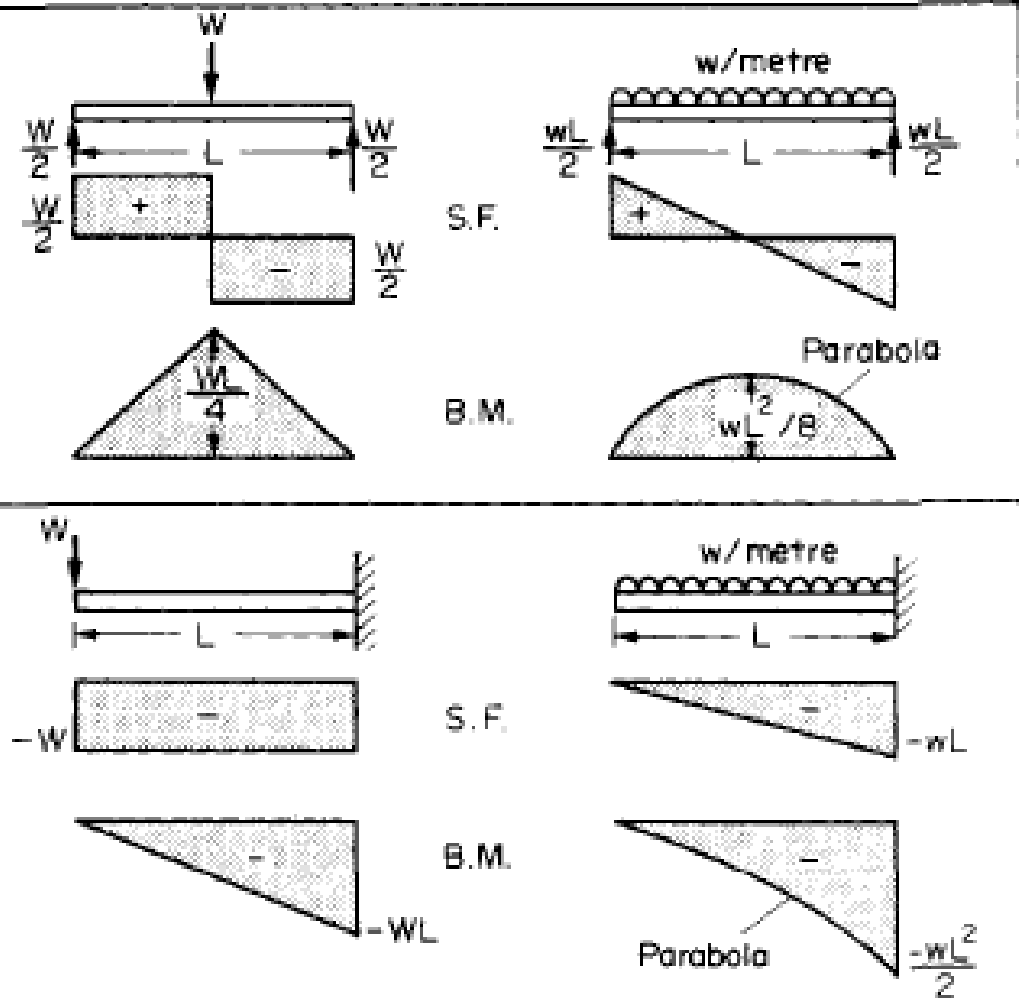
Effective Span = clear span + oce bearing.

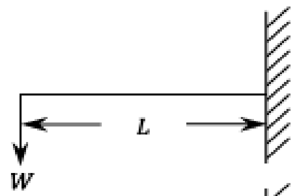
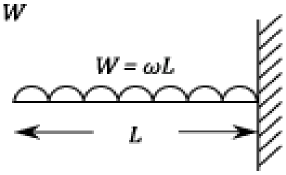
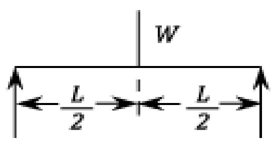
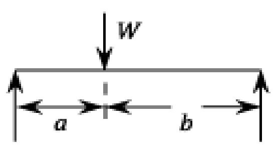
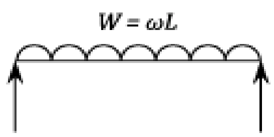
Fig. 17. Effective and clear span

2.5. Shear force

At any section in a beam carrying transverse loads the shearing force is defined as the algebraic sum of the forces taken on either side of the section. Similarly, the bending moment at any section is the algebraic sum of the moments of the forces about the section, again taken **on** either side. In order that the shearing-force and bending-moment values calculated **on** either side of the section shall have the same magnitude and sign, a convenient sign convention has to be adopted. Shearing-force (S.F.) and bending-moment (B.M.) diagrams show the variation of these quantities along the length of a beam for any fixed loading condition.

Shearing force and bending moment diagrams for standard cases



LOADING	$\hat{F}$	$\hat{M}$
	W	WL (Fixed End)
	W (Fixed End)	$\frac{WL}{2}$ (Fixed End)
	$\frac{W}{2}$	$\frac{WL}{4}$ (Centre)
	$\frac{Wb}{L}$	$\frac{Wab}{L}$ (Load)
	$\frac{W}{2}$ (Support)	$\frac{WL}{8}$ (Centre)

At every section in a beam carrying transverse loads there will be resultant forces on either side of the section which, for equilibrium, must be equal and opposite, and whose combined action tends to shear the section in one of the two ways. *The shearing force (S.F.) at the section is defined therefore as the algebraic sum of the forces taken on one side of the section.* Which side*is* chosen is purely a matter of convenience but in order that the value obtained on both sides shall have the same magnitude and sign a convenient sign convention has to be adopted.

a. Shearing force (S.F.) sign convention

Forces upwards ***to*** the left of a section or downwards to the right of the section are positive. Thus Fig. --a shows a positive S.F. system at X-X and Fig. ---b shows a negative S.F. system

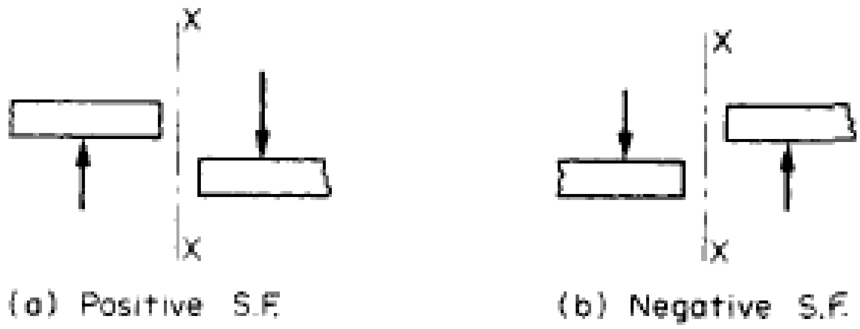


Fig. 20 shear force sign convection

In addition to the shear, every section of the beam will be subjected to bending, i.e. to a resultant **B.M.** which is the net effect of the moments of each of the individual loads. Again, for equilibrium, the values on either side of the section must have equal values. *The bending moment(B.M.) is defined therefore as the algebraic sum of the moments of the forces about the section, taken on either side of the section.* As for *S.F.,* a convenient sign convention must be adopted.

b. Bending moment (B.M.) sign convention

Clockwise moments to the left and counterclockwise to the right are positive. Thus Fig. a - shows a positive bending moment system resulting in *sagging* of the beam at X-X andFig. b- illustrates a negative **B.M.** system with its associated *hogging* beam.

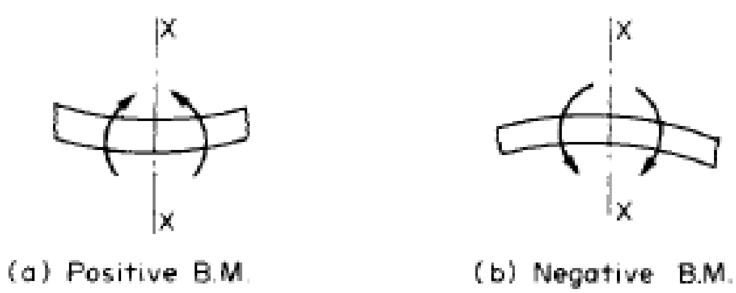


Fig. 21 Bending moment sign cnvection

It should be noted that whilst the above sign conventions for S.F. and **B.M.** are somewhat arbitrary and could be completely reversed, the systems chosen here are the only ones which yield the mathematically correct signs for slopes and deflections of beams in subsequent workand therefore are highly recommended.

c. Shearing force

The shearing force (SF) at any section of a beam represents the tendency for the portion of the beam on one side of the section to slide or shear laterally relative to the other portion. The diagram shows a beam carrying loadsW_1, W_2 and W_3. It is simply supported at two points where the reactions are R_1 and R_2. Assume that the beam is divided into two parts by a section XX. The resultant of the loads and reaction acting on the left of AA is F vertically upwards, and since the whole beam is in equilibrium, the resultant force to the right of AA must be F downwards. F is called the **Shearing Force** at the section AA. It has been defined as *The shearing force at any section of a beam is the algebraic sum of the lateral components of the forces acting on either side of the section.* Where forces are neither in the lateral or axial direction they must be resolvedin the usual way and only the lateral components are used to calculate the shear force.

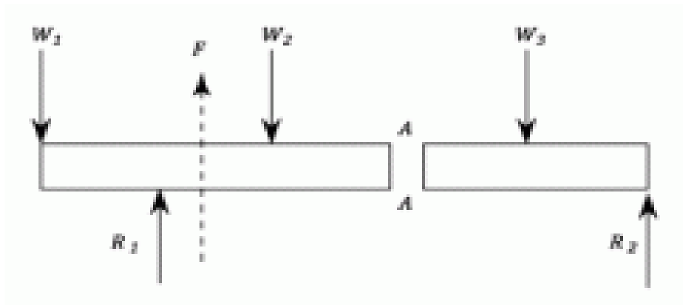


Fig 22 shear force through a section

d. Bending Moment

In a similar manner it can be seen that if the Bending moments (BM) of the forces to the left of AA are clockwise, then the bending moment of the forces to the right of AA must be anticlockwise.

Bending Moment at AA has been defined as the algebraic sum of the moments about the section of all forces acting on either side of the section. Bending moments are considered positive when the moment on the left portion is clockwise and on the right anticlockwise. This is referred to asa **sagging** bending moment as it tends to make the beam concave upwards at AA. A negative bending moment is termed **hogging**.

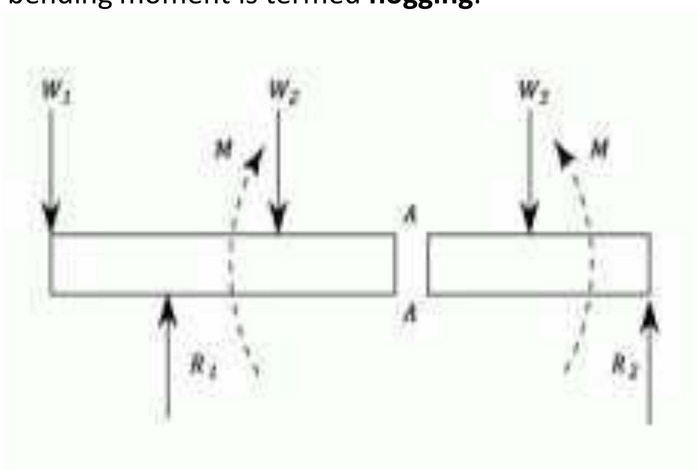


Fig. 23 mendinng moment through a section

RELATIONSHIP BETWEEN LOAD (w), SHEAR FORCE (F), AND BENDING MOMENT (M).

In the following diagram δx is the length of a small slice of a loaded beam at a distance x fromthe origin O

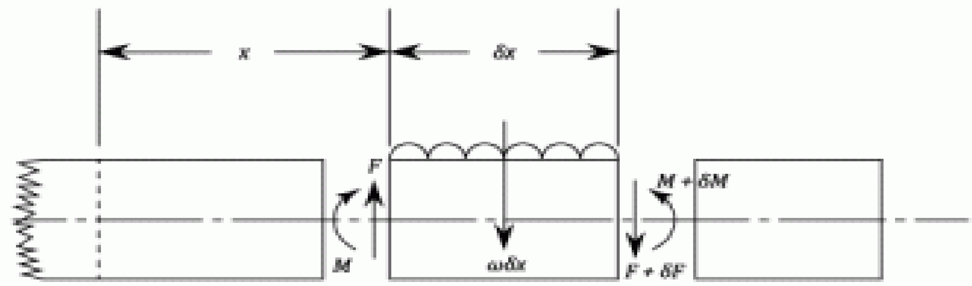


Fig. 24 Loaded beam of length x from origin O

Let the shearing force at the section x be F and at $x = \delta x \;\; be \;\; F + \delta F$. Similarly, the bending moment is M at x, and $M + \delta M \;\; at \;\; x + \delta x$. If w is the mean rate of loading of thelength δx then the total load is $w\delta x$, acting approximately (exactly if uniformly distributed) through the centre C. The element must be in equilibrium under the action of these forces and couples and the following equations can be obtained:-

Taking Moments about C:

$$M + F.\frac{\delta x}{2} + (F + \delta F)\,\frac{\delta x}{2} = M + \delta M \qquad \textbf{(1)}$$

Neglecting the product $\delta F.\delta x$ in the limit:

$$F = \frac{dM}{dx} \qquad \textbf{(2)}$$

Resolving vertically:

$$w\delta x + F + \delta F = F \qquad (3)$$

$$Or \qquad w = -\frac{dF}{dx} \qquad (4)$$

$$= -\frac{d^2M}{dx^2} \qquad from\ equation\ (2) \qquad (5)$$

From equation (2) it can be seen that if M is varying continuously, zero shearing force corresponds to either maximum or minimum bending moment. It can be seen from the examples

that "peaks" in the bending moment diagram frequently occur at concentrated loads or reactions,and these are not given by $F = \frac{dM}{dx} = 0$; although they may in fact represent the greatest bending moment on the beam. Consequently, it is not always sufficient to investigate the points of zero shearing force when determining the maximum bending moment.
At a point on the beam where the type of bending is changing from sagging to hogging, the bending moment must be zero, and this is called a point of *inflection* or *contraflexure*.
By integrating equation (2) between the x = a and x = b then:

$$M_b - M_a = \int_a^b F\, dx \qquad (6)$$

Which shows that the increase in bending moment between two sections is the area under the shearing force diagram.
Similarly integrating equation (4)

$$F_a - F_b = \int_a^b w\, dx \qquad (7)$$

equals the area under the load distribution diagram.
Integrating equation (5) gives:

$$M_a - M_b = \int \int_a^b w\, dx.dx \qquad (8)$$

These relations can be very valuable when the rate of loading cannot be expressed in an algebraicform as they provide a means of graphical solution.

2.3 Concentrated Loads

Tutorials.

Problem 1. A Cantilever of length *l* carries a concentrated load *W* at its free end. Draw theShear Force (SF) and Bending Moment (BM) diagrams.

Solution:

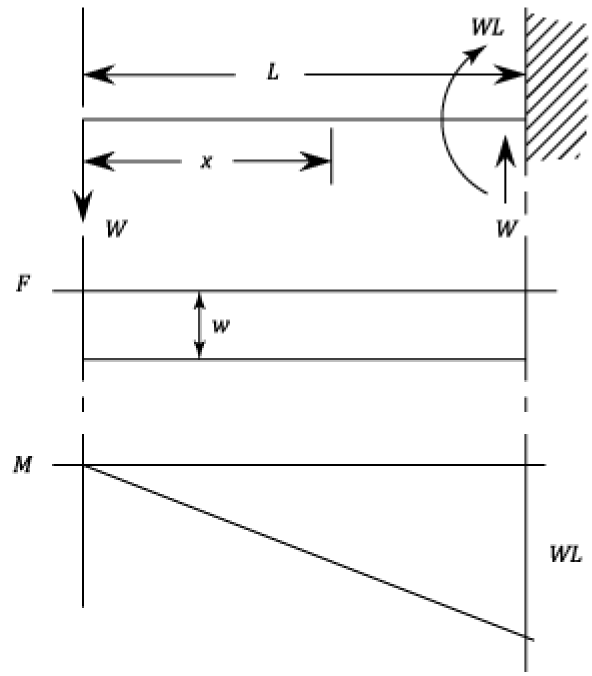
A Cantilever of length *l* carries a concentrated load *W* at its free end. Draw the Shear Force (SF)and Bending Moment (BM) diagrams.

Consider the forces to the left of a section at a distance *x* from the free end. Then F

= - W and is constant along the whole cantilever i.e. for all values of *x*
Taking Moments about the section gives M = - W *x* so that the maximum Bending Momentoccurs when *x* = *l* i.e. at the fixed end.

$$\hat{M} = W\,l \quad (Hogging) \qquad \textbf{(1)}$$

From equilibrium considerations it can be seen that the fixing moment applied at the built in endis WL and the reaction is W. Hence the **SF** and **BM** diagrams are as follows:



The following general conclusions can be drawn when only concentrated loads and reactions are involved.

- The shearing force suffers sudden changes when passing through a load point. Thechange is equal to the load.
- The bending moment diagram is a series of straight lines between loads. The slope oflines is equal to the shearing force between the loading points.

2.4 Uniformly Distributed Loads

Problem 2. Draw the SF and BM diagrams for a simply supported beam of length *l* carrying a uniformly distributed load *w* per unit length which occurs across the whole Beam.

Solution.

The Total Load carried is *wl* and by symmetry the reactions at both end supports are each w*l*/2If *x* is the distance of the section measured from the left-hand support then:

$$F = \frac{wl}{2} - wx = w\left(\frac{l}{2} - x\right)$$

(2)

This give a straight line graph equal to the rate of loading. The end values of Shearing Force are

$$\pm\frac{w\,l}{2}$$

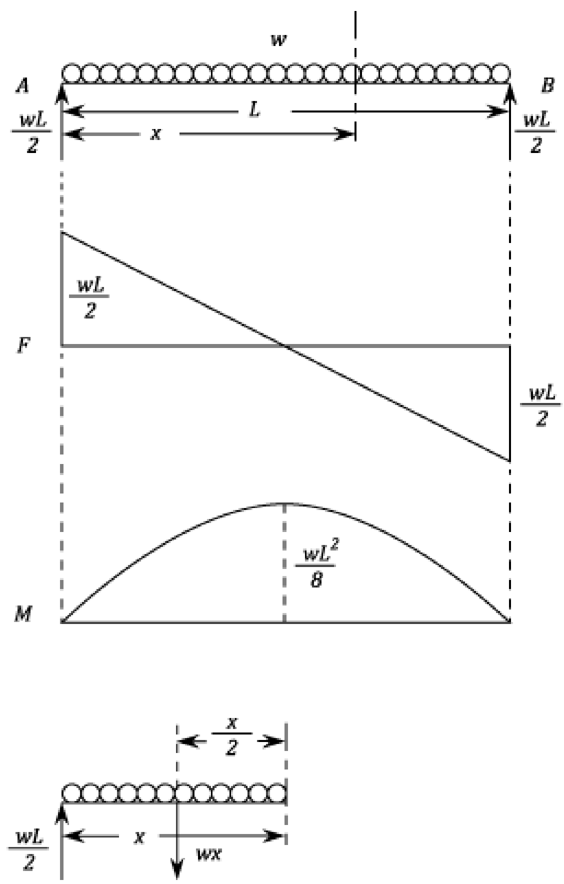
The Bending Moment at the section is found by assuming that the distributed load acts throughts center of gravity which is *x/2* from the section.

$$Hence \qquad M=\left(\frac{w\,l}{2}\right)x-(wx)\,\frac{x}{2}$$

(3)

$$=\left(\frac{wl}{2}\right)(l-x)$$

(4)



This is a parabolic curve having a value of zero at each end. The maximum is at the center and corresponds to zero shear force.

From Equation (2)

$$\hat{M}=\left(\frac{w\,l}{4}\right)\left(l-\frac{l}{2}\right)$$

(5)

Putting $x = l/2$

$$\hat{M} = \frac{w\,l^2}{8}$$

(6)

Combined Load

Problem 3. A Beam 25 m. long is supported at A and B and is loaded as shown. Sketch the

SF and BM diagrams and find (a) the position and magnitude of the maximum Bending Moment and (b) the position of the point of contraflexure.

<u>Solution</u>

Taking Moments about B

$$20\,R_a = 10 \times 15 + 2 \times 5 - 3 \times 5$$

(7)

(The distributed load is taken as acting at its centre of gravity.)

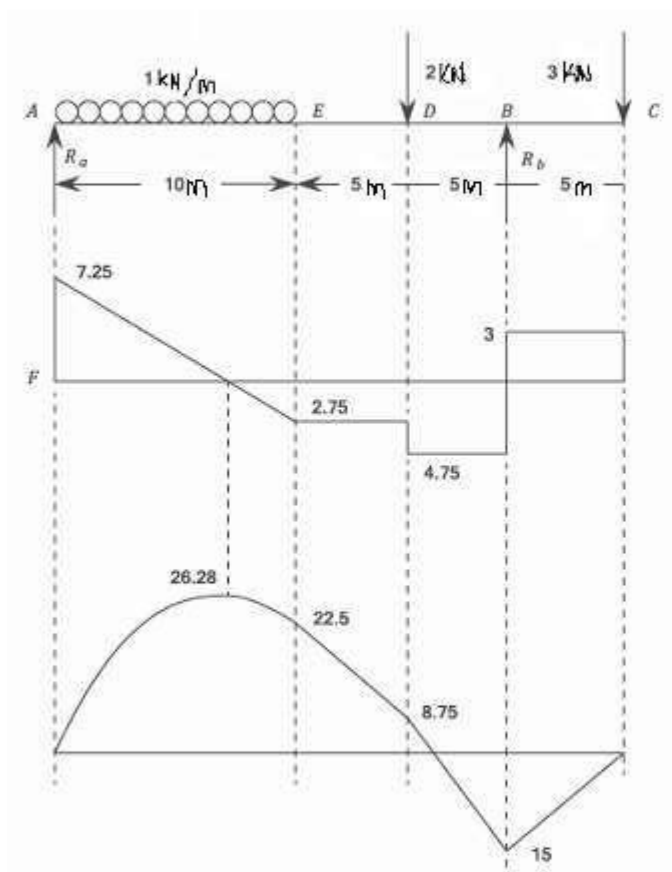
$\therefore\ R_a$ = 7.25 k N

$\therefore\quad R_b$ = Total Load - R_a = 10 + 2 + 3 – 7.25 = 7.75 k N

Starting at A, F = 7.25. As the section moves away from A F decreases at a uniform rate of w perunit length (i.e. f = 7.25 – w x) and reaches a value of -2.75 at E.

Between E and D, F is constant (There is no load on Ed) and at D it suffers a sudden decrease of2 kN (the load at D). Similarly there is an increase at B of 7.75 k N (the reaction at B).

This results in a value of F = 3k N at B which remains constant between B and C. Note this valueagrees with the load at C.



Bending Moment from A to E:

$$M = R_a\, x \cdot$$

This is a parabola which can be sketched by taking several values of x. B

eyond E the value of xfor the distributed load remains constant at 5 ft. from A

Between E and D

$$M = 7.25\, x - 10(X - 5) \ = -2.75x + 50$$

This produces a straight line between E and D. Similar equations apply for sections DB and BC.However it is only necessary to evaluate M at the points D and B since M is zero at C. The diagram consists in straight lines between these values.

At D

$$\mathrm{M} = -2.75 \ \times 15 + 50 = 8.75\ \mathrm{kN.m} \qquad \textbf{(10)}$$

Bending Moment from A to E:

$$M = R_a\, x - \frac{w\, x^2}{2} = 7.25\, x - \frac{x^2}{2} \quad since \quad (x = 1)$$

This is a parabola which can be sketched by taking several values of x.

Beyond E the value of xfor the distributed load remains constant at 5 ft. from A

Between E and D

$$M = 7.25\, x - 10(X - 5) \ = -2.75x + 50$$

This produces a straight line between E and D. Similar equations apply for sections DB and BC.However it is only necessary to evaluate M at the points D and B since M is zero at C. The diagram consists in straight lines between these values.

At D

$$\mathrm{M} = -2.75 \ \times 15 + 50 = 8.75\ \mathrm{kN.m}$$

At B

$$M = -3 \times 5 = -15 \text{ kN.m} \quad (13)$$

This last value was calculated for the portion BC

We were required to find the position and magnitude of the maximum BM. This occurs wherethe shearing force is zero.

i.e. .at 7.25 m. from A

$$\therefore \quad \mathbf{M = 7.25 \times 7.25 - \frac{7.25^2}{2} = 26.28\ k\ N.\ m} \qquad \textbf{(14)}$$

The point of contraflexure occurs when the bending moment is zero and this is between D and Bat:

$$\left(\frac{15}{15+8.75}\right) \times 5 = 3.16 \text{ m from B} \qquad \textbf{(15)}$$

2.5 Varying Distributed Loads

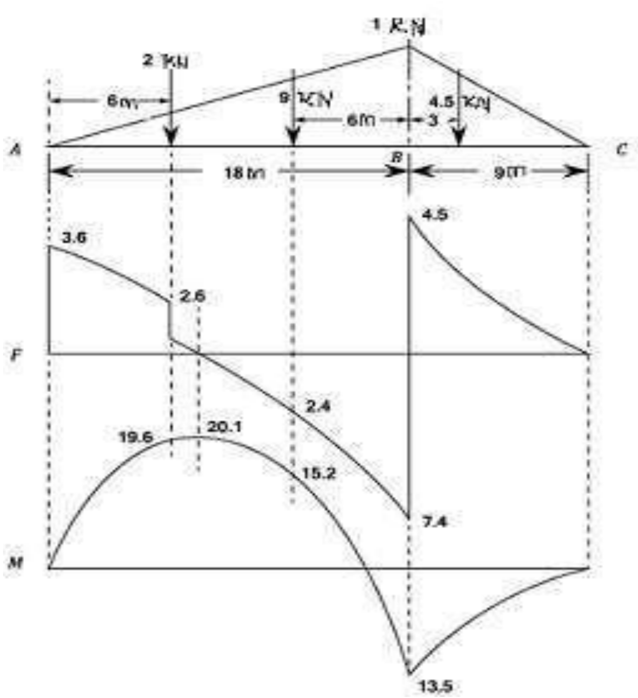
Problem

A Beam ABC, 27m long, is simply supported at A and B 18 m. across and carries a load of 2 kNat 6 m. from A together with a distributed load whose intensity varies in linear fashion from zeroat A and C to 1 kN/m. at B.

Workings

Draw the Shear Force and Bending Moment diagrams and calculate the position and magnitudeof the maximum B.M.

The Total Load on the beam (i.e. the load plus the mean rate of loading of 1/2 kN/m) is given by: $Load = 2 + \frac{1}{2} \times 27 = 15.5 \text{ kN}$



2.6 Graphical Solutions

This method may appear complicated but whilst the proof and explanation is fairly detailed, the application is simple and straight forward. The change of Bending Moment can be given by the double Integral of the rate of loading. This integration can be carried out by means of a **funicularpolygon**. Suppose that the loads carried on a simply supported beam are W_1, W_2, W_3, and W_4; and that R_1 and R_2 are the reactions at the supports. Letter the spaces between the loads and reactions A, B, C, D, E, and F.

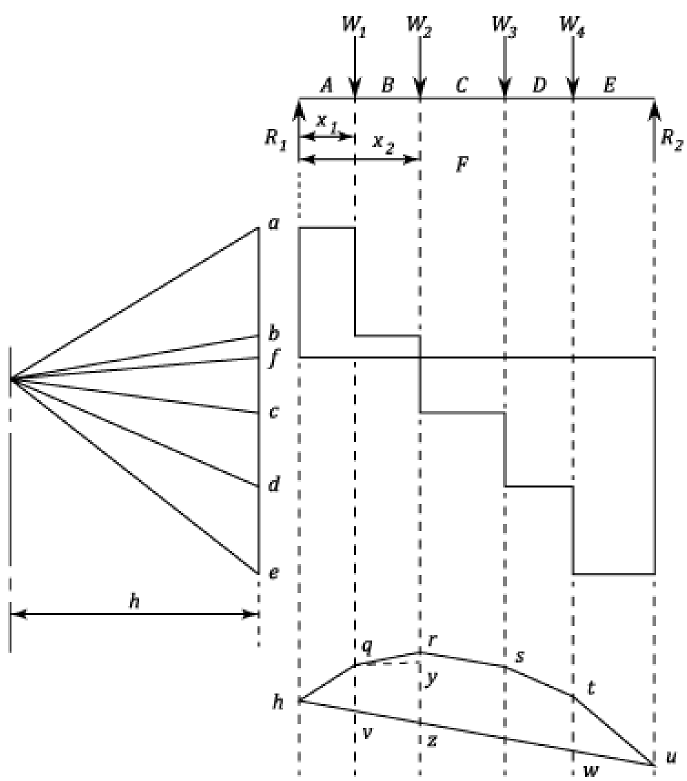
i. Draw to scale a vertical line such that:

$$a\,b = W_1; \quad b\,c = W_2; \quad c\,d = W_3; \quad and \quad d\,e = W_4. \tag{1}$$

ii. Now take any point "O" to the left of the line and join O to a, b, c, d, and e. Thisis called **The Polar Diagram**

iii. Commencing at any point p on the line of action of R_1 draw **pq** parallel to **Oa** in the space "A" , qr parallel to **Ob** in the space "B" and similarly rs, st, and tu. Draw Of parallel to pu. $W_1, W_2, W_3, and W_4, and that R_1 \; and \; R_2$

It will now be shown that **fa** represents R_1. Also, **pqrstu** is the Bending Moment diagram drawnon a base pu, M being proportional to the vertical ordinates. W_1 is represented by ab and acts through the point q; it can be replaced by forces **a O** along qp and Ob along qr.
Similarly, W_2 can be replaced by forces represented by **b O** along rq and Oc along rs, W_3 by **c O** along **s r** and **O d** along **s t** etc. All of these forces cancel each other out except **aO** along **qp** and**Od** along **te**, and these two forces must be in equilibrium with $R_1 \; and \; R_2$. This can only be so if R_1 is equivalent to a force **Oa** along **pq** and **fO** along up, R_2 being equivalent to **eO** along utand **Of** along **pu**. Hence, R_1 is represented by **fa** and R_2 by **ef.**



triangles **pqv** and **Oaf** are similar and hence:

$$q\,v = a\,f.\frac{pv}{Of}$$ **(2)**

$$Or \qquad qv \propto a\,f\,\frac{x_1}{h}$$

Where x_1 is the distance from W_1 from the left-hand end of the beam, and h is the length of the perpendicular from **O** on to **ae**. But $ay\,.\,x_1 \;\propto\; R_1\,x_1 \quad i.e.\,the\;BM\;at\;x_1$

Hence for a given position of the pole **O**, **qv** represents the B>M> at x_1 to a certain scale.

If **qy** is drawn parallel **tp pu**, then the triangle **qry** is similar to **Obf** and:

$$r\,y = b\,f\left(\frac{qy}{of}\right) = b\,f\left(\frac{x_2 - x_1}{h}\right)$$ **(3)**

$$\therefore \qquad rz = qv + ry = af\;\left(\frac{x_1}{h}\right) + bf\left(\frac{x_2\;-x_1}{h}\right)$$ **(4)**

$$which\;is\;\propto R_1x_1 = (R_1 - W_1)(x_2 - x_1) = R_1x_2 - W_1(x_2\;-X_1)$$ **(5)**

Which is the Bending Moment at x_2.

Similarly, the ordinates at the other load points give the Bending Moments at those points, the scale being determined as follows: If the load scale of the Polar Diagram is 1cm = S_1 kN then thelength scale along the beam is,. and the Bending Moment scale required is 1cm = S_3 kN m, then the length

$$qv \propto af.\frac{x_1}{h}$$ as shown above $$= \frac{W_1x_1}{s_1\,s_2h} = \frac{M}{s_1\,s_2\,h}$$

(6)

But,

$$qv = \frac{M}{s_3}$$ **(7)**

$$\therefore \qquad h = \frac{s_3}{s_2\,s_2}\;in.$$

If a base on the same level as f is drawn and the points a, b, c, d, and e are projected across from the Polar Diagram, then the Shearing Force diagram is obtained.
This method can be equally well used for distributed loads by dividing the loading diagram into strips and taking the load on a strip to act as if it were concentrated at its centre of gravity.
For cantilevers, if the Pole O is taken on the same horizontal level as the point a, then the base of the Bending Moment will be horizontal.

Shearing Force F

$$F = \frac{dM}{dx}$$

(8)

Mending
Moment M
Rate of
loading w

$$w = -\frac{dF}{dx} = -\frac{d^2M}{dx^2}$$

QUESTIONS :

1-(A) WHAT IS SHEAR FORCE ?

(B) WHAT IS BENDING MOMENT ?

(C) WHAT ARE THE SIGN CONVENTION OF SHEAR FORCE ?

(D) WHAT ARE THE SIGN CONVENTION OF BENDING MOMENT ?

(E) WHAT IS POINT LOAD ?

(F) WHAT IS U.D.L ?

(G) WHAT IS U.V.L ?

(H) WHAT IS POINT OF CONTRAFLEXURE ?

2-(A)CALCULATE MAXIMUM OF BENDING MOMENT AND SHEAR FORCE OF CANTILEVER BEAM CARRYING POINT LOAD ?

(B) CALCULATE MAXIMUM BENDING MOMENT AND SHEAR FORCE OF CANTILEVER BEAM CARRYING U.D.L. ?

(C) CALCULATE MAXIMUM BENDING MOMENT AND SHEAR FORCE OF SIMPLE SUPPORTED BEAM CARRYING POINT LOAD ?

(D) CALCULATE MAXIMUM BENDING MOMENT AND SHEAR FORCE OF SIMPLE SUPPORTED BEAM CARRYING U.D.L ?

(E) WHEN POINT OF CONTRAFLEXURE DEVELOPED ,SHOW WITH NEAT SKETCH OF SIMPLE SUPPORTED BEAM ?

INDETERMINATE BEAM(CH-7)

Statically Indeterminate Beams

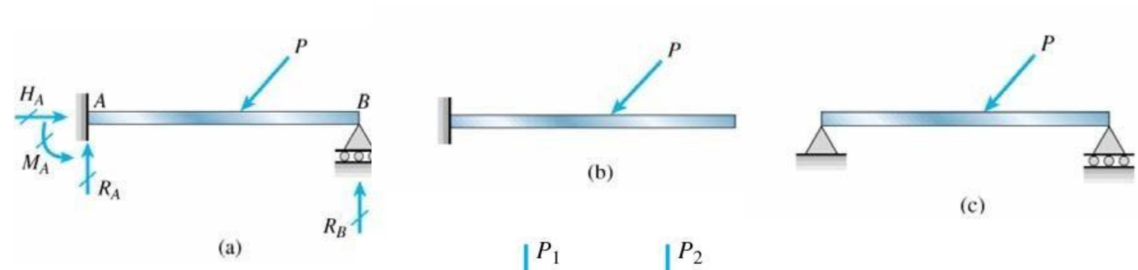
10.1 Introduction

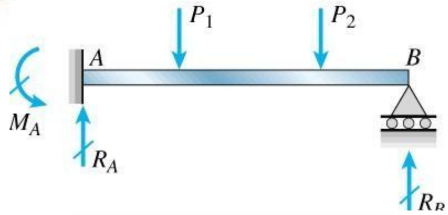
In this chapter we will analyze the beam in which the number ofreactions exceed the number of independent equations of equilibrium

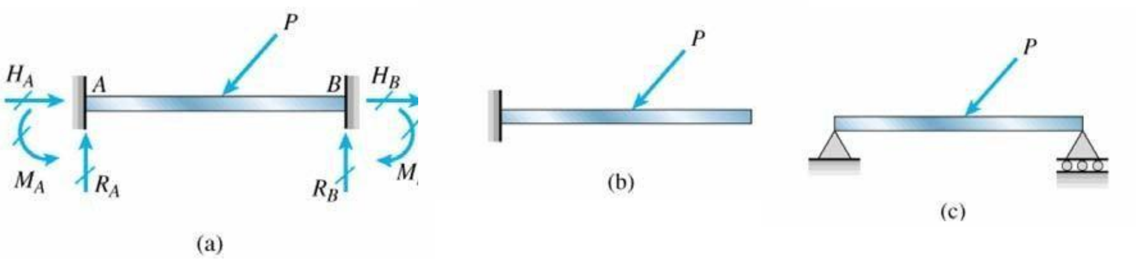
integration of the differential equation, method of superpositioncompatibility equation (consistence of deformation)

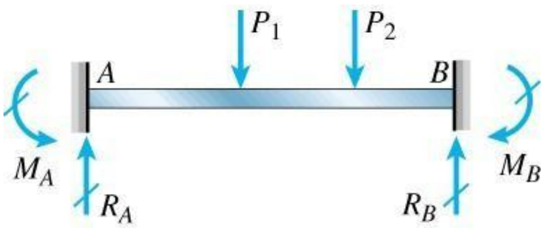
10.2 Types of Statically Indeterminate Beams

the number of reactions in excess of the number of equilibrium equationsis called the degree of static indeterminacy









$$R_A = qL - R_B \qquad M_A = \frac{CC}{2} - R_B L$$

and the bending moment of the beam is

$$M = R_A x - M_A - \frac{qx^2}{CC}$$

2

$$= qLx - R_B x - \frac{qL^2}{2} - R_B L - \frac{qx^2}{2}$$

$$EIv'' = M = qLx - R_B x - \frac{qL^2}{2} - R_B L - \frac{qx^2}{2}$$

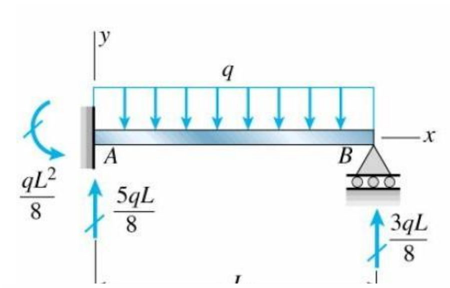
$$EIv' = \frac{qLx^2}{2} - \frac{R_B x^2}{2} - \frac{qL^2 x}{2} - R_B L x - \frac{qx^3}{6} + C_1$$

$$EIv = \frac{qLx^3}{6} - \frac{R_b x^3}{6} - \frac{qL^2x^2}{4} - \frac{R_B L x^2}{2} - \frac{qx^4}{24} + C_1 x + C_2$$

boundary conditions

$v(0) = 0 \qquad v'(0) = 0 \qquad v(L) = 0$

it is obtained

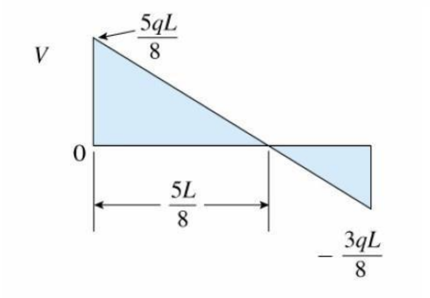


$C_1 = C_2 = 0 \qquad R_B = 3qL/8$

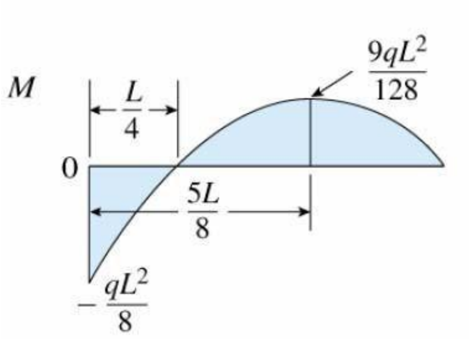
and $R_A = 5qL/8$

$M_A = qL^2/8$

the shear force and bending moment are

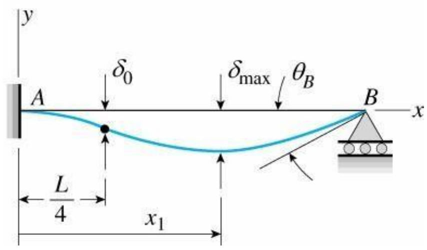


$$V = R_A - qx = \frac{5qL}{8} - qx$$



$$M = R_A x - M_A - \frac{qx^2}{2}$$

$$= \frac{5qLx}{8} - \frac{qL^2}{8} - \frac{qx^2}{2}$$



the maximum shear force is

$$V_{max} = 5qL/8 \quad \text{at the fixed end}$$

the maximum positive and negative moments are

$$M_{pos} = 9qL^2/128 \qquad M_{neg} = -qL^2/8$$

slope and deflection of the beam

$$v' = \frac{qx}{48EI}(-6L^2 + 15Lx - 8x^2)$$

$$v = -\frac{qx^2}{48EI}(3L^2 - 5Lx + 2x^2)$$

to determine the 6_{max}, set $v' = 0$

$$-6L^2 + 15Lx - 8x^2 = 0$$

we have $x_1 = 0.5785L$

$$6_{max} = -v(x_1) = 0.005416\frac{qL^4}{EI}$$

the point of inflection is located at $M = 0$, i.e. $x = L/4$

$$h < 0 \text{ and } M < 0 \text{ for } x < L/4$$

$$h > 0 \text{ and } M > 0 \text{ for } x > L/4$$

the slope at B is

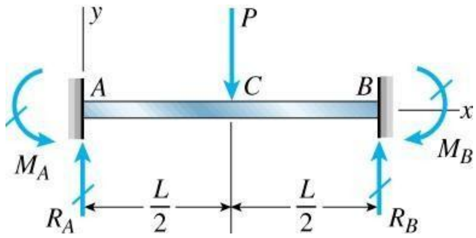
$$qL^3$$

$$\theta_B = (y')_{x=L} = \frac{CC}{48EI}$$

Example 10-2

a fixed-end beam *ABC* supports a concentrated load *P* at the midpoint

determine the reactions, shear forces, bending moments, slopes, and deflections



because the load *P* in vertical direction and symmetric

$$H_A = H_B = 0 \qquad R_A = R_B = P/2$$

$$M_A = M_B \quad \text{(1 degree of indeterminacy)}$$

$$M = \frac{Px}{2} - M_A \ (0 \leqq x \leqq L/2)$$

$$EIv'' = M = \frac{Px}{2} - M_A \quad (0 \leqq x \leqq L/2)$$

after integration, it is obtained

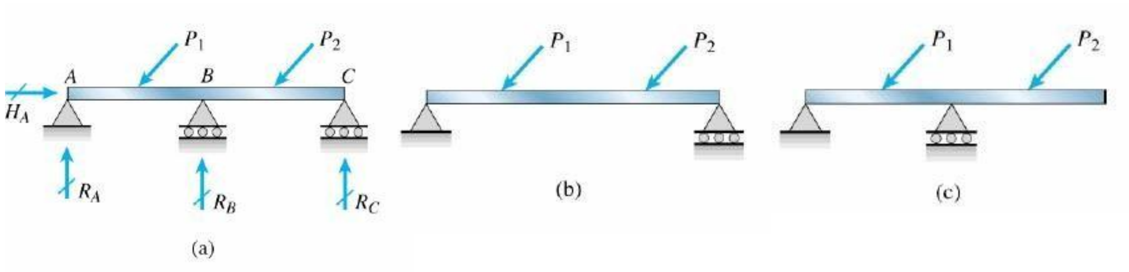
$$EIv' = \frac{Px^2}{4} - M_A x + C_1 \quad (0 \leqq x \leqq L/2)$$

$$EIv = \frac{Px^3}{12} - \frac{M_A x^2}{2} + C_1 x + C_2 \quad (0 \leqq x \leqq L/2)$$

boundary conditions

$v(0) = 0 \qquad v'(0) = 0$

symmetric condition



the excess reactions are called static redundants

the structure that remains when the redundants are released is calledreleased structure or the primary structure

10.3 Analysis by the Differential Equations of the Deflection Curve

$$EIv'' = M \qquad EIv''' = V \qquad EIv^{iv} = -q$$

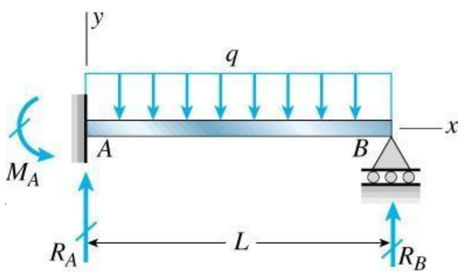
the procedure is essentially the same as that for a statically determinebeam and consists of writing the differential equation, integrating to obtain its general solution, and then applying boundary and other conditions to evaluate the unknown quantities, the unknowns consist of the redundantreactions as well as the constants of integration

this method have the computational difficulties that arise when a large number of constants to be evaluated, it is practical only for relatively simple case

Example 10-1

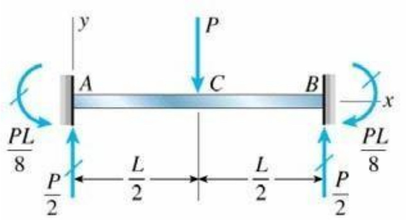
a propped cantilever beam *AB* supports a uniform load *q*

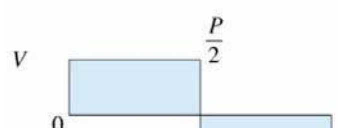
determine the reactions, shear forcesbending moments, slopes, and deflections



choose R_B as the redundant, then

$v'(0) = 0$





the constants C_1, C_2 and the moment M_A are obtained

$$C_1 = C_2 = 0$$

$$M_A = \frac{PL}{8} = M_B$$

the shear force and bending momentdiagrams can be plotted

thus the slope and deflection equations are

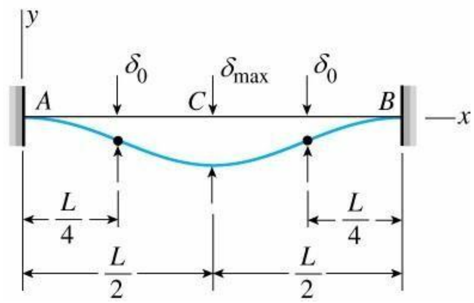
$$v' = -\frac{Px}{8EI}(L - 2x) \qquad (0 \leqq x \leqq L/2)$$

$$v = -\frac{Px^2}{48EI}(3L - 4x) \qquad (0 \leqq x \leqq L/2)$$

the maximum deflection occurs at the center

$$\delta_{max} = -v(L/2) = \frac{PL^3}{192EI}$$

the point of inflection occurs at the point where $M = 0$, i.e. $x = L/4$, the deflection at this point is



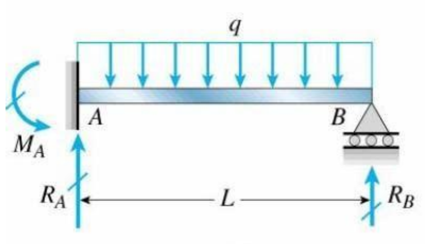
$$\delta = -v(L/4) = \frac{PL^3}{384EI}$$

which is equal $\delta_{max}/2$

10.4 Method of Superposition

1. selecting the reaction redundants
2. establish the force-displacement relations
3. consistence of deformation (compatibility equation)

consider a propped cantilever beam



(a)

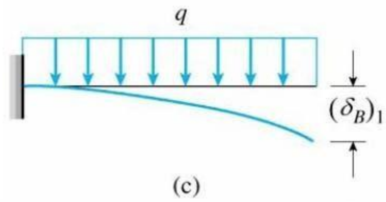
(i) select R_B as the redundant, then

$$R_A = qL - R_B \qquad M_A = \frac{qL^2}{2} - R_B L$$

(b)

force-displacement relation

$$(\delta_B)_1 = \frac{qL^4}{8EI} \qquad (\delta_B)_2 = \frac{R_B L^3}{3EI}$$

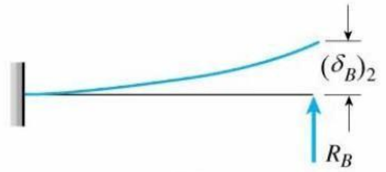


(c)

compatibility equation

$$\delta_B = (\delta_B)_1 - (\delta_B)_1 = 0$$

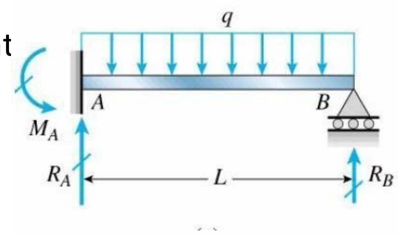
$$\frac{qL^4}{8EI} = \frac{R_B L^3}{3EI}$$



(d)

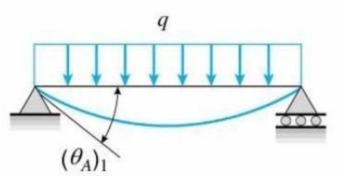
$$R_B = \frac{3qL}{8} \quad \Rightarrow \quad R_A = \frac{5qL}{8} \qquad M_A = \frac{\quad}{8}$$

(ii) select the moment M_A as the redundant

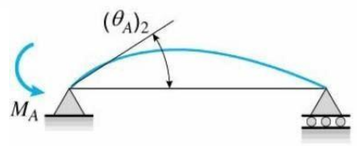


$$R_A = \frac{qL}{2} + \frac{M_A}{L} \qquad R_B = \frac{qL}{2} - \frac{M_A}{L}$$

(b)



(c)



(d)

force-displacement relation

$$(\theta_A)_1 = \frac{qL^3}{24EI} \qquad (\theta_A)_2 = \frac{M_A L}{3EI}$$

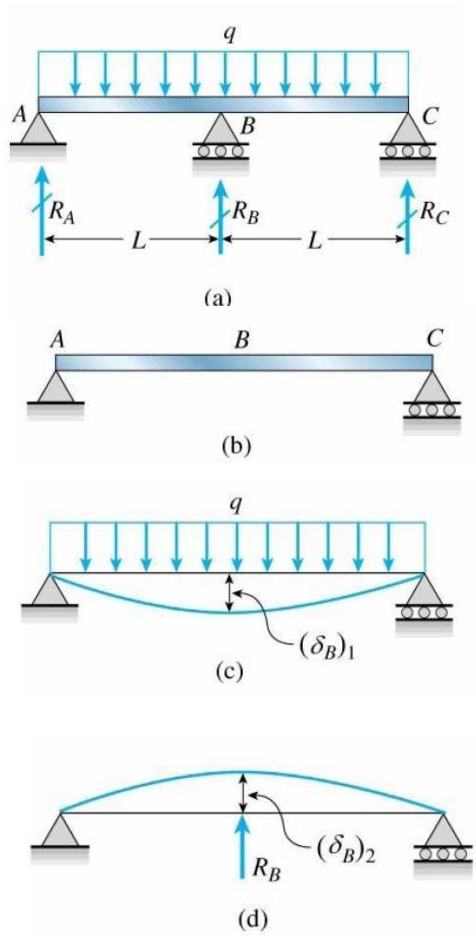
compatibility equation

$$\theta_A = (\theta_A)_1 - (\theta_A)_2 = \frac{qL^3}{24EI} - \frac{M_A L}{3EI} = 0$$

thus $M_A = qL^2/8$

and $R_A = 5qL/8 \qquad R_B = 3qL/8$

Example 10-3



a continuous beam *ABC* supports a uniform load *q*

determine the reactions

select R_B as the redundant, then

$$R_A = R_C = qL - \frac{qL}{2}$$

force-displacement relation

$$(6_B)_1 = \frac{5qL(2L)^4}{384EI} = \frac{5qL^4}{24EI}$$

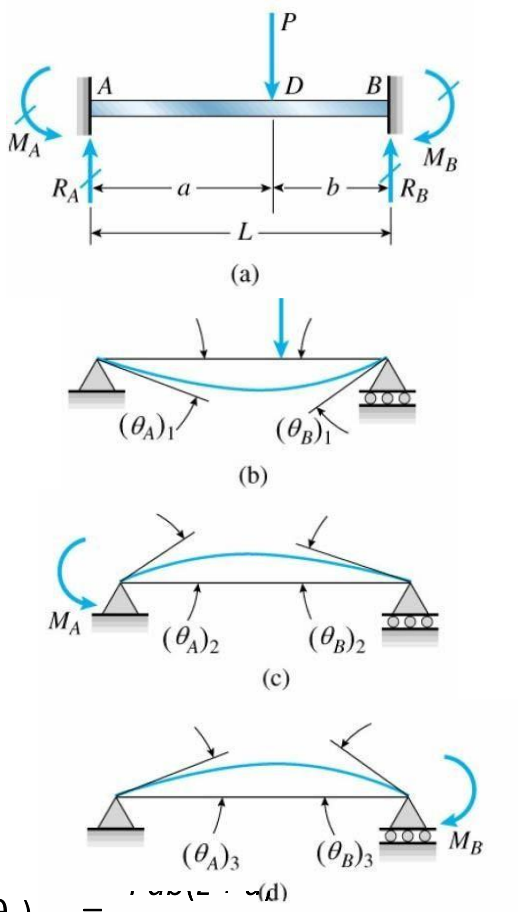
$$(6_B)_2 = \frac{R_B(2L)^3}{48EI} = \frac{R_B L^3}{6EI}$$

compatibility equation

$$6_B = (6_B)_1 - (6_B)_2 = \frac{5qL^4}{24EI} - \frac{R_B L^3}{6EI} = 0$$

thus $R_B = 5qL/4$

and $R_A = R_C = 3qL/8$



Example 10-4

a fixed-end beam AB is loaded by a force P acting at point D

determine reactions at the endsalso

determine 6_D

this is a 2-degree of indeterminacy problemselect M_A and M_B as the redundants

$$R_A = \frac{Pb}{L} + \frac{M_A}{L} - \frac{M_B}{L}$$

$$R_B = \frac{Pa}{L} - \frac{M_A}{L} + \frac{M_B}{L}$$

force-displacement relations

$$(\theta_A)_1 = \frac{Pab(L+b)}{6LEI} \qquad (\theta_B)_1 = \frac{Pab(L+a)}{6LEI}$$

$$(\theta_A)_2 = \frac{M_A L}{3EI} \qquad (\theta_B)_2 = \frac{M_A L}{6EI}$$

$$(\theta_A)_3 = \frac{M_B L}{6EI} \qquad (\theta_B)_3 = \frac{M_B L}{3EI}$$

compatibility equations

$$\theta_A = (\theta_A)_1 - (\theta_A)_2 - (\theta_A)_3 = 0$$

$$\theta_B = (\theta_B)_1 - (\theta_B)_2 - (\theta_B)_3 = 0$$

i.e.

$$\frac{M_A L}{3EI} + \frac{M_B L}{6EI} = \frac{Pab(L + b)}{6LEI}$$

$$\frac{M_A L}{6EI} + \frac{M_B L}{3EI} = \frac{Pab(L + a)}{6LEI}$$

solving these equations, we obtain

$$M_A = \frac{Pab^2}{L^2} \qquad M_B = \frac{Pa^2b}{L^2}$$

and the reactions are

$$R_A = \frac{Pb^2}{L^3}(L + 2a) \qquad R_B = \frac{Pa^2}{L^3}(L + 2b)$$

the deflection 6_D can be expressed as

$$6_D = (6_D)_1 - (6_D)_2 - (6_D)_3$$

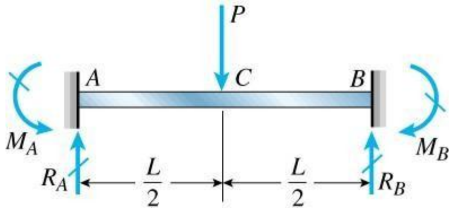
$$(6_D)_1 = \frac{Pa^2b^2}{3LEI}$$

$$(6_D)_2 = \frac{M_A ab}{6LEI}(L + b) = \frac{Pa^2b^3}{6L^3EI}(L + b)$$

$$(6_D)_3 = \frac{M_B ab}{6LEI}(L + a) = \frac{Pa^3b^2}{6L^3EI}(L + a)$$

thus $$6_D = \frac{Pa^3b^3}{3L^3EI}$$

if $a = b = L/2$



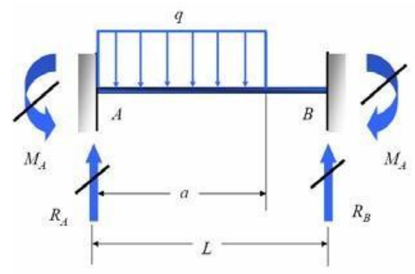
then $M_A = M_B = \frac{PL}{\ }$ $\qquad R_A = R_B = \frac{P}{\ }$

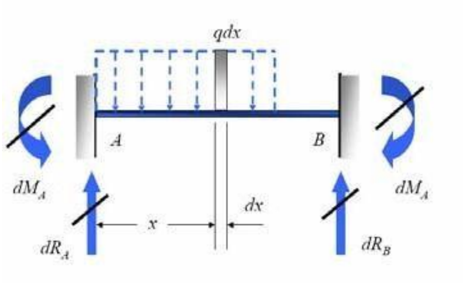
and $\delta_C = \frac{PL^3}{192EI}$

Example 10-5

a fixed-end beam AB supports a uniform load q acting over part of the span

determine the reactions of the beam





to obtain the moments caused by qdx, replace P to qdx, a to x, and b to $L - x$

$$dM_A = \frac{qx(L - x)^2 dx}{L^2}$$

$$dM_B = \frac{qx^2(L - x)dx}{L^2}$$

integrating over the loaded part

$$M_A = \int dM_A = \frac{q}{L^2}\int_0^a x(L - x)^2 dx = \frac{qa^2}{12L^2}(6L^2 - 8aL + 3a^2)$$

$$M_B = \int dM_B = \frac{q}{L^2}\int_0^a x^2(L - x)dx = \frac{qa^3}{12L^2}(4L^2 - 3a)$$

Similarly

$$dR_A = \frac{q(L - x)^2(L + 2x)dx}{L^3}$$

$$dR_B = \frac{qx^2(3L - 2x)dx}{L^3}$$

integrating over the loaded part

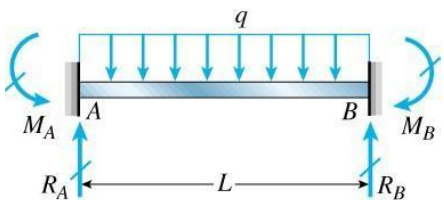
$$R_A = \int dR_A = \frac{q}{L^3}\int_0^a (L - x)^2(L + 2x)dx = \frac{qa}{2L^3}(2L^3 - 2a^2L + a^3)$$

$$R_B = \int dR_B = \frac{q}{L^3}\int_0^a x^2(3L - 2x)dx = \frac{qa^3}{2L^3}(2L - a)$$

for the uniform acting over the entire length, i.e. $a = L$

$$M_A = M_B = \frac{qL^2}{12}$$

$$R_A = R_B = \frac{qL}{2}$$



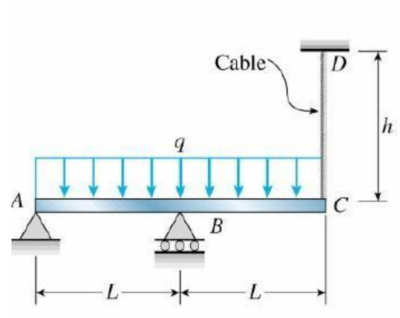
the center point deflections due to uniform load and the end moments are $5qL^4$ M_AL

$$(6_C)_1 = \frac{}{384EI} \qquad (6_C)_2 = \frac{(qL^2/12)L^2}{8EI} = \frac{qL^4}{8EI} = \frac{}{96EI}$$

$$6_C = (6_C)_1 - (6_C)_2 = \frac{qL^4}{384EI}$$

Example 10-6

a beam ABC rests on supports A and B and is supported by a cable at C

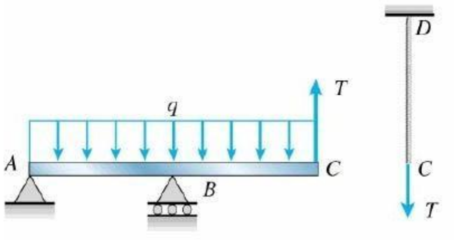


(a)

find the force T of the cable

take the cable force T as redundant the deflection $(6_c)_1$ due the uniform load can be found from example 9.9 with

$a = L$



(b)

$$(6_c)_1 = \frac{qL^4}{4E_bI_b}$$

the deflection $(6_c)_2$ due to a force T acting on C is obtained

use conjugate beam method

$$(6_c)_2 = M = \frac{TL^2}{3E_bI_b} L + \frac{TL}{E_bI_b} \frac{L}{2} \frac{2L}{3}$$

$$= \frac{2TL^3}{3E_bI_b}$$

the elongation of the cable is

$$(6_c)_3 = \frac{Th}{E_cA_c}$$

compatibility equation

$$(6_c)_1 - (6_c)_2 = (6_c)_3$$

$$\frac{qL^4}{4E_bI_b} - \frac{2TL^3}{3E_bI_b} = \frac{Th}{E_cA_c}$$

$$T = \frac{3qL^4E_cA_c}{8L^3E_cA_c + 12hE_bI_b}$$

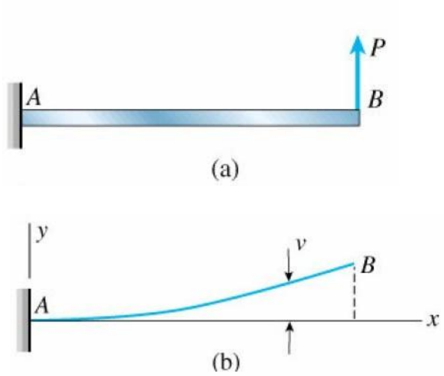
QUESTIONS

1. WHAT DO U MEAN BY INDETERMINATE BEAM ?
2. DEFINE PROPED CANTELEVER ?
3. WHAT ARE THE METHODS OF SUPER POSITION ?
4. WHAT ARE THE PRINCIPLE OF COMPATIBILITY ?
5. ANALYSIS OF PROPPED CANTILEVER ?
6. WHAT DO YOU MEAN BY TWO SPAN CONTINEOUS BEAM?
7. ADVANTAGES & DISADVANTAGES PROPPED CANTILEVER ?

Stresses in Beams (ch-3)

5.1 Introduction

Beam : loads acting transversely to the longitudinal axisthe loads create shear forces and bending moments, stresses and strains due to V and M are discussed in this chapter lateral loads acting on a beam cause the beam to bend, thereby deforming the axis of the beam into curve line, this is known as the **deflection curve** of the beam



the beams are assumed to be symmetric about x-y plane, i.e. y-axis is an axis of symmetric of the cross section, all loads are assumed to act in the x-y plane, then the bending deflection occurs in the same plane, it is known as the **plane of bending**

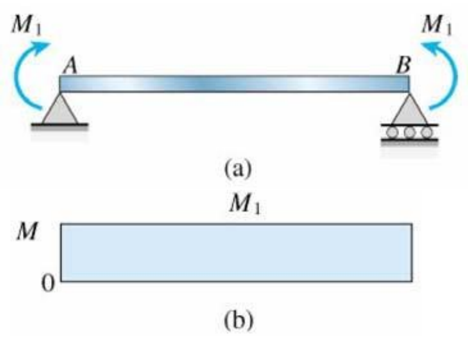
the **deflection** of the beam is the displacement of that point from its original position, measured in *y* direction

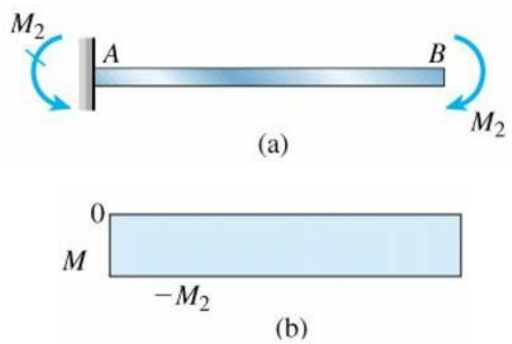
5.2 Pure Bending and Nonuniform Bending

pure bending :

$M = \text{constant} \qquad V = dM/dx = 0$

pure bending in simple beam and cantilever beam are shown



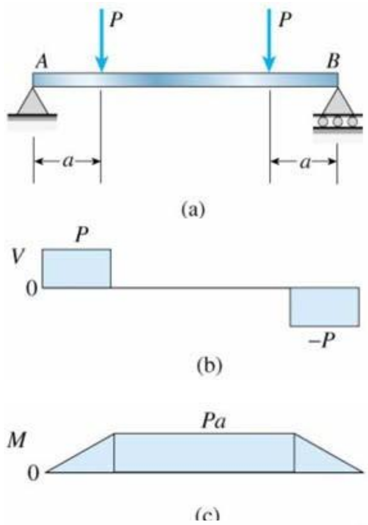


nonuniform bending :

$M \neq$ constant

$V = dM/dx \neq 0$

simple beam with central region in pure bending and end regions in nonuniform bending is shown



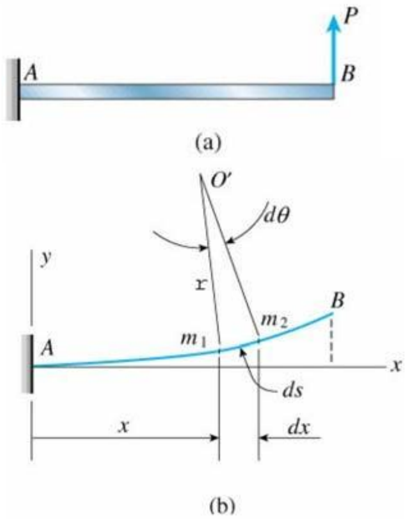
5.3 Curvature of a Beam

consider a cantilever beam subjected to a load P

choose 2 points m_1 and m_2 on the deflection curve, their normals intersect at point O', is called the center of curvature,

the distance m_1O' is called radius of curvature ρ, and the curvature κ is defined as

$$\kappa = 1/\rho$$



and we have $\rho\, d\theta = ds$

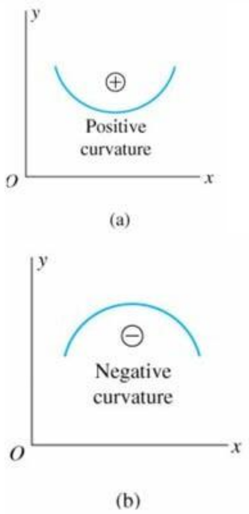
if the deflection is small $ds \approx dx$, then

$$\kappa = \frac{1}{\rho} = \frac{d\theta}{ds} = \frac{d\theta}{dx}$$

sign convention for curvature

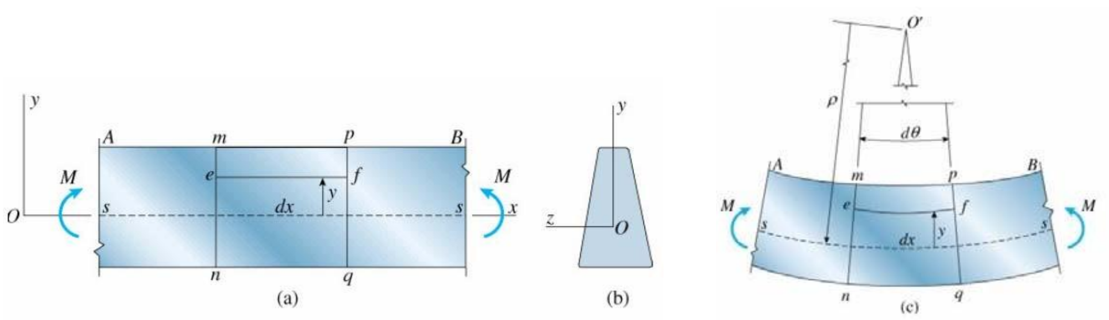
+ : beam is bent concave upward (convex downward)

- : beam is bent concave downward (convex upward)



5.4 Longitudinal Strains in Beams

consider a portion *ab* of a beam in pure bending produced by a positive bending moment *M*, the cross section may be of any shape provided it is symmetric about *y*-axis



under the moment *M*, its axis is bent into a circular curve, cross section *mn* and *pq* remain plane and normal to longitudinal lines (plane remains plane can be established by experimental result)

∵ the symmetry of the beam and loading, it requires that all elements of the beam deform in an identical manner (∴ the curve is circular), this are valid for any material (elastic or inelastic)

due to bending deformation, cross sections *mn* and *pq* rotate w.r.t. each other about axes perpendicular to the *xy* plane

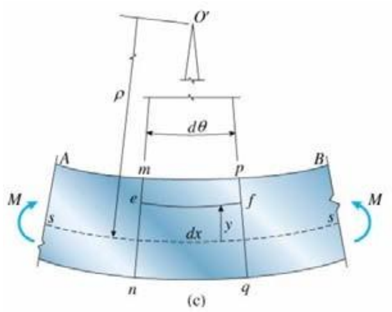
longitudinal lines on the convex (lower) side (*nq*) are elongated, and on the concave (upper) side (*mp*) are shortened

the surface *ss* in which longitudinal lines do not change in length is called the **neutral surface**, its intersection with the cross-sectional plane is called neutral axis, for instance, the *z* axis is the neutral axis of the cross section

in the deformed element, denote ρ the distance from *O'* to N.S. (or N.A.), thus

$$\rho \, d\theta = dx$$

consider the longitudinal line *ef*, the length L_1 after bending is



(c)

$$L_1 = (\rho - y)\, d\theta = dx - \frac{y}{\rho} dx$$

then

$$<_{ef} = L_1 - dx = - \frac{y}{\rho} dx$$

and the strain of line *ef* is

$$\varepsilon_x = \frac{<_{ef}}{dx} = - \frac{y}{\rho} = - h\, y$$

ε_x vary linear with y (the distance from N.S.)

$y > 0$ (above N. S.) $\varepsilon = -$

$y < 0$ (below N. S.) $\varepsilon = +$

the longitudinal strains in a beam are accompanied by transverse strainsin the y and z directions because of the effects of Poisson's ratio

Example 5-1

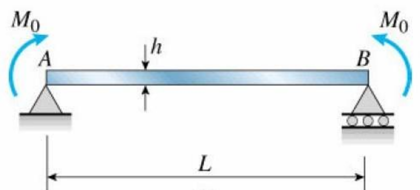
a simply supported beam *AB*,

$L = 4.9$ m $h = 300$ mm

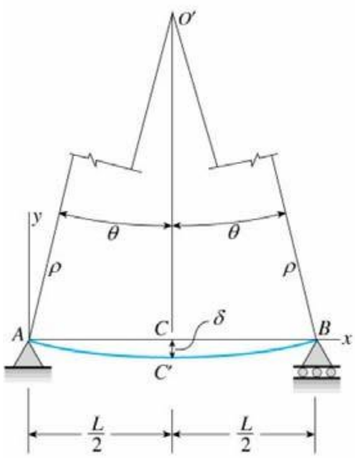
bent by M_0 into a circular arc

$\varepsilon_{bottom} = \varepsilon_x = 0.00125$

determine ρ, h, and 6 (midpoint deflection)



(a)



(b)

$$\rho = - \frac{y}{\varepsilon_x} = - \frac{-150}{0.00125}$$

$$= 120 \text{ m}$$

$$h = \frac{1}{\rho} = 8.33 \times 10^{-3}\ m^{-1}$$

$$6 = \rho\,(1 - \cos\theta)$$

∵ ρ is large, ∴ the deflection curve is very flat

then $$\sin\theta = \frac{L/2}{\rho} = \frac{8 \times 12}{2 \times 2{,}400} = 0.020$$

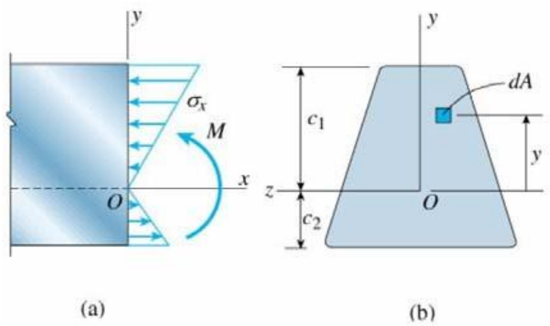
$$\theta = 0.02\ \text{rad} = 1.146^{o}$$

then $$6 = 120 \times 10^{3}\,(1 - \cos 1.146^{o}) = 24\ \text{mm}$$

5.4 Normal Stress in Beams (Linear Elastic Materials)

∵ ε_x occurs due to bending, ∴ the longitudinal line of the beam is subjected only to tension or compression, if the material is linear elastic

then $$\sigma_x = E\,\varepsilon_x = -E\,h\,y$$



σ vary linear with distance y from the neutral surface

consider a positive bending moment M applied, stresses are positive below N.S. and negative above N.S.

∵ no axial force acts on the cross section, the only resultant is M, thus two equations must satisfy for static equilibrium condition

i.e. $$X\,F_x = \int \sigma\,dA = -\int E\,h\,y\,dA = 0$$

∵ E and h are constants at the cross section, thus we have

$$\int y \, dA = 0$$

we conclude that the neutral axis passes through the controid of the cross section, also for the symmetrical condition in y axis, the y axis must pass through the centroid, hence, the origin of coordinates O is located at the centroid of the cross section

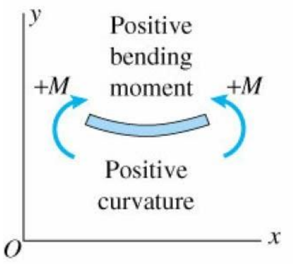
the moment resultant of stress σ_x is

$$dM = - \sigma_x \, y \, dA$$

then $$M = - \int \sigma_x \, y \, dA = \int E \kappa \, y^2 \, dA = E \kappa \int y^2 \, dA$$

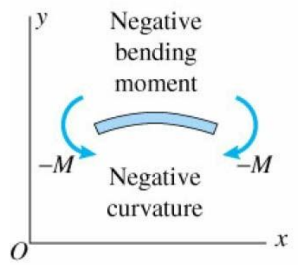
$$M = E \kappa I$$

where $I = \int y^2 \, dA$ is the moment of inertia of the cross-sectional area w. r. t. z axis



thus $$\kappa = \frac{1}{\rho} = \frac{M}{E I}$$

this is the **moment-curvature equation**, and EI is called flexural rigidity



$+ M$ => + curvature

$- M$ => - curvature

the normal stress is

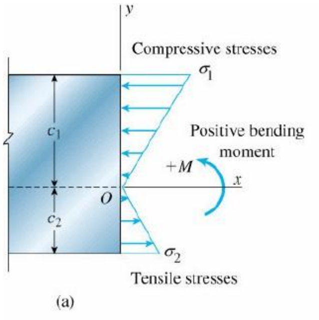
$$\sigma_x = - E \kappa y = - E y \left(\frac{M}{E I}\right) = - \frac{M y}{I}$$

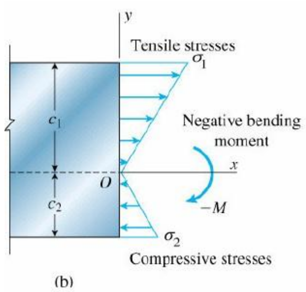
this is called the **flexure formula**, the stress σ_x is called bending stresses or flexural stresses

σ_x vary linearly with y

$\sigma_x \propto M \qquad \sigma_x \propto 1/I$

the maximum tensile and compressive stresses occur at the points located farthestfrom the N.A.





$$\sigma_1 = -\frac{M c_1}{I} = -\frac{M}{S_1}$$

$$\sigma_2 = \frac{M c_2}{I} = \frac{M}{S_2}$$

where $S_1 = \dfrac{I}{c_1}$, $S_2 = \dfrac{I}{c_2}$ are known as the section moduli

if the cross section is symmetric w.r.t. z axis (double symmetric cross section), then $c_1 = c_2 = c$

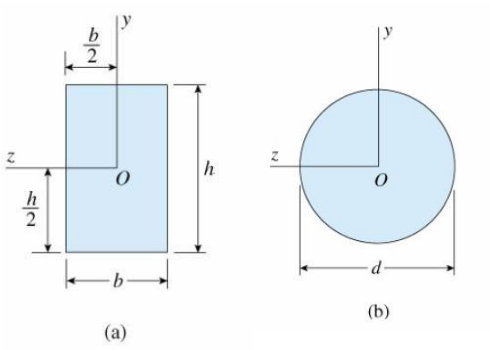
thus $S_1 = S_2$ and $\sigma_1 = -\sigma_2 = -\dfrac{M c}{I} = -\dfrac{M}{S}$

for rectangular cross section

$$I = \frac{b h^3}{12} \qquad S = \frac{b h^2}{6}$$

for circular cross section

$$I = \frac{\pi d^4}{64} \qquad S = \frac{\pi d^3}{32}$$



the preceding analysis of normal stress in beams concerned pure bending,no shear force

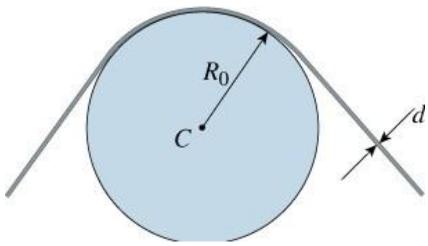
in the case of nonuniform bending ($V \neq 0$), shear force produces warping (out of plane distortion), plane section no longer remain plane after bending, but the normal stress σ_x calculated from the flexure formula are not significantly altered by the presence of shear force and warping

we may justifiably use the theory of pure bending for calculating σ_x even when we have nonuniform bending

the flexure formula gives results in the beam where the stress distribution is not disrupted by irregularities in the shape, or by discontinuous in loading (otherwise, stress concentration occurs)

example 5-2

a steel wire of diameter d = 4 mm is bent around a cylindrical drum of radius R_0 = 0.5 m



E = 200 GPa σ_{pl} = 1200 MPa

determine M and σ_{max}

the radius of curvature of the wire is

$$\rho = R_0 + \frac{d}{2}$$

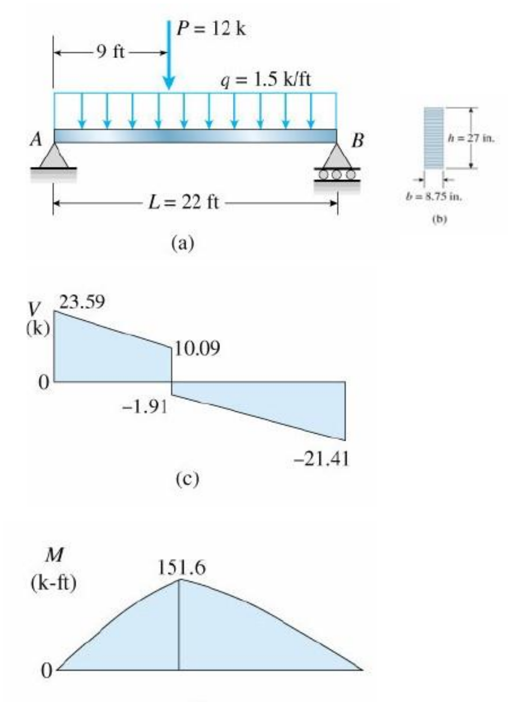
$$M = \frac{EI}{\rho} = \frac{2\,EI}{2\,R_0 + d} = \frac{\pi\,E\,d^4}{32(2R_0 + d)}$$

$$= \frac{\pi\,(200 \times 10^3)\,4^4}{32\,(2 \times 500 + 4)} = 5007\ \text{N-mm} = 5.007\ \text{N-m}$$

$$\sigma_{max} = \frac{M}{S} = \frac{M}{I/(d/2)} = \frac{M\,d}{2\,I} = \frac{2\,EI\,d}{2\,I\,(2\,R_0 + d)} = \frac{E\,d}{2\,R_0 + d}$$

$$= \frac{200 \times 10^3 \times 4}{2 \times 500 + 4} = 796.8 \text{ MPa} \quad < \quad 1{,}200 \text{ MPa (OK)}$$

Example 5-3



(a) (c) (d)

a simple beam *AB* of length L = 6.7 m

q = 22 kN/m $\quad$ P = 50 kN

b = 220 mm $\quad$ h = 700 mm

determine the maximum tensile and compressive stresses due to bending

firstly, construct the *V-dia* and *M-dia*

σ_{max} occurs at the section of

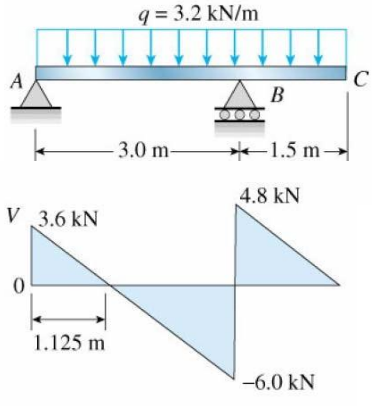
$M_{max}M_{max}$ = 193.9 kN-m

the section modulus S of the section is

$$S = \frac{b\,h^2}{6} = \frac{0.22 \times 0.7^2}{6} = 0.018 \text{ m}^3$$

$$\sigma_t = \sigma_2 = \frac{M}{S} = \frac{139.9 \text{ kN-m}}{0.018 \text{ m}^3} = 10.8 \text{ MPa}$$

$$\sigma_c = \sigma_1 = -\frac{M}{S} = -10.8 \text{ MPa}$$



(b)

Example 5-4

an overhanged beam *ABC* subjected uniform load of intensity q = 3.2 kN/m

for the cross section (channel section)

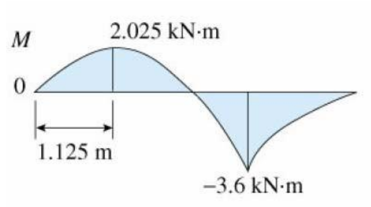
t = 12 mm b = 300 mm h = 80 mm

determine the maximum tensile and compressive stresses in the beam

construct the *V-dia.* and *M-dia.* first

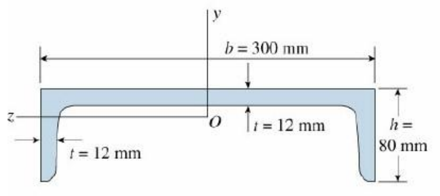
we can find $+M_{max} = 2.205$ kN-m

$-M_{max} = -3.6$ kN-m



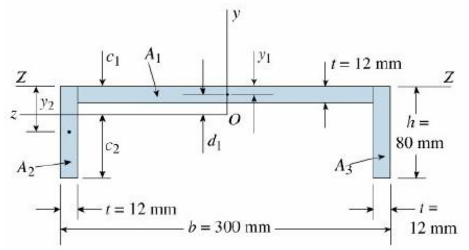
(c)

next, we want to find the N. A. of the section



(a)

	A(mm^2)	y(mm)	$A\,y$ (mm^3)
A_1	3,312	6	19,872
A_2	960	40	38,400
A_3	960	40	38,400
total	5,232		96,672



(b)

$$c_1 = \frac{\Sigma A_i y_i}{\Sigma A_i} = \frac{96,672}{5,232} = 18.48 \text{ mm}$$

$c_2 = h - c_1 = 61.52$ mm

moment of inertia of the section is

$$I_{z1} = I_{zc} + A_1 d_1^2$$

$$I_{zc} = \frac{1}{12}(b - 2t)\,t^3 = \frac{1}{12}\,276 \times 12^3 = 39744 \text{ mm}^4$$

$d_1 = c_1 - t/2 = 12.48$ mm

$I_{z1} = 39,744 + 3,312 \times 12.48^2 = 555,600$ mm^4

similarly $I_{z2} = I_{z3} = 956,000$ mm^4

then the centroidal moment of inertia I_z is

$$I_z = I_{z1} + I_{z2} + I_{z3} = 2.469 \times 10^6 \text{ mm}^4$$

$$S_1 = \frac{I_z}{c_1} = 133{,}600 \text{ mm}^3 \qquad S_2 = \frac{I_z}{c_2} = 40{,}100 \text{ mm}^3$$

at the section of maximum positive moment

$$\sigma_t = \sigma_2 = \frac{M}{S_2} = \frac{2.025 \times 10^3 \times 10^3}{40{,}100} = 50.5 \text{ MPa}$$

$$\sigma_c = \sigma_1 = -\frac{M}{S_1} = -\frac{2.025 \times 10^3 \times 10^3}{133{,}600} = -15.2 \text{ MPa}$$

at the section of maximum negative moment

$$\sigma_t = \sigma_1 = -\frac{M}{S_1} = -\frac{-3.6 \times 10^3 \times 10^3}{133{,}600} = 26.9 \text{ MPa}$$

$$\sigma_c = \sigma_2 = \frac{M}{S_2} = -\frac{-3.6 \times 10^3 \times 10^3}{40{,}100} = -89.8 \text{ MPa}$$

thus $(\sigma_t)_{max}$ occurs at the section of maximum positive moment$(\sigma_t)_{max}$

= 50.5 MPa

and $(\sigma_c)_{max}$ occurs at the section of maximum negative moment$(\sigma_c)_{max}$

= - 89.8 MPa

5.6 Design of Beams for Bending Stresses

design a beam : type of construction, materials, loads and environmentalconditions beam shape and size : actual stresses do not exceed the allowable stressfor the bending stress, the section modulus S must be larger than M / σ

i.e. $S_{min} = M_{max} / \sigma_{allow}$

σ_{allow} is based upon the properties of the material and magnitude of the desired factor of safety

if σ_{allow} are the same for tension and compression, doubly symmetric section is logical to choose

if σ_{allow} are different for tension and compression, unsymmetric cross section such that the distance to the extreme fibers are in nearly the same ratio as the respective allowable stresses

select a beam not only the required S, but also the smallest cross-sectional area

Beam of Standardized Shapes and Sizes

steel, aluminum and wood beams are manufactured in standard sizes steel : American Institute of Steel Construction (AISC)

Eurocode

e.g. wide-flange section *W* 30 x 211 depth = 30 in, 211 lb/ftHE

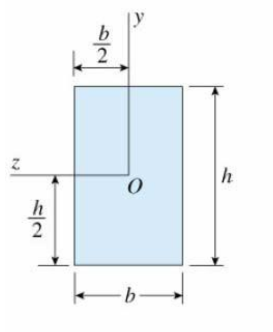
1000 B depth = 1000 mm, 314 kgf/m etc

other sections : *S* shape (I beam), *C* shape (channel section)

L shape (angle section)

aluminum beams can be extruded in almost any desired shape since thedies are relatively easy to make

wood beam always made in rectangular cross section, such as 4" x 8" (100 mm x 200 mm), but its actual size is 3.5" x 7.25" (97 mm x 195 mm) after it



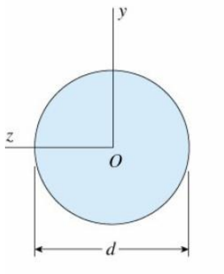
(a)

is surfaced

consider a rectangular of width b and depth h

$$S = \frac{b\,h^2}{6} = \frac{A\,h}{6} = 0.167\,A\,h$$

a rectangular cross section becomes more efficient as h increased, but very narrow section may fail because of lateral bucking



(b)

for a circular cross section of diameter d

$$S = \frac{\pi\,d^3}{32} = \frac{A\,d}{8} = 0.125\,A\,d$$

comparing the circular section to a square section of same area

$$h^2 = \pi\,d^2 / 4 \Rightarrow h = \sqrt{\pi}\,d / 2$$

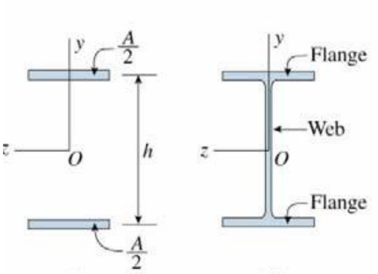
$$\frac{S_{square}}{S_{circle}} = \frac{0.167\,A\,h}{0.125\,A\,d} = \frac{0.167\ \sqrt{\pi}\,d / 2}{0.125\,d} = \frac{0.148}{0.125} = 1.184$$

∴ the square section is more efficient than circular section

the most favorable case for a given area A and depth h would have to distribute $A / 2$ at a distance $h / 2$ from the neutral axis, then

$$I = \frac{A}{2}\left(\frac{h}{2}\right)^2 \times 2 = \frac{A\,h^2}{4}$$

$$S = \frac{I}{h / 2} = \frac{A\,h}{2} = 0.5\,A\,h$$



(c) (d)

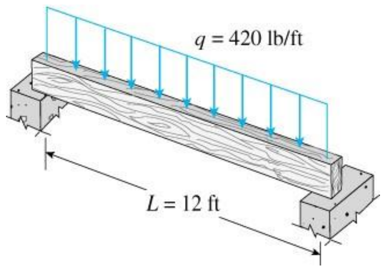
the wide-flange section or an I - section with most material in theflanges would be the most efficient section for standard wide-flange beams, S is approximately

$$S \approx 0.35\, A\, h$$

wide-flange section is more efficient than rectangular section of the same area and depth, ∵ much of the material in rectangular beam is located near the neutral axis where it is unstressed, wide-flange section have most of the material located in the flanges, in addition, wide-flange shape is wider and therefore more stable with respect to sideways bucking

Example 5-5

a simply supported wood beam carriesuniform load



$L = 3\text{ m} \qquad q = 4\text{ kN/m}$

σ_{allow} = 12 MPa   wood weights 5.4 kN/m^3

select the size of the beam

(a) calculate the required S

$$M_{max} = \frac{q\,L^2}{8} = \frac{(4\text{ kN/m})\,(3\text{ m})^2}{8} = 4.5\text{ kN-m}$$

$$S = \frac{M_{max}}{\sigma_{allow}} = \frac{4.5\text{ kN-m}}{12\text{ MPa}} = 0.375 \times 10^6\text{ mm}^3$$

(b) select a trial size for the beam (with lightest weight)

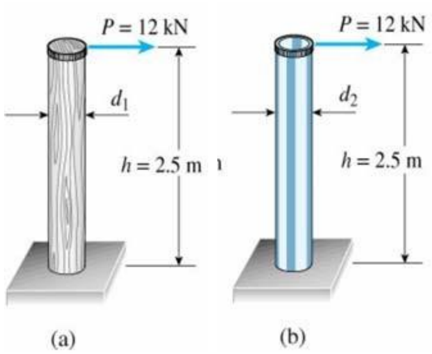
choose 75 x 200 beam, $S = 0.456 \times 10^6\text{ mm}^3$ and weight 77.11 N/m

(c) now the uniform load on the beam is increased to 77.11 N/m

$$S_{required} = (0.375 \times 10^6 \text{ mm}^3) \frac{4.077}{4.0} = 0.382 \times 10^6 \text{ mm}^3$$

(d) $S_{required} < S$ of 75 x 200 beam $(0.456 \times 10^6 \text{ mm}^3)$ (O.K.)

Example 5-6



a vertical post 2.5 m high support a lateralload

P = 12 kN at its upper end

(a) σ_{allow} for wood = 15 MPa

determine the diameter d_1

(b) σ_{allow} for aluminum tube = 50 MPa

determine the outer diameter d_2 if $t = d_2 / 8$

$$M_{max} = P\,h = 12 \times 2.5 = 30 \text{ kN-m}$$

(a) wood post

$$S_1 = \frac{\pi d_1^3}{32} = \frac{M_{max}}{\sigma_{allow}} = \frac{30 \times 10^3 \times 10^3}{15} = 2 \times 10^6 \text{ mm}^3$$

d_1 = 273 mm

(b) aluminum tube

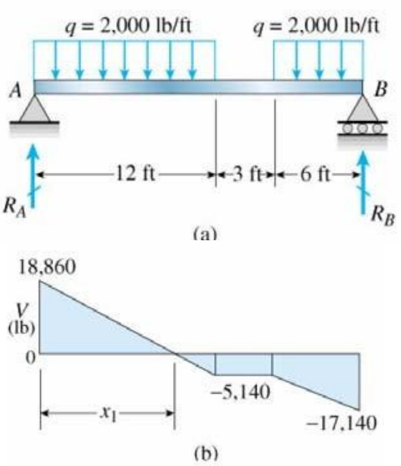
$$I_2 = \frac{\pi}{64} [d_2^4 - (d_2 - 2\,t)^4] = 0.03356\, d_2^4$$

$$S_2 = \frac{I_2}{c} = \frac{0.03356\, d_2^4}{d_2 / 2} = 0.06712\, d_2^3$$

$$S_2 = \frac{M_{max}}{\sigma_{allow}} = \frac{30 \times 10^3 \times 10^3}{50} = 600 \times 10^3 \text{ mm}^3$$

solve for d_2 => d_2 = 208 mm

Example 5-7



a simple beam *AB* of length 7 m q = 60 kN/m σ_{allow} = 110 MPa

select a wide-flange shape

firstly, determine the support reactions

R_A = 188.6 kN R_B = 171.4 kN

the shear force V for $0 \leqq x \leqq 4$ m is

$$V = R_A - q\,x$$

for $V = 0$, the distance x_1 is

$$x_1 = \frac{R_A}{q} = \frac{188.6 \text{ kN}}{60 \text{ kN/m}} = 3.143 \text{ m}$$

and the maximum moment at the section is

$$M_{max} = 188.6 \times 3.143 / 2 = 296.3 \text{ kN-m}$$

the required section moudlus is

$$S = \frac{M_{max}}{\sigma_{allow}} = \frac{296.3 \times 10^6 \text{ N-mm}}{110 \text{ MPa}} = 2.694 \times 10^6 \text{ mm}^3$$

from table *E*-1, select the HE 450 A section with S = 2,896 cm^3

the weight of the beam is 140 kg/m, now recalculate the reactions, M_{max}, and $S_{required}$, we have

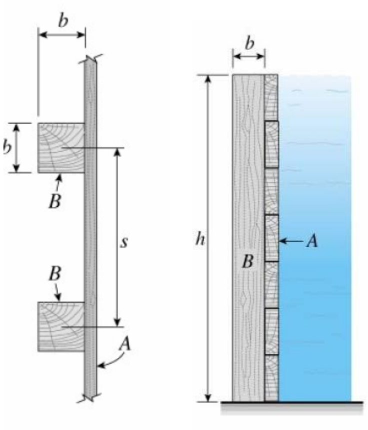
R_A = 193.4 kN R_B = 176.2 kN

V = 0 at x_1 = 3.151 m

=> M_{max} = 304.7 KN-m

$$S_{required} = \frac{M_{max}}{\sigma_{allow}} = 2,770 \text{ cm}^3 < 2,896 \text{ cm}^3 \quad \text{(O. K.)}$$

Example 5-8



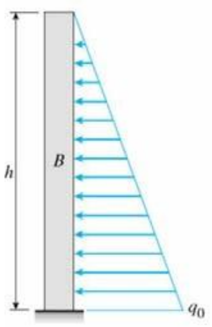
(a) Top view (b) Side view

the vertical posts B are supported planks A of the dam

post B are of square section b x b the spacing of the posts s = 0.8 m water level h = 2.0 m

σ_{allow} = 8.0 MPa

determine b



(c) Loading diagram

the post B is subjected to the water pressure (triangularly distributed)

the maximum intensity q_0 is

$$q_0 = h\,s$$

the maximum bending moment occurs at the base is

$$M_{max} = \frac{q_0 h}{2} \left(\frac{h}{3}\right) = \frac{h^3 s}{6}$$

$$\text{and} \quad S = \frac{M_{max}}{\sigma_{allow}} = \frac{h^3 s}{6\,\sigma_{allow}} = \frac{b^3}{6}$$

$$b^3 = \frac{h^3 s}{\sigma_{allow}} = \frac{9.81 \times 2^3 \times 0.8}{8 \times 10^6} = 0.007848 \text{ m}^3 = 7.848 \times 10^6 \text{ mm}^3$$

b = 199 mm use b = 200 mm

5.7 Nonprismatic Beams

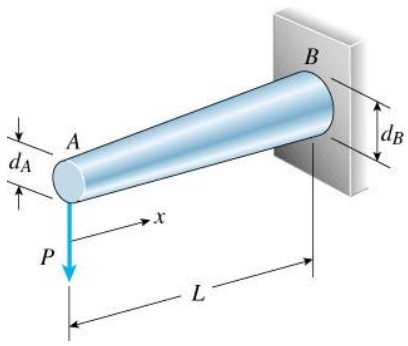
nonprismatic beams are commonly used to reduce weight and improve appearance, such beams are found in automobiles, airplanes, machinery,bridges, building etc.

$\sigma = M \ / \ S$, S varying with x, so we cannot assume that the maximum stress occur at the section with M_{max}

Example 5-9

a tapered cantilever beam *AB* of solid circular cross section supports a load *P* at the free end with $d_B \ / \ d_A = 2$

determine σ_B and σ_{max}



$$d_x = d_A + (d_B - d_A)\frac{x}{L}$$

$$S_x = \frac{\pi \, d_x^{3}}{32} = \frac{\pi}{32}\left[d_A + (d_B - d_A)\frac{x}{L}\right]^3$$

$\because$ $M_x = P\,x$, then the maximum bending stress at any cross section is

$$\sigma_1 = \frac{M_x}{S_x} = \frac{32\,P\,x}{\pi\,[d_A + (d_B - d_A)\,(x/L)]^3}$$

at support B, $d_B = 2\,d_A$, $x = L$, then

$$\sigma_B = \frac{4\,P\,L}{\pi\,d_A^{3}}$$

to find the maximum stress in the beam, take $d\sigma_1 \ / \ dx = 0$

$\Rightarrow x = L \ / \ 2$

at that section ($x = L/2$), the maximum is

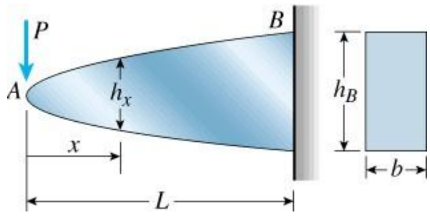
$$\sigma_{max} = \frac{128 P L}{27 \pi d_A^3} = 4.741 \frac{P L}{\pi d_A^3}$$

it is 19% greater than the stress at the built-in end

Example 5-10

a cantilever beam of length L support a load P at the free end

cross section is rectangular with constant width b, the height may vary such that $\sigma_{max} = \sigma_{allow}$ for every cross section (fully stressed beam)



determine the height of the beam

$$M = P x \qquad S = \frac{b h_x^2}{6}$$

$$\sigma_{allow} = \frac{M}{S} = \frac{P x}{b h_x^2 / 6} = \frac{6 P x}{b h_x^2}$$

solving the height for the beam, we have

$$h_x = \left(\frac{6 P x}{b \sigma_{allow}} \right)^{1/2}$$

at the fixed end ($x = L$)

$$h_B = \left(\frac{6 P L}{b \sigma_{allow}} \right)^{1/2}$$

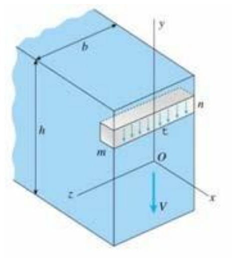
$$h_x = h_B \left(\frac{x}{L}\right)^{1/2}$$ the idealized beam has thethen parabolic shape

5-8 Shear Stress in Beam of Rectangular Cross Section

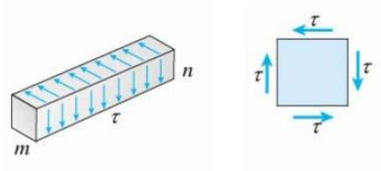
for a beam subjected to M and V with rectangular cross section having width b and height h, the shear stress $ acts parallel to the shear force V

assume that $ is uniform across the width of the beam consider a

beam section subjected the a shear force V, we isolate a small element mn, the shear stresses $ act vertically and accompanied horizontally as shown



(a)

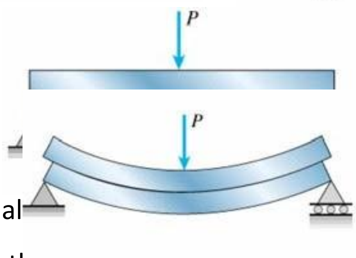


(b) (c)

∵ the top and bottom surfaces are free, then the shear stress must be vanish, i.e.

$ = 0 at y = ! $h/2$

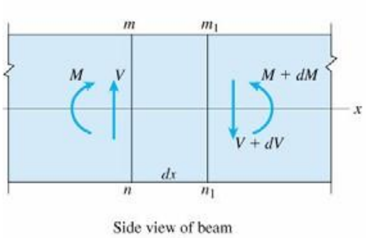
for two equal rectangular beams of height h subjected to a concentrated load P, if no friction between the beams, each beam will be in compression above its N.A., the lower longitudinal line of the upper beam will slide w.r.t. the upper line of the lower beam



(b)

for a solid beam of height $2h$, shear stress must exist al... thus single beam of depth $2h$ will much stiffer and stronger tl... h of depth h

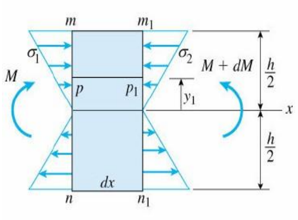
consider a small section of the beam subjected M and V in left face and $M + dM$ and $V + dV$ in right face



Side view of beam
(a)

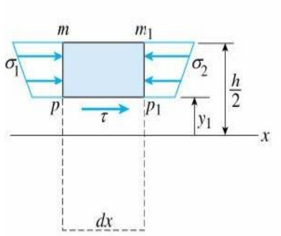
for the element mm_1p_1p, τ acts on p_1p and no stress on mm_1

if the beam is subjected to pure bending (M = constant), σ_x acting on mp and m_1p_1 must be equal, then $\tau = 0$ on pp_1



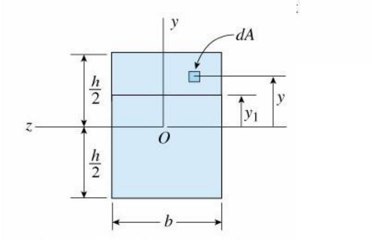
Side view of element
(b)

for nonuniform bending, M acts on mn an on m_1n_1, consider dA at the distance y form N.A., then on mn



$$\sigma_x \, dA = \frac{M y}{I} dA$$

hence the total horizontal force on mp is



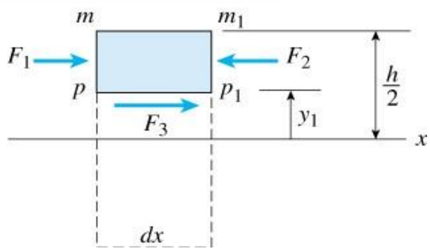
Cross section of beam at subelement
(d)

$$F_1 = \int \frac{M y}{I} dA$$

similarly

$$F_2 = \int \frac{(M + dM) y}{I} dA$$

and the horizontal force on pp_1 is



$$F_3 = \tau \, b \, dx$$

equation of equilibrium

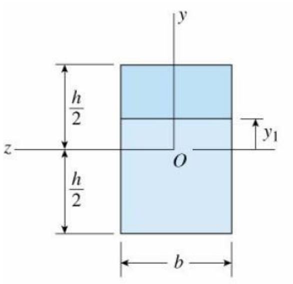
$$F_3 = F_2 - F_1$$

$$\tau\, b\, dx = \int \frac{(M + dM)\, y}{I}\, dA - \int \frac{M\, y}{I}\, dA$$

$$\tau = \frac{dM}{dx} \frac{1}{Ib} \int y\, dA = \frac{V}{I\, b} \int y\, dA$$

denote $Q = \int y\, dA$ is the first moment of the cross section areaabove the level y (area mm_1p_1p) at which the shear stress τ acts, then

$$\tau = \frac{V\, Q}{I\, b} \qquad \text{shear stress formula}$$



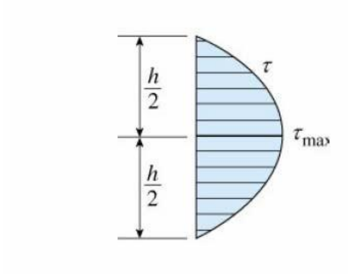
(a)

for V, I, b are constants, $\tau \sim Q$

for a rectangular cross section

$$Q = b\left(\frac{h}{2} - y_1\right)\left(y_1 + \frac{h/2 - y_1}{2}\right) = \frac{b}{2}\left(\frac{h^2}{4} - y_1^2\right)$$

then $$\tau = \frac{V}{2\, I}\left(\frac{h^2}{4} - y_1^2\right)$$



(b)

$\tau = 0$ at $y_1 = \pm h/2$, τ_{max} occurs at $y_1 = 0$ (N.A.)

$$\tau_{max} = \frac{V\, h^2}{8\, I} = \frac{V\, h^2}{8\, b\, h^3/12} = \frac{3\, V}{2\, A} = \frac{3}{2}\, \tau_{ave}$$

τ_{max} is 50% larger than τ_{ave}

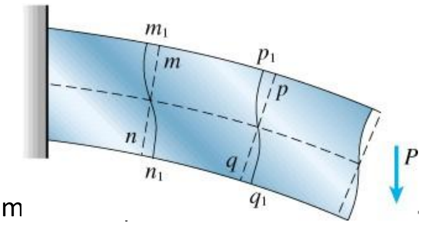
∵ V = resultant of shear stress, ∴ V and τ in the same direction

Limitations

the shear formula are valid only for beams of linear elastic material with small deflection

the shear formula may be consider to be exact for narrow beam (∵ $\$$ is assumed constant across b), when $b = h$, true $\$_{max}$ is about 13% larger than the value given by the shear formula

Effects of Shear Strains



∵ $\$$ vary parabolically from top to bottom, and

= $\$ / G$ must vary in the same manner

thus the cross sections were plane surfaces becom

occur on the surfaces, and maximum shear strain occurs onN.A.

∵ $_{max}$ = $\$_{max} / G$, if V remains constant along the beam, the warping of all sections is the same, i.e. $mm_1 = pp_1 = \cdots$, the stretching or shortening of the longitudinal lines produced by the bending moment is unaffected by the shear strain, and the distribution of the normal stress σ is the same as it is in pure bending

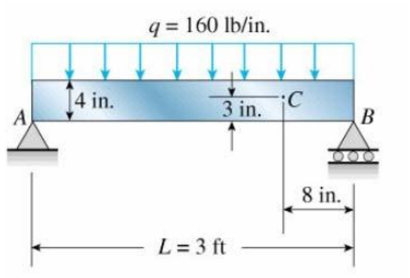
for shear force varies continuously along the beam, the warping of cross sections due to shear strains does not substantially affect the longitudinal strains by more experimental investigation

thus, it is quite justifiable to use the flexure formula in the case of nonuniform bending, except the region near the concentrate load acts of irregularly change of the cross section (stress concentration)

Example 5-11

a metal beam with span $L = 1$ m

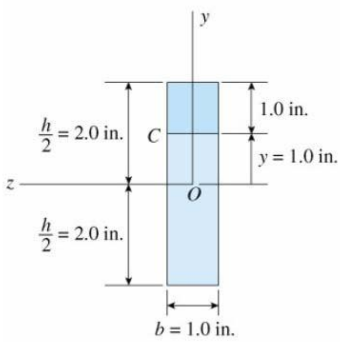
$q = 28$ kN/m $b = 25$ mm $h = 100$ mm



(a)

determine σ_C and $\$_C$ at point C

the shear force V_C and bending moment M_C at the section through C are found



(b)

$$M_C = 2.24 \text{ kN-m}$$

$$V_C = -8.4 \text{ kN}$$

the moment of inertia of the section is

$$I = \frac{b\,h^3}{12} = \frac{1}{12} \times 25 \times 100^3 = 2{,}083 \times 10^3 \text{ mm}^4$$

normal stress at C is

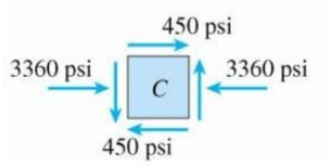
$$\sigma_C = -\frac{M\,y}{I} = -\frac{2.24 \times 106 \text{ N-mm} \times 25 \text{ mm}}{2{,}083 \times 10^3 \text{ mm}^4} = -26.9 \text{ MPa}$$

shear stress at C, calculate Q_C first

$A_C = 25 \times 25 = 625 \text{ mm}^2$ $\qquad y_C = 37.5$ mm

$$Q_C = A_C\,y_C = 23{,}400 \text{ mm}^3$$

$$\$_C = \frac{V_C\,Q_C}{I\,b} = \frac{8{,}400 \times 23{,}400}{2{,}083 \times 10^3 \times 25} = 3.8 \text{ MPa}$$



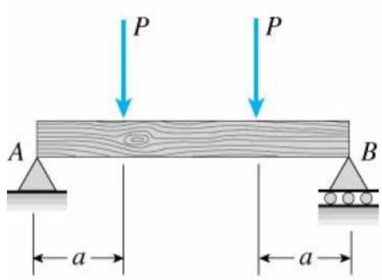
(c)

the stress element at point C is shown

Example 5-12

a wood beam AB supporting two concentrated loads P

$b = 100$ mm $\qquad h = 150$ mm



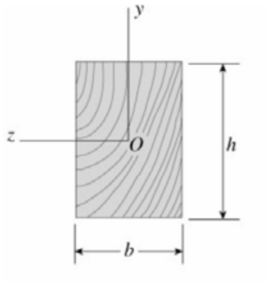
(a)

$a = 0.5$ m $\quad \sigma_{allow} = 11$ MPa $\quad \tau_{allow} = 1.2$ MPa

determine P_{max}

the maximum shear force and bending moment are

$$V_{max} = P \qquad M_{max} = P\,a$$



(b)

the section modulus and area are

$$S = \frac{b\,h^2}{6} \qquad A = b\,h$$

maximum normal and shear stresses are

$$\sigma_{max} = \frac{M_{max}}{S} = \frac{6\,P\,a}{b\,h^2} \qquad \tau_{max} = \frac{3\,V_{max}}{2\,A} = \frac{3\,P}{2\,b\,h}$$

$$P_{bending} = \frac{\sigma_{allow}\,b\,h^2}{6\,a} = \frac{11 \times 100 \times 150^2}{6 \times 500} = 8{,}250 \text{ N} = 8.25 \text{ kN}$$

$$P_{shear} = \frac{2\,\tau_{allow}\,b\,h}{3} = \frac{2 \times 1.2 \times 100 \times 150}{3} = 12{,}000 \text{ N} = 12 \text{ kN}$$

$\therefore \quad P_{max} = 8.25$ kN

8-9 Shear Stresses in Beam of Circular Cross Section

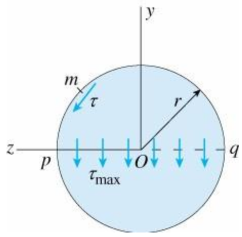
$$\tau = \frac{V\,Q}{I\,b} \qquad I = \frac{\pi\,r^4}{4} \quad \text{for solid section}$$

the shear stress at the neutral axis

$$Q = A\,\bar{y} = \left(\frac{\pi\,r^2}{2}\right)\left(\frac{4\,r}{3\,\pi}\right) = \frac{2\,r^3}{3} \qquad b = 2\,r$$

$$\tau_{max} = \frac{V(2r^3/3)}{(\pi r^4/4)(2r)} = \frac{4V}{3\pi r^2} = \frac{4V}{3A} = \frac{4}{3}\tau_{ave}$$

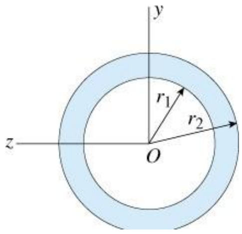
for a hollow circular cross section



$$I = \frac{\pi}{4}(r_2^4 - r_1^4) \qquad Q = \frac{2}{3}(r_2^3 - r_1^3)$$

$$b = 2(r_2 - r_1)$$

then the maximum shear stress at N.A. is



$$\tau_{max} = \frac{VQ}{Ib} = \frac{4V}{3A}\left(\frac{r_2^2 + r_2 r_1 + r_1^2}{r_2^2 + r_1^2}\right)$$

where $A = \pi(r_2^2 - r_1^2)$

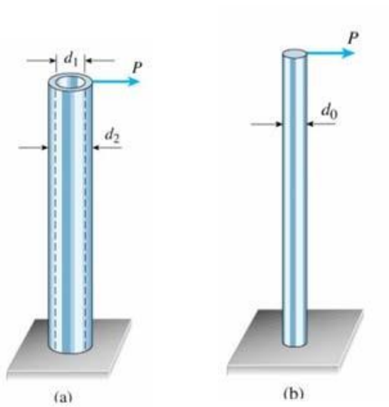
Example 5-13

a vertical pole of a circular tube

d_2 = 100 mm d_1 = 80 mm P = 6,675 N

(a) determine the τ_{max} in the pole

(b) for same P and same τ_{max}, calculate d_0 of a solid circular pole



(a) The maximum shear stress of a circulartube is

$$\tau_{max} = \frac{4P}{3\pi}\left(\frac{r_2^2 + r_2 r_1 + r_1^2}{r_2^4 - r_1^4}\right)$$

for P = 6,675 N r_2 = 50 mm r_1 = 40 mm

$$\tau_{max} = 4.68 \text{ MPa}$$

(b) for a solid circular pole, τ_{max} is

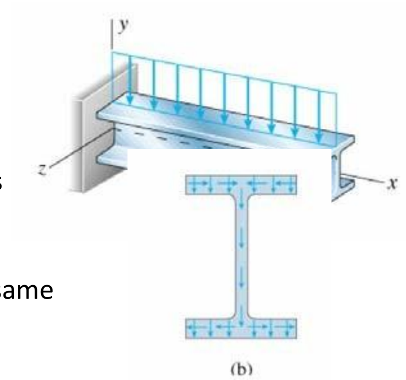
$$\tau_{max} = \frac{4P}{3\pi (d_0/2)^2}$$

$$d_0^2 = \frac{16P}{3\pi \tau_{max}} = \frac{16 \times 6{,}675}{3\pi \times 4.68} = 2.42 \times 10^{-3} \text{ m}^2$$

then d_0 = 49.21 mm

the solid circular pole has a diameter approximately 5/8 that of thetubular pole

5-10 Shear Stress in the Webs of Beams with Flanges



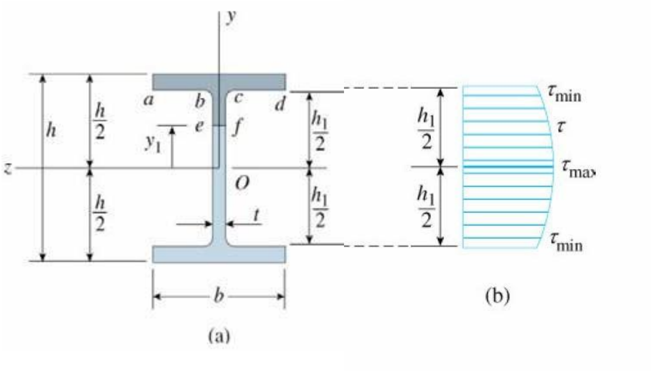
for a beam of wide-flange shape subjected to shear force V, shear stress is much more complicated

most of the shear force is carried by shearstresses in the web

consider the shear stress at ef, the same assumption as in the case in rectangular beam, i.e.

τ // y axis and uniformly distributed across t

$$\tau = \frac{VQ}{Ib}$$ is still valid with $b = t$



the first moment Q of the shaded area is divided into two parts, i.e. the upper flange and the area between bc and ef in the web

$$A_1 = b\left(\frac{h}{2} - \frac{h_1}{2}\right) \qquad A_2 = t\left(\frac{h_1}{2} - y_1\right)$$

then the first moment of A_1 and A_2 w.r.t. N.A. is

$$Q = A_1\left(\frac{h_1}{2} + \frac{h/2 - h_1/2}{2}\right) + A_2\left(y_1 + \frac{h_1/2 - y_1}{2}\right)$$

$$= \frac{b}{8}(h^2 - h_1^2) + \frac{t}{8}(h_1^2 - 4\,y_1^2)$$

$$\tau = \frac{V\,Q}{I\,b} = \frac{V}{8\,I\,t}\left[\frac{b}{8}(h^2 - h_1^2) + \frac{t}{8}(h_1^2 - 4\,y_1^2)\right]$$

where $$I = \frac{b\,h^3}{12} - \frac{(b - t)\,h_1^3}{12} = \frac{1}{12}(b\,h^3 - b\,h_1^3 + t\,h_1^3)$$

maximum shear stress in the web occurs at N.A., $y_1 = 0$

$$\tau_{max} = \frac{V}{8\,I\,t}(b\,h^2 - b\,h_1^2 + t\,h_1^2)$$

minimum shear stress occurs where the web meets the flange, $y_1 = \pm h_1/2$

$$\tau_{min} = \frac{V\,b}{8\,I\,t}(h^2 - h_1^2)$$

the maximum stress in the web is from 10% to 60% greater than theminimum stress

the shear force carried by the web consists two parts, a rectangle of area $h_1\,\tau_{min}$ and a parabolic segment of area $b\,h_1(\tau_{max} - \tau_{min})$

$$V_{web} = h_1\,\tau_{min} + b\,h_1(\tau_{max} - \tau_{min})$$

$$= \frac{t\,h_1}{3}\,(2\,\tau_{max} + \tau_{min})$$

V_{web} = 90% ~ 98% of total V

for design work, the approximation to calculate τ_{max} is

$$\tau_{max} = \frac{V}{t\,h_1} \qquad \begin{array}{l} \Leftarrow \text{ total shear force} \\ \Leftarrow \text{ web area} \end{array}$$

for typical wide-flange beam, error is within ! 10%

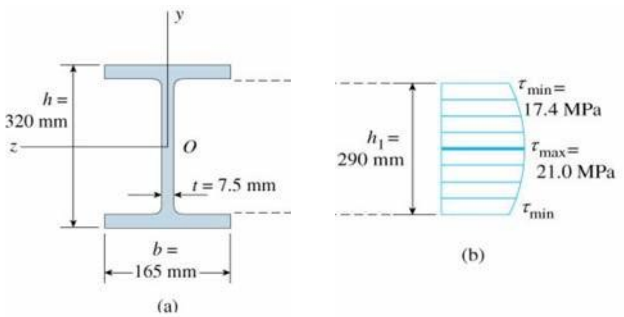
when considering y in the flange, constant τ across b cannot be made, e.g. at $y_1 = h_1/2$, t at ab and cd must be zero, but on bc, $\tau = \tau_{min}$

actually the stress is very complicated here, the stresses would become very large at the junction if the internal corners were square

Example 5-14

a beam of wide-flange shape with b = 165 mm, t = 7.5mm, h = 320 mm,and h_1 = 290 mm, vertical shear force V = 45 kN

determine τ_{max}, τ_{min} and total shear force in the web



the moment of inertia of the cross section is

$$I = \frac{1}{12}\,(b\,h^3 - b\,h_1^3 + t\,h_1^3) = 130.45 \times 10^6 \text{ mm}^4$$

the maximum and minimum shear stresses are

$$\tau_{max} = \frac{V}{8\,I\,t}\left(b\,h^2 - b\,h_1^2 + t\,h_1^2\right) = 21.0 \text{ MPa}$$

$$\tau_{min} = \frac{V\,b}{8\,I\,t}\left(h^2 - h_1^2\right) = 17.4 \text{ MPa}$$

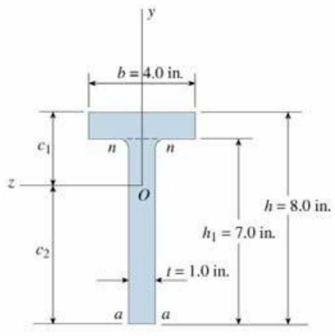
the total shear force is

$$V_{\text{web}} = \frac{t\,h_1}{3}\left(2\,\tau_{max} + \tau_{min}\right) = 43.0 \text{ kN}$$

tnd the average shear stress in the web is

$$\tau_{ave} = \frac{V}{t\,h_1} = 20.7 \text{ MPa}$$

Example 5-15

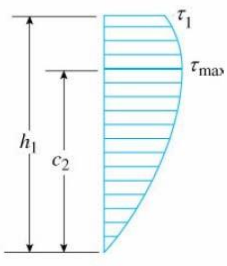


(a)

a beam having a *T*-shaped cross section b = 100 mm t = 24 mm h = 200 mm V = 45 kN

determine τ_{nn} (top of the web) and τ_{max}

$$c_1 = \frac{76 \times 24 \times 12 + 200 \times 24 \times 100}{76 \times 24 + 200 \times 24} = 75.77 \text{ mm}$$



(b)

$$c_2 = 200 - c_1 = 124.33 \text{ mm}$$

$$I = I_{aa} - A\,c_2^2$$

$$I_{aa} = \frac{b\,h^3}{3} - \frac{(b - t)\,h_1^3}{3} = 128.56 \times 10^6 \text{ mm}^3$$

$$A\,c_2^2 = 102.23 \times 10^6 \text{ mm}^3$$

$I \quad = \quad 26.33 \times 10^6 \text{ mm}^3$

to find the shear stress τ_1 (τ_{nn}), calculate Q_1 first

$Q_1 \quad = \quad 100 \times 24 \times (75.77 - 012) \quad = \quad 153 \times 10^3 \text{ mm}^3$

$$\tau_1 = \frac{V Q_1}{I t} = \frac{45 \times 10^3 \times 153 \times 10^3}{26.33 \times 10^6 \times 24} = 10.9 \text{ MPa}$$

to find τ_{max}, we want to find Q_{max} at N.A.

$Q_{max} \quad = \quad t\, c_2\, (c_2/2) \quad = 24 \times 124.23 \times (124.23/2) = 185 \times 10^3 \text{ mm}^3$

$$\tau_{max} = \frac{V Q_{max}}{I t} = \frac{45 \times 10^3 \times 185 \times 10^3}{26.33 \times 10^6} = 13.2 \text{ MPa}$$

5.11 Built-up Beams and Shear Flow

5.12 Beams with Axial Loads

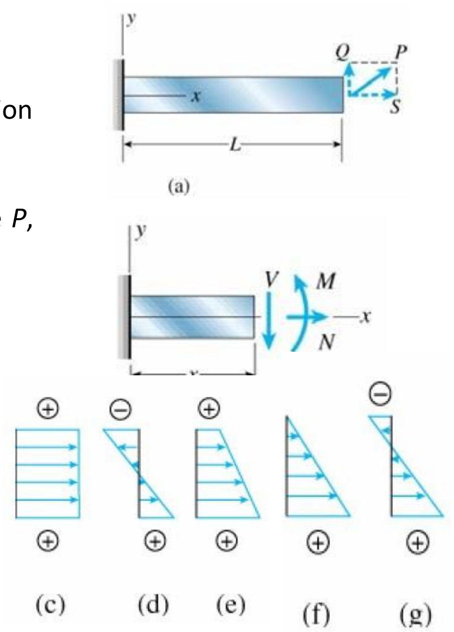
beams may be subjected to the simultaneous action of bending loads and axial forces,

e.g. cantilever beam subjected to an inclined force *P*, it may be resolved into twocomponents *Q* and *S*, then

$$M = Q\,(L - x) \qquad V = -\,Q \qquad N = S$$

and the stresses in beams are

$$\sigma = -\frac{M y}{I} \qquad \tau = \frac{V Q}{I b} \qquad \sigma = \frac{N}{A}$$

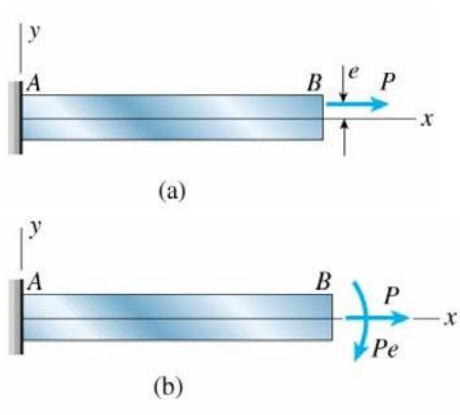


the final stress distribution can be obtained by combining the stresses associated with each stress resultant

$$\sigma = -\frac{M\,y}{I} + \frac{N}{A}$$

whenever bending and axial loads act simultaneously, the neutral axis nolonger passes through the centroid of the cross section

Eccentric Axial Loads



a load P acting at distance e from the x axis, e is called eccentricity

$$N = P \qquad M = -P\,e$$

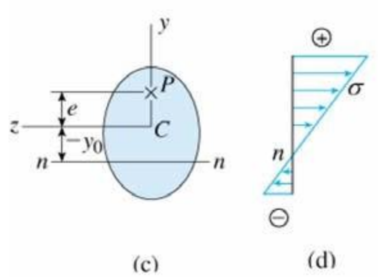
then the normal stress at any point is

$$\sigma = \frac{P\,e\,y}{I} + \frac{P}{A}$$

the position of the N.A. nn can be obtained by setting $\sigma = 0$

$$y_0 = -\frac{I}{A\,e}$$ minus sign shows the N.A. lies below z-axis

if e increased, N.A. moves closer to the centroid, if e reduced, N.A. moves away from the centroid

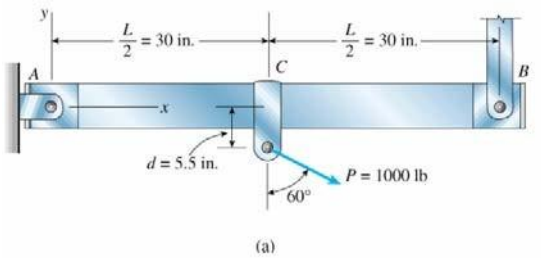


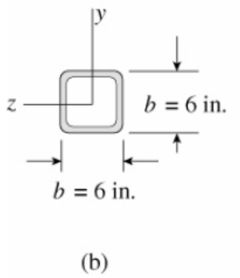
Example 5-15

a tubular beam ACB of length $L = 1.5$ m loaded by a inclined force P at mid length

P = 4.5 kN $\qquad d$ = 140 mm $\qquad b$ = 150 mm $\qquad A$ = 12,500 mm^2

$I \quad = \quad 33.86 \times 10^6 \text{ mm}^4$



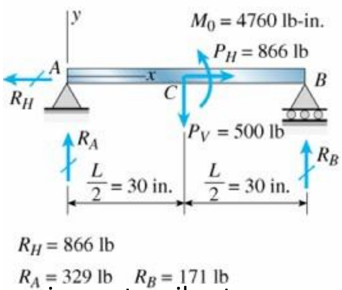


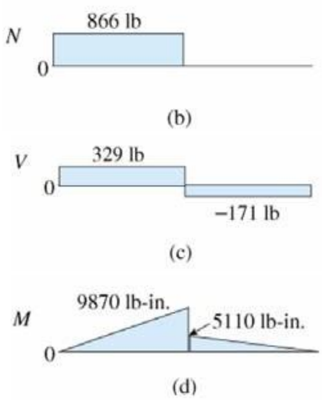
$$P_h \quad = \quad P \sin 60^o \quad = \quad 3{,}897 \text{ N}$$

$$P_v \quad = \quad P \cos 60^o \quad = \quad 2{,}250 \text{ N}$$

$$M_0 \quad = \quad P_h\, d \quad = \quad 3{,}897 \times 140 \quad = \quad 545.6 \times 10^3 \text{ N-mm}$$

the axial force, shear force and bending moment diagrams are sketchedfirst





the maximum tensile stress occurs atthe bottom of the beam, y = - 75 mm

$$(\sigma_t)_{max} \quad = \quad \frac{N}{A} \quad - \quad \frac{M\,y}{I} \quad = \quad \frac{3{,}897}{12{,}500} \quad - \quad \frac{1{,}116.8 \times 10^3\,(-75)}{33.86 \times 10^6}$$

$$= \quad 0.312 \quad + \quad 2.474 \quad = \quad 2.79 \text{ MPa}$$

the maximum compressive stress occurs at the top of the beam, y = 75mm

$$(\sigma_c)_{left} \quad = \quad \frac{N}{A} \quad - \quad \frac{M\,y}{I} \quad = \quad \frac{3{,}897}{12{,}500} \quad - \quad \frac{1{,}116.8 \times 10^3 \times 75}{33.86 \times 10^6}$$

$$= 0.312 - 2.474 = -2.16 \text{ MPa}$$

$$= \frac{N}{A} - \frac{M y}{I} = 0 - \frac{571.2\ 10^3 \times 75}{33.86 \times 10^6} \quad (\sigma_c)_{right}$$

$$= -1.265 \text{ MPa}$$

thus $(\sigma_c)_{max} = -2.16$ MPa occurs at the top of the beam to the left ofpoint *C*

QUESTIONS…..

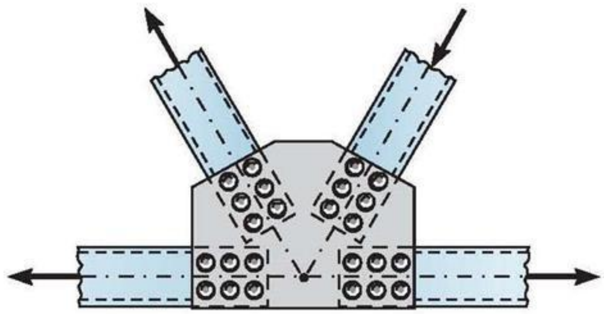
1- **Explain the theory of bending.**
2- **What are the assumptions in bending ?**
3- **Define moment of resistance ?**
4- **Explain the equation of flexure ?**
5- **Explain flexural distribution ?**
6- **What is flexural rigidity ?**
7- **What are the significance of section modulus ?**
8- **Stresses in shaft due to torsion ?**
9- **What are the assumptions of pure torsion ?**
10- **Calculation the equation of torsion ?**

<u>TRUSSES (CH-8)</u>

INTRODUCTION TO TRUSSES

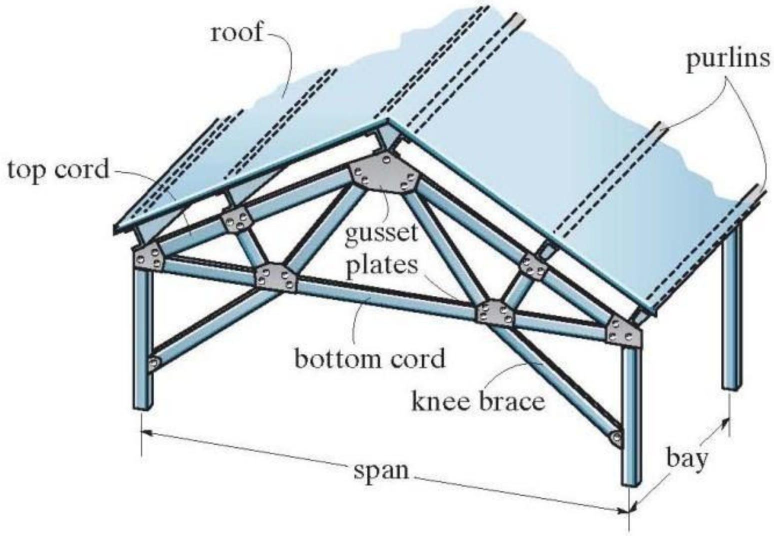
- A truss is one of the major types of engineering structures which provides a practical and economical solution for many engineering constructions, especially in the design of bridges and buildings that demand large spans.
- A truss is a structure composed of slender members joined together at their endpoints
- The joint connections are usually formed by bolting or welding the ends of themembers to a common plate called gusset

- Planar trusses lie in a single plane & is often used to support roof or bridges

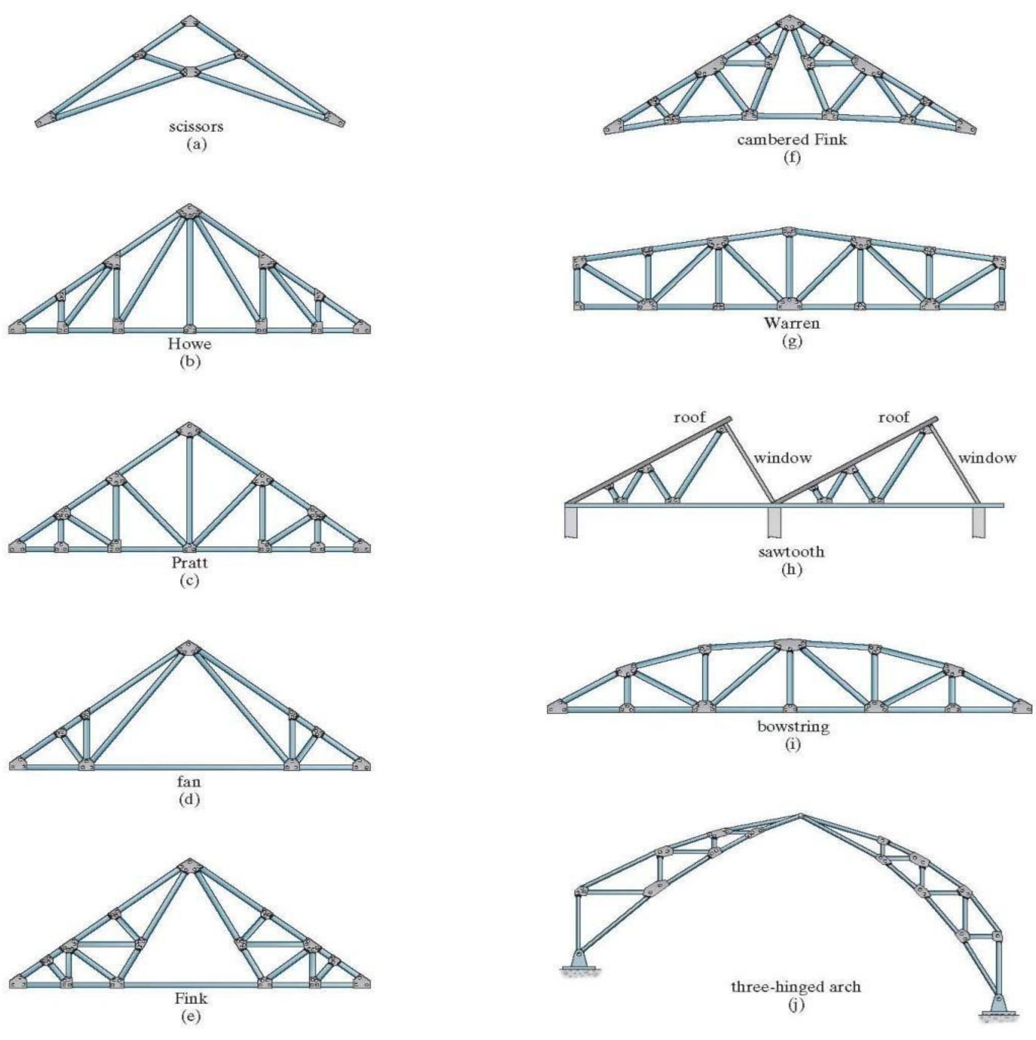


Common Types of Trusses

- Roof Trusses
  - They are often used as part of an industrial building frame
  - Roof load is transmitted to the truss at the joints by means of a series of purlins
  - To keep the frame rigid & thereby capable of resisting horizontal wind forces, knee braces aresometimes used at the supporting column

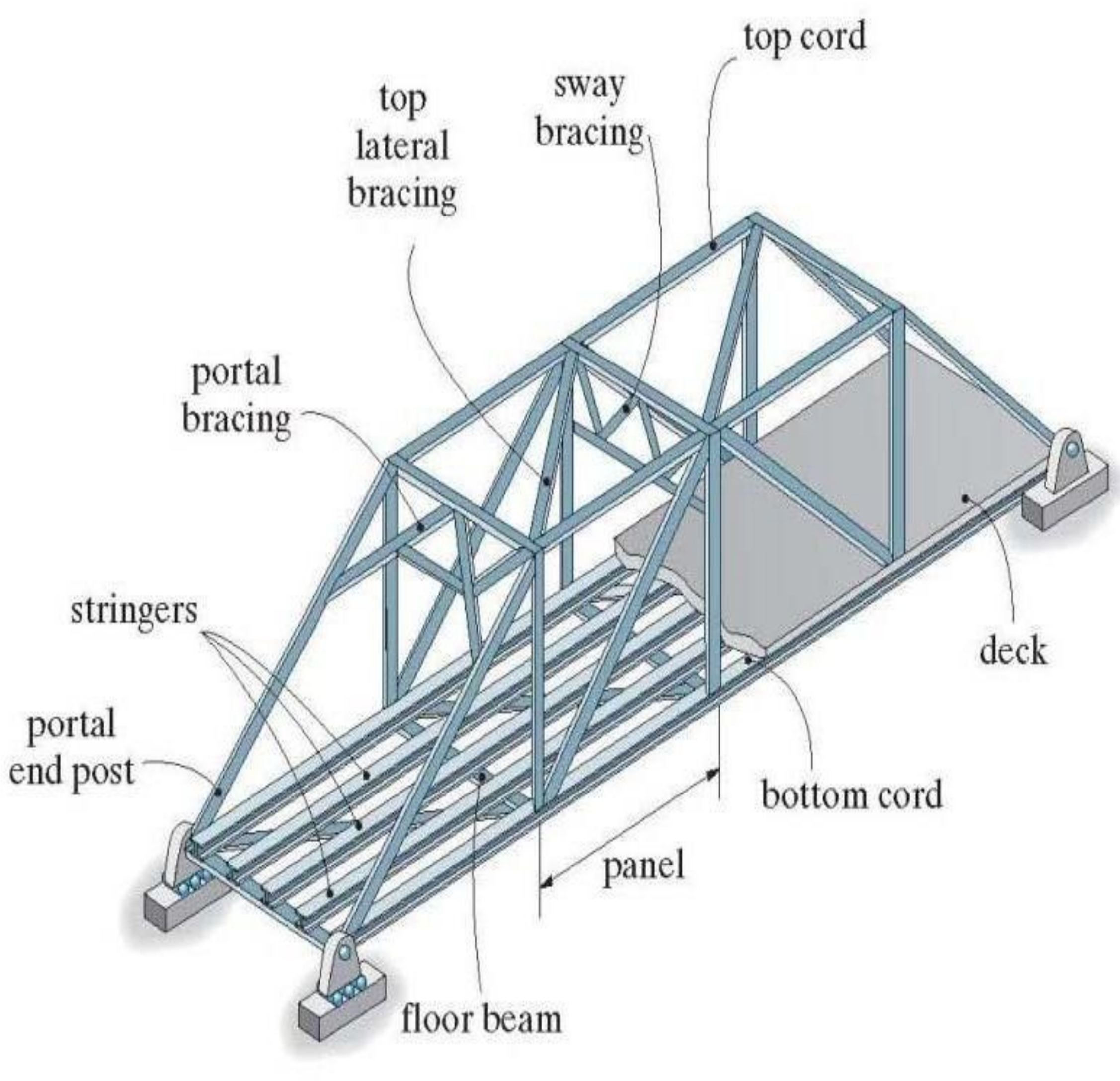


➤ Roof Trusses



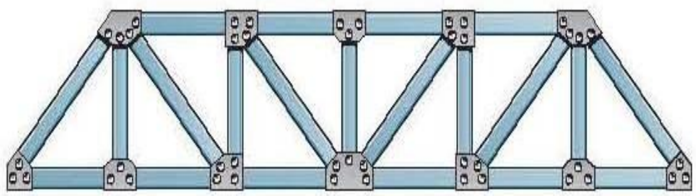
➢ Bridge Trusses

- The main structural elements of a typical bridge truss are shown in figure. Here it is seen that a load on the deck is first transmitted to stringers, then to floor beams, and finally to the joints ofthe two supporting side trusses.
- The top and bottom cords of these side trusses are connected by top and bottom lateral bracing, which serves to resist the lateral forces caused by wind and the sidesway caused by moving vehicles on the bridge.
- Additional stability is provided by the portal and sway bracing. As in the case of many long-span trusses, a roller is provided at one end of a bridge truss to allow for thermal expansion.

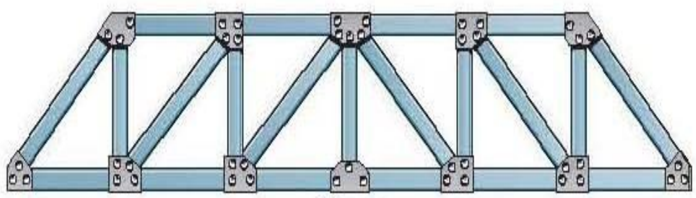


➢ Bridge Trusses

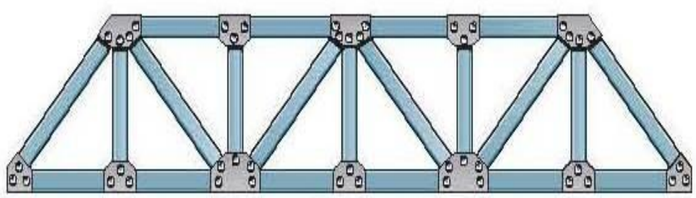
- In particular, the Pratt, Howe, and Warren trusses are normally used for spans upto 61 m in length. The most common form is the Warren truss with verticals.
- For larger spans, a truss with a polygonal upper cord, such as the Parker truss, isused for some savings in material.
- The Warren truss with verticals can also be fabricated in this manner for spans upto 91 m.



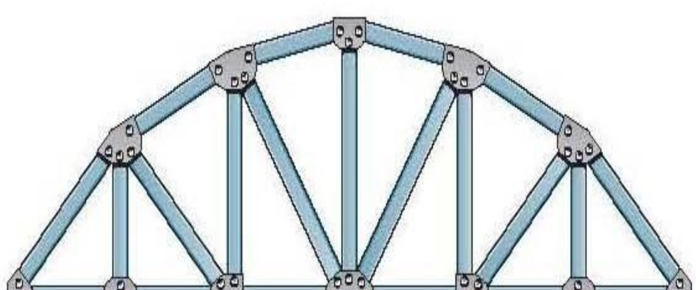
Pratt
(a)



Howe
(b)



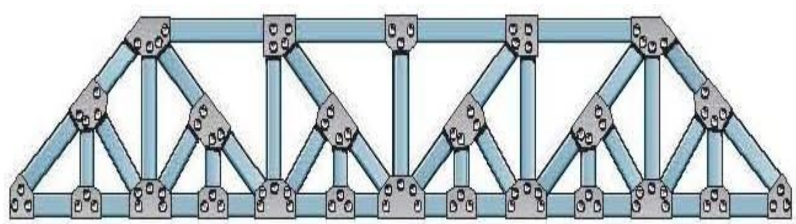
Warren (with verticals)
(c)



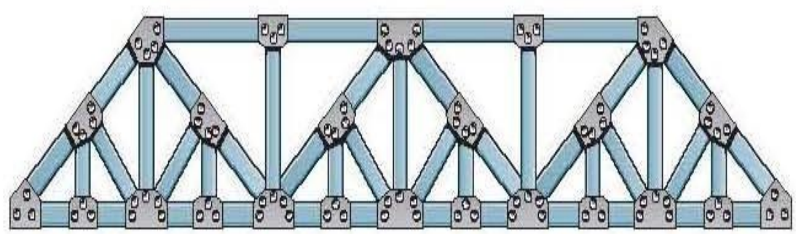
Parker
(d)

➢ Bridge Trusses

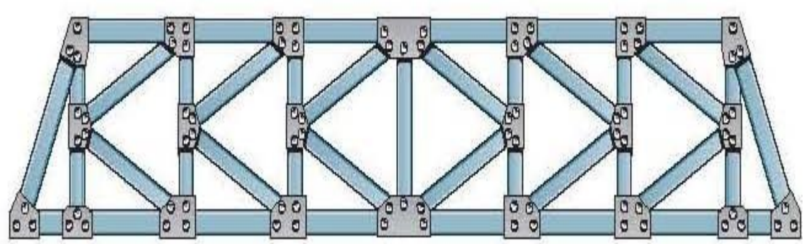
- The greatest economy of material is obtained if the diagonals have a slope between 45°and 60° with the horizontal. If this rule is maintained, then for spans greater than 91 m, the depth of the truss must increase and consequently the panels will get longer.
- This results in a heavy deck system and, to keep the weight of the deck within tolerable limits, subdivided trusses have been developed. Typical examples include the Baltimoreand subdivided Warren trusses.
- The K-truss shown can also be used in place of a subdivided truss, since it accomplishesthe same purpose.



Baltimore
(e)



subdivided Warren
(f)



K-truss
(g)

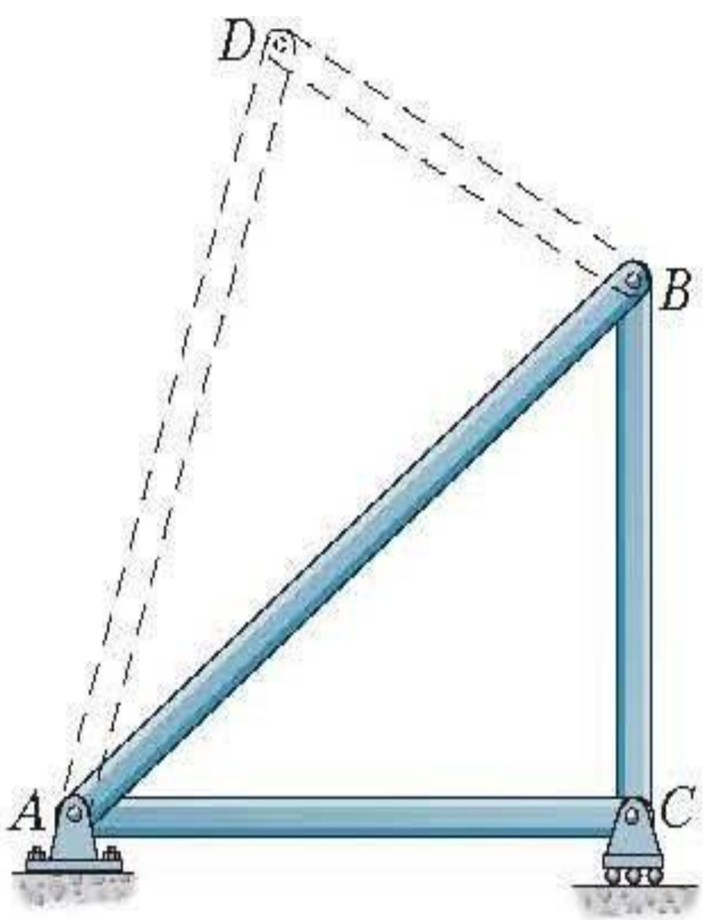
Assumptions for Design

- The members are joined together by smooth pins
- All loadings are applied at the joints

Due to the 2 assumptions, each truss member acts as an axial force member

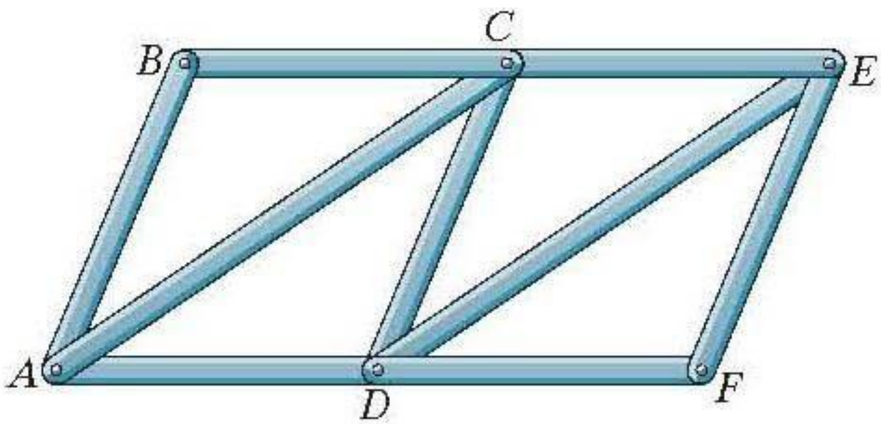
Classification of Coplanar Trusses

- **Simple , Compound or Complex Truss**
- **Simple Truss**
- **To prevent collapse, the framework of a truss must be rigid**
- **The simplest framework that is rigid or stable is a triangle**
- **The members are joined together by smooth pins**
- **All loadings are applied at the joints**

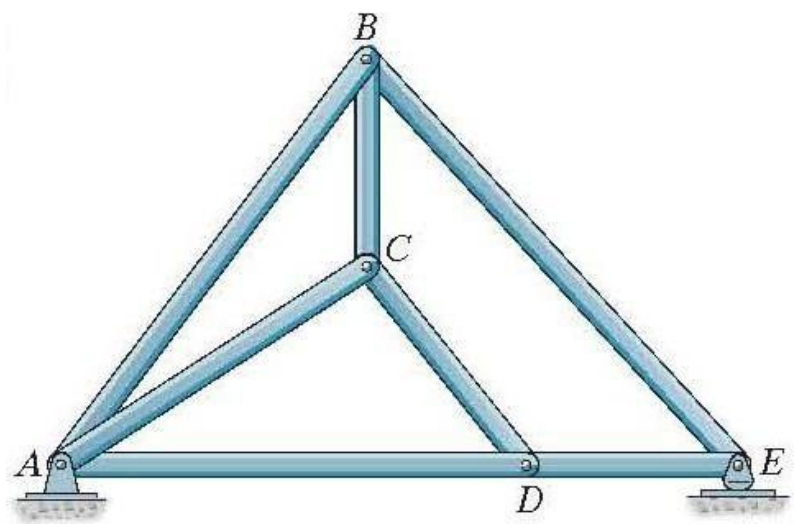


➢ Simple Truss

- The basic "stable" triangle element is ABC
- The remainder of the joints D, E & F are established in alphabetical sequence
- Simple trusses do not have to consist entirely of triangles



simple truss



simple truss

➢ Compound Truss

- It is formed by connecting 2 or more simple truss together
- Often, this type of truss is used to support loads acting over a larger span as it is cheaper toconstruct a lighter compound truss than a heavier simple truss

Types of compound truss:

- Type 1

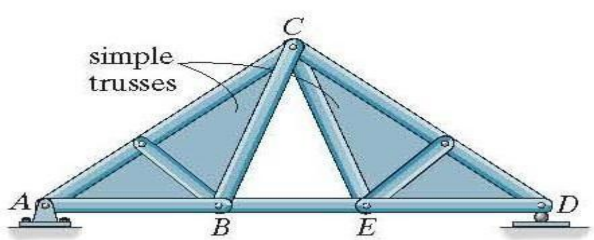
  The trusses may be connected by a common joint & bar

- Type 2

  The trusses may be joined by 3 bars

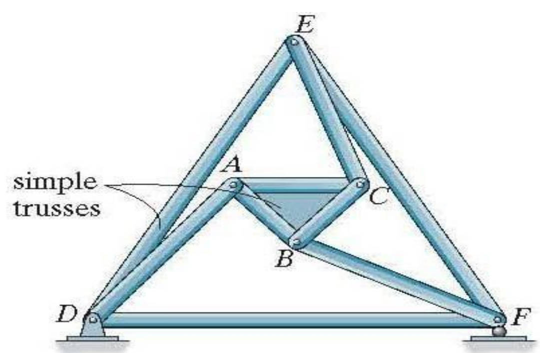
- Type 3

  The trusses may be joined where bars of a large simple truss, called the main truss, have been substituted by simple truss, called secondary trusses



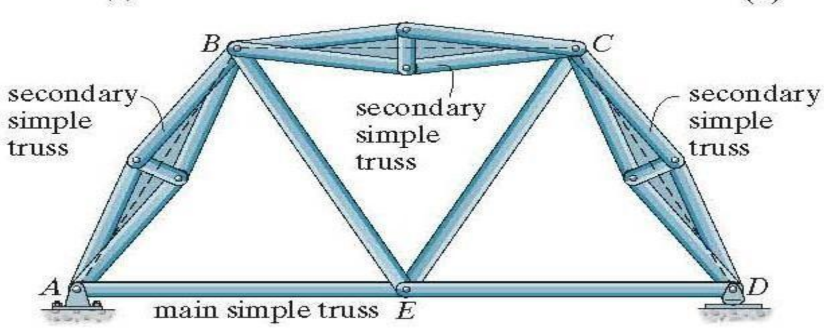
compound truss

(a)



compound truss

(b)

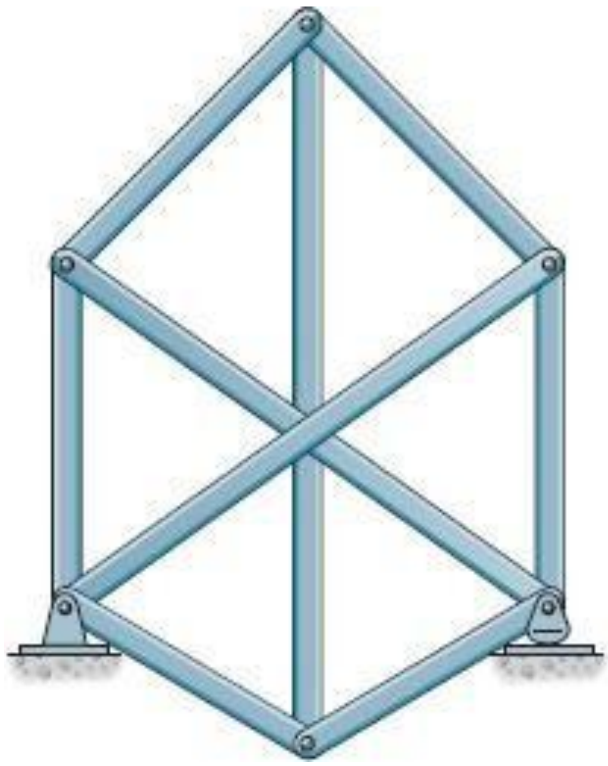


compound truss

(c)

- Complex Truss

- A complex truss is one that cannot be classified as being either simple or compound



Determinacy

- The total number of unknowns includes the forces in b number of bars of the truss andthe total number of external support reactions r.
- Since the truss members are all straight axial force members lying in the same plane, theforce system acting at each joint is coplanar and concurrent.
- Consequently, rotational or moment equilibrium is automatically satisfied at the joint (orpin).

Therefore only

$$\sum F_x = 0 \quad \text{and} \quad \sum F_y = 0$$

By comparing the total unknowns with the total number of available equilibrium equations, wehave:

$$b + r = 2j \quad \text{statically determinate}$$

$$b + r > 2j \quad \text{statically indeterminate}$$

- **Stability**

If b + r < 2j => collapse

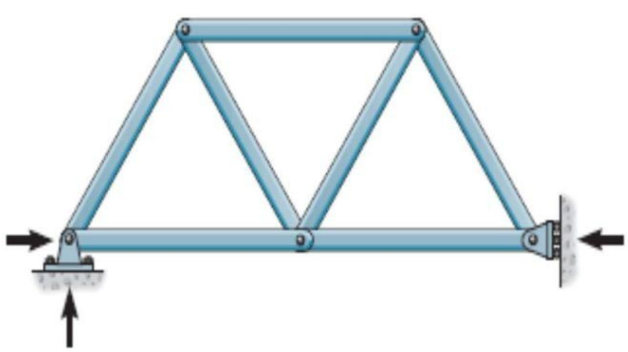
A truss can be unstable if it is statically determinate or statically indeterminate Stability

will have to be determined either through inspection or by force analysis

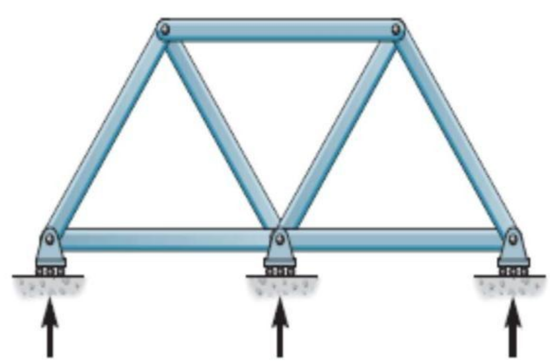
- **External Stability**

A structure is externally unstable if all of its reactions are concurrent or parallel

The trusses are externally unstable since the support reactions have lines of action that are either concurrent or parallel



unstable concurrent reactions



unstable parallel reactions

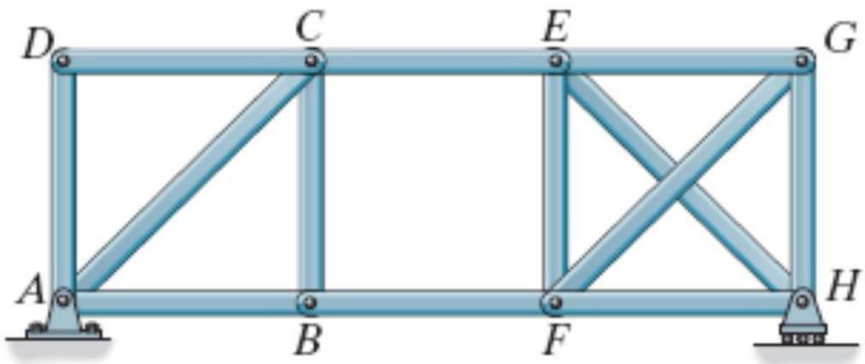
- Internal Stability

The internal stability can be checked by careful inspection of the arrangement of itsmembers

If it can be determined that each joint is held fixed so that it cannot move in a “rigid body”sense with respect to the other joints, then the truss will be stable

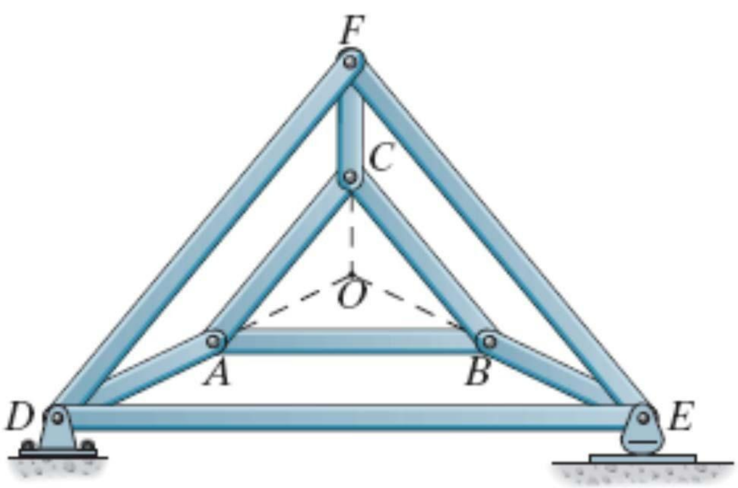
A simple truss will always be internally stable

If a truss is constructed so that it does not hold its joints in a fixed position, it will beunstable or have a “critical form”



To determine the internal stability of a compound truss, it is necessary to identify the wayin which the simple truss are connected together

The truss shown is unstable since the inner simple truss ABC is connected to DEF using 3bars which are concurrent at point O



Thus an external load can be applied at A, B or C & cause the truss to rotate slightlyFor

complex truss, it may not be possible to tell by inspection if it is stable

The instability of any form of truss may also be noticed by using a computer to solve the 2j simultaneous equations for the joints of the truss

If inconsistent results are obtained, the truss is unstable or have a critical form

DIFFERENCE BETWEEN FORCE & DISPLACEMENT METHODS

FORCE METHODS	DISPLACEMENT METHODS
1. Method of consistent deformation 2. Theorem of least work 3.Column analogy method 4. Flexibility matrix method	1. Slope deflection method 2.Moment distribution method3.Kani's method 4. Stiffness matrix method
Types of indeterminacy- static indeterminacy	Types of indeterminacy-indeterminacy kinematic
Governing equations-compatibility equations	Governing equations-equilibriumequations
Force displacement relations-flexibility Matrix	Force displacement relations- stiffness matrix

DETERMINATION OF THE MEMBER FORCES

- The Method of Joints
- The Method of Sections (Ritter Method)
- The Graphical Method (Cremona Method)

The Method of Joints

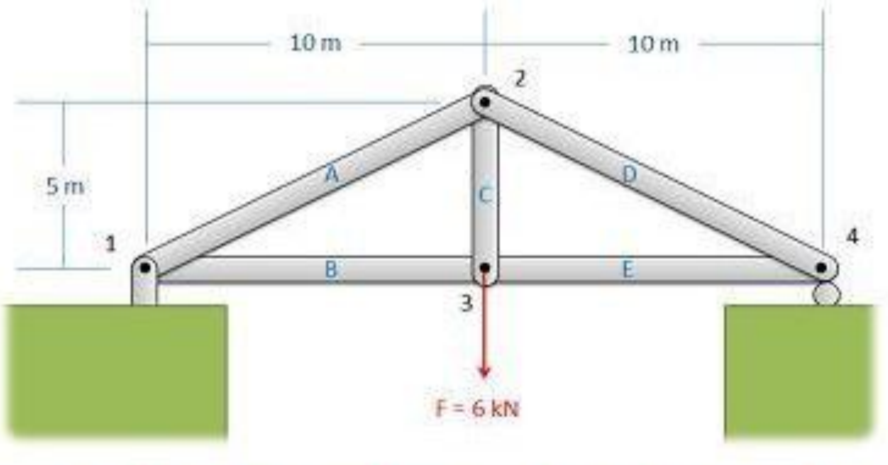
The method of joints is a process used to solve for the unknown forces acting on members of a truss. The method centers on the joints or connection points between the members, and it is usually the fastest and easiest way to solve for all the unknown forces in a truss structure

Using the Method of Joints:

The process used in the method of joints is outlined below:

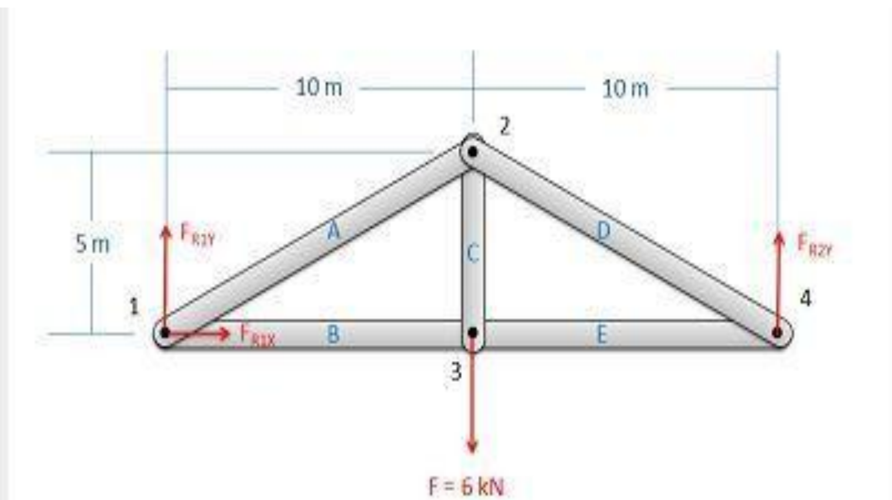
1. In the beginning it is usually useful to label the members and the joints in your truss. This will help you keep everything organized and consistent in later analysis. In this

book, the members will be labeled with letters and the joints will be labeled with numbers.



The first step in the method of joints is to label each joint and each member.

2. Treating the entire truss structure as a rigid body, draw a free body diagram, write out the equilibrium equations, and solve for the external reacting forces acting on the truss structure. This analysis should not differ from the analysis of a single rigid body.



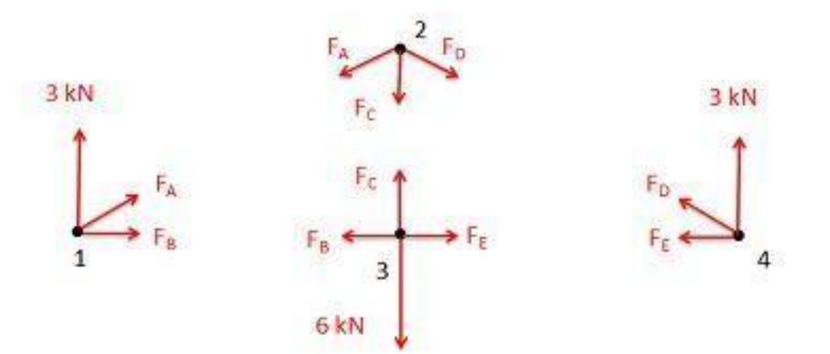
Treat the entire truss as a rigid body and solve for the reaction forces supporting the truss structure.

3. Assume there is a pin or some other small amount of material at each of the connection points between the members. Next you will draw a free body diagram for each

connection point.

Remember to include:

- Any external reaction or load forces that may be acting at that joint.
- A normal force for each two force member connected to that joint. Remember that for a two force member, the force will be acting along the line between the two connection points on the member. We will also need to guess if it will be a tensile or a compressive force. An incorrect guess now though will simply lead to a negative solution later on. A common strategy then is to assume all forces are tensile, then later in the solution any positive forces will be tensile forces and any negative forces will be compressive forces.
- Label each force in the diagram. Include any known magnitudes and directions and providevariable names for each unknown.



Drawing a free body diagram of each joint, we draw in the known forces as well as tensile forces from each two force member.

4. Write out the equilibrium equations for each of the joints. You should treat the joints as particles, so there will be force equations but no moment equations. With either two (for 2D problems) or three (for 3D problems) equations for each joint, this should give you a large number of equations.

$$\sum \mathbf{F} = 0$$

$$\sum F_x = 0 \quad \sum F_y = 0$$

In planar trusses, the sum of the forces in the x direction will be zero and the sum of the forces in the y direction will be zero for each of the joints.

$$\sum \mathbf{F} = 0$$

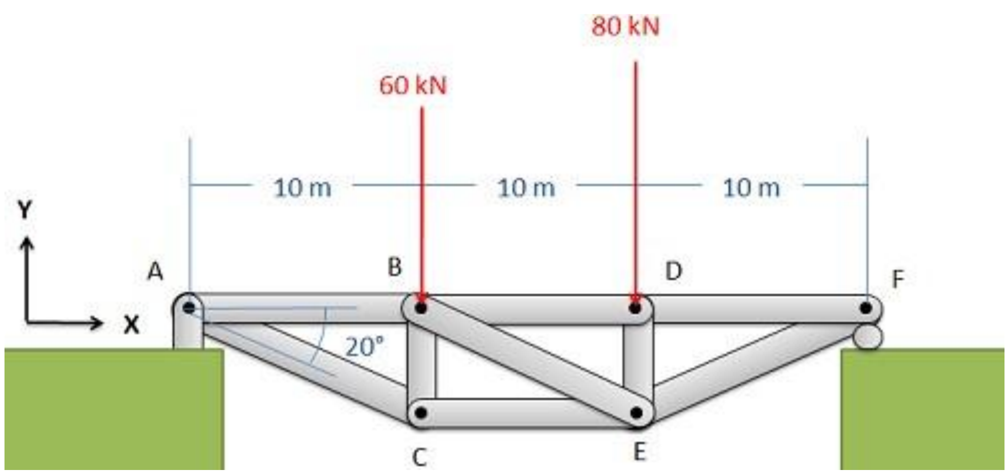
$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$

In space trusses, the sum of the forces in the x direction will be zero, the sum of the forces in the y direction will be zero, and the sum of forces in the z direction will be zero for each of the joints.

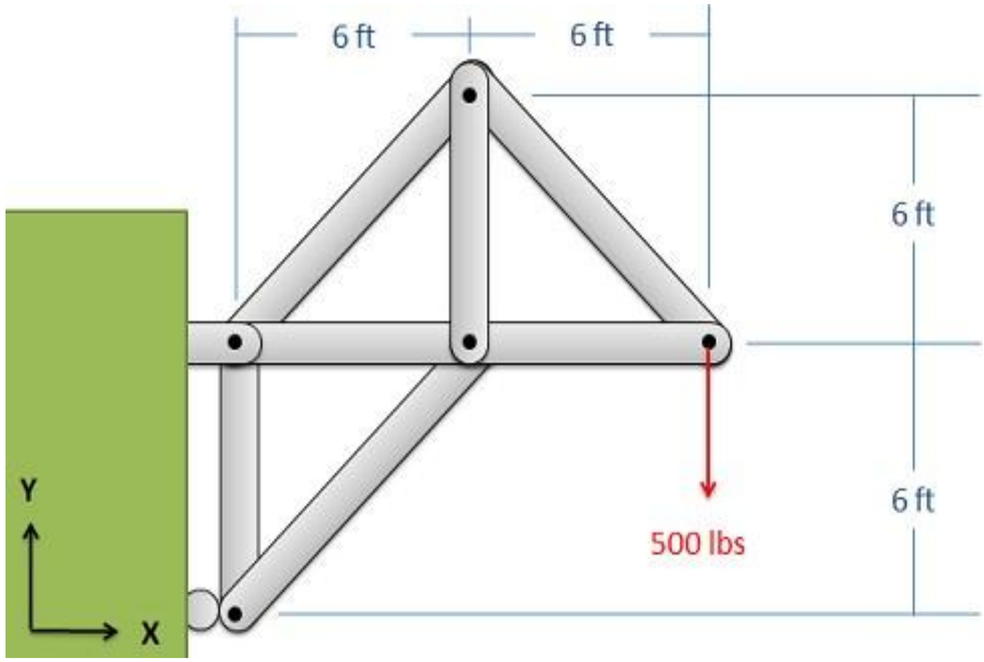
5.Finally, solve the equilibrium equations for the unknowns. You can do this algebraically, solving for one variable at a time, or you can use matrix equations to solve for everything at once. If you assumed that all forces were tensile earlier, remember that negative answers indicate compressive forces in the members.

PROBLEMS

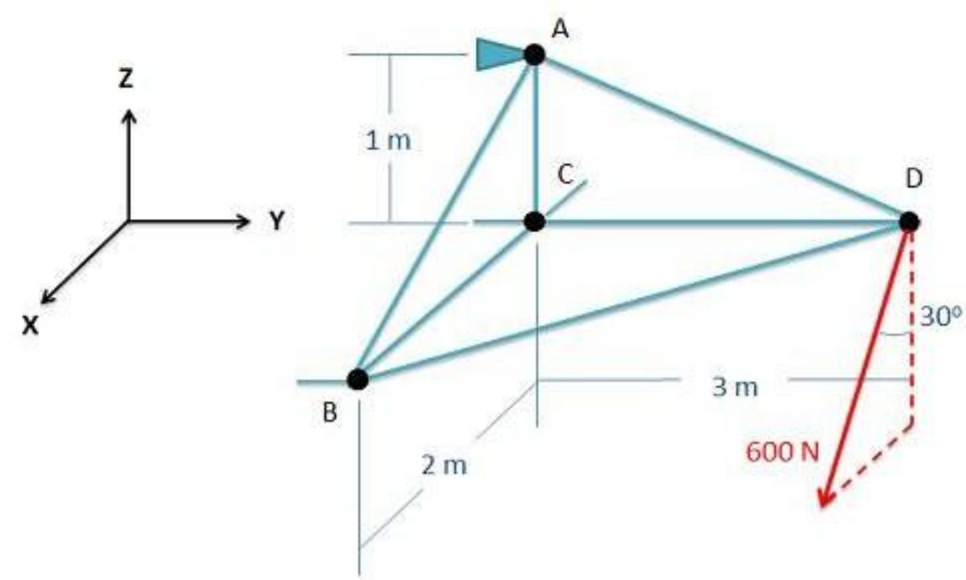
1. Find the force acting in each of the members in the truss bridge shown below. Remember tospecify if each member is in tension or compression



2. Find the force acting in each of the members of the truss shown below. Remember to specifyif each member is in tension or compression.



3. Find the force acting in each of the members of the truss shown below. Remember to specifyif each member is in tension or compression.



THE METHOD OF SECTIONS

APPLICATIONS

Long trusses are often used to construct bridges.
The method of joints requires that many joints be analyzed before we can determine the forces inthe middle part of the truss.

In the method of sections, a truss is divided into two parts by taking an imaginary "cut" (shown here as a-a) through the truss.
Since truss members are subjected to only tensile or compressive forces along their length, the internal forces at the cut member will also be either tensile or compressive with the same magnitude.

This result is based on the equilibrium principle and Newton's third law.

STEPS FOR ANALYSIS

- Decide how you need to "cut" the truss. This is based on where you need to determineforces, and, b) where the total number of unknowns does not exceed three (in general).
- Decide which side of the cut truss will be easier to work with (minimize the number ofreactions you have to find).
- If required, determine the necessary support reactions by drawing the FBD of the entiretruss and applying the E-of-E.

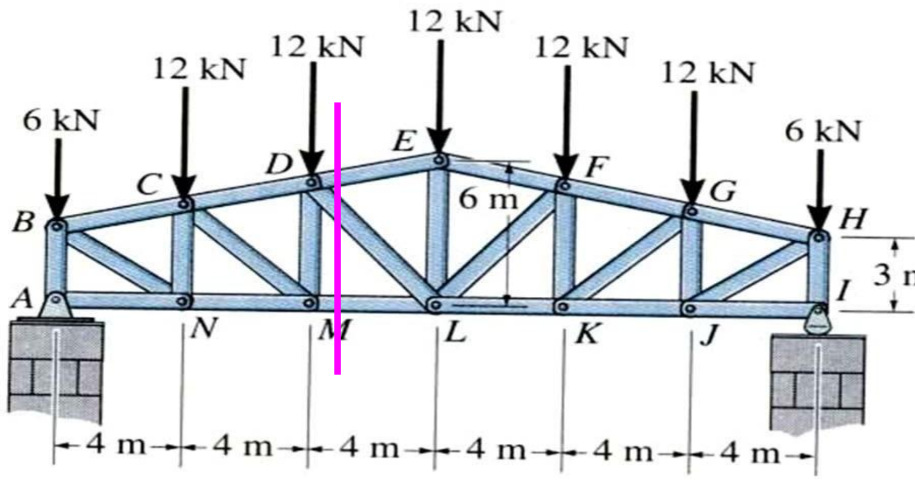
- Draw the FBD of the selected part of the cut truss. We need to indicate the unknown forces at the cut members. Initially we may assume all the members are in tension, as we did when using the method of joints. Upon solving, if the answer is positive, the member is in tension as per our assumption. If the answer is negative, the member mustbe in compression.

  - Apply the equations of equilibrium (E-of-E) to the selected cut section of the truss to solve for the unknown member forces. Please note that in most cases it is possible to write one equation to solve for one unknown directly.

PROBLEMS

Given: Loads as shown on the roof truss.

Find:The force in members DE, DL, and ML.



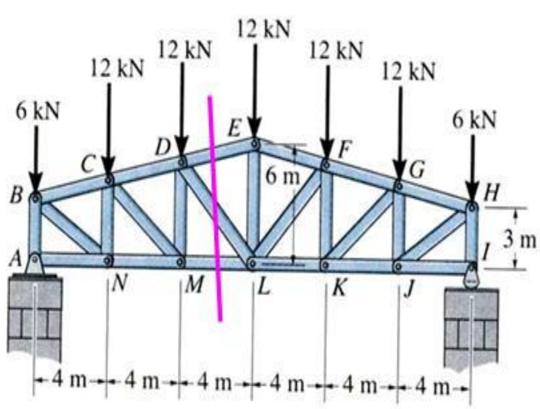
Plan:

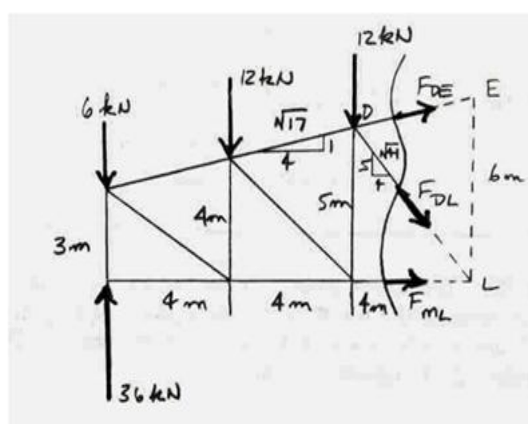
a) Take a cut through the members DE, DL, and ML.

b) Work with the left part of the cut section. Why?

c) Determine the support reaction at A.

d) Apply the EofE to find the forces in DE, DL, and ML.





Analyzing the entire truss, we get FX = AX = 0.By symmetry, the vertical support reactions are

AY = IY = 36 kN

+ MD = – 36 (8) + 6 (8) + 12 (4) + FML (5) = 0

FML = 38.4 kN(T)

+ML =–36 (12) + 6 (12) + 12 (8) + 12 (4) – FDE (4/17)(6) = 0

FDE =–37.11 kN or 37.1 kN (C)

+ FX =38.4 + (4/17) (–37.11)+ (4/41) FDL = 0

FDL = –3.84 kN or 3.84 kN (C)

INTERNAL FORCES

In order to obtain the internal forces at a specified point, we should make section cut perpendicular to the axis of the member at this point. This section cut divides the structure in two parts. The portion of the structure removed from the part in to consideration should be replaced by the internal forces. The internal forces ensure the equilibrium of the isolated part subjected to the action of external loads and support reactions. A free body diagram of either segment of the cut member is isolated and the internal loads could be derived by the six equations of equilibrium applied to the segment in to consideration.

ANALYSIS OF SPACE TRUSSES USING METHOD OF TENSION COEFFICIENTS

1. Tension Co-efficient Method

The tension co efficient for a member of a frame is defined as the pull or tension in that member is divided by its length.

t = T/l Where t = tension co efficient for the memberT=

Pull in the member

l = Length

2. Analysis Procedure Using Tension Co-efficient - 2D Frames

1. List the coordinates of each joint (node)of the truss.
2. Determine the projected lengths Xij and Yij of each member of the truss. Determine thesupport lengths lij of each member using the equation lij =?Xij2+Yij2

3. Resolve the the applied the forces at the joint in the X and Y directions. Determine thesupport reactions and their X and Y components.
4. Identify a node with only two unknown member forces and apply the equations of equilibrium. The solution yields the tension co efficient for the members at the node.
5. Select the next joint with only two unknown member forces and apply the equations ofequilibrium and apply the tension co efficient.
6. Repeat step 5 till the tension co efficient of all the members are obtained. 7. Compute themember forces from the tension co efficient obtained as above using

Tij= tijxlij

3. Analysis Procedure Using Tension Co-efficient - Space Frames

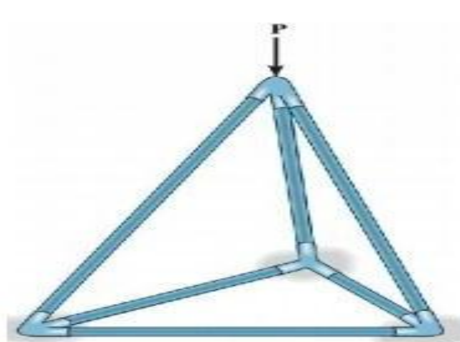
1. In step 2 above the projected lengths Zij in the directions are also computed. Determine thesupport lengths lij of each member using the equation lij =?Xij2+Yij2 +Zij2
2. In step 3 above the components of forces and reactions in the Z directions are also to bedetermined.
3. In step 4 and 5, each time, nodes with not more than three unknown member forces are to beconsidered.

Tetrahedron: simplest element of stable space truss (six members, four joints) expand byadding 3members and 1 joint each time

Determinacy and Stability b+r <3j unstable

b +r=3j statically determinate (check stability)

b+r>3j statically indeterminate (check stability)



<u>**INTERNAL FORCES**</u>

In order to obtain the internal forces at a specified point, we should make section cut perpendicular to the axis of the member at this point. This section cut divides the structure in two parts. The portion of the structure removed from the part in to consideration should be replaced by the internal forces. The internal forces ensure the equilibrium of the isolated part subjected to the action of external loads and support reactions. A free body diagram of either segment of the cut member is isolated and the internal loads could be derived by the six equations of equilibrium applied to the segment in to consideration.

UNIT-

II

ARCH

ES

Till now, we had been studying two-dimensional (plane) structures like beams, frames and trusses which were mostly linear in their geometry or comprised of elements which were formed out of straight lines. Now in this unit, we are being introduced to a class of structures which will be composed of curved-members instead of straight ones. The simplest member of this class is the arch. Arches as such are not a new mode of construction and have been in use as a load bearing structure since ancient times. Although it is more difficult to construct a curved structure like an arch, there are certain advantages, apart from their aesthetic look, which have made them popular among civil engineers and architects. It will be shown here in this unit that the bending moment in an arch section is generally less than that in a corresponding beam section, of similar span and loading.

Hence, the all-important bending stresses are less in arches. However, in an arch section, there is in additional normal thrust which is not present in beam sections (with transverse loading). But normally the net effect is not critical as concrete and masonry are usually stronger in compression and the total stresses are generally well within limits. So overall speaking, an arch is lighter and stronger than a similar or similarly-loaded beam.

Some kinds of arch used in civil engineering.

(a) Fixed Arch

(b) Linear Arch

(c) Trussed Arch.

An arch could be defined in simple terms as a two-dimensional structure element which is curved in elevation and is supported at ends by rigid or hinged supports which are capable of developing the desired thrust to resist the loads.

It could also be defined as a two-dimensional element which resists external loads through its profile. This is achieved by its characteristic horizontal reaction developed at the supports. The horizontal thrust causes hogging bending moment which tend to reduce the sagging moment due to loading and thus, the net bending moment is much smaller.

TWO HINGED

ARCHES

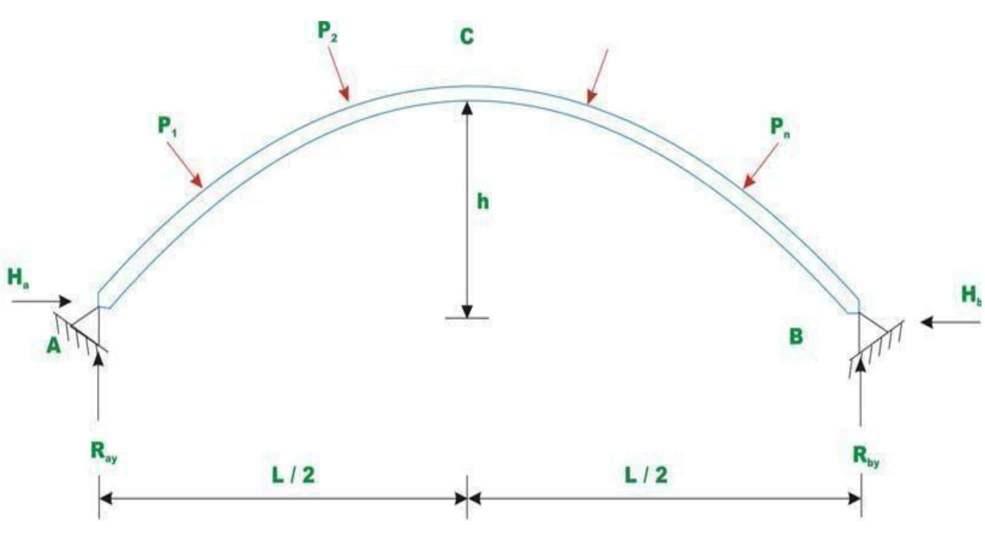
INTRODUCTION

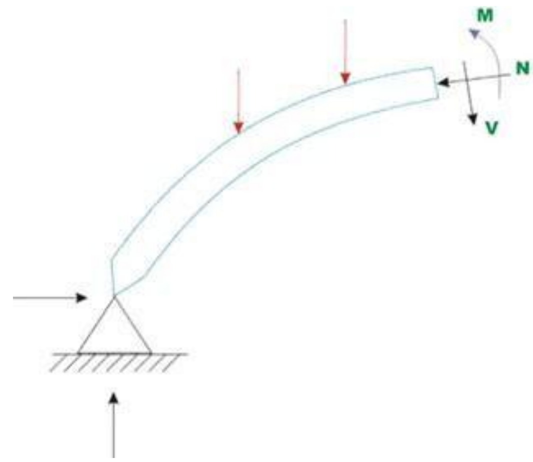
Mainly three types of arches are used in practice: three-hinged, two-hinged and hinge less arches. In the early part of the nineteenth century, three-hinged arches were commonly used for the long span structures as the analysis of such arches could be done with confidence. However, with the development in structural analysis, for long span structures starting from late nineteenth century engineers adopted two-hinged and hinge less arches.

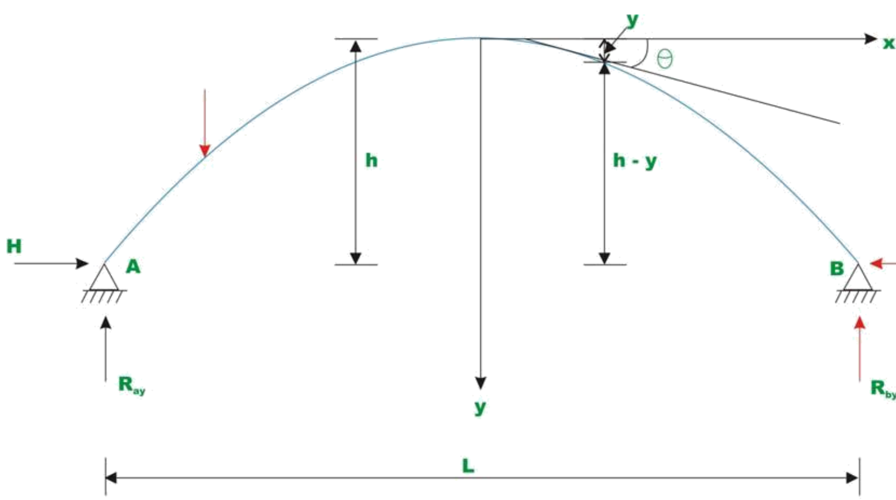
Two-hinged arch is the statically indeterminate structure to degree one. Usually, the horizontal reaction is treated as the redundant and is evaluated by the method of least work. In this lesson, the analysis of two-hinged arches is discussed and few problems are solved to illustrate the procedure for calculating the internal forces.

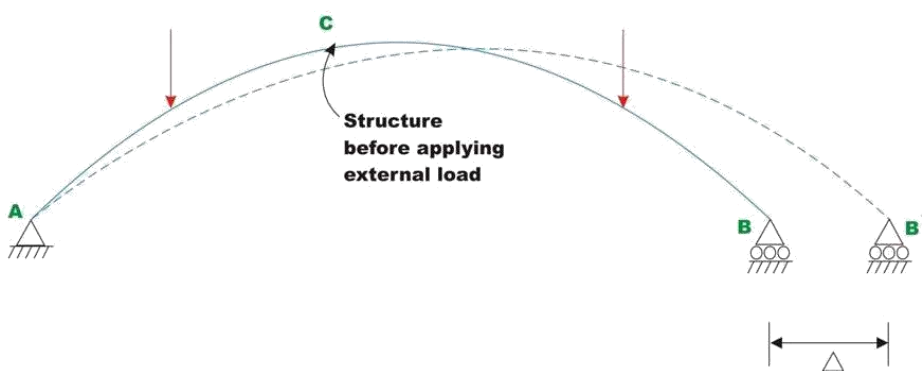
ANALYSIS OF TWO-HINGED ARCH

A typical two-hinged arch is shown in Figure below. In the case of two-hinged arch, we have four unknown reactions, but there are only three equations of equilibrium available. Hence, the degree of statically indeterminacy is one for two- hingedarch.




M
N
V


y
x
θ
h
h - y
H
A
B
H
R_{ay}
y
R_{by}
L


C
Structure
before applying
external load
A
B
B
△

The fourth equation is written considering deformation of the arch. The unknown redundant reaction $H_b$is calculated by noting that the horizontal displacement of hinge *B* is zero.

In general the horizontal reaction in the two hinged arch is evaluated by straightforward application of the theorem of least work, which states that the partial derivative of the strain energy of a statically indeterminate structure with respect to statically indeterminate action should vanish.

Hence to obtain, horizontal reaction, one must develop an expression for strain energy. Typically, any section of the arch (vide Fig 33.1b) is subjected to shear force*V,* bending moment *M* and the axial compression *N.* The strain energy due to bending $U_b$is calculated from the followingexpression.

$$U_b = \int_0^s \frac{M^2}{2EI} ds$$

The above expression is similar to the one used in the case of straight beams. However, in this case, the integration needs to be evaluated along the curved arch length. In the above equation, *s* is the length of the centerline of the arch, *I is* the moment of inertia of the arch cross section, *E*is the Young's modulus of the arch material.

The strain energy due to shear is small as compared to the strain energy due to bending and is usually neglected in the analysis. In the case of flat arches, the strain energy due to axial compression can be appreciable and is givenby,

$$U_a = \int_0^s \frac{N^2}{2AE} ds$$

The total strain energy of the arch is given by

$$U = \int_0^s \frac{M^2}{2EI} ds + \int_0^s \frac{N^2}{2AE} ds$$

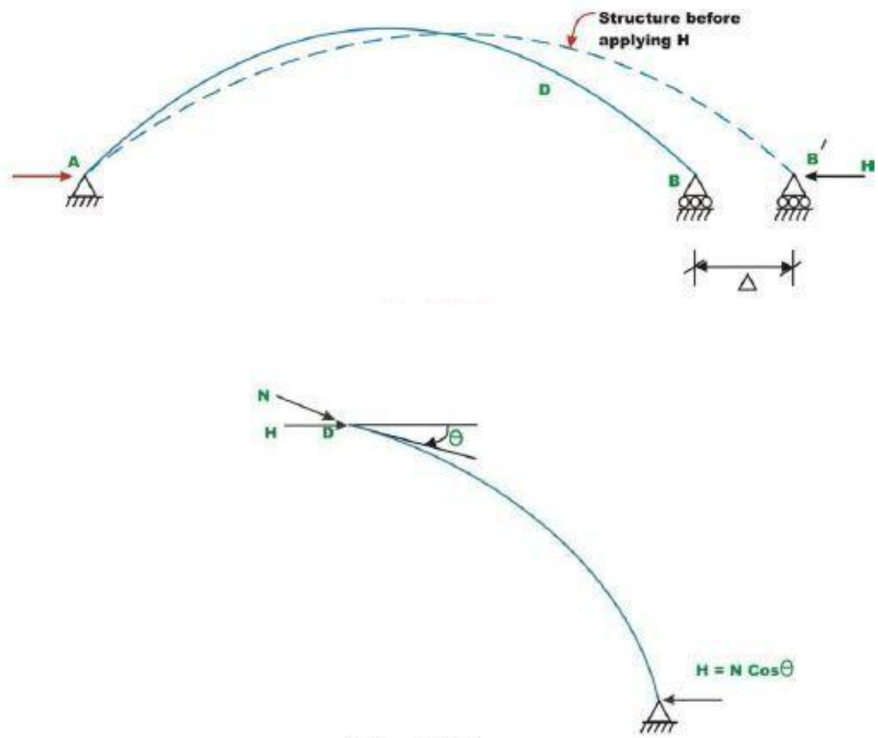
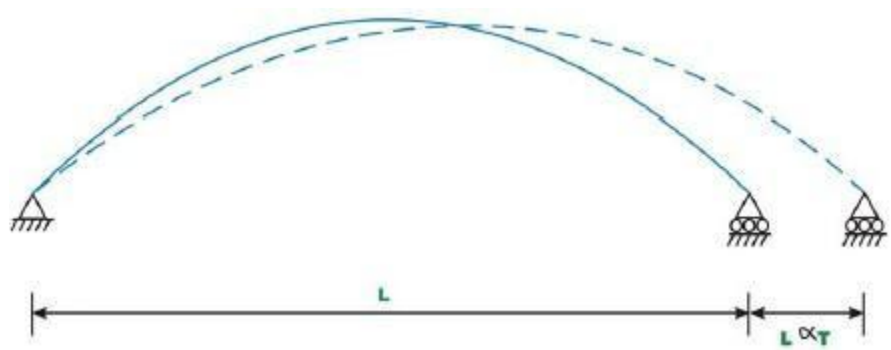
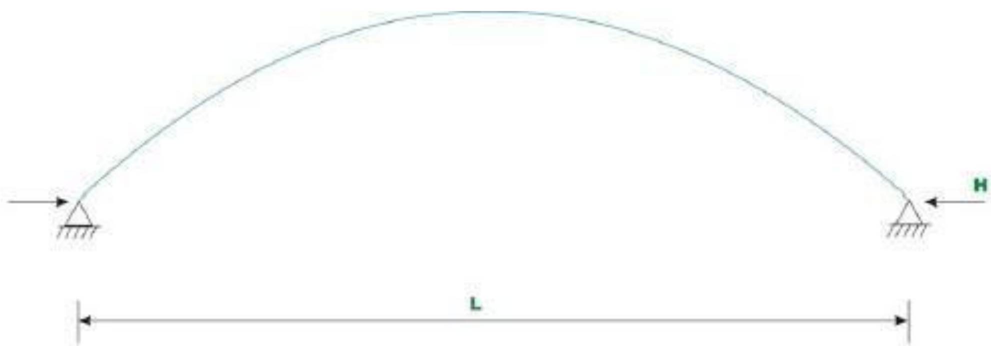


Fig. 33.2d.

TEMPERATURE EFFECT

Consider an unloaded two-hinged arch of span *L*. When the arch undergoes a uniform temperature change of *T*, then its span would increase by *C*°*TL*α if it were allowed to expand freely (vide Fig 33.3a). α is the co-efficient of thermal expansion of the arch material. Since the arch is restrained from the horizontal movement, a horizontal force is induced at the support as the temperature is increase





Now applying the Castigliano's first theorem,

Solving for *H*,

$$H = \frac{\alpha L T}{\int_0^s \frac{\tilde{y}^2}{EI} ds + \int_0^s \frac{\cos^2\theta}{EA} ds}$$

The second term in the denominator may be neglected, as the axial rigidity is quite high.

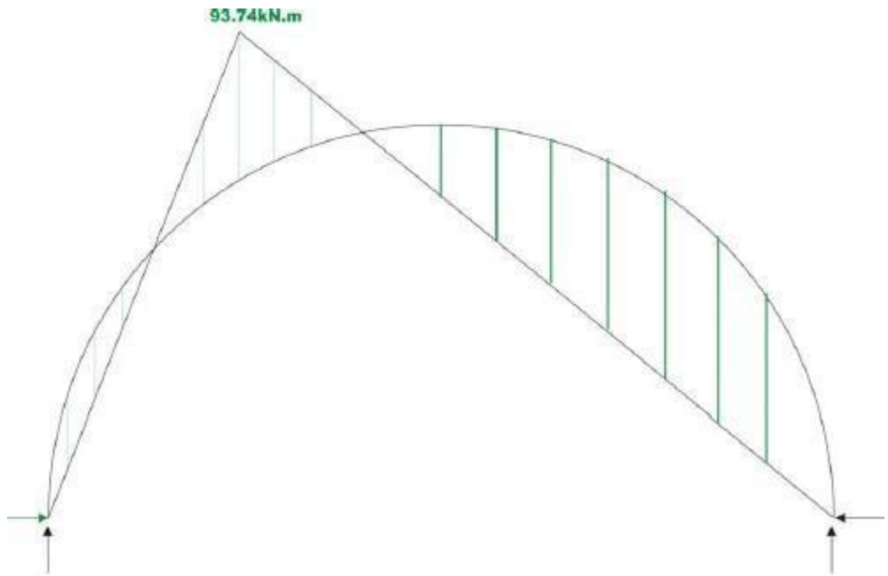
Neglecting the axial rigidity, the above equation can be written as

$$\frac{\partial U}{\partial H} = \alpha L T = \int_0^s \frac{H\tilde{y}^2}{EI} ds + \int_0^s \frac{H\cos^2\theta}{EA} ds$$

is given by,

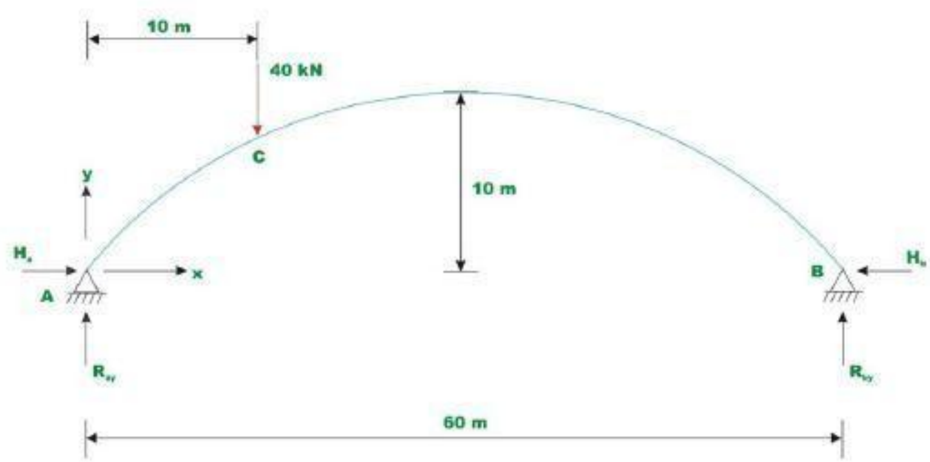
$$H = \frac{\alpha L T}{\int_0^s \frac{\tilde{y}^2}{EI} ds}$$

Using equations bending moment at any angle θ can be computed. The bending moment diagram is shown



A two hinged parabolic arch of constant cross section has a span of 60m and a rise of 10m. It is subjected to loading as shown . Calculate reactions of the arch if the temperature of the arch is raised by. Assume co-efficient of thermal expansion as

$$\alpha = 12 \times 10^{-6} / {}^\circ C.$$



Taking *A*as the origin, the equation of two hinged parabolic arch may be written

as,The given problem is solved in two steps.

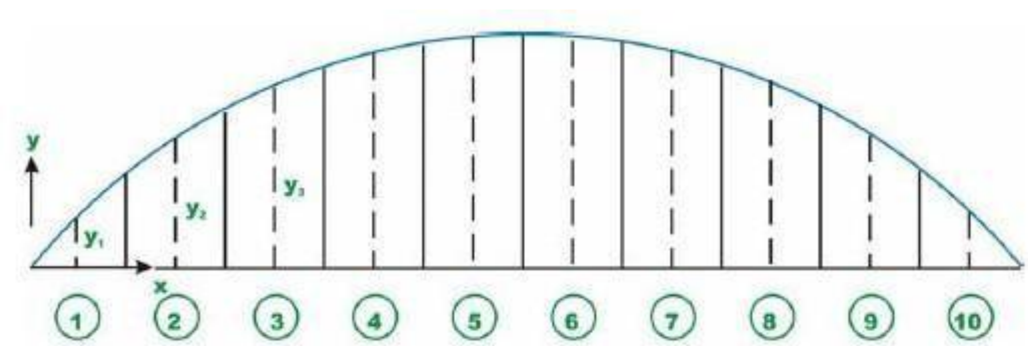
In the first step calculate the horizontal reaction due to 40kN load applied at

C.In the next step calculate the horizontal reaction due to rise in temperature.

Adding both, one gets the horizontal reaction at the hinges due to 40kN combined external loadingand temperature change. The horizontal reaction due to load may be calculated by the following equation,

Please note that in the above equation, the integrations are carried out along the *x*- axis instead of thecurved arch axis. The error introduced by this change in the variables in the case of flat arches is negligible. Using equation (1), the above equation (3) can be easily evaluated.

The vertical reaction *A* is calculated by taking moment of all forces about *B*. Hence,



	from A (m)		(kNm)		
1	3	1.9	99.99	1139.886	21.66
2	9	5.1	299.97	9179.082	156.06
3	15	7.5	299.95	13497.75	337.5
4	21	9.1	259.93	14192.18	496.86
5	27	9.9	219.91	13062.65	588.06
6	33	9.9	179.89	10685.47	588.06
7	39	9.1	139.87	7636.902	496.86
8	45	7.5	99.85	4493.25	337.5
9	51	5.1	59.83	1830.798	156.06
10	57	1.9	19.81	225.834	21.66
			$\sum$	75943.8	3300.3

$$H_1 = \frac{\sum (M_0)_i y_i (\Delta s)}{\sum (y)_i^2 (\Delta s)_i} = \frac{75943.8}{3200.3} = 23.73 \text{ kN} \qquad (10)$$

This compares well with the horizontal reaction computed from the exact integration.

Two-hinged arch is the statically indeterminate structure to degree one. Usually, the horizontal reaction is treated as the redundant and is evaluated by the method of least work. Towards this end, the strain energy stored in the two-hinged arch during deformation is given. The reactions developed due to thermal loadings are discussed. Finally, a few numerical examples are solvedto illustrate theprocedure.

UNIT- III

FORCE METHOD OF ANALYSIS OF INDETERMINATEBEAMS

INTRODUCTION TO FORCE AND DISPLACEMENT METHODS OF STRUCTURALANALYSIS

Since twentieth century, indeterminate structures are being widely used for its obvious merits. It may be recalled that, in the case of indeterminate structures either the reactions or the internal forces cannot be determined from equations of statics alone. In such structures, the number of reactions or the number of internal forces exceeds the number of static equilibrium equations. In addition to equilibrium equations, compatibility equations are used to evaluate the unknown reactions and internal forces in statically indeterminate structure.

In the analysis of indeterminate structure it is necessary to satisfy theequilibrium equations (implying that the structure is in equilibrium) compatibility equations(requirement if for assuring the continuity of the structure without any breaks) and force displacement equations (the way in which displacement are related to forces). We have two distinct method of analysis for statically indeterminate structure depending upon how the above equations are satisfied:

1. Force method of analysis

2. Displacement method of analysis

In the force method of analysis,primary unknown are forces.In this method compatibility equations are written for displacement and rotations (which are calculated by force displacement equations). Solving these equations, redundant forces are calculated.

Once the redundant forces are calculated, the remaining reactions are evaluated by equations of equilibrium. In the displacement method of analysis,the primary unknowns are the isplacements. In this method, first force -displacement relations are computed and subsequently equations are written satisfying the equilibrium conditions of the structure.

After determining the unknown displacements, the other forces are calculated satisfying the compatibility conditions and force displacement relations The displacement-based method is amenable to computer programming and hence the method is being widely used in the modern day structural analysis.

DIFFERENCE BETWEEN FORCE & DISPLACEMENT METHODS

FORCE METHODS	DISPLACEMENT METHODS
1. Method of consistent deformation 2. Theorem of least work 3. Column analogy method 4. Flexibility matrix method	1. Slope deflection method 2. Moment distribution method 3. Kani's method 4. Stiffness matrix method
Types of indeterminacy- static indeterminacy	Types of indeterminacy- kinematic indeterminacy
Governing equations-compatibility equations	Governing equations-equilibrium equations
Force displacement relations-flexibilitymatrix	Force displacement relations- stiffness matrix

All displacement methods follow the above general procedure. The Slope-deflection and moment distribution methods were extensively used for many years before the computer era. In the displacement method of analysis, primary unknowns are joint displacements which are commonly referred to as the degrees of freedom of the structure.

It is necessary to consider all the independent degrees of freedom while writing the equilibrium equations.These degrees of freedom are specified at supports, joints and at the free ends.

SLOPE DEFLECTION METHOD

In the slope-deflection method, the relationship is established between moments at the ends of the members and the corresponding rotations and displacements. The slope-deflection method can be used to analyze statically determinate and indeterminate beams and frames.

In this method it is assumed that all deformations are due to bending only. In other words deformations due to axial forces are neglected. In the force method of analysis compatibility equations are written in terms of unknown reactions. It must be noted that all the unknown reactions appear in each of the compatibility equations making it difficult to solve resulting equations.

The slope-deflection equations are not that lengthy in comparison. The basic idea of the slope deflection method is to write the equilibrium equations for each node in terms of the deflections and rotations. Solve for the generalized displacements. Using moment- displacement relations, moments are then known. The structure is thus reduced to a determinate structure. The slope-deflection method was originally developed by Heinrich Manderla and Otto Mohr for computing secondary stresses in trusses. The method as used today was presented by G.A.Maney in 1915 for analyzing rigid jointed structures.

All displacement methods follow the above general procedure. The Slope-deflection and moment distribution methods were extensively used for many years before the computer era. In the displacement method of analysis, primary unknowns are joint displacements which are commonly referred to as the degrees of freedom of the structure. It is necessary to consider all the independent degrees of freedom while writing the equilibrium equations.These degrees of freedom are specified at supports, joints and at the free ends.

FUNDAMENTAL SLOPE-DEFLECTION EQUATIONS:

The slope deflection method is so named as it relates the unknown slopes and deflections to the applied load on a structure. In order to develop general form of slope deflection equations, we will consider the typical span AB of a continuous beam which is subjected to arbitrary loading and has a constant EI. We wish to relate the beams internal end moments in terms of its three degrees of freedom, namely its angular displacements and linear displacement which could be caused by relative settlements between the supports.

Since we will be developing a formula, moments and angular displacements will be considered positive, when they act clockwise on the span. The linear displacement will be considered positive since this displacement causes the chord of the span and the span's chord angle to rotate clockwise.

The slope deflection equations can be obtained by using principle of superposition by considering separately the moments developed at each supports due to each of the displacements & then the loads.

GENERAL PROCEDURE OF SLOPE-DEFLECTION METHOD

- Find the fixed end moments of each span (both ends left & right).
- Apply the slope deflection equation on each span & identify the unknowns.
- Write down the joint equilibrium equations.
- Solve the equilibrium equations to get the unknown rotation & deflections.
    - Determine the end moments and then treat each span as simply supported beam subjected to given load & end moments so we can work out the reactions & draw the

bending moment & shear force diagram.

CONTINUOUS BEAMS

Introduction

Beams are made continuous over the supports to increase structural integrity. Acontinuous beam provides an alternate load path in the case of failure at a section. Inregions with high seismic risk, continuous beams and frames are preferred in buildingsand bridges. A continuous beam is a statically indeterminate structure.The advantages of a continuous beam as compared to a simply supported beam are asfollows.

1) For the same span and section, vertical load capacity is more.

2) Mid span deflection is less.

3) The depth at a section can be less than a simply supported beam for the samespan.

Else, for the same depth the span can be more than a simply supportedbeam.

⇒The continuous beam is economical in material.

4) There is redundancy in load path.

⇒ Possibility of formation of hinges in case of an extreme event.

5) Requires less number of anchorages of tendons.

6) For bridges, the number of deck joints and bearings are reduced.

⇒ Reduced maintenance

There are of course several disadvantages of a continuous beam as compared to asimplysupported beam.

1) Difficult analysis and design procedures.

2) Difficulties in construction, especially for precast members.

3) Increased frictional loss due to changes of curvature in the tendon profile.

4) Increased shortening of beam, leading to lateral force on the supporting columns.

5) Secondary stresses develop due to time dependent effects like creep andshrinkage, settlementof support and variation of temperature.

6) The concurrence of maximum moment and shear near the supports needsproper detailing ofreinforcement.

7) Reversal of moments due to seismic force requires proper analysis and design.

THE ANALYSIS OF CONTINUOUS BEAMS IS BASED ON ELASTIC THEORY.

For pre-stressed beams the following aspects are important.

1) Certain portions of a span are subjected to both positive and negative moments.

These moments are obtained from the envelop moment diagram.

2) The beam may be subjected to partial loading and point loading. The envelop moment diagrams are developed from "pattern loading". The pattern loading refers to the placement of live loads in patches only at the locations with positive or negative values of the influence line diagram for a moment at a particular location.

3) For continuous beams, pre-stressing generates reactions at the supports. These reactions cause additional moments along the length of a beam.

The analysis of a continuous beam is illustrated to highlight the aspects stated earlier. The bending moment diagrams for the following load cases are shown schematically in the following figures.

1) Dead load (DL)

2) Live load (LL) on every span

3) Live load on a single span.

For moving point loads as in bridges, first the influence line diagram is drawn. The influence line diagram shows the variation of the moment or shear for a particular location in the girder, due to the variation of the position of a unit point load. The vehicle load is placed based on the influence line diagram to get the worst effect. An influence line diagram is obtained by the Müller-Breslau Principle.

IS:456 - 2000, Clause 22.4.1, recommends the placement of live load as follows.

1) LL in all the spans.

2) LL in adjacent spans of a support for the support moment. The effect of LL in the alternatespans beyond is neglected.

3) LL in a span and in the alternate spans for the span moment.

The envelop moment diagrams are calculated from the analysis of each load case and their combinations. The analysis can be done by moment distribution method or by computer analysis.

In lieu of the analyses, the moment coefficients in Table 12 of IS: 456 - 2000 can be used under conditions of uniform cross-section of the beams in the several spans, uniform loads and similar lengths of span.

The envelop moment diagrams provide the value of a moment due to the external loads. It is to be noted that the effect of pre-stressing force is not included in the envelop moment diagrams.

Consider a continuous beam over several supports carrying arbitrary loads, w(x).Using the Moment-Area Theorem, we will analyze two adjoining spans of this beam to find the relationship between the internal moments at each of the supports and the loads applied to the beam. We will label the left, center ,and right supports of this two-span segment L, C, and R. The left span has length LL and flexural rigidity EIL; the right span has length LR and flexural rigidity EIR.

Applying the principle of superposition to this two-span segment, we can separate the moments caused by the applied loads from the internal moments at the supports. The two-span segment can be represented by two simply supported spans (with zero moment at L, C, and R) carrying the external loads plus two simply-supported spans carrying the internal moments ML,MC, and MR . The applied loads are illustrated below the beam, so as not to confuse the loads with the moment diagram. Note that we are being consistent with our sign convention: positive moments create positive curvature in the beam. The internal moments ML, MC, and MR are drawn in the positive directions. The areas under the moment diagrams due to the applied loads on the simply supported spans are AL and AR; ¯xL represents the distance from the left support to the centroid of AL, and ¯xR represents the distance from the right support to the centroid of AR, as shown. The moment diagrams due to the unknown moments, ML, MC, and MR are triangular, All displacement methods follow the above general procedure. The Slope-deflection and moment distribution methods were extensively used for many years before the computer era. In the displacement method of analysis, primary unknowns are joint displacements which are commonly referred to as the degrees of freedom of the structure. It is necessary to consider all the independent degrees of freedom while writing the equilibrium equations. These degrees of freedom are specified at supports, joints and at the free ends.

Examining the elastic curve of the continuous beam, we recognize that the rotation of the beam at the center support, θC, is continuous across support C. In other words, θC just to the left of point C is the same as θC

Just to the right of point C. This continuity condition may be expressed

$\Delta L\tan C\ LL = -\Delta R \tan C\ LR$, (1)

Where Δ Ltan C is the distance from the tangent at C to point L, and ΔR tan C is the distance from the tangent at C to point R.

Using the second Moment-Area Theorem, and assuming that the flexural rigidity (EI) is constant within each span, we can find the terms ΔLtanC, and ΔR tan C in terms of the unknown moments, ML, MC, and MR and the known applied loads.

To apply the three-moment equation numerically, the lengths, moments of inertia, and applied loads must be specified for each span. Two commonly applied loads are point loads and uniformly distributed loads.

For point loads PL and PR acting a distance xL and xR from the left and right supports respectively, the right hand side of the three-moment equation becomes the equation of three moments is set up for each pair of adjacent spans with all pairs considered in succession. Consequently the number of equations for a multi-span beam is equal to the degree of static indeterminacy.
All displacement methods follow the above general procedure. The Slope-deflection and moment distribution methods were extensively used for many years before the computer era. In the displacement method of analysis, primary unknowns are joint displacements which are

commonly referred to as the degrees of freedom of the structure. It is necessary to consider all the independent degrees of freedom while writing the equilibrium equations. These degrees of freedom are specified at supports, joints and at the free ends.

Application of the equation of three moments.

(1) First of all we determine the degree of static indeterminacy according to the formula

$$K = - m n ,$$

where K is the degree of static indeterminacy, m is the number of unknown reactions, n is the number of equations of static equilibrium. So, m n = = 4; 3 and

K =1. The fact of the beam being singly statically indeterminate gets obvious.

Bearing of Lintel

The bearing provided should be the minimum of following 3 cases.

i. 10 cm
ii. Height of beam
iii. 1/10th to 1/12th of span of the lintel.

Types of Lintel used in Building Construction

Lintels are classified based on the material of construction as:

1. Timber Lintel

In olden days of construction, Timber lintels were mostly used. But now a days they are replaced by several modern techniques, however in hilly areas these are using. The main disadvantages with timber are more cost and less durable and vulnerable to fire.

If the length of opening is more, then it is provided by joining multiple number of wooden pieces with the help of steel bolts In case of wider walls, it is composed of two wooden pieces kept at a distance with the help of packing pieces made of wood. Sometimes, these are strengthened by the provision of mild steel plates at their top and bottom, called as flitched lintels.

Stone Lintel

These are the most common type, especially where stone is abundantly available. The thickness of these are most important factor of its design. These are also provided over the openings in brick walls. Stone lintel is provided in the form of either one single piece or more than one piece.

The depth of this type is kept equal to 10 cm / meter of span, with a minimum value of 15 cm. They are used up to spans of 2 meters. In the structure is subjected to vibratory loads, cracks are formed in the stone lintel because of its weak tensile nature. Hence caution is needed.

Brick Lintel

These are used when the opening is less than 1m and lesser loads are acting. Its depth varies from 10 cm to 20 cm, depending up on the span. Bricks with frogs are more suitable than normal bricks because frogs when filled with mortar gives more shear resistance of end joints which is known as joggled brick lintel.

Reinforced Brick Lintel

These are used when loads are heavy and span is greater than 1m. The depth of reinforced brick lintel should be equal to 10 cm or 15 cm or multiple of 10 cm. the bricks are so arranged that 2 to 3 cm wide space is left length wise between adjacent bricks for the insertion of mild steel bars as reinforcement. 1:3 cement mortar is used to fill up the gaps.

Vertical stirrups of 6 mm diameter are provided in every 3rd vertical joint. Main reinforcement is provided at the bottom consists 8 to 10 mm diameter bars, which are cranked up at the ends.

Steel Lintel

These are used when the superimposed loads are heavy and openings are large. These consist of channel sections or rolled steel joists. We can use one single section or in combinations depending up on the requirement.

When used singly, the steel joist is either embedded in concrete or cladded with stone facing to keep the width same as width of wall. When more than one units are placed side by side, they are kept in position by tube separators.

Reinforced Cement Concrete Lintel

At present, the lintel made of reinforced concrete are widely used to span the openings for doors, windows, etc. in a structure because of their strength, rigidity, fire resistance, economy and ease in construction. These are suitable for all the loads and for any span. The width is equal to width of wall and depth depends on length of span and magnitude of loading.

Main reinforcement is provided at the bottom and half of these bars are cranked at the ends. Shear stirrups are provided to resist transverse shear

ARCH

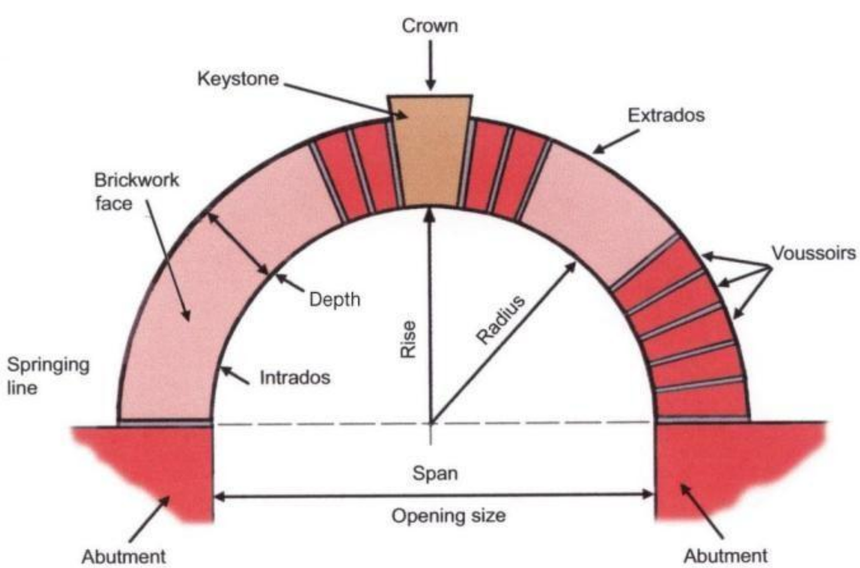
An arch is a structure constructed in curved shape with wedge shaped units (either bricks or stones), which are jointed together with mortar, and provided at openings to support the weight of the wall above it along with other superimposed loads.

Because of its shape the loads from above gets distributed to supports (pier or abutment).

Different Components of an Arch

The following are the different components of arches and terms used in arch construction:

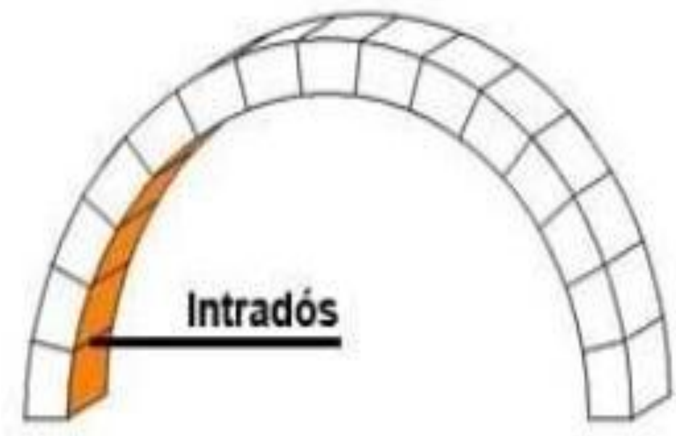
:



Intrados

The curve which bounds the lower edge of the arch OR The inner curve of an arch is called as intrados.

The distinction between soffit and intrados is that the intrados is a line, while the soffit is a surface.



Extrados

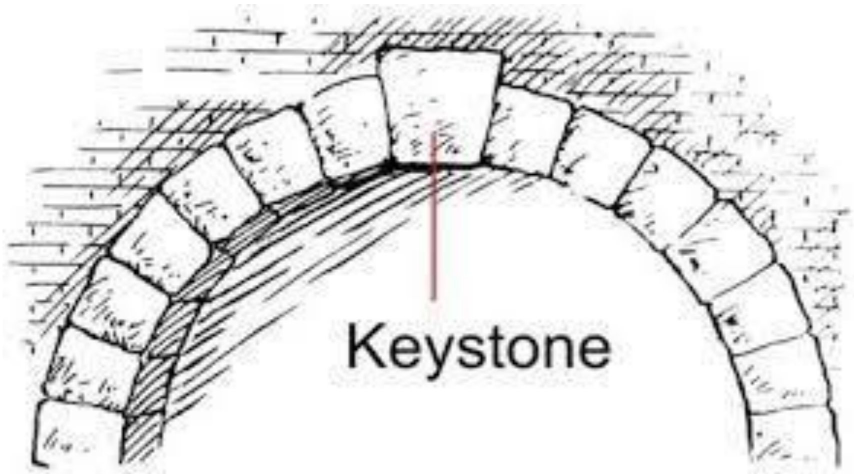
The outer curve of an arch is termed as extrados.

Crown

The apex of the arch's extrados. In symmetrical arches, the crown is at the mid span.

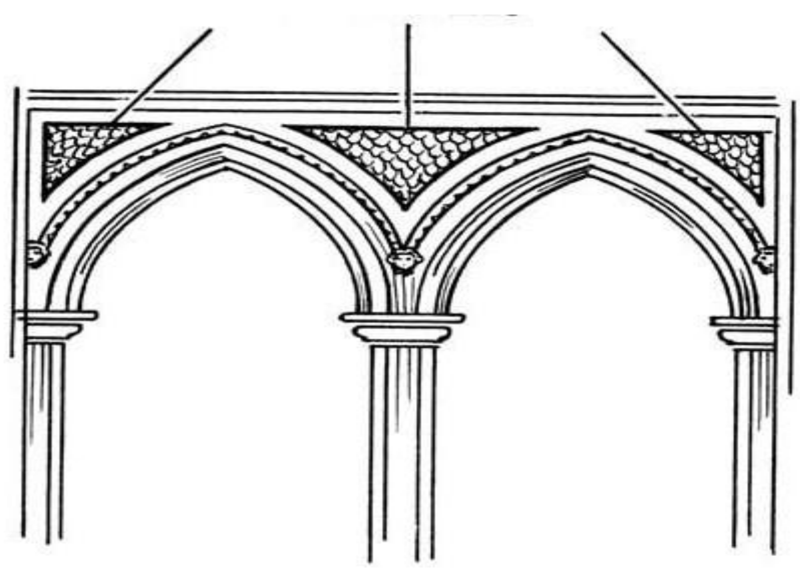
Keystone

The wedge shaped unit which is fixed at the crown of the arch is called keystone.



Spandrel in an Arch

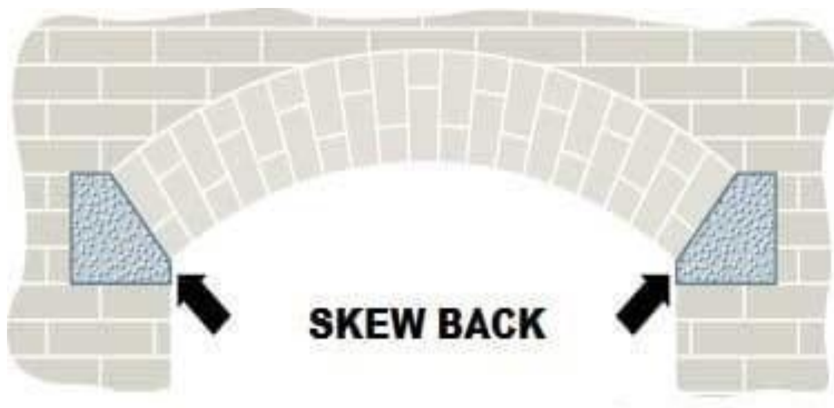
If two arches are constructed side by side, then a curved triangular space is formed between theextrados with the base as horizontal line through the crown. This space is called as spandrel.



Skew Back

The surface on which the arch joins the supporting abutment.

The upper surface of an abutment or pier from which an arch springs; its face is on a line radiating from the center of the arch.



Springing Points

The imaginary points which are responsible for the springing of curve of an arch arecalled as springing points.

Springing Line

The imaginary line joining the springing points of either ends is called as springing line.

Springer in Arches

The first voussoir at springing level which is immediately adjacent to the skewback is called as springer.

Haunch

The lower half of the arch between the crown and skewback is called haunch. Highlighted areain the below fig is haunch.Span of an Arch

The clear horizontal distance between the supports or abutments or piers is termed as span of anarch.

Rise of an Arch

The clear vertical distance between the highest point on the intrados and the springing line iscalled as rise.

Pier and Abutment of an Arch

The intermediate support of an arch is called as pier. The end support of an arch is called as abutment.

Muram or Mud Floors:

The ground floor having its topping consisting of muram or mud is called Muram or Mud FloorsThese

floors are easily and cheaply repairable Method of Construction:

- The surface of earth filling is properly consolidated
- 20cm thick layer of rubble or broken bats is laid, hand packed, wet and rammed
- 15cm thick layer of muram or good earth is laid
- 2.5cm thick layer of powdery variety of muram earth is uniformly spread
- The whole surface is well watered and rammed until the cream of muram earth rises tothe earth surface
- After 12 hours the surface is again rammed for three days.
- The surface is smeared with a thick paste of cow-dung and rammed for two days
- Thin coat of mixture of 4 parts of cow-dung and 1 part of Portland cement is evenlyapplied the surface is wiped clean by hand.
- For maintaining this type of floor properly, leaping is done once a week

Suitability: These floors are generally used for unimportant building in rural areas

Cement Concrete Floor:

The floor having its topping consisting of cement concrete is called Cement Concrete Floor or Conglomerate Floor Types of Cement Concrete Floor:

According to the method of finishing the topping, Cement Concrete Floor can be classified intothe following two types

1. Non-monolithic or bonded floor finish concrete floor
2. Monolithic floor finish concrete floor

Non-monolithic or bonded floor finish concrete floor:

The type of Cement Concrete Floor in which the topping is not laid monolithically with the base concrete is known as Non-monolithic or bonded floor finish concrete floor.

Method of Construction:

1. The earth is consolidated.
2. 10cm thick layer of clean sand is spread.
3. 10cm thick Lime Concrete (1:4:8) or Lean Cement Concrete (1:8:16) is laid thus forming base concrete
4. The topping {4cm thick Cement Concrete (1:2:4)} is laid on the third day of laying base cement concrete, thus forming Non-monolithic construction.

This type of construction is mostly adopted in the fieldThe

topping is laid by two methods:

I- Topping laid in single layer:

The topping consists of single layer of Cement Concrete (1:2:4), having its thickness 4cm

II- Topping laid two layers:

The topping consists of 1.5cm thick Cement Concrete (1:2:3), which is laid monolithically over 2.5cm thick Cement Concrete (1:3:6)

Monolithic Floor Finish Concrete Floor:

The Cement Concrete Floor in which the topping consisting of 2cm thick Cement Concrete (1:2:4) is laid monolithically with the Base Concrete is known as Monolithic Floor FinishConcrete Floor.

Method of Construction:

1. The surface of muram or earth filling is leveled, well watered and rammed
2. 10cm layer of clean and dry sand is spread over
3. When the sub soil conditions are not favorable and monolithic construction is desired,then, 5cm to 10cm thick hard core of dry brick or rubble filling is laid.
4. 10cm thick layer of Base Concrete consisting of Cement Concrete (1:4:8) or LeanCement Concrete (1:8:16) is laid.
5. The topping {2cm thick layer of Cement Concrete(1:2:4)} is laid after 45 minutes to 4 hours of laying Base Concrete.

Tile Floor: The floor having its topping consisting of tiles is called tile floor. Method ofConstruction:

1. The muram or earth filling is properly consolidated.
2. 10cm thick layer of dry clean sand is evenly laid
3. 10cm thick layer of Lime Concrete (1:4:8) or Lean Cement Concrete (1:8:16) is laid, compacted and cured to form a base concrete.
4. A thin layer of lime or cement mortar is spread with the help of screed battens.
5. Then the screed battens are properly leveled and fixed at the correct height.
6. When the surface mortar is harden sufficiently, 6mm thick bed of wet cement (1:5) is laidand then over this the specified tiles are laid.
7. The surplus mortar which comes out of the joints is cleaned off.
8. After 3 days, the joints are well rubbed with a corborundum stone to chip off all theprojecting edges.
9. Rubbing should not be done in case of glazed tiles.
10. The surface is polished by rubbing with a softer variety of a corborundum or a pumicestone.
11. The surface is finally washed with soap.

Suitability: This type of floor is suitable for courtyard of buildings. Glazed tiles are used inmodern buildings where a high class finish is desired.

Mosaic Floors:

The floors having its topping consisting of mosaic tiles or small regular cubes, square or hexagons, embedded into a cementing mixture is known as Mosaic Floors

Method of Construction:

1. The earth is consolidated.
2. 10cm thick layer of clean sand is spread.
3. 10cm thick Lime Concrete (1:4:8) or Lean Cement Concrete (1:8:16) is laid thus forming base concrete.
4. Over this base course 5cm thick Lime Mortar or Cement Mortar or Lime and Surkhi mortar (1:2) is laid.
5. The mortar is laid in small area so that the mortar may not get dried before finishing thewearing course.
6. 3mm thick cementing mixture is spread.
7. The cementing mixture consists of one part of pozzolana, one part of marble chips andtwo parts of slacked lime.
8. After nearing 4 hours, patterns are formed on the top of the cementing material.
9. Now the tiles of regular shaped marble cubes are hammered in the mortar along theoutline of the pattern.
10. The inner spaces are then filled with colored pieces of marble.
11. A roller 30cm in diameter and 50cm in length is passed gently over the surface.
12. Water is sprinkled to work up the mortar between the marble pieces.
13. The surface is then rubbed with pumice stone fixed to a wooden handle about 1.5m long.
14. The surface is then allowed to dry up for 2 weeks.

Double Flag Stone floor

Two layer of flagstones are used to build this type of floor, this is why it is called double flagstone floor.

Materials used to build this type of floor are –

- Flagstone (about 40 mm thickness)
- Rolled steel joist
- Rolled Steel beam (for span above 4 meter)

Procedure:

For span above 4 meter, a framework is built consist of rolled steel beam and rolled steel joist. To make formwork, beam are place at 10 feet centre to centre distance then joists are placed at right angle to the beam. And then two layers of flagstone are fixed with the joist. One layer is at top flanged of joist and another layer is at bottom flanged of joist. The gap between the two layers of flagstone is filled with earth or concrete before fixing the top layer of flagstone.

Jack arch floor

You'll find the following components/materials in this type of floor –

- Arch (brick arch or concrete arch)
- Rolled steel joist
- Rolled steel beam
- Wall
- Tie rod

Mechanism

Joists are placed on wall or beam and tied together with the tie rod. And then concrete arches orbrick arches are constructed and rest on lower flanged of Joists.

Non-Composite Floor

Non composite type of floors are those which are built using one material only. Mostly used material for non-composite floor is timber.

Timber floor can further be divided into 3 types

- Single joist floor
- Double joist floor

- Tripple joist floor,

Floor board: Floor board are fixed at the top of bridging joist. It acts as the wearing of the top surface of the floor.

Floor ceiling: To make the bottom of the floor flat and increasing the aesthetic look floor ceiling is provided. For this purpose plaster board or sheet of asbestors or some other suitable materials are used. Floor ceiling rests on bridging joist. To make the ceiling more durable and strong ceiling joist may be provided at the right angle to the bridging joist.

Single joist floor

In this type of floor single joist is placed below floor board. This joist is supported by wall-plateat both end.

Double joist floor

In this type of floor binders are provided to support the bridging joist. Binders are then rest onthe walls at both end.

Triple joist floor

Triple joist floor is also called framed timber floor. In this type of floor another member is added that is girder, which we didn't use in double joist floor.

These girders are placed on the wall to support the binders. And then joist are placed on the binders.

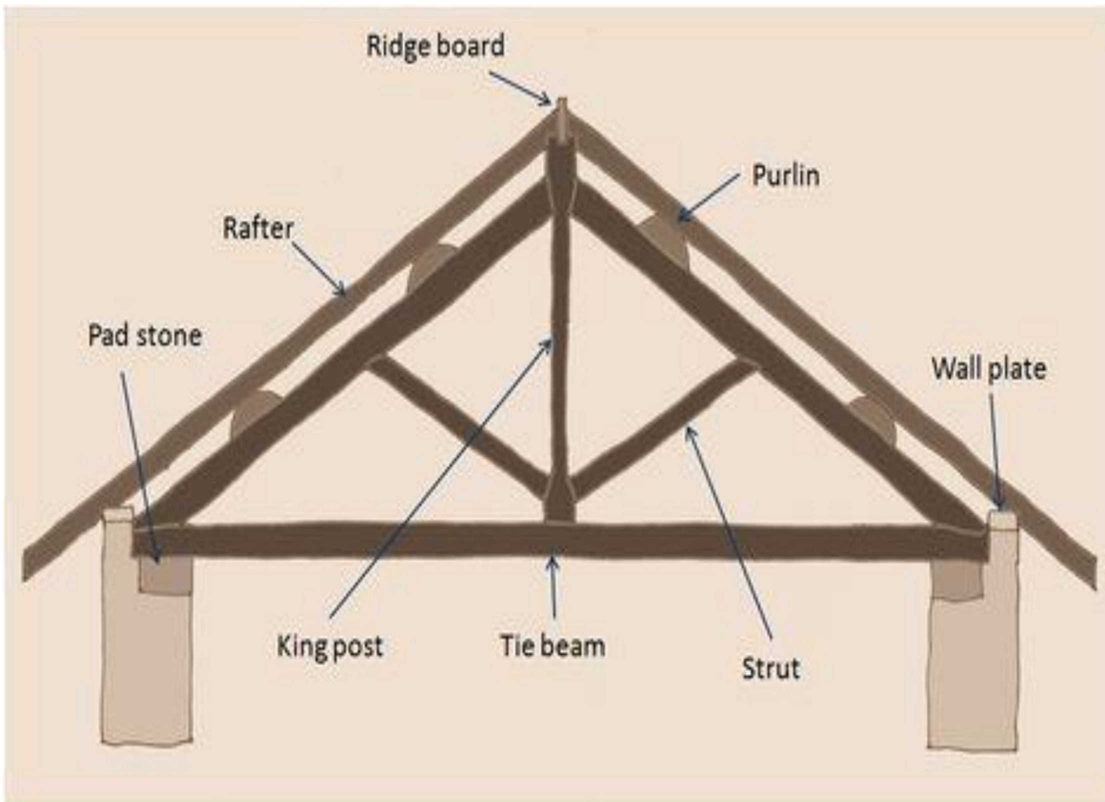
In this post, I did not include floors which are built on ground as a type of floor. Because, I think, it does not need serious mechanism to build a floor on ground. Because there is a ground itself to support the floor.

Roof truss

A roof truss is basically a structure that includes one or multiple triangular units that include straight slender members with their ends connected via nodes.

King Post Truss

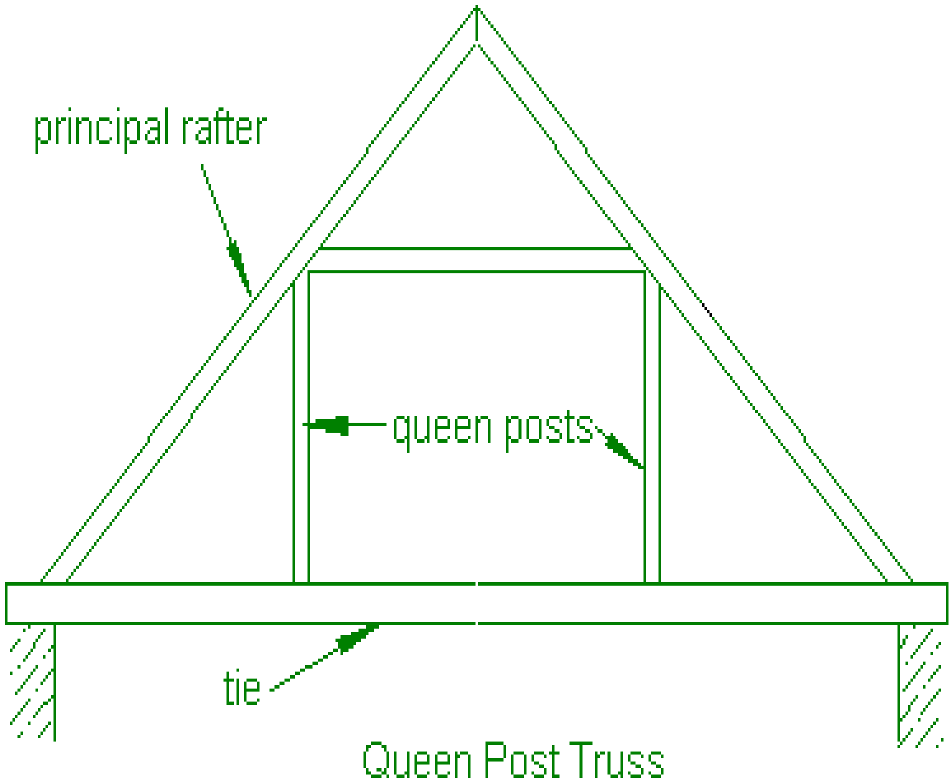
This particular truss is made out of wood most of the time, but it can also be built out of a combination of steel and wood. It all comes down to the architect and the building structure. The King Post Truss spans up to 8m, which makes it perfect for multiple types of houses, especially the smaller ones.



Queen Post Truss

The Queen Post Truss is designed to be a very reliable, simple and versatile type of roof truss that you can use at any given time.

It offers a good span, around 10m, and it has a simple design which makes it perfect for a wide range of establishments.



North Light Roof Truss

The North Light Roof Truss is suitable for the larger spans that go over 20m and get up to 30m. This happens because it's cheaper to add a truss that has a wide, larger set of lattice girders that include support trusses.

Mechanism

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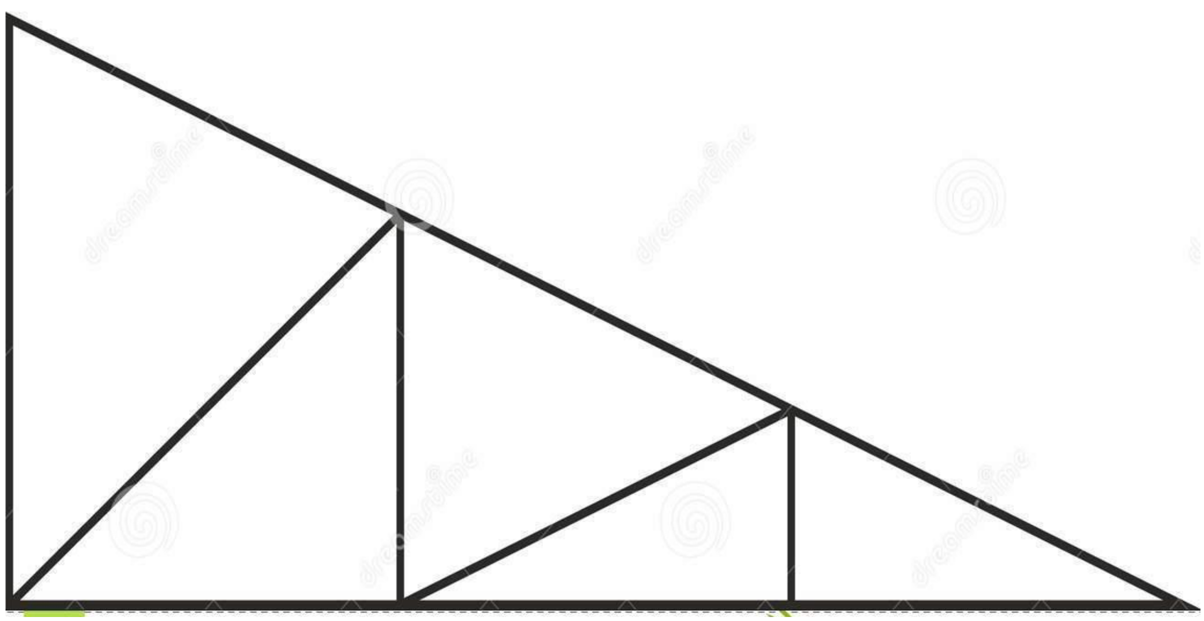

This method is one of the oldest, as well as most economical ones that you can find on the market, as it allows you to bring in proper ventilation. Plus, the roof has more resistance too because of that.

If you are looking for types of roof trusses design that bring in durability and versatility, this is a very good one to check out. You can use it for industrial buildings, but this truss also works for drawing rooms and in general those spaces that are very large.

UNIT- IV

DISPLACEMENT METHOD OF ANALYSIS: SLOPEDEFLECTION ANDMOMENT DISTRIBUTION

Since twentieth century, indeterminate structures are being widely used for its obvious merits. It may be recalled that, in the case of indeterminate structures either the reactions or the internal forces cannot be determined from equations of statics alone. In such structures, the number of reactions or the number of internal forces exceeds the number of static equilibrium equations. In addition to equilibrium equations, compatibility equations are used to evaluate the unknown reactions and internal forces in statically indeterminate structure. In the analysis of indeterminate structure it is necessary to satisfy theequilibrium equations (implying that the structure is in equilibrium) compatibility equations(requirement if for assuring the continuity of the structure without any breaks) and force displacement equations (the way in which displacement are related to forces). We have two distinct method of analysis for statically indeterminate structure depending upon how the above equations are satisfied:

1. Force method of analysis

2. Displacement method of analysis

In the force method of analysis, primary unknown are forces. In this method compatibility equations are written for displacement and rotations (which are calculated by force displacement equations). Solving these equations, redundant forces are calculated. Once the redundant forces are calculated, the remaining reactions are evaluated by equations of equilibrium. In the displacement method of analysis, the primary unknowns are the displacements. In this method, first force -displacement relations are computed and subsequently equations are written satisfying the equilibrium conditions of the structure. After determining the unknown displacements, the other forces are calculated satisfying the compatibility conditions and force displacement relations.

The displacement-based method is amenable to computer programming and hence the method is being widely used in the modern day structural analysis.

DIFFERENCE BETWEEN FORCE & DISPLACEMENT METHODS

FORCE METHODS	DISPLACEMENT METHODS
1. Method of consistent deformation 2. Theorem of least work 3. Column analogy method 4. Flexibility matrix method	1. Slope deflection method 2. Moment distribution method 3. Kani's method 4. Stiffness matrix method
Types of indeterminacy- static indeterminacy	Types of indeterminacy- kinematic indeterminacy
Governing equations-compatibility equations	Governing equations-equilibrium equations
Force displacement relations-flexibility matrix	Force displacement relations- stiffnessmatrix

All displacement methods follow the above general procedure. The Slope-deflection and moment distribution methods were extensively used for many years before the computer era. In the displacement method of analysis, primary unknowns are joint displacements which are commonly referred to as the degrees of freedom of the structure. It is necessary to consider all the independent degrees of freedom while writing the equilibrium equations.These degrees of freedom are specified at supports, joints and at the free ends.

SLOPE DEFLECTION METHOD

In the slope-deflection method, the relationship is established between moments at the ends of the members and the corresponding rotations and displacements. The slope-deflection method can be used to analyze statically determinate and indeterminate beams and frames.

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In this method it is assumed that all deformations are due to bending only. In other words deformations due to axial forces are neglected. In the force method of analysis compatibility equations are written in terms of unknown reactions. It must be noted that all the unknown reactions appear in each of the compatibility equations making it difficult to solve resulting equations. The slope-deflection equations are not that lengthy in comparison. The basic idea of the slope deflection method is to write the equilibrium equations for each node in terms of the deflections and rotations. Solve for the generalized displacements. Using moment-displacement relations, moments are then known.

The structure is thus reduced to a determinate structure. The slope-deflection method was originally developed by Heinrich Manderla and Otto Mohr for computing secondary stresses in trusses. The method as used today was presented by G.A.Maney in 1915 for analyzing rigid jointed structures.

FUNDAMENTAL SLOPE-DEFLECTION EQUATIONS:

The slope deflection method is so named as it relates the unknown slopes and deflections to the applied load on a structure. In order to develop general form of slope deflection equations, we will consider the typical span AB of a continuous beam which is subjected to arbitrary loading and has a constant EI. We wish to relate the beams internal end moments in terms of its three degrees of freedom, namely its angular displacements and linear displacement which could be caused by relative settlements between the supports. Since we will be developing a formula, moments and angular displacements will be considered positive, when they act clockwise on the span. The linear displacement will be considered positive since this displacement causes the chord of the span and the span's chord angle to rotate clockwise. The slope deflection equations can be obtained by using principle of superposition by considering separately the moments developed at each supports due to each of the displacements & then the loads.

GENERAL PROCEDURE OF SLOPE-DEFLECTION METHOD

- Find the fixed end moments of each span (both ends left & right).
- Apply the slope deflection equation on each span & identify the unknowns.
- Write down the joint equilibrium equations.
- Solve the equilibrium equations to get the unknown rotation & deflections.
- Determine the end moments and then treat each span as simply supported beamsubjected to given load & end moments so we can work out the reactions & drawthe bending moment &shear force diagram.

Loads and displacements are vector quantities and hence a proper coordinate system is required to specify their correct sense of direction. Consider a planar truss, In this truss each node is identified by a number and each member is identified by a number enclosed in a circle. The displacements and loads acting on the truss are defined with respect to global co-ordinate system *xyz.*

The same co ordinate system is used to define each of the loads and displacements of all loads. In a global co-ordinate system, each node of a planer truss can have only two displacements: one along *x* -axis and another along *y*-axis. The truss shown in figure has eight displacements. Each displacement (degree of freedom) in a truss is shown by a number in the figure at the joint.

The direction of the displacements is shown by an arrow at the node. However out of eight displacements, five are unknown. The displacements indicated by numbers 6, 7 and 8 are zero due to support conditions.

The displacements denoted by numbers 1-5 are known as unconstrained degrees of freedom of the truss and displacements denoted by 6-8 represent constrained degrees of freedom. In this course, unknown displacements are denoted by lower numbers and the known displacements are denoted by higher code numbers.

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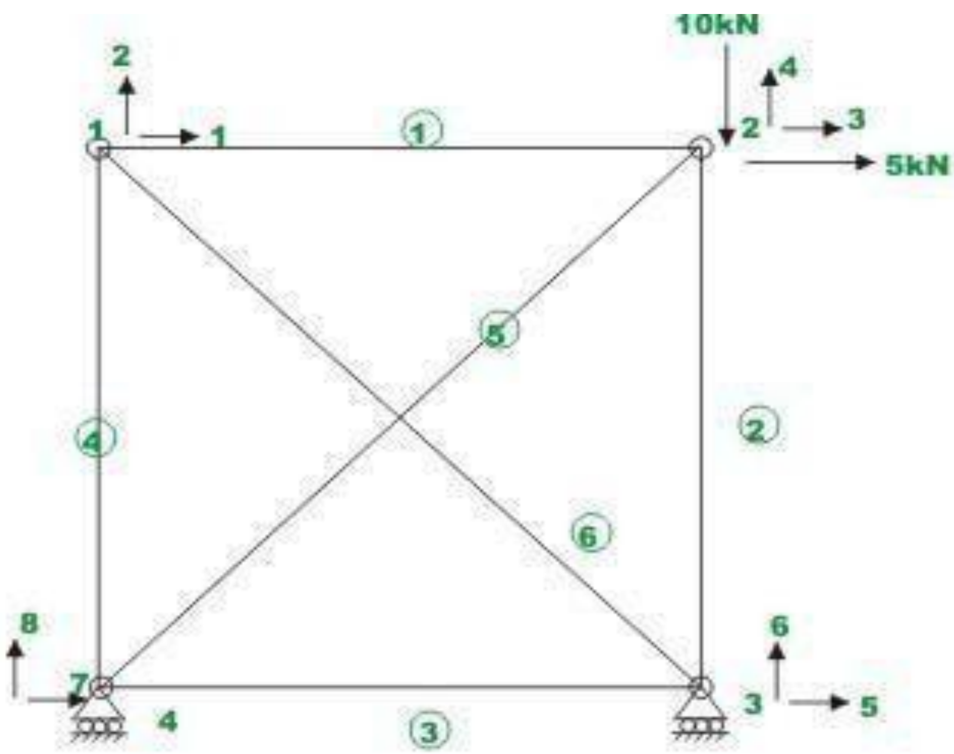
The displacements and loads acting on the truss are defined with respect to global co- ordinate system *xyz.*



MEMBER STIFFNESS MATRIX INTRODUCTION

To analyse the truss shown in, the structural stiffness matrix *K* need to be evaluated for the given truss. This may be achieved by suitably adding all the member stiffness matrices *k*', which is used to express the force-displacement relation of the member in local co-ordinate system. Since all members are oriented at different directions, it is required to transform member displacements and forces from the local co-ordinate system to global co-ordinate system so that a global load-displacement relation may be written for the complete truss.

MEMBER STIFFNESS MATRIX ANALYSIS

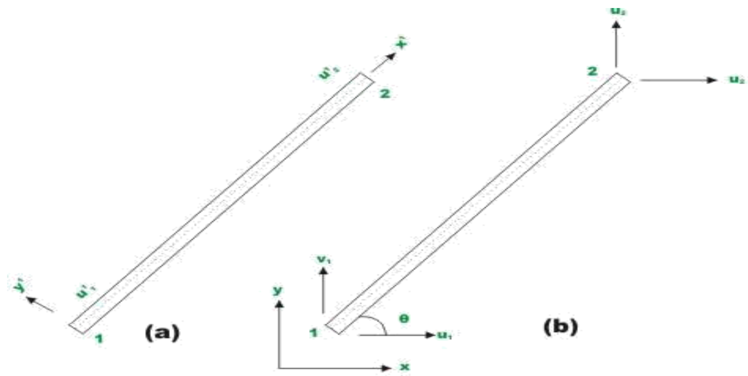
Consider a member of the truss in local co-ordinate system *x*'*y*'. As the loads are applied along the centroidal axis, only possible

All displacement methods follow the above general procedure. The Slope-deflection and moment distribution methods were extensively used for many years before the computer era. In the displacement method of analysis, primary unknowns are joint displacements which are commonly referred to as the degrees of freedom of the structure. It is necessary to consider all the independent degrees of freedom while writing the equilibrium equations.These degrees of freedom are specified at supports, joints and at the free ends.

refers to node 1 of the truss member and subscript 2 refers to node 2 of the truss member.Give displacement u'_1 at node 1 of the member in the positive *x*' direction, keeping all other displacements to zero. This displacement in turn **TRANSFORMATION FROM LOCAL TO GLOBAL CO-ORDINATE SYSTEM.**

Displacement Transformation Matrix

A truss member is shown in local and global co ordinate system in Figure. Let *x*' *y*' *z*'be in local co ordinate system and*xyz* be the global co ordinate system.



The nodes of the truss member be identified by 1 and 2. Let u'_1 and u'_2 be the displacement of nodes 1 and 2 in local co ordinate system. In global co ordinate system, each node has two degrees of freedom. Thus, u_1, v_1 and u_2, v_2 are the nodal displacements at nodes 1 and 2 respectively along x - and y - directions.

Let the truss member be inclined to x axis by ϑ as shown in figure. It is observed from the figure that u'_1 is equal to the projection of u_1 on x' axis plus projection of v_1 on x'-axis. Thus, (vide Fig. 24.7)

$u'_1 = u_1\cos\vartheta + v_1\sin\vartheta$

$u'_2 = u_2\cos\vartheta + v_2\sin\vartheta$ This may be written as

$$\begin{Bmatrix} u'_1 \\ u'_2 \end{Bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{Bmatrix}$$

$$\begin{Bmatrix} u'_1 \\ u'_2 \end{Bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{Bmatrix}$$

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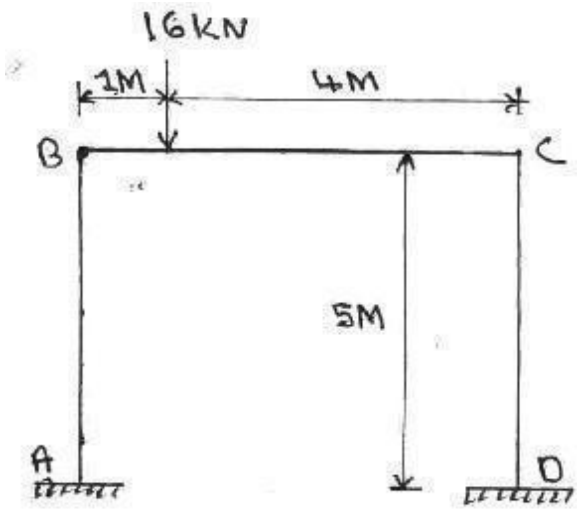
MOMENT DISTRIBUTION METHOD FOR FRAMES WITH SIDESWAY

Frames that are non-symmetrical with reference to material property or geometry (different lengths and I values of column) or support condition or subjected to non-symmetrical loading have a tendency to side sway.

Analyze the frame shown in figure by moment distribution method. Assume EI is constant.

Non SwayAnalysis:



First consider the frame without side sway

$$M_{FAB} = M_{FBA} = M_{FCD} = 0$$

$$M_{FBC} = -\frac{16\times1\times4^2}{5^2} = -10.24 KNm$$

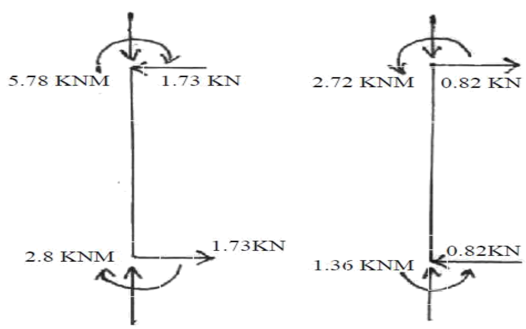
$$M_{FCB} = \frac{16\times4\times1^2}{5^2} = 2.56 KNm$$

DISTRIBUTION FACTOR

Jt.	Member	Relative stiffness K	ΣK	$DF = \frac{K}{\sum K}$
B	BA BC	I/5 = 0.2 I I/5 = 0.2 I	0.4 I	0.5 0.5
C	CB CD	I/5 = 0.2I I/5 = 0.2I	0.4I	0.5 0.5

DISTRIBUTION OF MOMENTS FOR NON-SWAY ANALYSIS

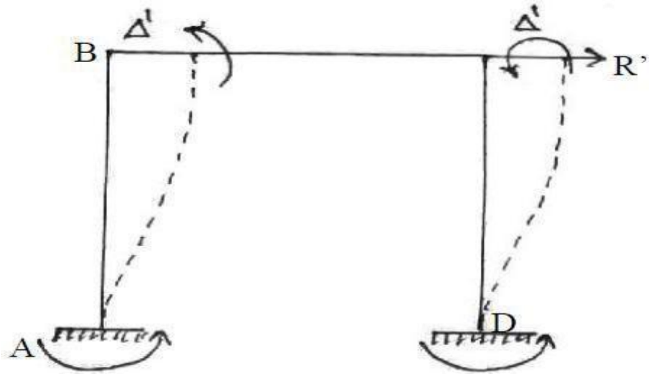
Joint	A		B		C	D
Member	AB	BA	BC	CB	CD	DC
D.F	0	0.5	0.5	0.5	0.5	0
FEM	0		-10.24	2.56	0	0
Balance		5.12	5.12	-1.28	-1.28	
CO	2.56		-0.64	2.56		-0.64
Balance		0.32	0.32	-0.08	-0.08	
CO	0.16		-0.64	0.16		-0.64
Balance		0.32	0.32	-0.08	-0.08	
C.O	0.16		-0.04	0.16		-0.04
Balance		0.02	0.02	-0.08	-0.08	
C.O	0.01					-0.04
Final moments	**2.89**	**5.78**	**-5.78**	**2.72**	**-2.72**	**-1.36**



By seeing of the FBD of columns R = 1.73 – 0.82

(Using F_x =0 for entire frame) = 0.91 KN ⟵

Now apply R = 0.91 KN acting opposite as shown in the above figure for the sway analysis. Sway analysis: For this we will assume a force R' is applied at C causing the frame to deflect as shown in the following figure.



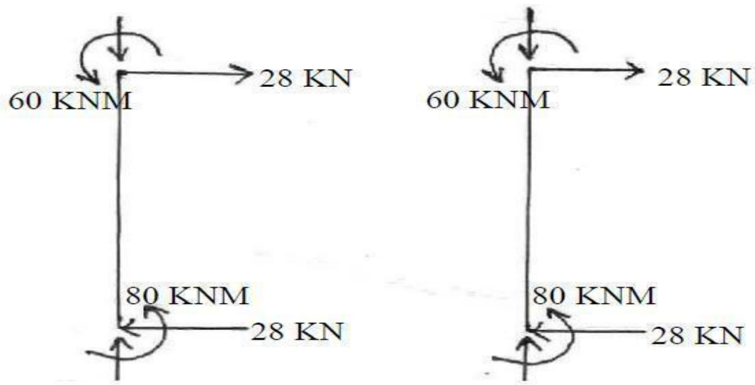
Since both ends are fixed, columns are of same length & I and assuming joints B & C are temporarily restrained from rotating and resulting fixed end momentare assumed.

$$M'_{AB} = M'_{BA} = M'_{CD} = M'_{DC} = \frac{6EI}{L^2}\Delta$$
$$M'_{BA} = -100KNm$$
$$M'_{AB} = M'_{CD} = M'_{DC} = -100KNm$$

Moment distribution table for sway analysis:

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D.F	01	0.5	0.5	0.5	0.5	0
FEM	−100	−100	0	0	−100	−100
Balance		50	50	50	50	
CO	25		25	25		25
Balance		−12.5	−12.5	−12.5	−12.5	
CO	-6.25		−6.25	-6.25		−6.25
Balance		3.125	3.125	3.125	3.125	
C.O	1.56		1.56	1.56		1.56
Balance		−0.78	−0.78	−0.78	−0.78	
C.O	-0.39		−0.39	−0.39		0.39
Balance		0.195	0.195	0.195	0.195	
C.O	0.1					0.1
Final moments	**- 80**	**- 60**	**60**	**60**	**– 60**	**– 80**

Free body diagram of columns



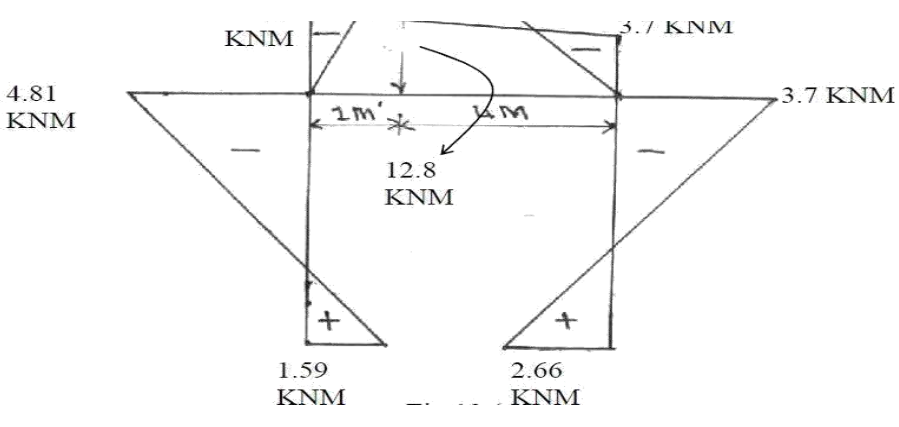
Using F x = 0 for the entire frame R = 28 + 28 = 56 KN

Hence R′ = 56KN creates the sway moments shown in above moment distribution table. Corresponding moments caused by R = 0.91KN can be determined by proportion. Thus final moments are calculated by adding non sway moments and sway.

Moments calculated for R = 0.91KN, as shown below.

$$M_{AB} = 2.89 + \frac{0.91}{56}(-80) = 1.59KNm$$
$$M_{BA} = 5.78 + \frac{0.91}{56}(-60) = 4.81KNm$$

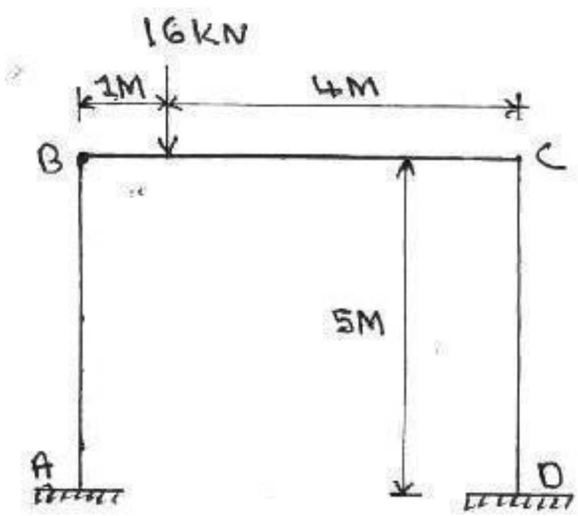
$$M_{BC} = -5.78 + \frac{0.91}{56}(60) = -4.81KNm$$
$$M_{CB} = 2.72 + \frac{0.91}{56}(60) = 3.7KNm$$
$$M_{CD} = -2.72 + \frac{0.91}{56}(-60) = -3.7KNm$$
$$M_{DC} = -1.36 + \frac{0.91}{56}(-80) = -2.66KNm$$



BMD

Frames that are non-symmetrical with reference to material property or geometry (different lengths and I values of column) or support condition or subjected to non-symmetrical loading have a tendency to side sway.

Analyze the frame shown in figure by moment distribution method. Assume EI is constant.



Non SwayAnalysis:

First consider the frame without side sway

$$M_{FAB=} M_{FBA=} M_{FCD=} 0$$

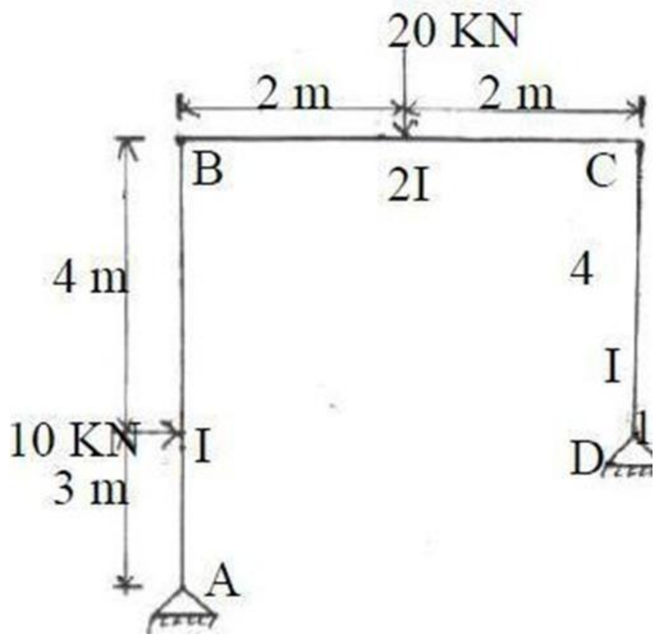
$$M_{FBC=} -\frac{16\times1\times4^2}{5^2}=-10.24KNm$$

$$M_{FCB=} \frac{16\times4\times1^2}{5^2}=2.56KNm$$

Hence $R^{'}$ = 56KN creates the sway moments shown in above moment distribution table. Corresponding moments caused by R = 0.91KN can be determined by proportion. Thus final moments are calculated by adding non sway moments and sway.

Frames that are non-symmetrical with reference to material property or geometry (different lengths and I values of column) or support condition or subjected to non-symmetrical loading have a tendency to side sway.

Q) Analysis the rigid frame shown in figure by moment distribution method and draw BMD



A. Non Sway Analysis:

First consider the frame held from side sway

FEMS

$$M_{FAB} = -\frac{10 \times 3 \times 4^2}{7^2} = -9.8KNm$$

$$M_{FBA} = \frac{10 \times 4 \times 3^2}{7^2} = 7.3KNm$$

$$M_{FBC} = -\frac{20 \times 4}{8} = -10KNm$$

$$M_{FCB} = \frac{20 \times 4}{8} = 10KNm$$

$$M_{FCD} = M_{FDC} = 0$$

Calculation of distribution factors:

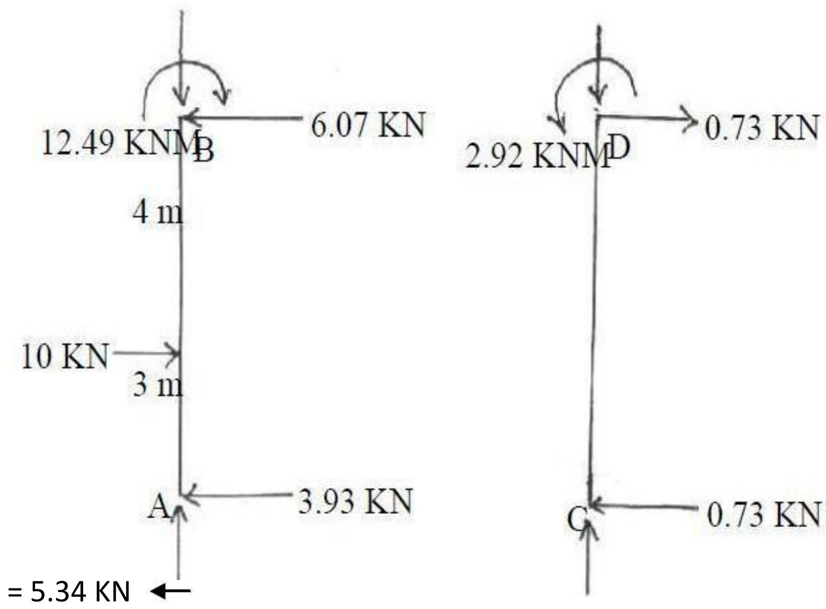
Joint	Member	Relative stiffness k	Σk	DF = $\frac{K}{\sum K}$
B	BA	$\frac{3}{4}\times\frac{I}{7} = 0.11I$	0.61 I	0.18
	BC	2I/4 = 0.5I		0.82
C	CB	2I/4 = 0.5I		0.72
	CD	$\frac{3}{4}\times\frac{I}{4} = 0.19\ I$	0.69 I	0.28

DISTRIBUTION FACTOR

DISTRIBUTION OF MOMENTS FOR NON-SWAY ANALYSIS

Joint	A	B		C		D	
Member	AB	BA	BC	CB	CD	DC	
D.F	1	0.18	0.82	0.72	0.28	1	
FEM	-9.8	7.3	-10	10	0	0	
Release jt. 'D'	+9.8						
CO		4.9					
Initial moments	0	12.2	-10	10	0	0	
Balance		-0.4	-1.8	-7.2	-2.8		
CO			-3.6	-0.9			
Balance		0.65	2.95	0.65	0.25		
C.O			0.33	1.48			
Balance		-0.06	-0.27	-1.07	-0.41		
C.O			-0.54	-0.14			
Balance		0.1	0.44	0.1	0.04		
Final moments	**0**	**12.49**	**-12.49**	**2.92**	**-2.92**	**0**	

FREE BODY DIAGRAM OF COLUMNS



Applying Fx = 0 for frame as a Whole, R = 10 – 3.93 – 0.73Now

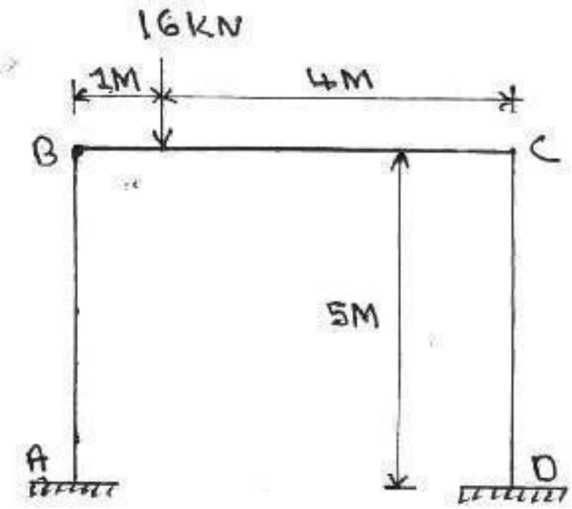
apply R = 5.34KN acting opposite

Frames that are non-symmetrical with reference to material property or geometry (different lengths and I values of column) or support condition or subjected to non-symmetrical loading have a tendency to side sway.

All displacement methods follow the above general procedure. The Slope-deflection and moment distribution methods were extensively used for many years before the computer era. In the displacement method of analysis, primary unknowns are joint displacements which are commonly referred to as the degrees of freedom of the structure. It is necessary to consider all the independent degrees of freedom while writing the equilibrium equations.These degrees of freedom are specified at supports, joints and at the free ends.

Frames that are non-symmetrical with reference to material property or geometry (different lengths and I values of column) or support condition or subjected to non-symmetrical loading have a tendency to side sway.

Analyze the frame shown in figure by moment distribution method. Assume EIis constant.



Non SwayAnalysis:

First consider the frame without side sway

$$M_{FAB}= M_{FBA} = M_{FCD}= 0$$

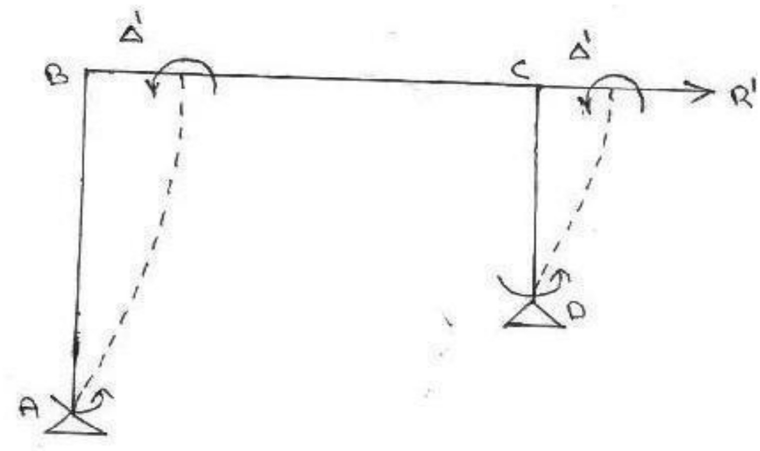
$$M_{FBC}= -\frac{16\times1\times4^2}{5^2}=-10.24KNm$$

$$M_{FCB}= \frac{16\times4\times1^2}{5^2}=2.56KNm$$

Frames that are non-symmetrical with reference to material property or geometry (different lengths and I values of column) or support condition or subjected to non-symmetrical loading have a tendency to side sway.

It is necessary to consider all the independent degrees of freedom while writing the equilibrium equations.These degrees of freedom are specified at supports, joints and at thefree ends.

Sway analysis: For this we will assume a force R′ is applied at C causing the frame todeflect as shown in figure



Since ends A & D are hinged and columns AB & CD are of different lengths

$M'_{BA} = -\frac{3EI}{L_1^2}\Delta'$, $M'_{CD} = -\frac{3EI}{L_2^2}\Delta'$,

$$\frac{M'_{BA}}{M'_{CD}} = \frac{\frac{3EI}{L_1^2}\Delta'}{\frac{3EI}{L_2^2}\Delta'} = \frac{L_2^2}{L_1^2} = \frac{4^2}{7^2} = \frac{16}{49}$$

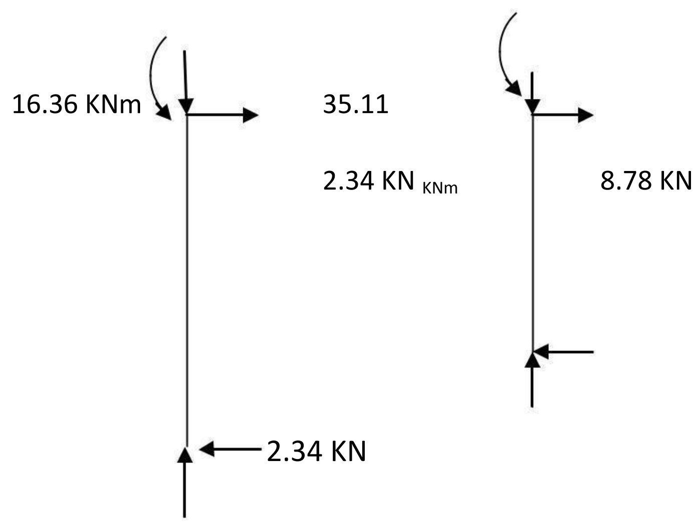
Assume

$$M'_{BA} = -16KNm, M'_{AB} = 0$$
$$M'_{CD} = -49KNm, \ M'_{DC} = 0$$

MOMENT DISTRIBUTION FOR SWAY ANALYSIS

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D.F	1	0.18	0.82	0.72	0.28	1
FEM	0	-16	0	0	-49	0
Balance		2.88	13.12	35.28	13.72	
CO			17.64	6.56		
Balance		-3.18	-14.46	-4.72	-1.84	
CO			-2.36	-7.23		
Balance		0.42	1.94	5.21	2.02	
C.O			2.61	0.97		
Balance		-0.47	-2.14	-0.7	-0.27	
C.O			0.35	-1.07		
Balance		0.06	0.29	0.77	0.3	
C.O			0.39	0.15		
Balance		-0.07	-0.32	-0.11	-0.04	
Final moments	**0**	**-16.36**	**16.36**	**35.11**	**-35.11**	**0**

FREE BODY DIAGRAMS OF COLUMNS AB &CD



Using Fx = 0 for the entire frame $R^{'}$=

11.12 kN →

Hence $R^{'}$= 11.12 KN creates the sway moments shown in the above moment distributiontable. Corresponding moments caused by R = 5.34 kN can be determined by proportion. Thus final moments are calculated by adding non-sway moments and sway moments determined for R = 5.34 KN as shown below.

$$M_{AB} = 0$$

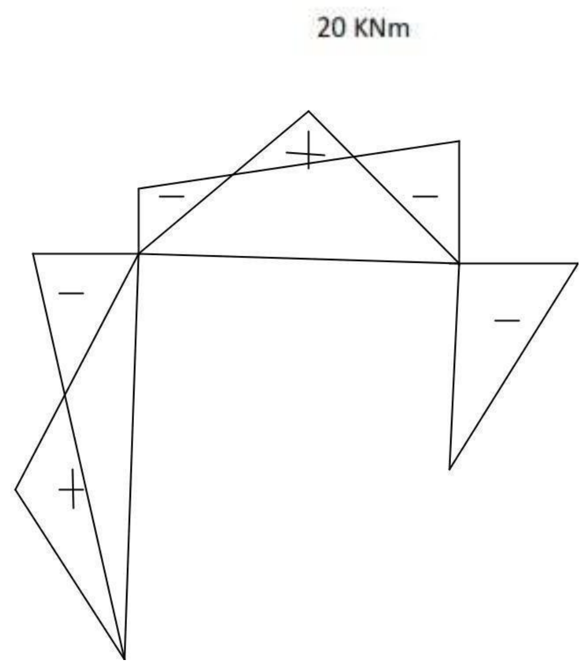
$$M_{BA} = 12.49 + \frac{5.34}{11.12}(-16.36) = 4.63 KNm$$

$$M_{BC} = -12.49 + \frac{5.34}{11.12}(16.36) = -4.63 KNm$$

$$M_{CB} = 2.92 + \frac{5.34}{11.12}(35.11) = 19.78 KNm$$

$$M_{CD} = -2.92 + \frac{5.34}{11.12}(-35.11) = -19.78 KNm$$

$$M_{DC} = 0$$



B.M.D

APPROXIMATE LATERAL LOAD ANALYSIS BY PORTAL METHOD

Portal Frame

Portal frames, used in several Civil Engineering structures like buildings, factories, bridges have the primary purpose of transferring horizontal loads applied at their tops to their foundations. Structural requirements usually necessitate the use of statically indeterminate layout for portal frames, and approximate solutions are often used in their analyses.

Assumptions for the Approximate Solution

In order to analyze a structure using the equations of statics only, the number of independent force components must be equal to the number of independent equations of statics.

If there are *n* more independent force components in the structure than there are independent equations of statics, the structure is statically indeterminate to the *nth* degree. Therefore to obtain an approximate solution of the structure based on statics only, it will be necessary to make *n* additional independent assumptions. A solution based on statics will not be possible by making fewer than *n* assumptions, while more than *n* assumptions will not in general be consistent.

Thus, the first step in the approximate analysis of structures is to find its degree of statically indeterminacy (dosi) and then to make appropriate number of assumptions. For example, the dosi of portal frames shown in (i), (ii), (iii) and (iv) are 1, 3, 2 and 1respectively. Based on the type of frame, the following assumptions can be made forportal structures with a vertical axis of symmetry that are loaded horizontally atthetop

1. The horizontal support reactions areequal
2. There is a point of inflection at the center of the unsupported height of each fixedbasedcolumn

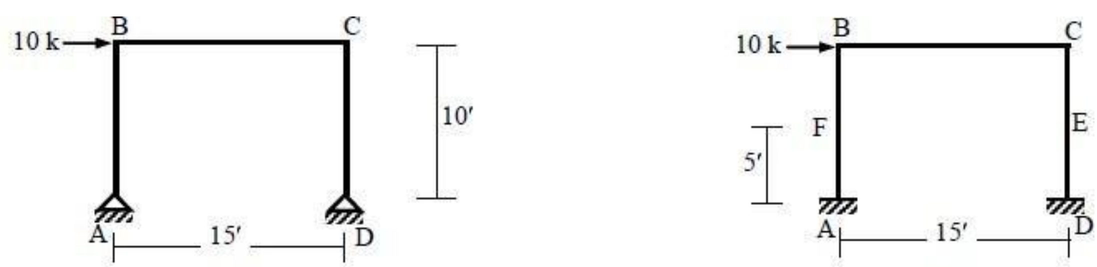
Assumption 1 is used if dosi is an odd number (i.e., = 1 or 3) and Assumption 2 isused if dosi 1.

Some additional assumptions can be made in order to solve the structure approximately for different loading and support conditions.

3. Horizontal body forces not applied at the top of a column can be divided into twoforces (i.e., applied at the top and bottom of the column) based on simple supports
4. For hinged and fixed supports, the horizontal reactions for fixed supports can beassumed to be four times the horizontal reactions for hinged supports

Example

Draw the axial force, shear force and bending moment diagrams of the frames loadedas shown below.



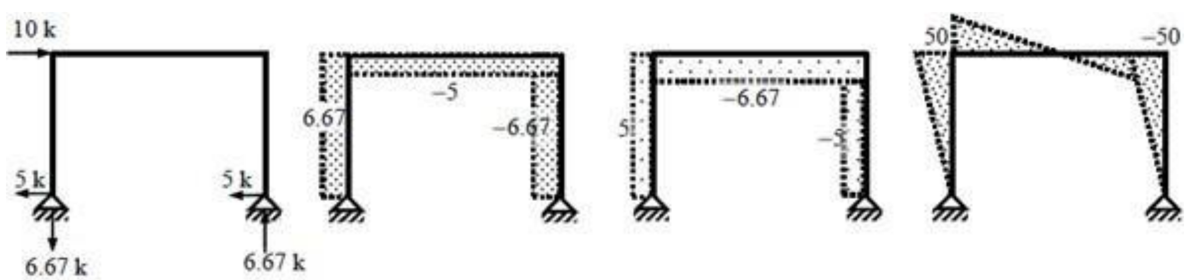
(i) (ii)

Solution

(i) For this frame, dosi = $3 \times 3 + 4 - 3 \times 4 = 1$; i.e., Assumption 1 $\Rightarrow H_A = H_D = 10/2 = 5$ k

$\therefore \sum M_A = 0 \Rightarrow 10 \times 10 - V_D \times 15 = 0 \Rightarrow V_D = 6.67$ k

$\therefore \sum F_y = 0 \Rightarrow V_A + V_D = 0 \Rightarrow V_A = -6.67$ k



Reactions AFD (k) SFD (k) BMD

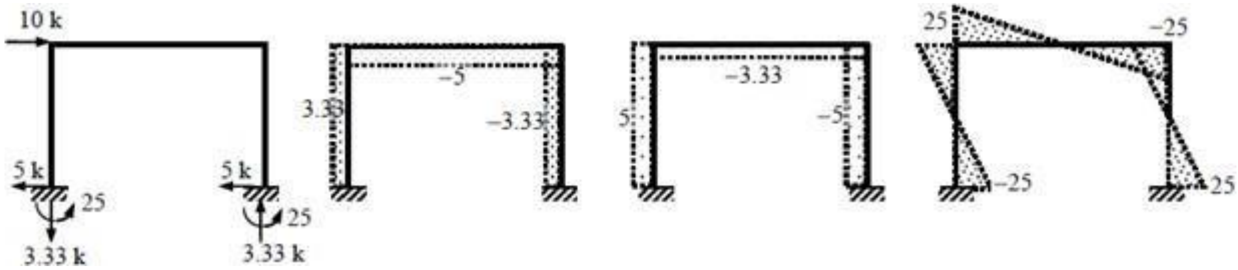
(ii) dosi = $3 \times 3 + 6 - 3 \times 4 = 3$

Assumption 1 $\Rightarrow H_A = H_D = 10/2 = 5$ k, Assumption 2 $\Rightarrow BM_E = BM_F = 0$

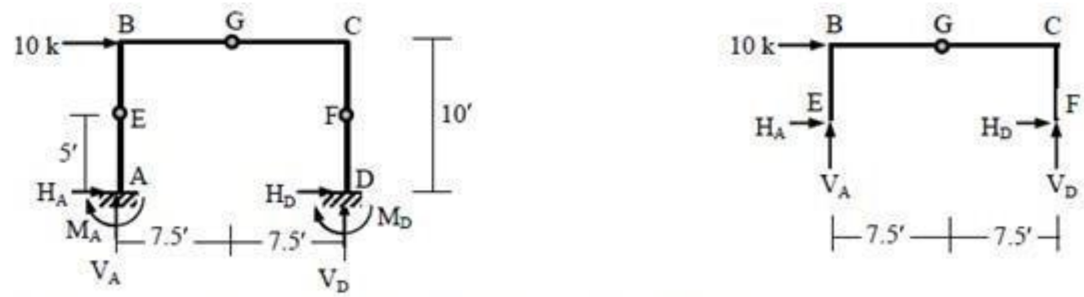
$\therefore BM_F = 0 \Rightarrow H_A \times 5 + M_A = 0 \Rightarrow M_A = -25$ k-ft; Similarly $BM_E = 0 \Rightarrow M_D = -25$

$\therefore \sum M_A = 0 \Rightarrow -25 - 25 + 10 \times 10 - V_D \times 15 = 0 \Rightarrow V_D = 3.33$ k

$\therefore \sum F_y = 0 \Rightarrow V_A + V_D = 0 \Rightarrow V_A = -3.33$ k



(iii)



dosi = $3 \times 4 + 6 - 3 \times 5 - 1 = 2$; ∴ Assumption 1 and 2 ⇒ $BM_E = BM_F = 0$

∴ $BM_E = 0$ (bottom) ⇒ $-H_A \times 5 + M_A = 0 \Rightarrow M_A = 5H_A$; Similarly $BM_F = 0 \Rightarrow M_D = 5H_D$

Also $BM_E = 0$ (free body of EBCF) ⇒ $10 \times 5 - V_D \times 15 = 0 \Rightarrow V_D = 3.33$ k

∴ $\Sigma F_y = 0 \Rightarrow V_A + V_D = 0 \Rightarrow V_A = -V_D = -3.33$ k

$BM_G = 0$ (between E and G) ⇒ $V_A \times 7.5 - H_A \times 5 = 0 \Rightarrow H_A = -5$ k ⇒ $M_A = 5H_A = -25$

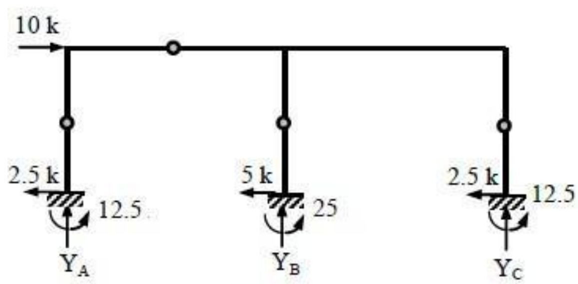
$\Sigma F_x = 0$ (entire structure) ⇒ $H_A + H_D + 10 = 0 \Rightarrow -5 + H_D + 10 = 0 \Rightarrow H_D = -5$ k ⇒ $M_D = 5H_D = -25$

(iv) dosi = $3 \times 5 + 9 - 3 \times 6 = 6$ ⇒ 6 Assumptions needed to solve the structure

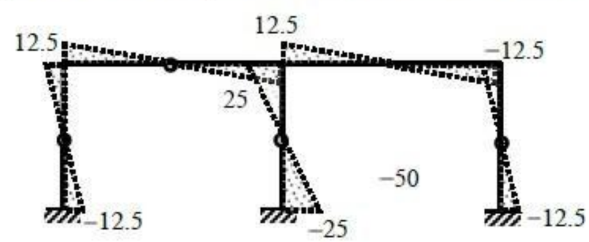
Assumption 1 and 2 ⇒ $H_A : H_B : H_C = 1 : 2 : 1 \Rightarrow H_A = 10/4 = 2.5$ k, $H_B = 5$ k, $H_C = 2.5$ k

∴ $M_A = M_C = 2.5 \times 5 = 12.5$ k-ft, $M_B = 5 \times 5 = 25$

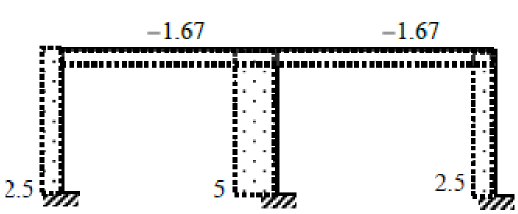
The other 4 assumptions are the assumed internal hinge locations at midpoints of columns and one beam



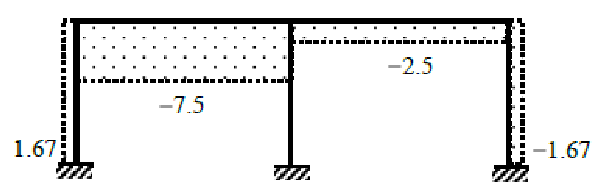
Reactions



BMD



SFD (k)



AFD (k)

Analysis of Multi-storied Structures by Portal Method

Approximate methods of analyzing multi-storied structures are important because such structures are statically highly indeterminate. The number of assumptions that must be made to permit an analysis by statics alone is equal to the degree of statically indeterminacy of the structure.

Assumptions

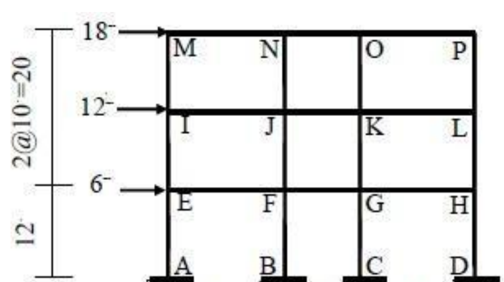
The assumptions used in the approximate analysis of portal frames can be extended for the lateral load analysis of multi-storied structures. The *Portal Method* thus formulated is based on three assumptions

1. The shear force in an interior column is twice the shear force in an exterior column.
2. There is a point of inflection at the center of eachcolumn.
3. There is a point of inflection at the center of eachbeam.

Assumption 1 is based on assuming the interior columns to be formed by columns of two adjacent bays or portals. Assumption 2 and 3 are based on observing the deflected shape of thestructure.

Example

Use the Portal Method to draw the axial force, shear force and bending moment diagramsof the three-storied frame structure loaded as shown below.



Column shear forces are at the ratio of 1:2:2:1.

∴ Shear force in (V) columns IM, JN, KO, LP are $[18 \times 1/(1+2+2+1) =]\ 3$, $[18 \times 2/(1+2+2+1) =]\ 6$, 6, 3 respectively. Similarly,

$V_{EI} = 30\times1/(6) = 5$, $V_{FJ} = 10$, $V_{GK} = 10$, $V_{HL} = 5$; and

$V_{AE} = 36\times1/(6) = 6$, $V_{BF} = 12$, $V_{CG} = 12$, $V_{DH} = 6$

Bending moments are

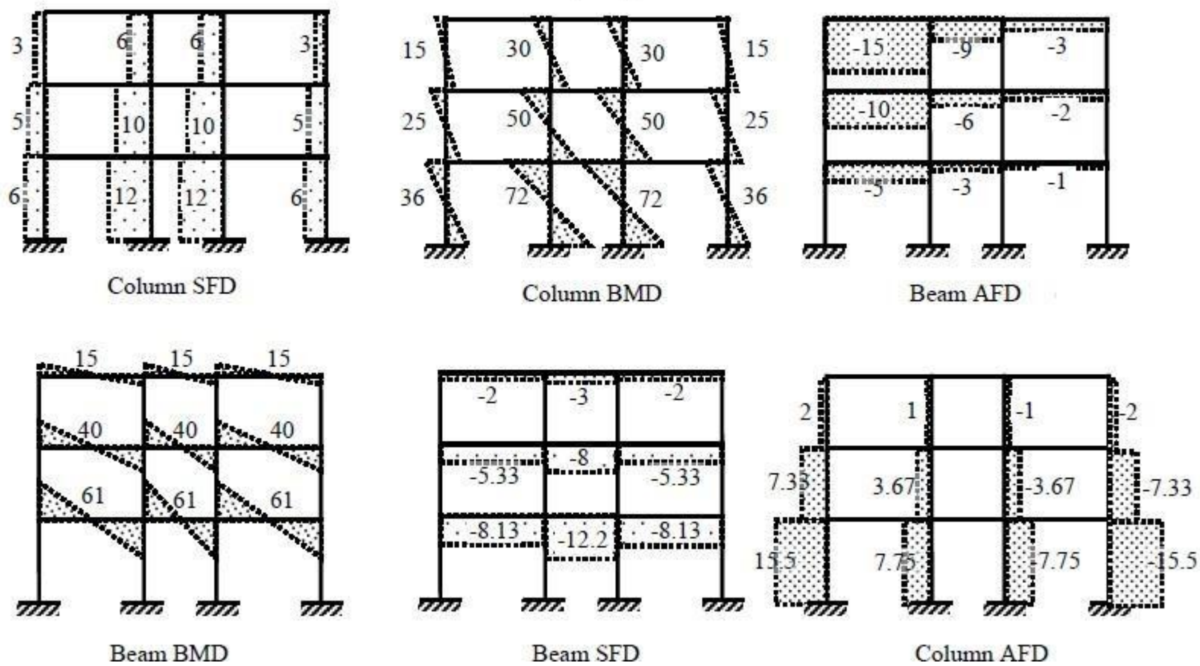
$M_{IM} = 3\times10/2 = 15$, $M_{JN} = 30$, $M_{KO} = 30$, $M_{LP} = 15$

$M_{EI} = 5\times10/2 = 25$, $M_{FJ} = 50$, $M_{GK} = 50$, $M_{HL} = 25$

$M_{AE} = 6\times10/2 = 30$, $M_{FJ} = 60$, $M_{GK} = 60$, $M_{HL} = 30$

The assumptions used in the approximate analysis of portal frames can be extended for the lateral load analysis of multi-storied structures.

The rest of the calculations follow from the free-body diagrams



Analysis of Multi-storied Structures by Cantilever Method

Although the results using the *Portal Method* are reasonable in most cases, the method suffers due to the lack of consideration given to the variation of structural response due to the difference between sectional properties of various members.

The *Cantilever Method* attempts to rectify this limitation by considering the cross- sectional areas of columns in distributing the axial forces in various columns of a story.

Assumptions

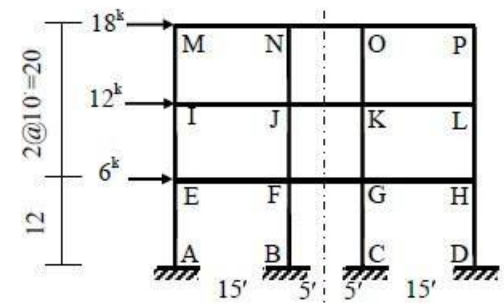
The Cantilever Method is based on three assumptions

1. The axial force in each column of a storey is proportional to its horizontal distancefrom the centroidal axis of all the columns of thestorey.
2. There is a point of inflection at the center of eachcolumn.
3. There is a point of inflection at the center of eachbeam.

Assumption 1 is based on assuming that the axial stresses can be obtained by a method analogous to that used for determining the distribution of normal stresses on a transverse section of a cantilever beam. Assumption 2 and 3 are basedon observing the deflected shape of the structure.

Example

Use the Cantilever Method to draw the axial force, shear force and bending moment diagrams of the three -storied frame structure loaded as shown below.



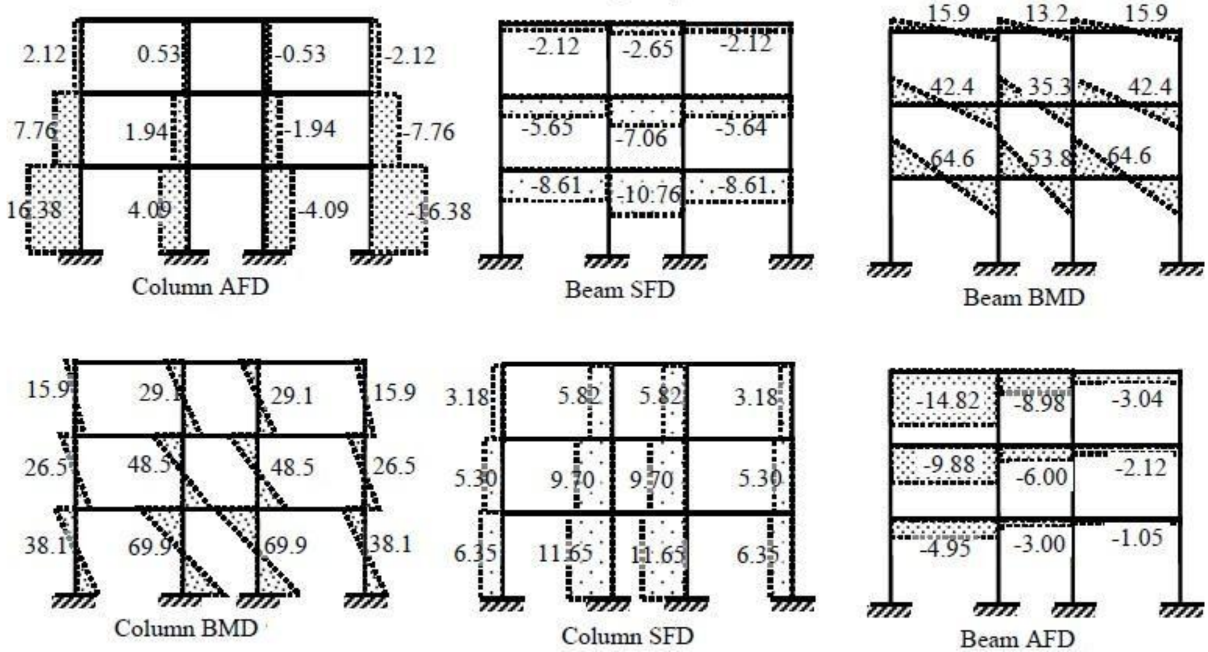
The dotted line is the column centerline (at all floors)

∴ Column axial forces are at the ratio of 20: 5: −5: −20.

∴ Axial force in (P) columns IM, JN, KO, LP are $[18 \times 5 \times 20/\{20^2 + 5^2 + (-5)^2 + (-20)^2\} =]$ 2.12, $[18 \times 5 \times 5/(20^2 + 5^2 + (-5)^2 + (-20)^2\} =]$ 0.53, −0.53, −2.12 respectively.

Similarly, $P_{EI} = 330\times20/(850) = 7.76$, $P_{FJ} = 1.94$, $P_{GK} = -1.94$, $P_{HL} = -7.76$; and $P_{AE} = 696\times20/(850) = 16.38$, $P_{BF} = 4.09$, $P_{CG} = -4.09$, $P_{DH} = 16.38$

The rest of the calculations follow from the free-body diagrams



Introducing direction cosines $l \cos\vartheta$; $m \sin\vartheta$; the above equation is written as

$$\begin{Bmatrix} u'_1 \\ u'_2 \end{Bmatrix} = \begin{bmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{Bmatrix} \qquad (24.10a)$$

Or, $$\{u'\} = [T]\{u\} \qquad (24.10b)$$

In the above equation $T$is the displacement transformation matrix whichtransforms the fourglobal displacement components to two displacement component in local coordinate system.

UNIT -V

INFLUENCE LINES AND MOVING LOADS

Influence Lines

An influence line represents the variation of either the reaction, shear, moment or deflection at a specific point in a member as a concentrated force moves over the member. Example bridges, industrial crane rails, conveyors, etc

Influence lines are important in the design of structures that resist large live loads. ¬In our work up to this point, we have discussed analysis techniques for structures subjected to dead or fixed loads.

We learned that shear and moment diagrams are important in determining the maximum internal force in a structure. ¬If a structure is subjected to a live or moving load, the variation in shear and moment is best described using influence lines.

Since beams or girders are usually major load– carrying members in large structures, it is important to draw influence lines for reaction, shear, and moment at specified points. ¬ Once an influence line has been drawn, it is possible to locate the live loads on the beam so that the maximum value of the reaction, shear, or moment is produced. ¬ This is very important in the design procedure.

Concentrated Force - Since we use a unit force (a dimensionless load), the value of the function (reaction, shear, or moment) can be found by multiplying the ordinate of the influence line at the position x by the magnitude of the actual force P.

One can tell at a glance where the moving load should be placed on the structure so that it creates the greatest influence at the specified point.

Influence lines for statically determinate structures are piecewise linear.

statically indeterminate example

shear& moment diagrams:

effect of <u>fixed</u> loads at <u>all points</u> along the axis of the member , influence lines:

effect of a moving load only at a specified point on the member

Rounded Aggregate

The rounded aggregates are completely shaped by attrition and available in the form of seashore gravel. Rounded aggregates result the minimum percentage of voids (32 – 33%) hence gives more workability. They require lesser amount of water-cement ratio. They are not considered for high strength concrete because of poor interlocking behavior and weak bond strength.

Irregular Aggregates

The irregular or partly rounded aggregates are partly shaped by attrition and these are available in the form of pit sands and gravel. Irregular aggregates may result 35- 37% of voids. These will give lesser workability when compared to rounded aggregates. The bond strength is slightly higher than rounded aggregates but not as required for high strength concrete.

Angular Aggregates

The angular aggregates consist well defined edges formed at the intersection of roughly planar surfaces and these are obtained by crushing the rocks. Angular aggregates result maximum percentage of voids (38-45%) hence gives less workability. They give 10-20% more compressive strength due to development of stronger aggregate-mortar bond. So, these are useful in high strength concrete manufacturing.

Flaky Aggregates

When the aggregate thickness is small when compared with width and length of that aggregate it is said to be flaky aggregate. Or in the other, when the least dimension of aggregate is less than the 60% of its mean dimension then it is said to be flaky aggregate.

Elongated Aggregates

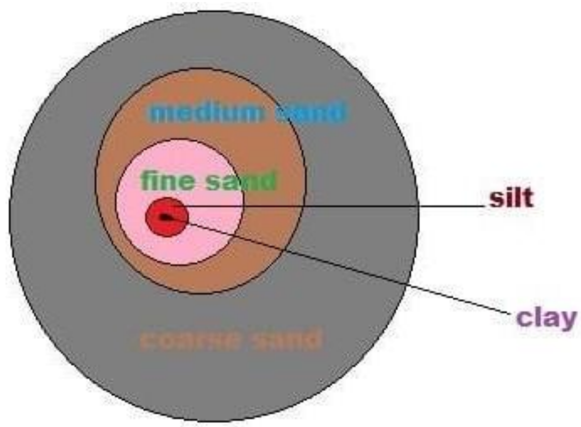
When the length of aggregate is larger than the other two dimensions then it is called elongated aggregate or the length of aggregate is greater than 180% of its mean dimension.

Flaky and Elongated Aggregates

When the aggregate length is larger than its width and width is larger than its thickness then it is said to be flaky and elongated aggregates. The above 3 types of aggregates are not suitable for concrete mixing. These are generally obtained from the poorly crushed rocks.

Classification of Aggregates Based on Size

Aggregates are available in nature in different sizes. The size of aggregate used may be related to the mix proportions, type of work etc. the size distribution of aggregates is called grading of aggregates.



Following are the classification of aggregates based on size:

Aggregates are classified into 2 types according to size

1. Fine aggregate
2. Coarse aggregate

Fine Aggregate

When the aggregate is sieved through 4.75mm sieve, the aggregate passed through it called as fine aggregate. Natural sand is generally used as fine aggregate, silt and clay are also come under this category. The soft deposit consisting of sand, silt and clay is termed as loam. The purpose of the fine aggregate is to fill the voids in the coarse aggregate and to act as a workability agent.

Coarse Aggregate

When the aggregate is sieved through 4.75mm sieve, the aggregate retained is called coarse aggregate. Gravel, cobble and boulders come under this category. The maximum size aggregate used may be dependent upon some conditions. In general, 40mm size aggregate used for normal strengths and 20mm size is used for high strength concrete. the size range of various coarse aggregates given below.

Grading of Aggregates

Grading is the particle-size distribution of an aggregate as determined by a sieve analysis usingwire mesh sieves with square openings. As per IS:2386(Part-1)

Fine aggregate—6 standard sieves with openings from 150 µm to 4.75 mm.

Coarse aggregate—5 sieves with openings from 4.75mm to 80 mm.

Gradation (grain size analysis)

Grain size distribution for concrete mixes that will provide a dense strong mixture. Ensure thatthe voids between the larger particles are filled with medium particles. The remaining voids arefilled with still smaller particles until the smallest voids are filled with a small amount of fines. Ensure maximum density and strength using a maximum density curve

Good Gradation:

Concrete with good gradation will have fewer voids to be filled with cement paste (economicalmix)

Concrete with good gradation will have fewer voids for water to permeate (durability)

Particle size distribution affects:

1. Workability
2. Mixproportioning

Fine Aggregate effect on concrete:

2. Over sanded (More than requiredsand)

- Over cohesivemix.
- Water reducers may be lesseffective.
- Air entrainment may be moreeffective.

3. Under sanded (deficit ofsand)

- Prone to bleed andsegregation.
- May get high levels of waterreduction.
- Air entrainers may be lesseffective.

Shape and surface texture of aggregates:

The shape of aggregate is an important characteristic since it affects the workability of concrete.

It is difficult to measure the shape of irregular shaped aggregates. Not only the type of parent rock but also the type of crusher used also affects the shape of the aggregate produced.

Good Granite rocks found near Bangalore will yield cuboidal aggregates. Many rocks contain planes of jointing which is characteristics of its formation and hence tend to yield more flaky aggregates.

The shape of the aggregates produced is also dependent on type of crusher and the reduction ratio of the crusher.

Quartzite which does not possess cleavage planes tend to produce cubical shape aggregates.

From the standpoint of economy in cement requirement for a given water cement ratio rounded aggregates are preferable to angular aggregates.

On the other hand, the additional cement required for angular aggregates is offset to some extent by the higher strengths and some times greater durability as a result of greater Interlocking texture of the hardened concrete.

Flat particles in concrete will have objectionable influence on the workability of concrete, cement requirement, strength and durability.

In general excessively flaky aggregates make poor concrete.

While discussing the shape of the aggregates, the texture of the aggregate also enters the discussion because of its close association with the shape.

Generally round aggregates are smooth textured and angular aggregates are rough textured. Therefore some engineers argue against round aggregates from the point of bond strength between aggregates and cement.

But the angular aggregates are superior to rounded aggregates from the following two points:

Angular aggregates exhibit a better interlocking effect in concrete, which property makes it superior in concrete used for road and pavements.

The total surface area of rough textured angular aggregate is more than smooth rounded aggregates for the given volume.

By having greater surface area, the angular aggregates may show higher bond strength than rounded aggregates.

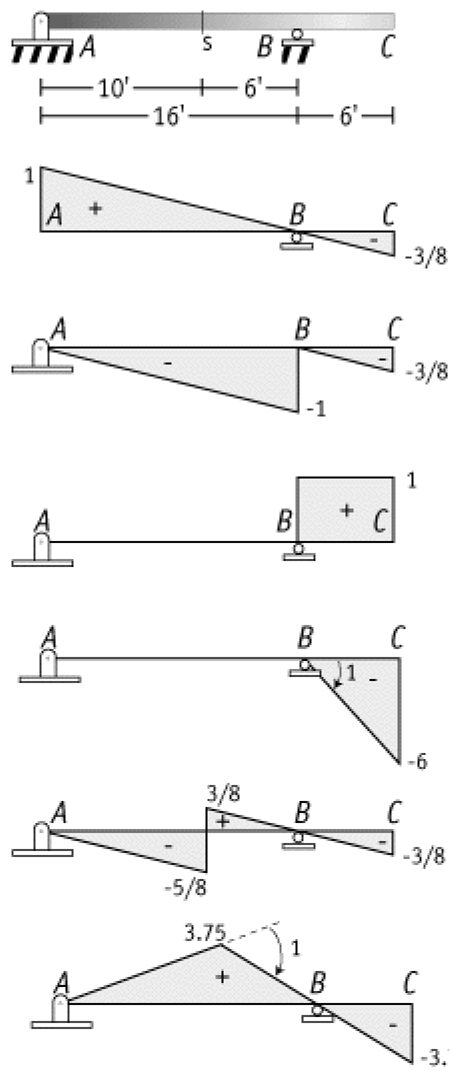
The shape of the aggregates becomes all the more important in case of high strength and high performance concrete where very low water/cement ratio is required to be used . In such cases cubical aggregates are required for better workability.

Surface texture is the property, the measure of which depends upon the relative degree to which particle surface are polished or dull, smooth or rough.

Surface texture depends upon hardness, grain size, pore structure, structure of the rock and the degree to which the forces acting on it have smoothened the surface or roughened. Experience and laboratory experiments have shown that the adhesion between cement paste and the aggregate is influenced by several complex factors in

Procedure of Analysis

1. tabulate values
2. influence-line equations



Assumptions for the Approximate Solution

In order to analyze a structure using the equations of statics only, the number of independent force components must be equal to the number of independent equations of statics.

If there are *n* more independent force components in the structure than there are independent equations of statics, the structure is statically indeterminate to the *nth* degree. Therefore to obtain an approximate solution of the structure based on statics only, it will be necessary to make *n* additional independent assumptions. A solution based on statics will not be possible by making fewer than *n* assumptions, while more than *n* assumptions will not in general be consistent. Thus, the first step in the approximate analysis of structures is to find its degreeof statically indeterminacy (dosi) and then to make appropriate number ofassumptions.

For example, the dosi of portal frames shown in (i), (ii), (iii) and (iv) are 1, 3, 2 and 1 respectively. Based on the type of frame, the following assumptions can be made for portal structures with a vertical axis of symmetry that are loaded horizontally at thetop

The horizontal support reactions areequal

There is a point of inflection at the center of the unsupported height of each fixed basedcolumn

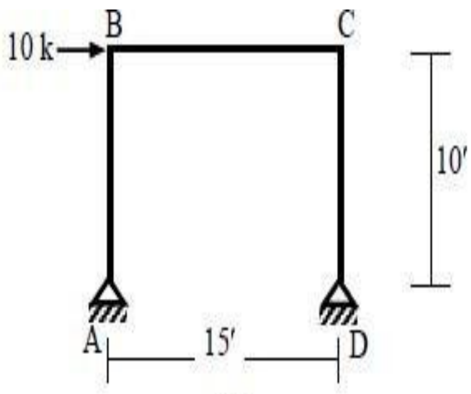
Assumption 1 is used if dosi is an odd number (i.e., = 1 or 3) and Assumption 2is used if dosi 1.

Some additional assumptions can be made in order to solve the structureapproximately for different loading and support conditions.

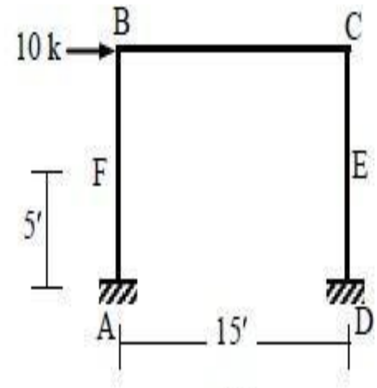
Horizontal body forces not applied at the top of a column can be divided into twoforces (i.e., applied at the top and bottom of the column) based on simplesupports

For hinged and fixed supports, the horizontal reactions for fixed supports can be assumed to be four times the horizontal reactions for hinged supports Example Draw the axial force, shear force and bending moment diagrams of the frames loaded as shown below.

Therefore to obtain an approximate solution of the structure based on statics only, it will be necessary to make *n* additional independent assumptions. A solution based on statics will not be possible by making fewer than *n* assumptions, while more than *n* assumptions will not in general beconsistent.



(i)



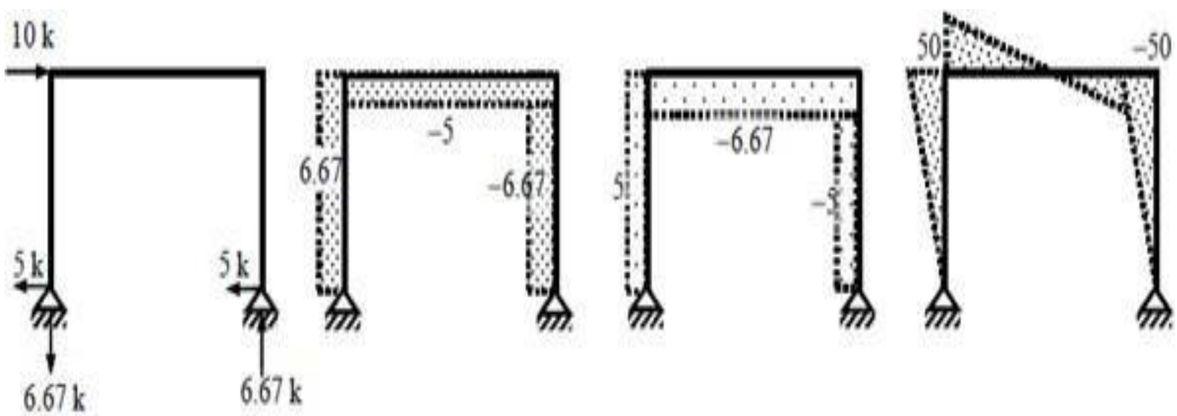
(ii)

Solution

(i) For this frame, dosi = 3 × 3 + 4 − 3 × 4 = 1; i.e., Assumption 1 $\Rightarrow H_A = H_D = 10/2 = 5$ k

$\therefore \sum M_A = 0 \Rightarrow 10 \times 10 - V_D \times 15 = 0 \Rightarrow V_D = 6.67$ k

$\therefore \sum F_y = 0 \Rightarrow V_A + V_D = 0 \Rightarrow V_A = -6.67$ k



Reactions AFD (k) SFD (k) BMD

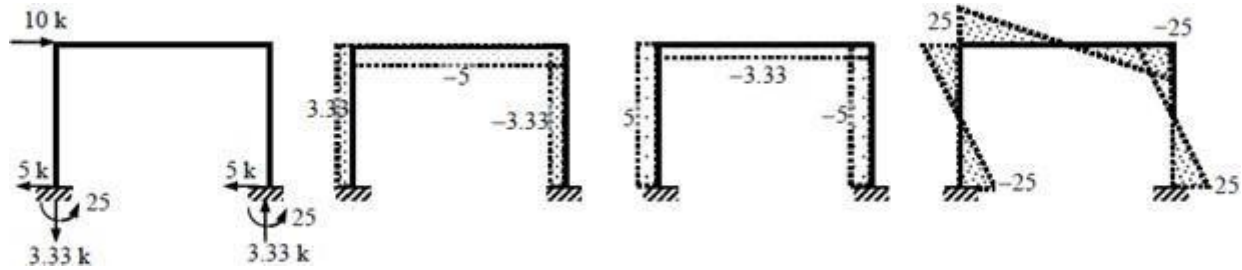
(ii) dosi = 3 × 3 + 6 − 3 × 4 = 3

Assumption 1 $\Rightarrow H_A = H_D = 10/2 = 5$ k, Assumption 2 $\Rightarrow BM_E = BM_F = 0$

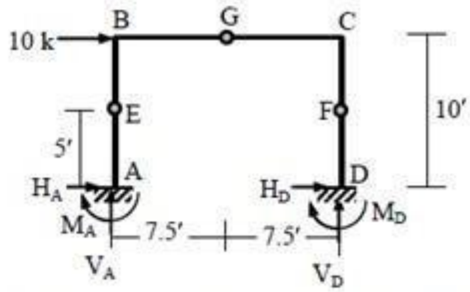
$\therefore BM_F = 0 \Rightarrow H_A \times 5 + M_A = 0 \Rightarrow M_A = -25$ k-ft; Similarly $BM_E = 0 \Rightarrow M_D = -25$

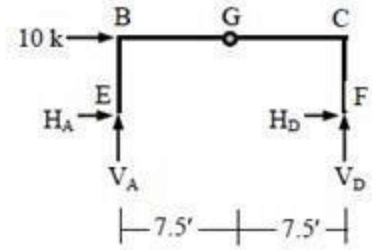
$\therefore \sum M_A = 0 \Rightarrow -25 - 25 + 10 \times 10 - V_D \times 15 = 0 \Rightarrow V_D = 3.33$ k

$\therefore \sum F_y = 0 \Rightarrow V_A + V_D = 0 \Rightarrow V_A = -3.33$ k



(iii)



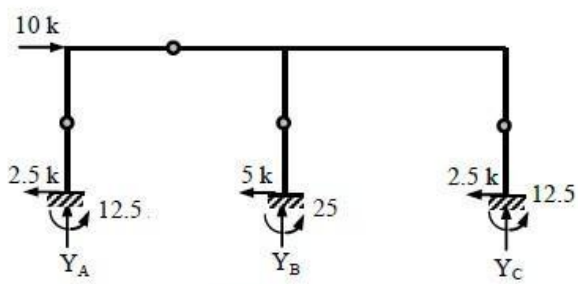


dosi = 3 × 4 + 6 – 3 × 5 – 1 = 2; ∴ Assumption 1 and 2 ⇒ $BM_E = BM_F = 0$
∴ $BM_E = 0$ (bottom) ⇒ $-H_A \times 5 + M_A = 0 \Rightarrow M_A = 5H_A$; Similarly $BM_F = 0 \Rightarrow M_D = 5H_D$
Also $BM_E = 0$ (free body of EBCF) ⇒ $10 \times 5 - V_D \times 15 = 0 \Rightarrow V_D = 3.33$ k
∴ $\Sigma F_y = 0 \Rightarrow V_A + V_D = 0 \Rightarrow V_A = -V_D = -3.33$ k

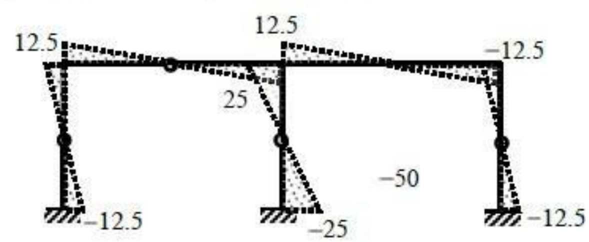
$BM_G = 0$ (between E and G) ⇒ $V_A \times 7.5 - H_A \times 5 = 0 \Rightarrow H_A = -5$ k ⇒ $M_A = 5H_A = -25$
$\Sigma F_x = 0$ (entire structure) ⇒ $H_A + H_D + 10 = 0 \Rightarrow -5 + H_D + 10 = 0 \Rightarrow H_D = -5$ k ⇒ $M_D = 5H_D = -25$

(iv) dosi = 3 × 5 + 9 – 3 × 6 = 6 ⇒ 6 Assumptions needed to solve the structure
Assumption 1 and 2 ⇒ $H_A: H_B: H_C = 1: 2: 1 \Rightarrow H_A = 10/4 = 2.5$ k, $H_B = 5$ k, $H_C = 2.5$ k
∴ $M_A = M_C = 2.5 \times 5 = 12.5$ k-ft, $M_B = 5 \times 5 = 25$

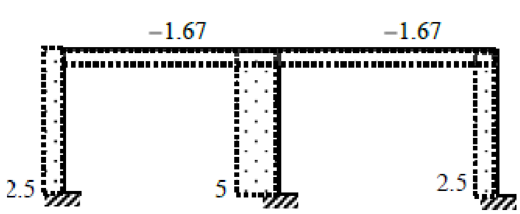
The other 4 assumptions are the assumed internal hinge locations at midpoints of columns and one beam



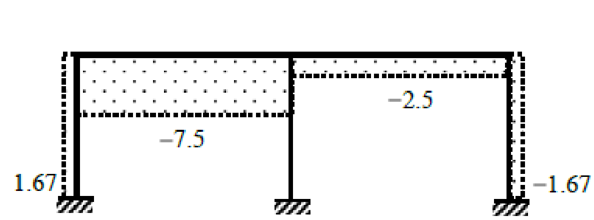
Reactions



BMD



SFD (k)



AFD (k)

Analysis of Multi-storied Structures by Portal Method

Approximate methods of analyzing multi-storied structures are important because such structures are statically highly indeterminate. The number of assumptions that must be made to permit an analysis by statics alone is equal to the degree of statically indeterminacy of the structure.

Assumptions

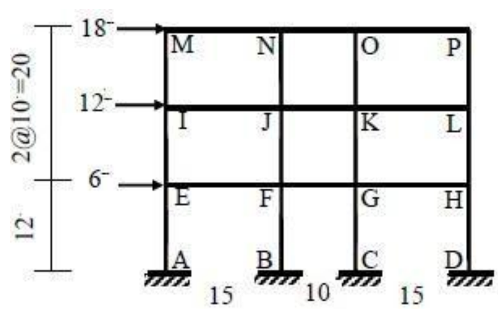
The assumptions used in the approximate analysis of portal frames can be extended for the lateral load analysis of multi-storied structures. The *Portal Method* thus formulated is based on three assumptions

4. The shear force in an interior column is twice the shear force in an exterior column.
5. There is a point of inflection at the center of eachcolumn.
6. There is a point of inflection at the center

Assumption 1 is based on assuming the interior columns to be formed by columns of two adjacent bays or portals. Assumption 2 and 3 are based on observing the deflected shape of thestructure.

Example

Use the Portal Method to draw the axial force, shear force and bending moment diagramsof the three-storied frame structure loaded as shown below.



Column shear forces are at the ratio of 1:2:2:1.

∴ Shear force in (V) columns IM, JN, KO, LP are $[18 \times 1/(1+2+2+1) =]\ 3$, $[18 \times 2/(1+2+2+1) =]\ 6$, 6, 3 respectively. Similarly,

$V_{EI} = 30\times1/(6) = 5$, $V_{FJ} = 10$, $V_{GK} = 10$, $V_{HL} = 5$; and

$V_{AE} = 36\times1/(6) = 6$, $V_{BF} = 12$, $V_{CG} = 12$, $V_{DH} = 6$

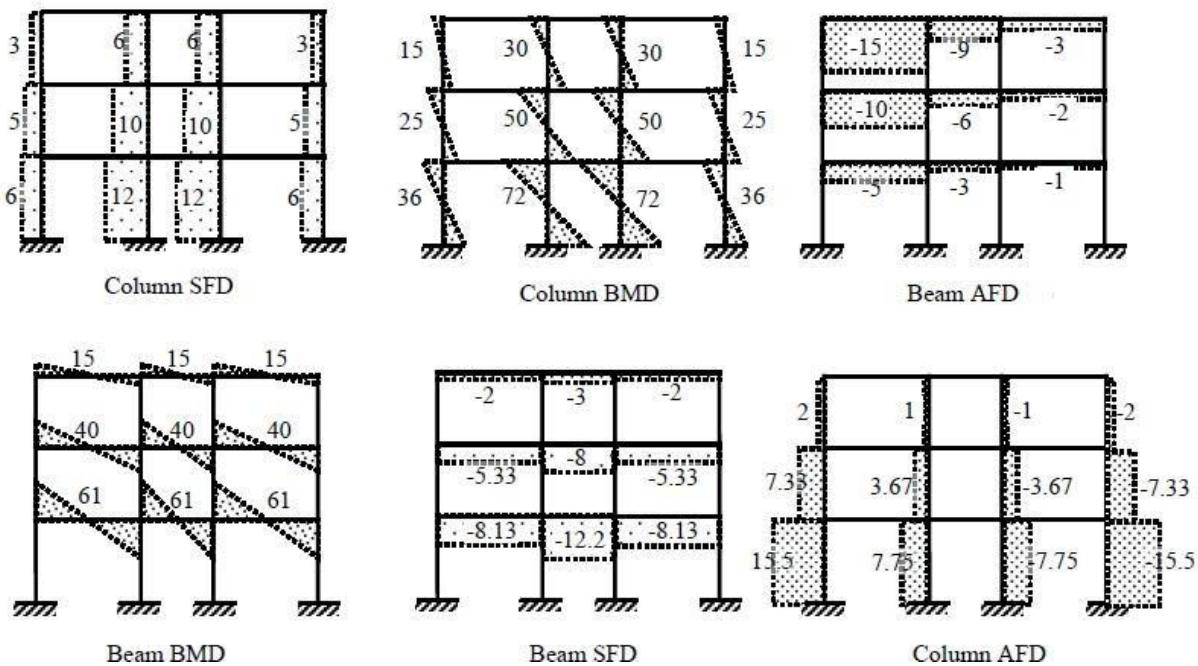
Bending moments are

$M_{IM} = 3\times10/2 = 15$, $M_{JN} = 30$, $M_{KO} = 30$, $M_{LP} = 15$

$M_{EI} = 5\times10/2 = 25$, $M_{FJ} = 50$, $M_{GK} = 50$, $M_{HL} = 25$

$M_{AE} = 6\times10/2 = 30$, $M_{FJ} = 60$, $M_{GK} = 60$, $M_{HL} = 30$

The rest of the calculations follow from the free-body diagrams



Analysis of Multi-storied Structures by Cantilever Method

Although the results using the *Portal Method* are reasonable in most cases, the method suffers due to the lack of consideration given to the variation of structural response due to the difference between sectional properties of various members. The *Cantilever Method* attempts to rectify this limitation by considering the cross- sectional areas of columns in distributing the axial forces in various columns of a story.

Assumptions

The Cantilever Method is based on three assumptions

The axial force in each column of a storey is proportional to its horizontal distance from the centroidal axis of all the columns of thestorey.

There is a point of inflection at the center of eachcolumn.

There is a point of inflection at the center of eachbeam. Assumption 1 is based on assumingthat the axial stresses can be obtained by a method analogous to that used for determining the distribution of normal stresseson a transverse section of a cantilever beam. Assumption

2 and 3 are based on observing the deflected shape of the structure.

Although the results using the *Portal Method* are reasonable in most cases, the method suffers due to the lack of consideration given to the variation of structural response due to the difference between sectional properties of various members. The *Cantilever Method* attempts to rectify this limitation by considering the cross- sectional areas of columns in distributing the axial forces in various columns of a story.

Approximate methods of analyzing multi-storied structures are important because such structures are statically highly indeterminate. The number of assumptions that must be made to permit an analysis by statics alone is equal to the degree of statically indeterminacyof the structure.

Although the results using the *Portal Method* are reasonable in most cases, the method suffers due to the lack of consideration given to the variation of structural response due tothe difference between sectional properties of various members

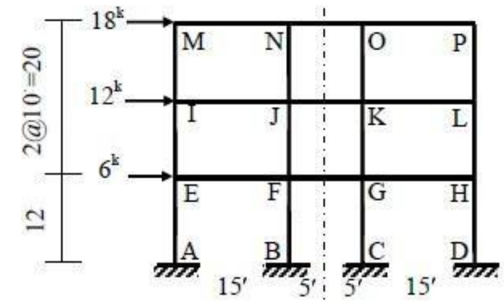
If there are *n* more independent force components in the structure than there are independent equations of statics, the structure is statically indeterminate to the *nth* degree. Therefore to obtain an approximate solution of the structure based on statics only, it will benecessary to make *n* additional independent assumptions. A solution based on statics will not be possible by making fewer than *n*

Since beams or girders are usually major load– carrying members in large structures, it is important to draw influence lines for reaction, shear, and moment at specified points. ¬ Once an influence line has been drawn, it is possible to locate the live loads on the beam so that the maximum value of the reaction, shear, or moment is produced. ¬ This is very important in the design procedure.

Concentrated Force - Since we use a unit force (a dimensionless load), the value of the function (reaction, shear, or moment) can be found by multiplying the ordinate of the influence line at the position x by the magnitude of the actual force P.

Example

Use the Cantilever Method to draw the axial force, shear force and bending moment diagrams of the three -storied frame structure loaded as shown below.



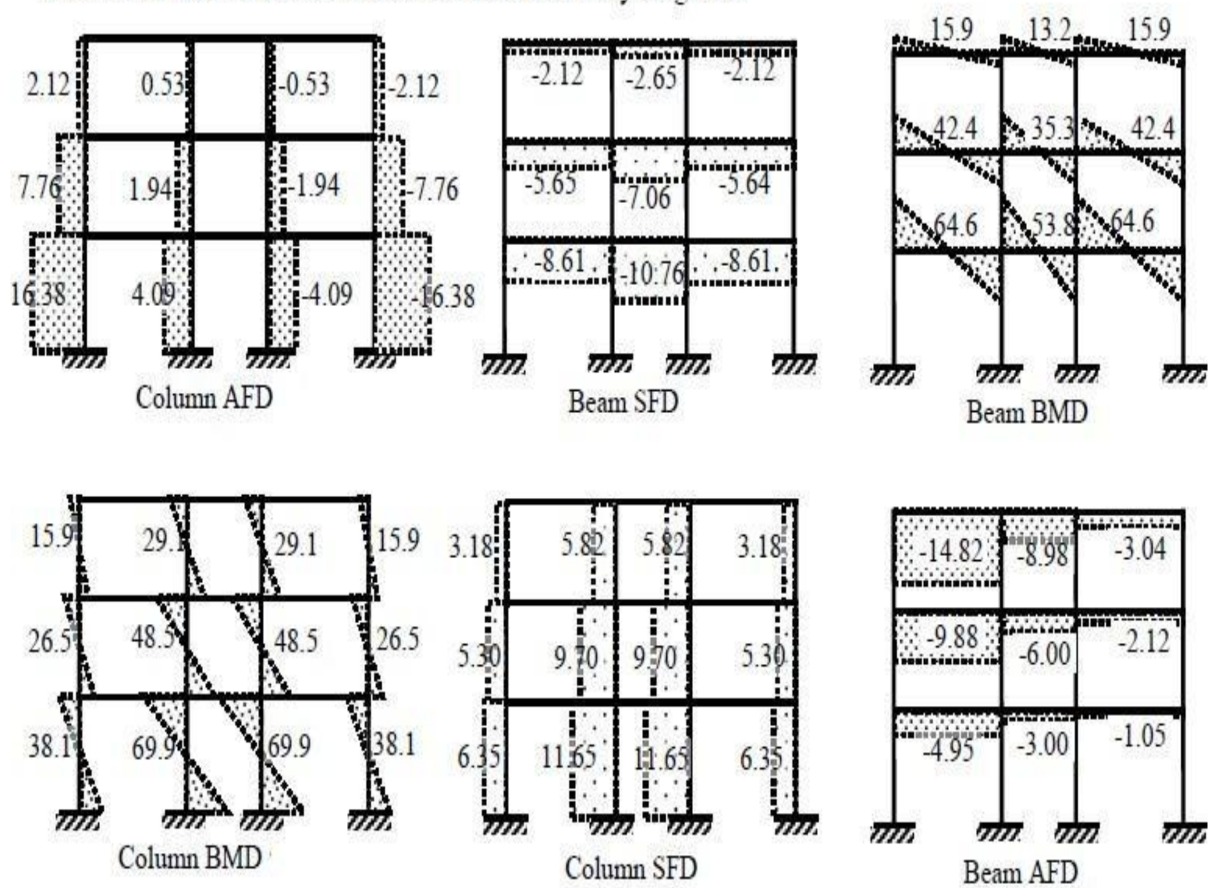
The dotted line is the column centerline (at all floors)

∴ Column axial forces are at the ratio of 20: 5: −5: −20.

∴ Axial force in (P) columns IM, JN, KO, LP are $[18 \times 5 \times 20/\{20^2 + 5^2 + (-5)^2 + (-20)^2\} =]$ 2.12 , $[18 \times 5 \times 5/(20^2 + 5^2 + (-5)^2 + (-20)^2\} =]$ 0.53, −0.53, −2.12 respectively.

Similarly, $P_{EI} = 330\times20/(850) = 7.76$, $P_{FJ} = 1.94$, $P_{GK} = -1.94$, $P_{HL} = -7.76$; and $P_{AE} = 696\times20/(850) = 16.38$, $P_{BF} = 4.09$, $P_{CG} = -4.09$, $P_{DH} = 16.38$

The rest of the calculations follow from the free-body diagrams



Introducing direction cosines l cosϑ;m sinϑ; the above equation is written as

$$\begin{Bmatrix} u'_1 \\ u'_2 \end{Bmatrix} = \begin{bmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{Bmatrix}$$

Or, $$\{u'\} = [T]\,\{u\}$$

In the above equation $T$is the displacement transformation matrix which transforms the four global displacement components to two displacement component in local coordinatesystem

GENERAL PROCEDURE OF SLOPE-DEFLECTION METHOD

- Find the fixed end moments of each span (both ends left & right).
- Apply the slope deflection equation on each span & identify the unknowns.
- Write down the joint equilibrium equations.
- Solve the equilibrium equations to get the unknown rotation & deflections.
- Determine the end moments and then treat each span as simply supported beamsubjected to given load & end moments so we can work out the reactions & drawthe bending moment & shear force diagram.

Loads and displacements are vector quantities and hence a proper coordinate system is required to specify their correct sense of direction. Consider a planar truss, In this truss each node is identified by a number and each member is identified by a number enclosed in a circle. The displacements and loads acting on the truss are defined with respect to global co- ordinate system *xyz.*

ASSUMPTIONS FOR THE APPROXIMATE SOLUTION

In order to analyze a structure using the equations of statics only, the number of independent force components must be equal to the number of independent equations of statics.

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Thus, the first step in the approximate analysis of structures is to find its degree of statically indeterminacy (dosi) and then to make appropriate number of assumptions.

For example, the dosi of portal frames shown in (i), (ii), (iii) and (iv) are 1, 3, 2 and 1 respectively. Based on the type of frame, the following assumptions can be made for portal structures with a vertical axis of symmetry that are loaded horizontally at thetop

The horizontal support reactions areequal

There is a point of inflection at the center of the unsupported height of each fixed basedcolumn

Assumption 1 is used if dosi is an odd number (i.e., = 1 or 3) and Assumption 2 is usedif dosi 1.

Some additional assumptions can be made in order to solve the structureapproximately for different loading and support conditions.

Horizontal body forces not applied at the top of a column can be divided into twoforces (i.e., applied at the top and bottom of the column) based on simple supports

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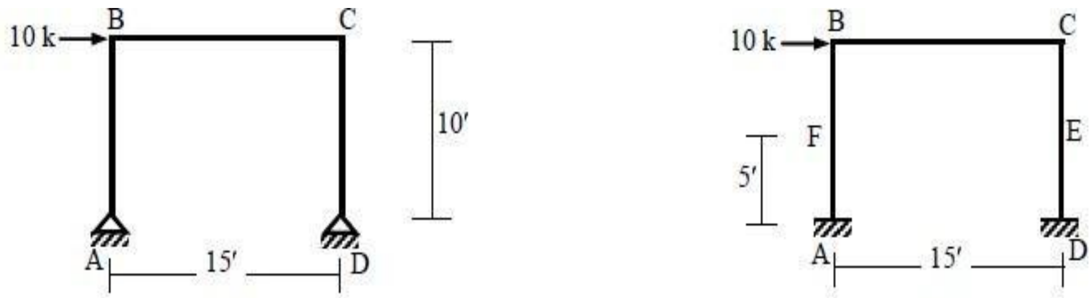
Draw the axial force, shear force and bending moment diagrams of the frames loadedas shown below.



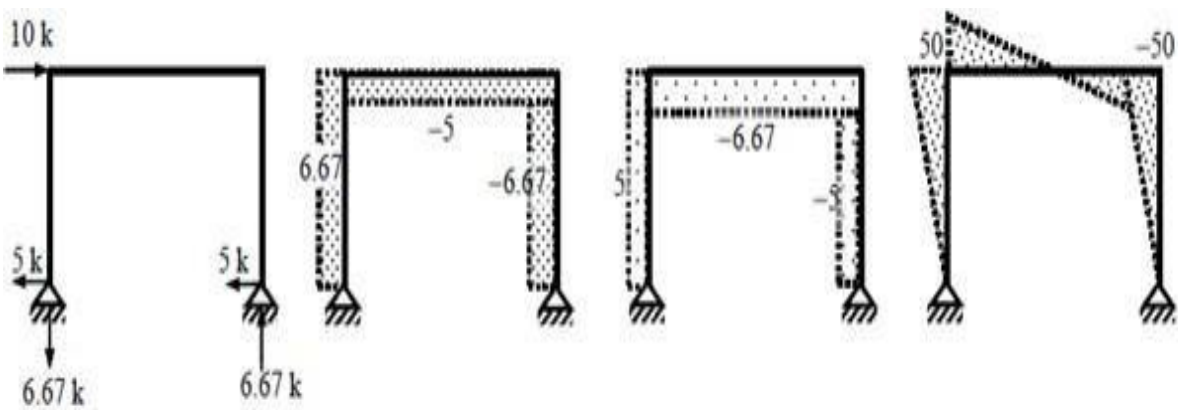
(i) (ii)

Solution

(i) For this frame, dosi = 3 × 3 + 4 – 3 × 4 = 1; i.e., Assumption 1 $\Rightarrow H_A = H_D = 10/2 = 5$ k

$\therefore \sum M_A = 0 \Rightarrow 10 \times 10 - V_D \times 15 = 0 \Rightarrow V_D = 6.67$ k

$\therefore \sum F_y = 0 \Rightarrow V_A + V_D = 0 \Rightarrow V_A = -6.67$ k



Reactions AFD (k) SFD (k) BMD

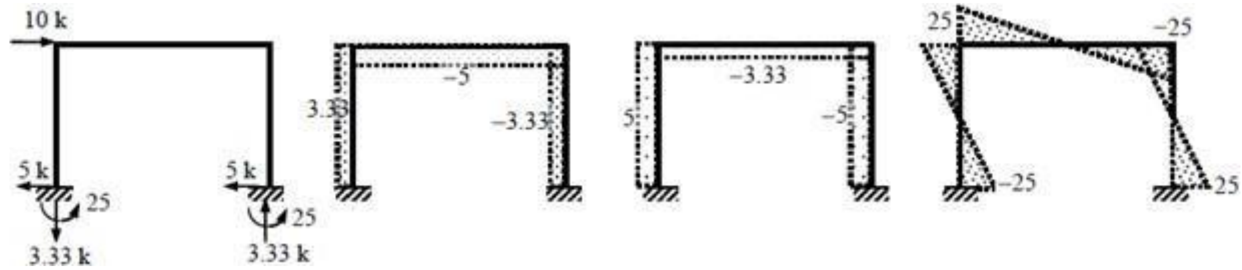
(ii) dosi = 3 × 3 + 6 – 3 × 4 = 3

Assumption 1 $\Rightarrow H_A = H_D = 10/2 = 5$ k, Assumption 2 $\Rightarrow BM_E = BM_F = 0$

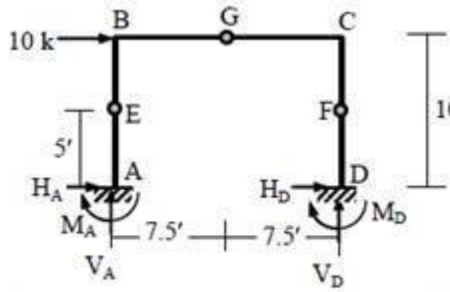
$\therefore BM_F = 0 \Rightarrow H_A \times 5 + M_A = 0 \Rightarrow M_A = -25$ k-ft; Similarly $BM_E = 0 \Rightarrow M_D = -25$

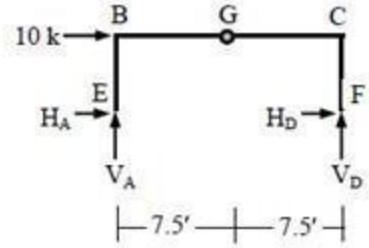
$\therefore \sum M_A = 0 \Rightarrow -25 - 25 + 10 \times 10 - V_D \times 15 = 0 \Rightarrow V_D = 3.33$ k

$\therefore \sum F_y = 0 \Rightarrow V_A + V_D = 0 \Rightarrow V_A = -3.33$ k



(iii)





dosi = 3 × 4 + 6 − 3 × 5 − 1 = 2; ∴ Assumption 1 and 2 ⇒ $BM_E = BM_F = 0$

∴ $BM_E = 0$ (bottom) ⇒ $-H_A \times 5 + M_A = 0 \Rightarrow M_A = 5H_A$; Similarly $BM_F = 0 \Rightarrow M_D = 5H_D$

Also $BM_E = 0$ (free body of EBCF) ⇒ $10 \times 5 - V_D \times 15 = 0 \Rightarrow V_D = 3.33$ k

∴ $\Sigma F_y = 0 \Rightarrow V_A + V_D = 0 \Rightarrow V_A = -V_D = -3.33$ k

$BM_G = 0$ (between E and G) ⇒ $V_A \times 7.5 - H_A \times 5 = 0 \Rightarrow H_A = -5$ k ⇒ $M_A = 5H_A = -25$

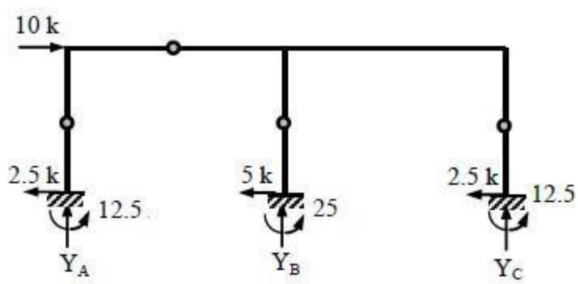
$\Sigma F_x = 0$ (entire structure) ⇒ $H_A + H_D + 10 = 0 \Rightarrow -5 + H_D + 10 = 0 \Rightarrow H_D = -5$ k ⇒ $M_D = 5H_D = -25$

(iv) dosi = 3 × 5 + 9 − 3 × 6 = 6 ⇒ 6 Assumptions needed to solve the structure

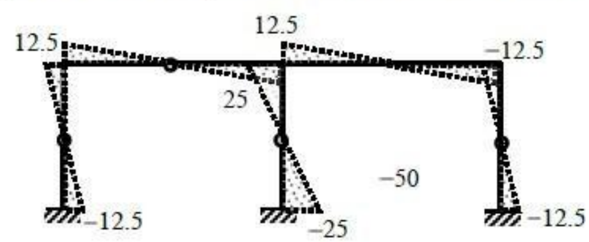
Assumption 1 and 2 ⇒ $H_A : H_B : H_C = 1 : 2 : 1 \Rightarrow H_A = 10/4 = 2.5$ k, $H_B = 5$ k, $H_C = 2.5$ k

∴ $M_A = M_C = 2.5 \times 5 = 12.5$ k-ft, $M_B = 5 \times 5 = 25$

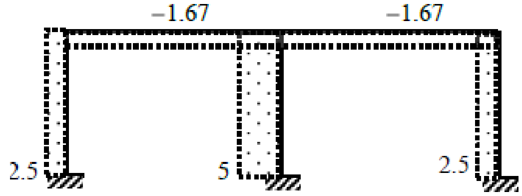
The other 4 assumptions are the assumed internal hinge locations at midpoints of columns and one beam



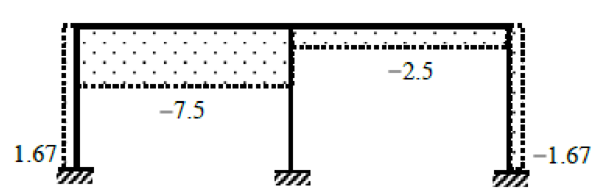
Reactions



BMD



SFD (k)



AFD (k)

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Approximate methods of analyzing multi-storied structures are important because such structures are statically highly indeterminate. The number of assumptions that must be made to permit an analysis by statics alone is equal to the degree of statical indeterminacy of the structure.

Assumptions

The assumptions used in the approximate analysis of portal frames can be extended for the lateral load analysis of multi-storied structures. The *Portal Method* thus formulated is based on three assumptions

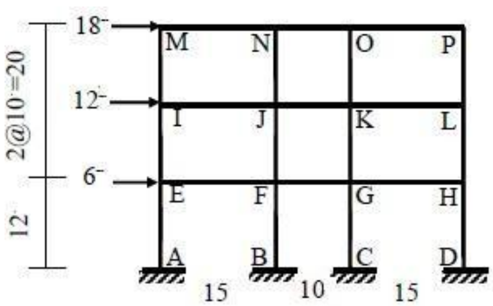
The shear force in an interior column is twice the shear force in an exterior column. There is a point of inflection at the center of eachcolumn.

There is a point of inflection at the center of eachbeam.

Assumption 1 is based on assuming the interior columns to be formed by columns of two adjacent bays or portals. Assumption 2 and 3 are based on observing the deflected shape of thestructure.

Example

Use the Portal Method to draw the axial force, shear force and bending moment diagrams of the three-storied frame structure loaded as shown below.



Column shear forces are at the ratio of 1:2:2:1.

∴ Shear force in (V) columns IM, JN, KO, LP are $[18 \times 1/(1+2+2+1) =]\ 3$, $[18 \times 2/(1+2+2+1) =]\ 6$, 6, 3 respectively. Similarly,

$V_{EI} = 30\times1/(6) = 5$, $V_{FJ} = 10$, $V_{GK} = 10$, $V_{HL} = 5$; and

$V_{AE} = 36\times1/(6) = 6$, $V_{BF} = 12$, $V_{CG} = 12$, $V_{DH} = 6$

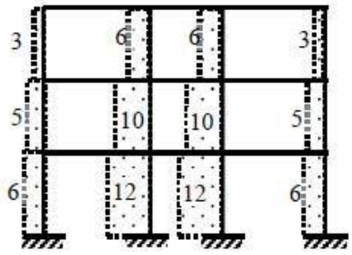
Bending moments are

$M_{IM} = 3\times10/2 = 15$, $M_{JN} = 30$, $M_{KO} = 30$, $M_{LP} = 15$

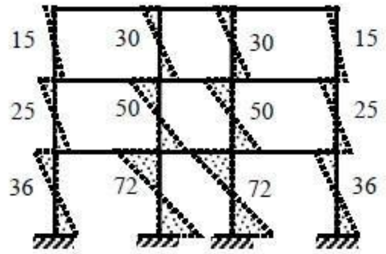
$M_{EI} = 5\times10/2 = 25$, $M_{FJ} = 50$, $M_{GK} = 50$, $M_{HL} = 25$

$M_{AE} = 6\times10/2 = 30$, $M_{FJ} = 60$, $M_{GK} = 60$, $M_{HL} = 30$

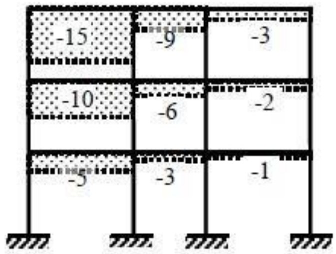
The rest of the calculations follow from the free-body diagrams



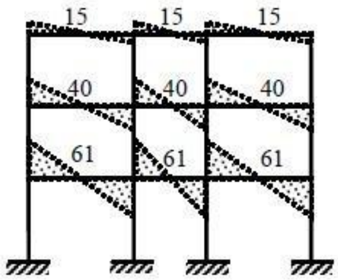
Column SFD



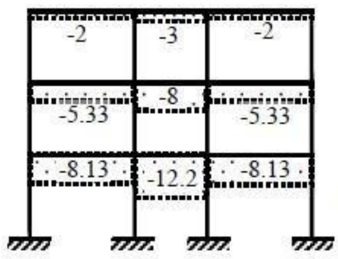
Column BMD



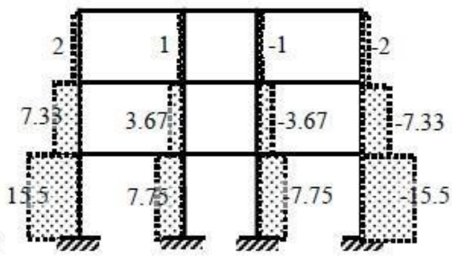
Beam AFD



Beam BMD



Beam SFD



Column AFD

Analysis of Multi-storied Structures by Cantilever Method

Although the results using the *Portal Method* are reasonable in most cases, the method suffers due to the lack of consideration given to the variation of structural response due to the difference between sectional properties of various members. The *Cantilever Method* attempts to rectify this limitation by considering the cross-sectional areas of columns in distributing the axial forces in various columns of a story.

Assumptions

The Cantilever Method is based on three assumptions

The axial force in each column of a storey is proportional to its horizontal distance fromthe centroidal axis of all the columns of thestorey.

There is a point of inflection at the center of eachcolumn.There is a point of inflection at the center of eachbeam.

QUESTIONS…..

1- **Describe types of trusses ?**
2- **Explain statically determinant & indeterminant trusses ?**

3- **What is degree of indeterminancy ?**

4- **Describe stable and unstable trusses ?**

5- **Write advantages of trusses ?**

6- **Define method of joint ?**

7- **Describe method of section used in truss ?**